Feshbach projection operator method and few-body reaction models in Baer-Kouri-Levin-Tobocman many-body scattering theory

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A generalization of the Feshbach projection operator method to include rearrangement in the context of the Baer, Kouri, Levin, and Tobocman many-body scattering theory has been proposed recently. The formalism provides a simplified set of connected kernel equations for an approximate transition operator matrix and an explicit procedure for relating the approximate transition operators to the exact ones. In this paper we extend the flexibility of the aforementioned generalization by making it possible to allow the approximate transition operators to couple fewer partitions than the exact ones. Particular attention is given to the case where the approximate transition operator matrix is the solution of coupled-reaction-channel–type equations for transitions between few cluster configurations of the system. It is also shown that all the equations derived here have connected kernels after iterations.

[NUCLEAR REACTIONS Multipartition generalization of the Feshbach projection operator method applied to BKLT many-body scattering theory. Approximate few-cluster-model scattering equations developed.]

I. INTRODUCTION

Several theories have been formulated in recent years which provide mathematically well defined formalisms for analyzing the N-body scattering problem.[1] The complexity of the equations is sufficiently great to rule out exact treatments of many-body systems in the immediate future. Consequently, there has been interest in developing approximate versions of these N-body formalisms which would be more amenable to calculation.[2] These approximate theories generally have the form of a coupled-reaction-channel formalism[3] where the wave function is restricted to a small number of few-cluster configurations.

It is desirable to be able to formulate the approximate version of the N-body scattering theory in such a way that the approximate version is imbedded in the exact theory. By this we mean that either there is a well defined procedure for modifying the approximate version so that the difference from the exact theory can be made arbitrarily small or that there exists a well defined procedure for using the result of the approximate version to calculate the exact transition amplitudes. An approximation procedure of this sort has been proposed by Polyzou and Redish[4] in the context of the Bencze, Redish, and Sloan N-body scattering theory.

Tobocman[5] has shown that the Baer, Kouri, Levin, and Tobocman (BKLT) N-body scattering formalism[6] can be transformed into an approximate theory of the coupled-reaction-channels–type by means of a multipartition generalization of the Feshbach projection operator method.[7] However, the equation Tobocman gives for the relationship between the approximate and exact transition operators is an integral equation having a kernel which will not be connected if the approximate version includes three-or-more-body breakup channels. This shortcoming of the method was eliminated in a reformulation by Raphael, Tandy, and Tobocman (RTT).[8] However, both treatments have another undesirable feature, namely, all (two-cluster) partitions must be included in the approximate version as well as the exact one in order for the dynamical equations to have connected kernels. This paper generalizes the method of RTT so that the number of partitions coupled in the approximate version of the theory is fairly arbitrary. We show that our equations are of the connected kernel type and that they can be used to generate a sequence of approximations which approach the exact solution.

In Sec. II we present the antisymmetrized BKLT theory N-body scattering equations. The Feshbach projection operator reduction of these equations is carried out in Sec. III. Several alternative ways of using the projected BKLT formalism are described in Sec. IV. The partition coupling array used in the projected BKLT formalism is discussed in Sec. V. The choice of projection operators is discussed in Sec. VI. Section VII is devoted to a proof of the connectivity of the reduced equations for the modified wave operator. In Sec. VIII we show that the choice of projectors and partition coupling which makes the reduced equations connected also ensures the connectedness of the em-

bedding equations. Our results are summarized and discussed in Sec. IX, and a simple α-triton cluster model example is given in the Appendix B.

II. ANTISYMMETRIZED BKLT N-BODY SCATTERING FORMALISM

Consider a system of N particles initially regarded as distinguishable. For each partition $\alpha(1), \alpha(2), \ldots, \alpha(N_\alpha), \beta(1), \beta(2), \ldots, \omega(N_\omega - 1), \omega(N_\omega)$ of the particles into a pair of clusters there exists a decomposition of the Hamiltonian into the sum of two terms:

$$H = H_{\alpha(1)} + V_{\alpha(1)} = H_{\alpha(2)} + V_{\alpha(2)} = \ldots . \tag{1}$$

The partition Hamiltonian $H_{\alpha(i)}$ contains the kinetic energy and the intracluster interactions, while the residual interaction $V_{\alpha(i)}$ is the sum of the intercluster interactions for the two-cluster partition $\alpha(i)$. The partitions labeled by the same Greek letter and differing only by the index that appears in the parentheses belong to the family of partitions whose members are physically indistinguishable in that they differ only by the exchange of identical particles between clusters. The Greek letters then serve to identify physically distinguishable families of partitions. N_α will represent the number of distinct permutations of identical particles which can result from exchanges between the two clusters of the α family of partitions. The Greek alphabet $\alpha, \beta, \ldots, \psi, \omega$ will serve to represent a particular (arbitrary) ordering of the partition families.

The transition operator for scattering from a partition $\alpha(i)$ channel to a partition $\gamma(j)$ channel is

$$T_{\gamma(j)\alpha(i)} = V_{\gamma(j)} + V_{\gamma(j)} \mathcal{G} V_{\alpha(i)}, \tag{2a}$$

$$\mathcal{G} = (E + i\epsilon - H)^{-1}. \tag{2b}$$

The consequence of exchange symmetry is the replacement of these transition operators by

$$\tilde{T}_{\gamma(1)\alpha(1)} = N_\gamma^{1/2} N_\alpha^{-1/2} \sum_{n=1}^{N_\alpha} (-1)^{\sigma_{\alpha(n)}} T_{\gamma(1)\alpha(n)} P_{\alpha(n)} \tag{3}$$

and the use of the eigenstates of $H_{\alpha(1)}, H_{\beta(1)}, \ldots, H_{\omega(1)}$ only as asymptotic states for the calculation of transition amplitudes $\mathcal{T}_{ca}$:

$$\mathcal{T}_{ca} = \langle \phi_{c(1)}^{(-)} | \tilde{T}_{\gamma(1),\alpha(1)} | \phi_{a(1)}^{(+)} \rangle, \tag{4a}$$

$$(E - H_{\alpha(i)}) \phi_{a(i)}^{(\pm)} = 0, \tag{4b}$$

$$(E - H_{\gamma(i)}) \phi_{c(i)}^{(\pm)} = 0. \tag{4c}$$

In Eq. (3) $P_{\alpha(n)}$ is the permuation operator that transforms partition $\alpha(1)$ into $\alpha(n)$,

$$P_{\alpha(n)} \phi_{a(1)} = \phi_{a(n)}, \tag{5}$$

and $\sigma_{\alpha(n)}$ is the number of fermion exchanges required by $P_{\alpha(n)}$. The above formulation of exchange symmetry is sufficiently general to include the case of a system comprised of more than one type of identical particles.

Our starting point will be one of the antisymmetrized BKLT equations given by Goldflam and Tobocman (GT)[9]:

$$\tilde{T}_{\alpha(1)\mu(1)} = \sum_\gamma (M_{\alpha\gamma} \Lambda_{\gamma\mu} + U_\alpha W_{\alpha\gamma} G_{\gamma(1)} \tilde{T}_{\gamma(1)\mu(1)}). \tag{6a}$$

Using matrix notation, this is written

$$\tilde{T} = M\Lambda + UWG\tilde{T}. \tag{6a'}$$

In the above equations

$$G_{\gamma(1)} = (E + i\epsilon - H_{\gamma(1)})^{-1}, \tag{6b}$$

$$U_\alpha = V_{\alpha(1)} G_{\alpha(2)} V_{\alpha(2)} \ldots G_{\alpha(N_\alpha)} V_{\alpha(N_\alpha)}, \tag{6c}$$

$$W_{\alpha\gamma} = (N_\alpha / N_\gamma)^{1/2} \hat{\delta}_{\alpha\gamma}, \tag{6d}$$

$$M_{\alpha\gamma} = (V_{\alpha(1)} G_{\alpha(2)} + V_{\alpha(1)} G_{\alpha(2)} V_{\alpha(2)} G_{\alpha(3)} + \ldots + V_{\alpha(1)} G_{\alpha(2)} V_{\alpha(2)} \ldots G_{\alpha(N_\alpha)} V_{\alpha(N_\alpha)} G_{\gamma(1)}) W_{\alpha\gamma}, \tag{6e}$$

$$\Lambda_{\gamma\mu} = \sum_{n=1}^{N_\mu} (-1)^{\sigma_{\mu(n)}} G_{\mu(n)}^{-1} P_{\mu(n)} \delta_{\gamma\mu}. \tag{6f}$$

The quantity $\hat{\delta}_{\alpha\gamma}$ is defined to be equal to 1 when γ follows α in the sequence $\alpha, \beta, \gamma, \ldots, \psi, \omega, \alpha, \ldots$, and it equals zero otherwise.

The inhomogeneity in the above equations differs from the one given by GT, the latter being in error. The factors $G_{\alpha(i)} G_{\mu(n)}^{-1}$ appearing in $M\Lambda$ can be transformed to more convenient expressions as follows:

$$\begin{aligned} G_{\alpha(i)} G_{\mu(n)}^{-1} &= G_{\alpha(i)} (\mathcal{G}^{-1} + V_{\mu(n)}) \\ &= G_{\alpha(i)} (G_{\alpha(i)}^{-1} - V_{\alpha(i)} + V_{\mu(n)}) \\ &= 1 + G_{\alpha(i)} (V_{\mu(n)} - V_{\alpha(i)}). \end{aligned} \tag{7}$$

Alternatively, the Lippmann identity

$$G_{\alpha(i)} G_{\mu(n)}^{-1} \phi_{\mu(n)} = \delta_{\alpha\mu} \delta_{in} \phi_{\mu(n)} \tag{7a}$$

can be used. This reduces the sum for $M\Lambda$ to a single term, and we obtain a modified $\tilde{T}$ matrix equation of the form

$$\tilde{T}_{\alpha(1)\mu(1)} = \sum_\gamma U_\alpha W_{\alpha\gamma} (\delta_{\gamma\mu} + G_{\gamma(1)} \tilde{T}_{\gamma(1)\mu(1)}), \tag{8}$$

which is on-shell equivalent to Eq. (6a). However, if $\phi_{\mu(n)}$ is represented by an expression which is not an exact eigenfunction of $H_{\mu(n)}$, then this substitution may give poor results. This was found to be true for a particular case studied by RTT.

In the following we shall omit the (1) index on the partition unless necessary. Thus $G_\alpha \equiv G_{\alpha(1)}$, etc., unless otherwise required.

The nonantisymmetrized version of the theory is recovered from these equations by simply setting $N_\alpha = 1$ for all α.

In Appendix A we discuss some of the questions that arise in the implementation of the antisymmetrized BKLT equations.

III. REDUCTION OF THE BKLT EQUATIONS WITH PROJECTION OPERATORS

The fact that the inhomogeneity $M\Lambda$ differs from the interaction UW represents an obstacle to the application of the Feshbach projection operator formalism to our coupled integral equations. To circumvent this difficulty we follow the procedure used by RTT to replace T by an operator K which is the solution of an equation in which UW is the inhomogeneity. Let K be defined by

$$K = UW + UWG^{(0)}K = (1 + KG^{(0)})UW. \tag{9}$$

Then K and $\tilde{T}$ are related to each other in the following way:

$$\begin{aligned}\tilde{T} &= (1 + KG^{(0)})(M\Lambda + UWG\tilde{T}) - KG^{(0)}\tilde{T} \\ &= (1 + KG^{(0)})M\Lambda + K(G - G^{(0)})\tilde{T}.\end{aligned} \tag{10}$$

The procedure just described is an example of the subtraction technique discussed by Kowalski.[10]

Now we have a couple of options we can exercise in the choice of $G^{(0)}$. We can take $G^{(0)} = G$, in which case the second term in Eq. (10) vanishes and $\tilde{T}$ is calculated from K by means of a simple quadrature. Alternatively, $G^{(0)}$ may be chosen to be the principal value counterpart of G, in which case T is calculated from the on-the-energy-shell part of K by integration and a matrix inversion in channel space.

Next we introduce a reduction procedure to simplify the coupled integral equations for the reactance operator K. This is done as suggested by Tobocman[5] by replacing the Green's function operaoperators $G_{\alpha(i)}$ and $G^{(0)}_{\alpha(i)}$ by $G_{\alpha(i)}\mathcal{P}_{\alpha(i)}$ and $G^{(0)}_{\alpha(i)}\mathcal{P}_{\alpha(i)}$, respectively. Here $\mathcal{P}_{\alpha(i)}$ projects the partition Green's function operator $G^{(0)}_{\alpha(i)}$ onto a finite number of partition $\alpha(i)$ channels. However, as a part of the reduction procedure we make the additional step of replacing the partition coupling array W by an alternative one $\mathcal{W}$. Thus the reduced reactance operator $\mathcal{K}$ is defined to be the solution of

$$\mathcal{K} = \mathcal{U}\mathcal{W} + \mathcal{U}\mathcal{W}G^{(0)}\mathcal{P}\mathcal{K}, \tag{11}$$

where

$$\mathcal{U}_\alpha = V_{\alpha(1)}G_{\alpha(2)}\mathcal{P}_{\alpha(2)}V_{\alpha(2)} \cdots G_{\alpha(N_\alpha)}\mathcal{P}_{\alpha(N_\alpha)}V_{\alpha(N_\alpha)}. \tag{12}$$

Similar introductions of projectors into the BKLT equations have been carried out by Tobocman,[11] Kouri and Levin,[12] and by Greben and Levin.[13] However, the projectors employed here are somewhat more general than those used in these references in that we do not restrict ourselves to projection onto two-cluster channels only.

The purpose of using the reduced partition coupling array $\mathcal{W}$ to replace W is to remove the coupling to those partitions which are eliminated by the projection operator matrix $\mathcal{P}$. Thus it becomes possible to have fewer partitions included in the coupled equations for the reduced reactance operator matrix $\mathcal{K}$ than in the coupled equations for K. The elements of $\mathcal{K}$ that refer to the excluded channels are set equal to zero except for the diagonal elements which are set equal to $\mathcal{U}$. The explicit expressions required for $\mathcal{W}$ and $\mathcal{P}$ in order to ensure connectedness will be discussed in Secs. V and VI.

The final step in our analysis is the establishment of a relationship between the exact reactance operator matrix K and the reduced one $\mathcal{K}$. To do this we introduce the wave operators $\mathcal{J}, J$ which are solutions of

$$\mathcal{J} = 1 + \mathcal{U}\mathcal{W}\mathcal{P}G^{(0)}\mathcal{J} = 1 + \mathcal{J}\mathcal{U}\mathcal{W}\mathcal{P}G^{(0)} \tag{13}$$

and

$$J = 1 + UWG^{(0)}J = 1 + JUWG^{(0)}. \tag{14}$$

Combining Eqs. (13) and (14) we obtain the *embedding equation*

$$J = \mathcal{J} + \mathcal{J}(UW - \mathcal{U}\mathcal{W}\mathcal{P})G^{(0)}J. \tag{15}$$

These modified wave operators are related to the K matrices $K, \mathcal{K}$ by

$$K = JUW, \tag{16a}$$

$$\mathcal{K} = \mathcal{J}\mathcal{U}\mathcal{W}. \tag{16b}$$

We will show that this provides a connected kernel formalism for calculating the exact reactance operator matrix K from a modified wave operator matrix $\mathcal{J}$. Equation (13) provides a connected kernel formalism for calculating the modified wave operator matrix $\mathcal{J}$. The connectivity of Eqs. (13)–(15) depends on the choice of the projection operator matrix $\mathcal{P}$ and the reduced partition coupling array $\mathcal{W}$. The properties of these quantities and their choice are discussed in Secs. V and VI.

Our choice of the letters $\mathcal{J}$ and J to represent wave operators is different from the more conventional choice of the Greek letter Ω for that purpose.

IV. APPLICATIONS OF THE PROJECTED BKLT FORMALISM

One can imagine several alternative procedures for employing the projected BKLT formalism to

generate a sequence of successively improved reactance operator matrices. One alternative is to have $\mathcal{P}$ be nonzero for all partitions and have $\mathcal{P}$ exclude only the elastic channel and no more than a few others. Then $\mathcal{W}$ is set equal to W in Eqs. (15) and (16b) with the result

$$J=\mathcal{J}+\mathcal{J}(UW-\mathcal{U}W\mathcal{P})G^{(0)}J \tag{17a}$$

or

$$K=\mathcal{J}UW+\mathcal{J}(UW-\mathcal{U}W\mathcal{P})G^{(0)}K. \tag{17b}$$

For the nonantisymmetrized version of the theory where $\mathcal{U}=U$ this set of equations would be relatively simple, coupling no more than a few channels to the elastic channel. The true complexity of the problem would be buried in the modified wave operator matrix $\mathcal{J}$. Then one could start with $\mathcal{J}=1$ as the lowest order approximation to $\mathcal{J}$, and use Eq. (13) for $\mathcal{J}$ to generate successively improved representations for $\mathcal{J}$. In this approach $\mathcal{J}(UW-\mathcal{U}W\mathcal{P})$ appears as a kind of generalized optical potential. The procedure just described is similar to one proposed by Kowalski, Siciliano, and Thaler.[14]

Alternatively, one could have $\mathcal{P}$ project onto what are deemed to be the most important few-cluster channels. Then Eq. (13) would be a set of coupled-reaction-channel equations for the modified wave operator matrix $\mathcal{J}$.[15] The result of a calculation of $\mathcal{J}$ then would represent the shape elastic and direct interaction contributions. The reactance operator matrix K would be calculated by using $\mathcal{J}$ Eqs. (15) and (16a). The formal solution of these equations is

$$\begin{aligned}K&=[1-\mathcal{J}(UW-\mathcal{U}\mathcal{W}\mathcal{P})G^{(0)}]^{-1}\mathcal{J}UW\\&=\mathcal{J}UW+\mathcal{J}(UW-\mathcal{U}\mathcal{W}\mathcal{P})G^{(0)}\\&\quad\times[1-\mathcal{J}(UW-\mathcal{U}\mathcal{W}\mathcal{P})G^{(0)}]^{-1}\mathcal{J}UW.\end{aligned} \tag{18}$$

The inverse appearing in the second term might then be evaluated approximately by using a finite basis of states suitable for representing compound nucleus configurations. Then the second term on the right of Eq. (18) could be identified with the compound nucleus contribution to the reactance while the first term would be the shape elastic plus direct interaction part.

Another way to proceed is to have a sequence of successively more encompassing projectors $\mathcal{P}^{(1)}, \mathcal{P}^{(2)}, \ldots$ such that $\mathcal{P}^{(i)}\mathcal{P}^{(j)}=\mathcal{P}^{(j)}$ if $i\geq j$. These are used to define a sequence of more and more complex sets of equations with the solutions of one set of equations being the input into the next more complex set. To introduce a more convenient notation for this purpose we rewrite Eqs. (16a) and (14) in the form

$$K^{(j)}=J^{(j)}U^{(j)}W^{(j)}\equiv K, \tag{19a}$$

$$J^{(j)}=1+U^{(j)}W^{(j)}\mathcal{P}^{(j)}G^{(0)}J^{(j)}\equiv J, \tag{19b}$$

and we rewrite Eqs. (15) and (16b) in the form

$$K^{(j-1)}=J^{(j-1)}U^{(j-1)}W^{(j-1)}\equiv\mathcal{K}, \tag{19c}$$

$$J^{(j-1)}=1+U^{(j-1)}W^{(j-1)}\mathcal{P}^{(j-1)}G^{(0)}J^{(j-1)}\equiv\mathcal{J}. \tag{19d}$$

Then Eq. (15) becomes

$$\begin{aligned}J^{(j)}&=J^{(j-1)}\\&\quad+J^{(j-1)}(U^{(j)}W^{(j)}\mathcal{P}^{(j)}\\&\qquad-U^{(j-1)}W^{(j-1)}\mathcal{P}^{(j-1)})G^{(0)}J^{(j)}.\end{aligned} \tag{20}$$

One way to use this version of the projected BKLT theory would be to let $\mathcal{P}^{(1)}$ be the projector onto all two-body channels, let $\mathcal{P}^{(2)}$ be the projector onto all two-body and three-body channels, let $\mathcal{P}^{(3)}$ be the projector onto all two-body, three-body, and four-body channels, and so on. Then starting with a coupled two-body channel formalism, one could address a sequence of successively more complex sets of coupled equations. This program is similar to the one outlined by Redish[16] for the Bencze, Redish, and Sloan N-body formalism.

Another way to use this version of the projected BKLT theory would be to let $\mathcal{P}^{(1)}$ be the projector onto all N-body channels, let $\mathcal{P}^{(2)}$ be the projector onto all N-body and $(N-1)$-body channels, and so on. Then $\mathcal{P}^{(N-2)}$ would be the projector onto all channels except the two-body channels and $\mathcal{P}^{(N-1)}$ would be the projector onto all channels. Then one could start a calculation by setting $J^{(n)}=1$ for some n intermediate between 1 and $N-1$. Then this is inserted into the embedding equation for $J^{(n+1)}$. The solution is inserted into the embedding equation for $J^{(n+2)}$, and the process is continued until $J^{(N-1)}$ is calculated. This amounts to approximating transition operator $K^{(n)}$ by the inhomogeneity $U^{(m)}W^{(m)}$ and then solving the subsequent (i.e., $m<n$) equations exactly.

V. CHOICE OF THE PARTITION COUPLING ARRAY

The partition coupling array W is defined in Eq. (6d). It is seen that only the elements $W_{\alpha\beta}, W_{\beta\gamma}, W_{\gamma\delta}, \ldots, W_{\psi\omega}, W_{\omega\alpha}$ are nonvanishing. By cycling in this manner through all possible families of *two-cluster* partitions we are assured of having a connected kernel formalism for the reactance operator matrix K.

Now let us consider the reduced partition coupling array $\mathcal{W}$ that appears in Eq. (12) for the reduced reactance $\mathcal{K}$. If all the projectors $\mathcal{P}_\alpha, \mathcal{P}_\beta, \ldots, \mathcal{P}_\omega$ are nonvanishing, then $\mathcal{W}$ must be chosen to be identical with W to ensure connectivity. Under these circumstances our formalism reduces to that of RTT.[8]

Next consider the case where some of the projectors $\mathcal{P}_\gamma$ vanish. Let B_R be the set of distinguishable two-cluster partitions β for which the projectors $\mathcal{P}_\beta$ are nonvanishing, let B be the set of all distinguishable two-cluster partitions $\alpha, \beta, \ldots$, etc., and let $\hat{B}_R$ be the complement of B_R in B:

$$B = B_R \cup \hat{B}_R . \tag{21}$$

The matrix $\mathcal{W}$ is then chosen so that it couples sequentially all the partitions belonging to B_R and is the identity matrix[17] in the part of partition space spanned by the members of $\hat{B}_R$. Using the Greek alphabet to specify a particular arbitrary ordering we can choose

$$\alpha, \beta, \ldots, \mu \in B_R , \tag{22a}$$

$$\nu, \xi, \ldots, \omega \in \hat{B}_R . \tag{22b}$$

The reduced partition coupling array $\mathcal{W}$ is then written

$$\mathcal{W}_{\alpha\gamma} = (\hat{\delta}_{\alpha\gamma}^{(B_R)} + \delta_{\alpha\gamma}^{(\hat{B}_R)})(N_\alpha / N_\gamma)^{1/2} , \tag{23}$$

where $\hat{\delta}_{\alpha\gamma}^{(B_R)} = \hat{\delta}_{\alpha\gamma}$ for $\alpha, \gamma \in B_R$ and $\delta_{\alpha\gamma}^{(\hat{B}_R)} = \delta_{\alpha\gamma}$ for $\alpha, \gamma \in \hat{B}_R$ and both are zero otherwise. Note that $W_{\alpha\gamma} = \mathcal{W}_{\alpha\gamma}$ for $\alpha, \gamma \in B_R$ except for one element.

For example, for a system of three distinguishable partitions α, β, γ we can take $\alpha, \beta \in B_R$. Then

$$\mathcal{W} = \begin{pmatrix} 010 \\ 100 \\ 001 \end{pmatrix} \quad \text{and} \quad W = \begin{pmatrix} 010 \\ 001 \\ 100 \end{pmatrix} .$$

Substituting Eq. (23) for $\mathcal{W}$ into Eq. (11) gives a set of coupled integral equations for $\mathcal{K}_{\alpha\beta}$; $\alpha, \beta \in B_R$, and $\mathcal{K}_{\sigma\sigma'} = \delta_{\sigma\sigma'} \mathfrak{U}_\sigma$ for $\sigma, \sigma' \in B_R$. All other elements of $\mathcal{K}$ vanish. Thus there is no coupling between B_R and $\hat{B}_R$.[17]

VI. CHOICE OF THE PROJECTORS

For the case where some of the projectors $\mathcal{P}_\gamma$ vanish we have chosen the reduced partition coupling array $\mathcal{W}$ so that there is no coupling to the partition families that have been eliminated (i.e., where $\mathcal{P}_\gamma = 0$). With such a choice of $\mathcal{W}$ and $\mathcal{P}$ one must take care not to destroy the connectedness of the kernel of the equation for the reduced reactance and the embedding equation by either including too few partitions in the coupling scheme or by taking the projectors to be too encompassing.

Before discussing the sufficient conditions for connectedness of our equation we shall define our symbols more carefully. To do this we start by noting that in the BKLT equations, only the two-cluster partitions $\beta \in B$ are coupled. The Hamiltonian H_β corresponding to such a partition β can be split into three parts:

$$H_\beta = H_{\beta_1} + H_{\beta_2} + T_{\text{rel}} = H_{\text{int}} + T_{\text{rel}} , \tag{24}$$

where H_{β_1}, H_{β_2} describe the internal motion of clusters β_1, β_2, and T_{rel} is the kinetic energy operator for the relative motion of the two clusters. The spectrum of the partition Hamiltonian H_β includes states in which clusters β_1 and β_2 are in unbound configurations as well as states in which β_1, β_2, or both are bound.

A channel is defined by the set of quantum numbers required to specify an eigenstate of a partition Hamiltonian H_β. To do this one starts with a partition $\alpha \subseteq \beta$ composed of n clusters $A_1, A_2, \ldots, A_n$. Each cluster A_i is understood to be in a bound state with quantum numbers a_i. Each set a_i includes the negative binding energy E_i. The set of quantum numbers a_i, $i = 1, \ldots, n$ defines a *cluster state*. We refer to the a_i as the *internal quantum numbers* of the channel.

To complete the identification of a channel it is necessary to specify the *external quantum numbers* in addition to the internal ones. These describe the motion of the clusters relative to each other. There are a number of ways to associate a relative motion state with these $(n-1)$ vector degrees of freedom. A possible choice of external quantum numbers is to associate the kinetic energy $\mathcal{E}_q > 0$, the orbital angular momentum l_q, and its projection m_q with the qth *independent* vector coordinate. An *asymptotic channel state* is then defined as the scattering eigenstate of H_β specified by the quantum numbers a_i, $\mathcal{E}_q$, l_q, m_q; $i = 1, \ldots, n$; $q = 1, \ldots, n-1$. Such an eigenstate of H_β is said to be of connectivity α.

The conservation of energy requires

$$E = \sum_{i=1}^{n} E_i + \sum_{q=1}^{n-1} \mathcal{E}_q . \tag{25}$$

Each of the quantum numbers contributing to the identification of a channel (i.e., an asymptotic channel state) belongs to a discrete set except for the relative motion energies $\mathcal{E}_q$. For a two-cluster channel, $\mathcal{E}_q$ is fixed uniquely by Eq. (25). So the set of two-cluster channels is denumerable. However, the n-cluster ($n \geq 3$) channels form a continuum for each given total energy E.

In the BKLT theory, associated with each two-cluster partition there is an infinite set of channels. This set includes a discrete set of two-cluster channels and a continuum of three-or-more-cluster channels. For the two-cluster channels, both clusters are in a bound state. For the breakup channels one or both of the clusters are in an unbound state.

For a given two-cluster channel there is a unique two cluster partition $\beta \in B$ to which it belongs.[18] For a three-or-more-cluster channel associated

with the partition $\alpha = A_1 A_2 \cdots A_n$ there will be several two cluster partitions to which this channel may be assigned. These will be all the two cluster partitions β such that $\alpha \subseteq \beta$, that is, all the two cluster partitions that are identical to α or can form α by some division of their members.

Following Polyzou and Redish[4] we define the reaction mechanism[19] $\mathfrak{a}_R$ to be the collection of cluster states which are deemed physically important. These are the states we want to include in the calculation of the reduced wave matrix $\mathcal{J}$. Each cluster state $a \in \mathfrak{a}_R$ corresponds to a partition α. We define the reaction set A_R to be the set of all partitions α such that there exists at least one cluster state $a \in \mathfrak{a}_R$ corresponding to it. We then require that the projectors $\mathcal{P}_\beta$ be nonvanishing for the set B_R of all distinguishable two-cluster partitions $\beta \in B$ such that there exists a partition $\alpha \in A_R$ such that $\alpha \subseteq \beta$. We also require that none of the projectors $\mathcal{P}_\beta$ projects onto any channel that he has a cluster state associated with a partition $\gamma \notin A_R$.

We will show that these requirements are sufficient to ensure the connectedness of the kernels of our dynamical equations after iteration. The requirement that B_R includes all the two-cluster partitions consistent with our choice of the reaction mechanism is the analog of the requirement that all two-cluster partitions be included in the (unprojected) BKLT equations to ensure connectedness. It has been pointed out that such a requirement is necessary for the uniqueness and unitarity of the BKLT equations.[20] The same arguments can also be used to show that projected BKLT equations will be unitary on the restricted channel space for this choice of B_R.[21]

VII. CONNECTIVITY OF THE REDUCED EQUATIONS

With these definitions we are ready to discuss the sufficient condition for the connectedness of the kernel of the modified wave operator equations. We will show that the nth iterate kernel of Eq. (13) is a connected operator if for each partition $\alpha \in A_R$ the set B_R includes all two-cluster partitions $\beta \in B$ such that $\alpha \subseteq \beta$. Here n denotes the dimension of B_R. By connectedness of the kernel of Eq. (13) we mean that the matrix elements $[(\mathfrak{U}\mathcal{W}\mathcal{P}G^{(0)})^n]_{\beta\gamma}$ $\beta,\gamma \in B_R$, of the nth iterate of the kernel are connected operators. For β or $\gamma \in \hat{B}_R$ the kernel matrix element $[(\mathfrak{U}\mathcal{W}\mathcal{P}G^{(0)})^n]_{\beta\gamma}$ vanishes.

When the coupled equations for the approximate wave operator $\mathcal{J}$, Eq. (13), are decoupled by iteration,[22] we get an integral equation for each element of $\mathcal{J}$.[23] The kernel for each of these equations is a product of the form[24]

$$C = \prod_{\beta \in B_R} \mathcal{P}_\beta G_\beta^{(0)} \mathfrak{U}_\beta . \tag{26}$$

The order of the factors is different in different equations but that is irrelevant for the discussion of connectivity. Let us consider the products

$$C_1 = \prod_{\beta \in B_R} G_\beta^{(0)} \mathfrak{U}_\beta \tag{27}$$

and

$$C_2 = \prod_{\beta \in B_R} \mathcal{P}_\beta . \tag{28}$$

These can be expressed as sums of terms of different connectivity. Let $\gamma_1(\gamma_2)$ be the connectivity of any particular term in the sum for $C_1(C_2)$. It is clear that the product $C = C_1 C_2$ must then contain a term of connectivity $\gamma = \gamma_1 \cup \gamma_2$, the union of γ_1 and γ_2. The union $\gamma_1 \cup \gamma_2$ is the finest partition γ, that is, the one with most clusters, such that $\gamma_1 \subseteq \gamma$ and $\gamma_2 \subseteq \gamma$. If C is a connected operator, then γ must be the one-cluster partition for all γ_1, γ_2.

Suppose now that the cluster state associated with one of the channels is composed of clusters $A_1, A_2, \ldots, A_n$. We label this n-cluster partition α. The product C_2 will then contain a term of connectivity α. Since we required that B_R contain all partitions $\beta \supseteq \alpha$, it must also contain the partition $\beta_1 = A_1(A_2 A_3 \ldots A_n)$. Since $\mathcal{W}$ cycles through all partitions in B_R, C_1 must contain a factor $\mathfrak{U}_{\beta_1}$ which has every one of its terms connecting a particle from A_1 to a particle external to A_1. Since A_1 is a stable cluster (a bound state), *all* particles in the cluster A_1 are therefore connected to this particle external to A_1. Suppose this external particle belongs to A_2. Since A_2 is a stable cluster we conclude that this term of $\mathfrak{U}_{\beta_1}$ connects all particles in cluster A_1 to all particles in cluster A_2.

If partition α has only two clusters ($n = 2$), then the connectedness of C is established. If $n > 2$, then B_R must also contain the partition $\beta_2 = (A_1 A_2)(A_3 A_4 \cdots A_n)$ and C_1 must also contain the factor $\mathfrak{U}_{\beta_2}$. Each term of $\mathfrak{U}_{\beta_2}$ connects a particle in $(A_1 A_2)$ to an external particle, say one in A_3. Since A_3 is a stable cluster, it follows that $A_1 A_2 A_3$ is connected. If $n = 3$ the proof is complete. For $n > 3$ the proof follows by induction.

VIII. CONNECTIVITY OF THE EMBEDDING EQUATION

Having assured ourselves that there is an appropriate choice of $\mathcal{W}$ and $\mathcal{P}$ for which the kernel of the equation for the reduced reactance $\mathcal{J}$ is connected, we now demonstrate that this choice will also ensure the connectivity of the kernel of the

embedding equation, Eq. (15). If Eq. (15) is iterated $(N-1)$ times (N being the number of all distinguishable two-cluster partitions possible in the system), we find

$$J=\sum_{s=0}^{N-1}\Gamma^{s}\mathcal{J}+\Gamma^{N}J, \tag{29a}$$

where

$$\Gamma=\mathcal{J}(UW-\mathcal{U}\mathcal{W}\mathcal{P})G^{(0)}. \tag{29b}$$

From the discussion of Sec. VII we know that $(\mathcal{U}\mathcal{W}\mathcal{P}G^{(0)})^{n}$ is a connected operator where n is the dimension of B_R. A similar proof also shows that the operator $(UWG^{(0)})^{N}$ is connected. The second statement is just the statement of the connectedness of the original BKLT equations.[6] From the definition of $\mathcal{W}$ in Eq. (23) we have

$$[(\mathcal{U}\mathcal{W}\mathcal{P}G^{(0)})^{n}]_{\alpha_1\alpha}$$

$$=\mathcal{U}_{\alpha_1}\mathcal{P}_{\alpha_2}G^{(0)}_{\alpha_2}\mathcal{U}_{\alpha_2}\mathcal{P}_{\alpha_3}\cdots\mathcal{U}_{\alpha_n}\mathcal{P}_{\alpha_1}G^{(0)}_{\alpha_1}\delta^{(B_R)}_{\alpha_1\alpha}, \tag{30}$$

and from the definition of W in Eq. (6d) we have

$$[(UWG^{(0)})^{N}]_{\alpha_1\alpha}=\delta^{(B)}_{\alpha_1\alpha}U_{\alpha_1}G^{(0)}_{\alpha_2}U_{\alpha_2}\cdots U_{\alpha_N}G^{(0)}_{\alpha_1}. \tag{31}$$

On the right of Eq. (30) the subscripts cycle through all the partitions in B_R and on the right of Eq. (31) the subscripts cycle through all the possible distinguishable two-cluster partitions of the many-body system.

We now show that Eq. (29) has a connected kernel. From Eq. (13) we observe that

$$\mathcal{J}=\sum_{s=0}^{\infty}(\mathcal{U}\mathcal{W}\mathcal{P}G^{(0)})^{s}. \tag{32}$$

It follows that Γ^{N} is an infinite series each term of which is a product of N or more of $UWG^{(0)}$ and $\mathcal{U}\mathcal{W}\mathcal{P}G^{(0)}$. As a result of the partition coupling structure of the arrays W and $\mathcal{W}$, the nonvanishing terms in the series expansion of Γ^{N} have a particular structure. We will describe this structure with the help of Fig. 1.

Figure 1 is an $N\times N$ matrix of points. We have circled the points coupled by the full partition coupling array W and placed crosses at the points coupled by the reduced partition coupling array $\mathcal{W}$. These connections are indicated by the solid and broken lines, respectively.

A particular term of order $M\geq N$ will contain, say, a factors $\mathcal{U}_{\beta_i}\mathcal{P}_{\beta_{i+1}}G^{(0)}_{\beta_{i+1}}$ and b factors $U_{\alpha_j}G^{(0)}_{\alpha_{j+1}}$, where $\beta_i\in B_R$ and $\alpha_j\in B$ for $i=1,\ldots,n$ and $j=1,\ldots,N$. We also take $n+i=i$ and $N+j=j$. Starting at a point $\gamma_j\gamma_{j+1}$ on the diagram we move clockwise, making each step along a solid or a broken line segment, until M steps are made. Moving along a solid line away from $\gamma_j\gamma_{j+1}$ gives the factor $U_{\gamma_j}G^{(0)}_{\gamma_{j+1}}$, whereas following the broken line segment instead gives $\mathcal{U}_{\gamma_j}\mathcal{P}_{\gamma_{j+1}}G^{(0)}_{\gamma_{j+1}}$. We note that we must pass through all the circles or all the crosses at least once since $M\geq N$. This means that we cycle either through all the partitions $\beta\in B_R$ or through all the partitions $\alpha\in B$. In the latter case the term must include all the factors $U_\alpha G_\alpha$ on the right of Eq. (31) which is sufficient to make that term a connected operator.

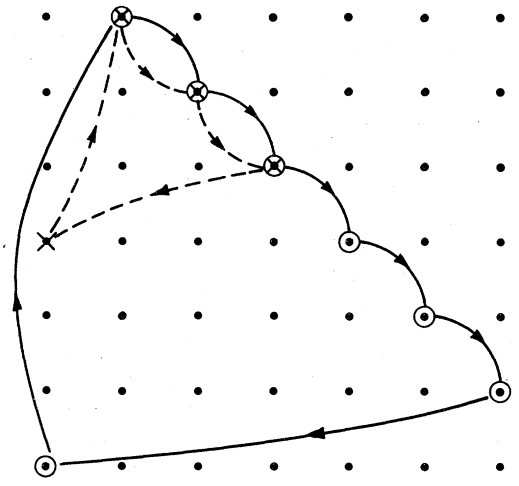

FIG. 1. Diagram for constructing the terms of $[\mathcal{J}(UW-\mathcal{U}\mathcal{W}\mathcal{P})]^{N}$ for $N=7$ and $n=4$.

If in a particular term we cycle through all the partitions $\beta\in B_R$, that term must contain the factors $\mathcal{U}_{\beta_1},\mathcal{U}_{\beta_2},\ldots,\mathcal{U}_{\beta_n},G_{\beta_1},G_{\beta_1},\ldots,G_{\beta_n}$, and $\mathcal{P}_{\beta_1}$. This set of factors differs from that which appears in Eq. (30) in that $\mathcal{P}_{\beta_1}$ replaces the product $\mathcal{P}_{\beta_1}\mathcal{P}_{\beta_2}\cdots\mathcal{P}_{\beta_n}$.[25] We now show that a term of this form is connected.

Suppose at first that our reaction mechanism consists of a single cluster state corresponding to the multicluster partition α. Then each of the projectors $\mathcal{P}_{\beta_i}$ where $\beta_i\in B_R$ will have a connectivity α, and the product $\mathcal{P}_{\beta_1}\mathcal{P}_{\beta_2}\cdots\mathcal{P}_{\beta_n}$ will have connectivity α as well. Consequently, the product $(\prod_{i=1}^{n}U_{\beta_i}G^{(0)}_{\beta_i})\mathcal{P}_{\beta_j}$ must have the same connectivity as the product in Eq. (30) and thus must be a connected operator. Therefore, for the case of a single partition α in the reaction set A_R every term in $(\Gamma^{N})_{\alpha\beta}$ must be connected.

Consider now a reaction mechanism consisting of several cluster states corresponding to the multicluster partitions $\alpha_1,\ldots,\alpha_m$. Suppose that $\alpha_j\subseteq\beta_i$. By the single cluster state argument, the product $(\prod_{\beta\in B_R}\mathcal{U}_\beta G^{(0)}_\beta)\mathcal{P}_{\beta_i}$ contains a fully connected factor multiplied by some extra factors of U_β,G_β. Since the above holds for every $\alpha_j\subseteq\beta_i$, the operator $(\Gamma^{N})_{\alpha\beta}$ must be fully connected for any α,β.

This completes the proof of connectivity of Eq. (15). Together with the connectedness proof of

Sec. VII we have shown that the projected BKLT theory based on Eqs. (12)–(16) has a connected kernel structure.

IX. CONCLUSION

We have shown how the coupled integral equations of the BKLT N-body scattering formalism can be reduced to equations of the same type as the coupled-reaction-channels formalism by means of a generalization of the Feshbach projection operators. In addition, we have provided explicit expressions relating the solutions of the reduced equations to the exact transition operators or to the solutions of a less reduced set of equations. Exchange symmetry effects are explicitly included.

Throughout the derivation care has been taken to ensure the connectedness of all equations. In particular, we have shown that the reduced equations for the modified wave operator and the equations embedding the approximation in the exact theory both have connected kernels after iteration. The connectedness of our equations then allows their solution by constructive numerical methods.

The reduced equations given here provide a method for treating approximately few-cluster reactions within the framework of the many-body connected kernel formulation of scattering theory. In this respect, our work parallels that of Polyzou and Redish,[4] with the specific application to the BKLT many-body equations. It should be noted that the present approach appears to be easily generalizable to include other partition coupling schemes by a suitable redefinition of the partition coupling arrays, W,$\mathfrak{W}$ and the projectors.

The present formulation has various possible applications. An example of a possible application is the cluster model for α-triton scattering given in the Appendix B. It is seen that the theory provides a substantial reduction in the number of coupled equations in comparison to the exact formalism. Furthermore, since the BKLT theory is capable of dealing with many-body forces and nonlocal potentials, the present formulation can be taken to include such forces.

The BKLT equations used in our discussion were the antisymmetrized form of the Kouri-Levin–type equations due to Goldflam and Tobocman.[9] Our analysis can be carried out in a similar fashion for other versions of the BKLT formalism. The projection operator reduction of the nonantisymmetrized BKLT formalism equations is recovered from the above results by simply setting $N_\alpha = N_\beta = \ldots = N_\omega = 1$.

In place of the Goldflam and Tobocman antisymmetrized BKLT formalism equations we could have started with the antisymmetrized BKLT formalism equations due to Tobocman.[26] These have the same form as Eq. (6) with just the following changes in definition:

$$U_\alpha = V_{\alpha(1)} A_\alpha^\dagger , \tag{33a}$$

$$A_\alpha = N_\alpha^{-1} \sum_{n=1}^{N_\alpha} (-1)^{\sigma_{\alpha(n)}} P_{\alpha(n)} , \tag{33b}$$

$$M_{\alpha\gamma} = U_\alpha W_{\alpha\gamma} G_{\gamma(1)} , \tag{33c}$$

$$\Lambda_{\gamma\mu} = A_\gamma^\dagger N_\gamma^{1/2} G_{\mu(1)}^{-1} N_\mu^{1/2} . \tag{33d}$$

Thus the results derived above can be applied to these equations just by making these changes in definition. The Tobocman equations are much simpler than the Goldflam and Tobocman ones, but they are not of the connected kernel type.[27] However, if one always makes the approximation of neglecting three-or-more-body channels, then the resulting equations will be connected.

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APPENDIX A. SOLUTION OF THE BKLT EQUATIONS FOR REALISTIC CASES

First of all, one must recognize that evaluation of the antisymmetrized BKLT equations presents no difficulties that are not already present in the original unantisymmetrized BKLT equations. The antisymmetrized equations are the result of making a finite number of iterations and forming linear combinations of subsets of the original finite set of BKLT equations. Such a finite sequence of iterations will in general be used in solving the BKLT equations since they are thereby decoupled from each other.

Like any other N-body scattering formalism, the BKLT equations require the solution of all the $N-1, N-2, \ldots$, etc. problems as input to the N-body equations. This is simple enough for $N=3$; for N greater than three we must be prepared to make approximations and use models.

In the BKLT and all other minimally coupled theories, the solution of the equations is expressed in terms of the T-operator matrix elements with respect to the eigenstates (i.e., the asymptotic channel states) of the two-cluster Hamiltonians H_β. Also, the partition Green's functions G_β are expressed (via their spectral decomposition) in terms of these eigenstates. Ultimately, we are required to evaluate the matrix elements of the residual interaction V_β of the form

(a) $\langle \phi_\alpha(a) | V_\alpha | \phi_\beta(b) \rangle$, $\beta \neq \alpha$

(b) $\langle\phi_\alpha(a)|V_\alpha|\phi_\alpha(a')\rangle$,

where (a), (b) denotes the relevant quantum numbers of the channel. To evaluate such matrix elements the knowledge of the asymptotic channel states $\phi_\alpha(a)$ is necessary. Here we may use various models of nuclear structure, e.g., the shell model. Such matrix elements are the same as those required in distorted wave Born approximation calculations for inelastic and rearrangement scattering and folding model potential calculations of elastic scattering. Techniques for evaluating such matrix elements are well developed.

The effect of the antisymmetrization is the reduction on the number of such matrix elements, thus decreasing the amount of computational labor needed to solve the problem. In particular, the particle identity allows us to express all matrix elements $\langle\phi_{\alpha(i)}(a)|V_{\alpha(i)}|\phi_{\alpha(i+1)}(a')\rangle$ in terms of the matrix element $\langle\phi_{\alpha(1)}(a)|V_{\alpha(1)}|\phi_{\alpha(2)}(a')\rangle$. Thus, for any partition α, the net reduction in the number of matrix elements of V_α needed for the solution is $\frac{1}{2}N_\alpha(N_\alpha+1)$. Similar reduction obtains for all other matrix elements. If N_α is not too large ($N_\alpha \leq 5$) it appears that the GT (Ref. 9) version of the antisymmetrized BKLT equations is numerically tractable.

APPENDIX B. CLUSTER MODEL EXAMPLE

In this appendix we shall work out a few details of a simple cluster model within the framework of the present formalism. We start by considering an effective four-body problem: α + three nucleons, where all the nucleons are treated as distinguishable and we shall assume that the α particle exists in the system as an inert core.

In the full BKLT equations for a four-body problem, all the two-cluster partitions must be included. These are

$$B \equiv \{\alpha(n_1n_2p),\ (\alpha n_1)(n_2p),\ (\alpha n_2)(n_1p),\ (\alpha n_1p)n_2,\ (\alpha n_2p)n_1,\ (\alpha n_1n_2)p,\ (\alpha p)(n_1n_2)\}. \tag{B1}$$

The first partition corresponds to the $\alpha+t$ channels, the next two to the ^{5}He$+d$ channels, the next two to ^{6}Li$+n$, the sixth partition corresponds to the ^{6}He$+p$ channels, and the last to ^{5}Li$+(nn)$ scatterings. Within the assumption of an inert α particle, the complete set of partitions occurring in the final state is

$$A = \{\alpha(n_1n_2p),\ (\alpha n_1)(n_2p),\ (\alpha n_2)(n_1p),\ (\alpha n_1p)n_2,\ (\alpha n_2p)n_1,\ (\alpha n_1n_2)p,\ (\alpha p)(n_1n_2),\ \alpha n_1(n_2p),\ \alpha n_2(n_1p),\ \alpha p(n_1n_2),$$
$$(\alpha p)n_1n_2,\ (\alpha n_1)pn_2,\ (\alpha n_2)pn_1,\ \alpha pn_1n_2\}, \tag{B2}$$

where the parentheses indicate a stable cluster (i.e., bound state of the particles in the parentheses). The three- and four-body final states appearing in A enter in the BKLT via the continuum of the two-cluster states associated with the two-cluster partitions $\beta \in B$.

Let us now choose the reaction set, the set A_R of partitions $\alpha \in A$ to which belong the channel states (i.e., the eigenstates of H_α) which we select as most important from all those which can be formed in the $\alpha+t$ reaction. To be specific, assume that only the following final partitions can occur,

$$A_R \equiv \{\alpha(n_1n_2p),\ (\alpha n_1)(n_2p),\ (\alpha n_2)(n_1p),\ (\alpha n_1p)n_2,\ (\alpha n_2p)n_1,\ (\alpha n_1)n_2p,\ (\alpha n_2)n_1p,\ (\alpha n_1n_2)p\}; \tag{B3}$$

i.e., we allow three-body breakup but not the four-body final states.

To this set A_R corresponds the BKLT reaction set

$$B_R \equiv \{\forall\beta \in B,\ \exists\alpha \in A_R,\ \alpha \subseteq \beta\}$$
$$\equiv \{\alpha(n_1n_2p),\ (\alpha n_1)(n_2p),\ (\alpha n_2)(n_1p),\ (\alpha n_1p)n_2,\ (\alpha n_2p)n_1,\ (\alpha n_1n_2)p\}, \tag{B4}$$

where all two cluster partitions from B which contain at least one partition $\alpha \in A_R$ have been included to ensure connectedness. Thus B_R contains six partitions out of the seven possible. We now construct the projectors in the following manner. For each partition $\alpha \in A_R$ there is a set of quantum numbers $a = \{E, a_1, \ldots, a_n\}$ corresponding to the internal states of each of the clusters and the total energy. From the collection of eigenstates of H_β, $\beta \in B_R$, we select only the states with asymptotic n-cluster states corresponding to partition $\alpha \subseteq \beta$.

In particular, since our reaction mechanism does not contain the breakup states of the triton, the projector $P_{\alpha t}$ is

$$P_{\alpha t} = \int d^3\vec{k}\,|\phi_\alpha\rangle|\phi_t\rangle|\vec{k}\rangle\langle\vec{k}|\langle\phi_\alpha|\langle\phi_t| \times \text{(weight factor)}. \tag{B5}$$

Note that only the ground state of the triton appears once we have assumed that α is inert.

Similarly, the ^{5}He $d = (\alpha n)(np)$ projector is

$$P_{^5\mathrm{He}d}=\sum\int d^3\vec{k}\,|\phi_{^5\mathrm{He}}\rangle|\phi_d\rangle|\vec{k}\rangle\langle\vec{k}|\langle\phi_d|\langle\phi_{^5\mathrm{He}}|\times(\text{weight factor})\,. \tag{B6}$$

The sum over the internal states of ^{5}He is limited to the selected bound states, while the sum over the internal deuteron states includes the bound and breakup states selected as important for the problem.

In this manner, we can construct the projectors for each two-cluster partition $\beta\in B_R$ from a given set of important states. The expressions for the projectors are understood as compact representations of, e.g., the coordinate space projectors of the form

$$P_{^5\mathrm{He}d}=\sum_{ij}\int d^3r\,|\phi_{^5\mathrm{He}}(i)\phi_d(j)\delta(\vec{r}_{^5\mathrm{He}d}-\vec{r}))B(i,j)\langle\phi_{^5\mathrm{He}}(i)\phi_d(j)\delta(\vec{r}_{^5\mathrm{He}d}-\vec{r})|\,, \tag{B7}$$

where $B(i,j)=1$ for included states and zero otherwise. For the problem described here we can now take the partition coupling array $\mathcal{W}$ to be

$$\mathcal{W}_{\beta_1\beta_2}=\mathcal{W}_{\beta_2\beta_3}=\cdots=\mathcal{W}_{\beta_6\beta_1}=1=\mathcal{W}_{\beta_7\beta_7}\,,$$

where $\beta_7\notin\hat{B}_R$, $\beta_1,\ldots,\beta_6\in B_R$. Of course any other choice of $\mathcal{W}$ which cyclically couples the partitions $\beta_1,\ldots,\beta_6\in B_R$ can be used to write Eqs. (12)–(16).

Before writing down explicitly our set of coupled equations, we carry out a partial antisymmetrization of the problem. We will assume that α behaves as an inert particle and antisymmetrize only the external nucleons. Then the partitions β_2 and β_3, β_4 and β_5 are equivalent, and we obtain the following set of equations ($\beta_1=\alpha t$, $\beta_2=\beta_3=$ ^{5}He d, $\beta_4=\beta_5=$ ^{6}Li h, $\beta_6=$ ^{6}He p):

$$\begin{aligned}
\tilde{T}_{\beta_2\beta_1}&=V_{\beta_2}G_{\beta_3}(1+V_{\beta_3}G_{\beta_4})G_{\beta_1}{}^{-1}+V_{\beta_2}G_{\beta_3}V_{\beta_3}G_{\beta_4}\tilde{T}_{\beta_4\beta_1}\,,\\
\tilde{T}_{\beta_4\beta_1}&=V_{\beta_4}G_{\beta_5}(1+V_{\beta_5}G_{\beta_6})G_{\beta_1}{}^{-1}+V_{\beta_4}G_{\beta_5}V_{\beta_6}G_{\beta_6}\tilde{T}_{\beta_6\beta_1}\,,\\
\tilde{T}_{\beta_6\beta_1}&=V_{\beta_6}+V_{\beta_6}G_{\beta_1}\tilde{T}_{\beta_1\beta_1}\,,\qquad \tilde{T}_{\beta_1\beta_1}=V_{\beta_1}G_{\beta_1}(G_{\beta_1}{}^{-1}+\tilde{T}_{\beta_2\beta_1})\,.
\end{aligned} \tag{B8}$$

The implementation of equations of this type is discussed in Appendix A. In Eq. (B8) the propagators are assumed implicitly to contain the appropriate projectors.

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[18]The term "partition" used here denotes a particular grouping of N particles into n distinct groups or clusters. Some authors have used the term "channel" for this purpose. We, however, use the term channel to identify a particular asymptotic state, that is, a channel is characterized by both the internal and external quantum numbers defined above. This corresponds to the usage found in reaction theory literature. The few-body scattering theory literature more often uses channel to identify what we have called a cluster state so that it is characterized by the internal

quantum numbers only.

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[23]The fact that the equations decouple after iteration is a special feature of the BKLT many-body equations. For other choices of W and $\mathbf{w}$ the present proof does not apply.

[24]In the theory given here we replace the cluster propagators G_α by $\sigma_\alpha G_\alpha$ everywhere in the theory. Other forms of the theory are possible. For example we can write $\mathcal{X} = U\mathbf{w} + \sigma U \mathbf{w} G^{(0)} \mathcal{X}$. In the distinguishable particles case the two theories are identical. However, for identical particles the latter theory is not self-consistent.

[25]Although factors U_{β_i} can appear in place of factors $\mathfrak{u}_{\beta_i}$, this cannot decrease the connectivity of the product. Hence we will treat $\mathfrak{u}_{\beta_i}$ and U_{β_i} as identical in the proof of connectivity.

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