Average polarization of the recoil ^{12}B from muon capture by ^{12}C

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It is shown, in the "elementary-particle" treatment, that, if the source to produce the background ^{12}B(g.s.) comes *only* from the cascade processes $\mu^- {}^{12}C(g.s.) \to \nu_\mu {}^{12}B^*(2.62) \to \nu_\mu {}^{12}B(g.s.)\,\gamma$, the average polarization of the recoil ^{12}B(g.s.) produced by the *direct* polarized-muon capture $\mu^- {}^{12}C(g.s.) \to \nu_\mu {}^{12}B(g.s.)$ can be extracted reliably from the observed angular distribution of the electrons in the subsequent beta decays of ^{12}B(g.s.). Using the recent data of Possoz *et al.*, our result for the average polarization P_{av} of the recoil ^{12}B(g.s.) from the *direct* polarized-muon capture is $P_{av} = 0.47 \pm 0.05$, which agrees very well with the prediction of the standard picture, namely, conservation of vector current, partial conservation of axial vector current, and absence of second-class axial currents.

[RADIOACTIVITY $\mu^- {}^{12}C(g.s.) \to \nu_\mu {}^{12}B^*(2.62)$; effects on the average polarization of ^{12}B(g.s.); CVC, PCAC, and no second-class axial vector currents.]

As a pioneering attempt to pin down experimentally the value of the pseudoscalar form factor and so to test the validity of partial conservation of axial vector current (PCAC), Possoz *et al.*[1] measured the "apparent" average polarization P_{av}^{appr} of the recoil ^{12}B(g.s.) from polarized-muon capture by ^{12}C(g.s.) and subtracted the contribution due to those recoil ^{12}B(g.s.) which were produced *indirectly*, e.g., $\mu^- {}^{12}C(g.s.) \to \nu_\mu {}^{12}B^* \to \nu_\mu {}^{12}B(g.s.)\gamma$. In this way, they extracted a value of the average polarization P_{av} in the *direct* polarized-muon capture, i.e., $\mu^- {}^{12}C(g.s.) \to \nu_\mu {}^{12}B(g.s.)$:

$$P_{av} = 0.55 \pm 0.05 \tag{1}$$

from the observed value of P_{av}^{appr}:

$$P_{av}^{appr} = 0.463 \pm 0.040\,. \tag{2}$$

The major source for the difference between P_{av} and P_{av}^{appr} comes from the $B^*(2.62; J^\pi = 1^-, I = 1)$ contribution, viz.:

$$\mu^- {}^{12}C(g.s.) \to \nu_\mu {}^{12}B^*(2.62) \searrow {}^{12}B(g.s.)\gamma\,, \tag{3}$$

since the corresponding capture rate[2]

$$\Gamma(\mu^- {}^{12}C(g.s.) \to \nu_\mu {}^{12}B^*(2.62)) = (0.84 \pm 0.09) \times 10^3\ \text{sec}^{-1} \tag{4}$$

is the only nonnegligible muon capture to the excited state of ^{12}B. As pointed out by Kobayashi *et al.*,[3] the correction due to the cascade processes of Eq. (3) varies substantially with the nuclear wave function of ^{12}B*(2.62). Instead of Eq. (1), they obtained[3]

$$P_{av} = 0.48 \pm 0.05\,. \tag{5}$$

In this paper, a model-independent analysis of the correction due to the cascade processes of Eq. (3) is presented making use of the "elementary-particle" treatment (EPT).[4-6] Using as the basic inputs the data of Eqs. (2) and (4), the observed γ transition rate $\Gamma({}^{12}C^*(17.23) \to {}^{12}C(g.s.)\gamma)$, and the relative sign of the covariant form factors $F_E(0)$ and $F_T(0)$ as inferred from a general consideration of the nuclear wave functions, we demonstrate that the correction due to the cascade processes of Eq. (3) can be evaluated in a reliable manner. Our result for the average polarization P_{av} of the recoil ^{12}B(g.s.) from the *direct* polarized-muon capture is

$$P_{av} = 0.47 \pm 0.05 \tag{6}$$

which confirms the result of Kobayashi *et al.* In addition, it is also pointed out that, if the Cohen-Kurath wave functions for ^{12}C(g.s.) and ^{12}B(g.s.) are invoked in the impulse-approximation calculation, the lower value of P_{av} [Eqs. (5) and (6)] is in fact favored strongly by the prediction of the standard picture, namely, conservation of vector current (CVC), PCAC, and the absence of second-class axial vector currents.

I. BRIEF REVIEW OF DIRECT POLARIZED-MUON CAPTURE

With $V_\lambda(x)$ and $A_\lambda(x)$ the hadronic weak polar and axial vector currents, the covariant nuclear form factors defined in the "elementary particle" treatment are given by[5]

$$\langle {}^{12}B(p^{(f)}, \xi^{(f)}) \,|\, V_\lambda(0) \,|\, {}^{12}C(p^{(i)}) \rangle = -\sqrt{2}\,\epsilon_{\lambda\kappa\rho\eta}\,\xi_\kappa^{(f)*}\,\frac{q_\rho}{2m_p}\,\frac{Q_\eta}{2M}\,F_M(q^2)\,, \tag{7a}$$

$$\langle {}^{12}\mathrm{B}(p^{(f)},\xi^{(f)})\,|\,A_\lambda(0)\,|\,{}^{12}\mathrm{C}(p^{(i)})\rangle = \sqrt{2}\left(\xi_\lambda^{(f)*}F_A(q^2) + q_\lambda \frac{q\cdot\xi^{(f)*}}{m_\pi^2}F_P(q^2) - \frac{Q_\lambda}{2M}\frac{q\cdot\xi^{(f)*}}{2m_p}F_E(q^2)\right), \qquad (7b)$$

where

$$q_\lambda \equiv (p^{(f)} - p^{(i)})_\lambda,\quad Q_\lambda \equiv (p^{(f)} + p^{(i)})_\lambda\,,$$

$$M \equiv \tfrac{1}{2}(M_i + M_f) = \tfrac{1}{2}[M({}^{12}\mathrm{C}) + M({}^{12}\mathrm{B})]\,,$$

$$\xi^{(f)*} \equiv (\vec{\xi}^{(f)*}, i\xi_0^{(f)*}),\quad \xi^{(f)*}\cdot\xi^{(f)} \equiv \vec{\xi}^{(f)*}\cdot\vec{\xi}^{(f)} - \xi_0^{(f)*}\xi_0^{(f)} = 1\,,$$

and

$$\xi^{(f)*}\cdot p^{(f)} = 0\,.$$

In terms of these form factors, the transition amplitude $\mathcal{T}$ for muon capture *directly* to the ground state of ^{12}B:

$$\mu^-(p^{(\mu)},s^{(\mu)}) + {}^{12}\mathrm{C}(p^{(i)}) \to \nu_\mu(p^{(\nu)},s^{(\nu)}) + {}^{12}\mathrm{B}(p^{(f)},\xi^{(f)}) \qquad (8)$$

is given by

$$\mathcal{T} = (G/\sqrt{2})\langle {}^{12}\mathrm{B}(p^{(f)},\xi^{(f)})\,|\,[V_\lambda(0) + A_\lambda(0)]\,|\,{}^{12}\mathrm{C}(p^{(i)})\rangle i u^{(\nu)\dagger}(p^{(\nu)},s^{(\nu)})\gamma_4\gamma_\lambda(1+\gamma_5)u^{(\mu)}(p^{(\mu)},s^{(\mu)}) \qquad (9)$$

which, to a sufficient approximation, reduces to

$$\mathcal{T}(s^{(\mu)},s^{(\nu)},\xi^{(f)*}) = (G/\sqrt{2})v^{(\nu)\dagger}(1-\vec{\sigma}\cdot\hat{\nu})\vec{\xi}^{(f)*}\cdot[-G_V i\vec{\sigma}\times\hat{\nu} + (G_A - G_V)\vec{\sigma} + (G_P + G_V)\hat{\nu}]v^{(\mu)}\,. \qquad (10)$$

Here, in Eq. (10), $v^{(\nu)}$ and $v^{(\mu)}$ are two-component Pauli spinors, $\vec{\sigma}$ are two-by-two Pauli matrices to be sandwiched between $v^{(\nu)\dagger}$ and $v^{(\mu)}$, and $\hat{\nu}$ is the unit vector defined by the neutrino momentum. Further, $G_{V,A,P}$ are convenient combinations of the covariant form factors as introduced by Primakoff:

$$G_V = -F_M(q^2)(E^{(\nu)}/2m_p)\,, \qquad (11a)$$

$$G_A = -F_A(q^2) - F_M(q^2)(E^{(\nu)}/2m_p)\,, \qquad (11b)$$

$$G_P = F_P(q^2)\frac{m_\mu E^{(\nu)}}{m_\pi^2} - F_E(q^2)\frac{E^{(\nu)}}{2m_p} - F_M(q^2)\frac{E^{(\nu)}}{2m_p} \qquad (11c)$$

with[5]

$$q^2 = (p^{(f)} - p^{(i)})^2 = (p^{(\mu)} - p^{(\nu)})^2 = -m_\mu^2 + 2m_\mu E^{(\nu)} = 0.740 m_\mu^2\,,$$

$$E^{(\nu)} = (m_\mu - \Delta) - \frac{(m_\mu - \Delta)^2}{2M({}^{12}\mathrm{B})} = 91.41\ \mathrm{MeV}\,, \qquad (11d)$$

$$\Delta \equiv M({}^{12}\mathrm{B}) - M({}^{12}\mathrm{C}) = 13.881\ \mathrm{MeV}\,.$$

Denoting the direction of the muon spin by $\hat{n}$, we therefore obtain

$$\begin{aligned} G^{-2}\sum_{s^{(\nu)}}|\mathcal{T}(s^{(\mu)},s^{(\nu)},\xi^{(f)*})|^2 = {} & G_A^2(1+\hat{n}\cdot\hat{\nu}) + (-2G_AG_P + G_P^2)\hat{\nu}\cdot\vec{\xi}^{(f)}\hat{\nu}\cdot\vec{\xi}^{(f)*} - G_P^2\hat{n}\cdot\hat{\nu}\hat{\nu}\cdot\vec{\xi}^{(f)}\hat{\nu}\cdot\vec{\xi}^{(f)*} \\ & + (2G_AG_V - G_V^2)i\vec{\xi}^{(f)}\times\vec{\xi}^{(f)*}\cdot\hat{\nu} + G_V^2\hat{n}\cdot\hat{\nu}i\vec{\xi}^{(f)}\times\vec{\xi}^{(f)*}\cdot\hat{\nu} + (G_A^2 - G_V^2)i\vec{\xi}^{(f)}\times\vec{\xi}^{(f)*}\cdot\hat{n} \\ & - (G_A^2 - G_AG_P)(\hat{n}\cdot\vec{\xi}^{(f)}\hat{\nu}\cdot\vec{\xi}^{(f)*} + \hat{n}\cdot\vec{\xi}^{(f)*}\hat{\nu}\cdot\vec{\xi}^{(f)}) \\ & + (G_V^2 - G_AG_P)(i\hat{n}\cdot\vec{\xi}^{(f)}\times\hat{\nu}\hat{\nu}\cdot\vec{\xi}^{(f)*} - i\hat{n}\cdot\vec{\xi}^{(f)*}\times\hat{\nu}\hat{\nu}\cdot\vec{\xi}^{(f)})\,; \end{aligned} \qquad (12)$$

whence

$$G^{-2}\int\frac{d^2\Omega^{(\nu)}}{4\pi}\sum_{s^{(\nu)}}|\mathcal{T}(s^{(\mu)},s^{(\nu)},\xi^{(f)*})|^2 = \tfrac{1}{3}(3G_A^2 - 2G_AG_P + G_P^2) + i\vec{\xi}^{(f)}\times\vec{\xi}^{(f)*}\cdot\hat{n}\tfrac{1}{3}(3G_A^2 - 2G_AG_P)\,, \qquad (13a)$$

$$G^{-2}\int\frac{d^2\Omega^{(\nu)}}{4\pi}\frac{1}{2}\sum_{s^{(\mu)}}\sum_{s^{(\nu)}}\sum_{\xi^{(f)}}|\mathcal{T}(s^{(\mu)},s^{(\nu)},\xi^{(f)*})|^2 = 3G_A^2 - 2G_AG_P + G_P^2\,. \qquad (13b)$$

With the effect of the initial-state Coulomb interaction factorized in a way analogous to the factorization of the Fermi functions in the β-decay processes, the capture rate can be cast into the following form[5]:

$$\Gamma(\mu^- \, ^{12}\mathrm{C(g.s.)} \to \nu_\mu \, ^{12}\mathrm{B(g.s.)}) \equiv \Gamma_0(3G_A{}^2 - 2G_A G_P + G_P{}^2)\,,$$

$$\Gamma_0 \equiv \left(\frac{GE^{(\nu)}}{\pi}\right)^2 \left(1 + \frac{E^{(\nu)}}{m_\mu + M_f}\right)^{-1} C(^{12}\mathrm{C}) \times \left(\frac{Z(^{12}\mathrm{C})}{137} \frac{m_\mu M_i}{m_\mu + M_i}\right)^3,$$

$$Z(^{12}\mathrm{C}) = 6, \quad C(^{12}\mathrm{C}) = 0.841\,. \tag{14}$$

Further, we choose the $\hat{z}$ axis for the quantization of the spin of the recoil ^{12}B so that $\vec{\xi}^{(f)}(\pm 1) = (\mp 1/\sqrt{2})(\hat{x} \pm i\hat{y})$ and $\vec{\xi}^{(f)}(0) = \hat{z}$ (with $\hat{x},\hat{y},\hat{z}$ orthogonal) correspond, respectively, to the $J_z = \pm 1, 0$ substates. We then determine the three populations $h_{\pm 1}, h_0$ corresponding to the $J_z = \pm 1, 0$ substates by substitution of $\vec{\xi}^{(f)}(\pm 1) = (\mp 1/\sqrt{2})(\hat{x} \pm i\hat{y})$ and $\vec{\xi}^{(f)}(0) = \hat{z}$ into Eqs. (12) and (13a). In the case of polarized-muon capture, the average polarization P_{av} and average alignment A_{av} are calculated from Eq. (13a) by choosing $\hat{z} = \hat{n}$. In this way, we obtain

$$P_{\mathrm{av}} \equiv \frac{h_{+1} - h_{-1}}{h_{+1} + h_{-1} + h_0} = \frac{2}{3}\left(1 - \frac{G_P{}^2}{3G_A{}^2 - 2G_A G_P + G_P{}^2}\right), \tag{15a}$$

$$A_{\mathrm{av}} \equiv \frac{h_{+1} + h_{-1} - 2h_0}{h_{+1} + h_{-1} + h_0} = 0\,. \tag{15b}$$

In the case of unpolarized muon capture, we can also choose $\hat{z} = \hat{\nu}$ in Eq. (12) (with all the $\hat{n}$ terms dropped out) and calculate the *longitudinal* polarization P_L and *longitudinal* alignment A_L:

$$P_L = \frac{2(2G_A G_V - G_V{}^2)}{3G_A{}^2 - 2G_A G_P + G_P{}^2}\,, \tag{16a}$$

$$A_L = \frac{2(2G_A G_P - G_P{}^2)}{3G_A{}^2 - 2G_A G_P + G_P{}^2}\,. \tag{16b}$$

In principle, P_{av}, A_{av}, P_L, and A_L are measured by detecting the angular distribution of the electrons from the subsequent β decay of the recoil ^{12}B(g.s.) (with respect to the $\hat{z}$ axis defined by $\hat{n}$ or $\hat{\nu}$). In practice, the nonzero value of A_L enters into the angular distribution by a multiplication factor of $\alpha_- E_e(\frac{3}{2}\cos^2\vartheta_e - \frac{1}{2})$ and so is too small to be of any interest.[5] Nonetheless, the experimental determination of the capture rate [Eq. (14)], average polarization P_{av} [Eq. (15a)], and longitudinal polarization P_L [Eq. (16a)] already allow us to solve $G_{V,A,P}$; the solution is unique if the standard signs of $G_{V,A,P}$, i.e., $G_V < 0$, $G_A < 0$, and $G_P < 0$, are invoked.

To complete our review of *direct* polarized-muon capture, we append here three brief comments on elementary-particle treatment (EPT):

(1) In view of $\xi^{(f)*} \cdot \xi^{(f)} = 1$ and $\xi^{(f)*} \cdot p^{(f)} = 0$, the small recoil effect due to $\xi_0^{(f)*} \neq 0$ in the rest frame of the initial ($\mu^- \, ^{12}$C) state has been neglected in Eq. (10). By restricting ourselves to the rest frame of the recoil ^{12}B(g.s.), we can see immediately that the addition to Eq. (10) of the terms proportional to $m_\mu/(m_\mu + M_i)$ are needed to accomodate such recoil effect. Accordingly, the resultant modifications to Eqs. (14), (15a), and (16a) are negligible in comparison with the current experimental precision of at best 5%.

(2) The factorization hypothesis that the effect due to the initial-state Coulomb interaction be factorized almost completely is nothing more than the factorization of the Fermi functions in the β-decay processes. Since the failure of such factorization reflects a loss in the connection between the $\mu^- \, ^{12}$C bound-state problem and the $\mu^- \, ^{12}$C scattering problem, an appreciable breakdown of the factorization hypothesis seems very unlikely as long as the covariant form factors have been defined properly [Eqs. (7a) and (7b)].

(3) The definition of the covariant form factors given by Eqs. (7a) and (7b) is the most general one as long as the linearity in the polarization four-vector $\xi_\lambda^{(f)*}$ is required. Since the polarization four-vector $\xi_\lambda^{(f)*}$ refers to the intrinsic spin space, the linearity in $\xi_\lambda^{(f)*}$ is equivalent to the statement that the nucleon transition matrix elements be linear in both the initial and final Dirac spinors.

INVESTIGATION OF $\mu^- \, ^{12}\mathrm{C\,(g.s.)} \to \nu_\mu \, ^{12}\mathrm{B}^*\,(2.62)$

To investigate $\mu^- \, ^{12}\mathrm{C(g.s.)} \to \nu_\mu \, ^{12}\mathrm{B}^*(2.62)$, we introduce the following covariant form factors[6]:

$$\langle ^{12}\mathrm{B}^*(p^{(n)}, \xi^{(n)}) \,|\, V_\lambda(0) \,|\, ^{12}\mathrm{C}(p^{(i)}) \rangle = -\sqrt{2}\, i \left(\xi_\lambda^{(n)*} F_V(q^2) + q_\lambda \frac{q \cdot \xi^{(n)*}}{m_\pi{}^2} F_S(q^2) - \frac{Q_\lambda}{2M} \frac{q \cdot \xi^{(n)*}}{2m_p} F_T(q^2) \right) \tag{17a}$$

$$\langle ^{12}\mathrm{B}^*(p^{(n)}, \xi^{(n)}) \,|\, A_\lambda(0) \,|\, ^{12}\mathrm{C}(p^{(i)}) \rangle = \sqrt{2}\, i \epsilon_{\lambda\kappa\rho\eta} \xi_\kappa^{(n)*} \frac{q_\rho}{2m_p} \frac{Q_\eta}{2M} F_E(q^2) \tag{17b}$$

with $q_\lambda = (p^{(n)} - p^{(i)})_\lambda$, $Q_\lambda = (p^{(n)} + p^{(i)})_\lambda$, $M = \frac{1}{2}(M_i + M_n)$, $\xi^{(n)*}\cdot\xi^{(n)} = 1$, and $\xi^{(n)*}\cdot p^{(n)} = 0$. The overall phase factors have been chosen such that, by time reversal invariance, all the nuclear form factors $F_{V,S,T,E}(q^2)$ of Eqs. (17a) and (17b), together with $F_{M,A,P,E}(q^2)$ of Eqs. (7a) and (7b), are real if the nuclear wave functions of ^{12}C(g.s.), ^{12}B*(2.62), and ^{12}B(g.s.) are of a common phase.[6] Hereafter the notations q_λ, Q_λ, M, and $F_E(q^2)$ refer to the $0^+ \to 1^-$ transitions unless specified otherwise. Therefore, the transition amplitude for muon capture to the 2.62 MeV excited state of ^{12}B:

$$\mu^-(p^{(\mu)},s^{(\mu)}) + {}^{12}\mathrm{C}(p^{(i)}) \to \nu_\mu(p^{(\nu)},s^{(\nu)}) + {}^{12}\mathrm{B}^*(p^{(n)},\xi^{(n)}) \tag{18}$$

is given by

$$\mathcal{T}(1^-) = (G/\sqrt{2})\langle {}^{12}\mathrm{B}^*(p^{(n)},\xi^{(n)})\,|\,[V_\lambda(0) + A_\lambda(0)]\,|\,{}^{12}\mathrm{C}(p^{(i)})\rangle i u^{(\nu)\dagger}(p^{(\nu)},s^{(\nu)})\gamma_4\gamma_\lambda(1+\gamma_5)u^{(\mu)}(p^{(\mu)},s^{(\mu)}), \tag{19}$$

which, to a sufficient approximation, reduces to

$$\mathcal{T}(1^-;s^{(\mu)},s^{(\nu)},\xi^{(n)*}) = -i(G/\sqrt{2})v^{(\nu)\dagger}(1 - \vec{\sigma}\cdot\hat{\nu})\vec{\xi}^{(n)*}\cdot[-H_A i\vec{\sigma}\times\hat{\nu} + (H_V - H_A)\vec{\sigma} + (H_S + H_A)\hat{\nu}]v^{(\mu)}. \tag{20}$$

Here $v^{(\nu)}$, $v^{(\mu)}$, and $\vec{\sigma}$ are defined as in Eq. (10) and $H_{A,V,S}$ are given by

$$H_A = -F_E(q^2)(E^{(\nu)}/2m_p), \tag{21a}$$

$$H_V = -F_V(q^2) - F_E(q^2)(E^{(\nu)}/2m_p), \tag{21b}$$

$$H_S = F_S(q^2)\,\frac{m_\mu E^{(\nu)}}{m_\pi^{\ 2}} - F_T(q^2)\,\frac{E^{(\nu)}}{2m_p} - F_E(q^2)\,\frac{E^{(\nu)}}{2m_p}, \tag{21c}$$

with[7]

$$q^2 = (p^{(n)} - p^{(i)})^2 = -m_\mu^{\ 2} + 2m_\mu E^{(\nu)} = 0.681 m_\mu^{\ 2},$$

$$E^{(\nu)} = (m_\mu - \Delta^*) - \frac{(m_\mu - \Delta^*)^2}{2M(^{12}\mathrm{B}^*)} = 88.80\ \mathrm{MeV}, \tag{21d}$$

$$\Delta^* \equiv M(^{12}\mathrm{B}^*) - M(^{12}\mathrm{C}) = 16.502\ \mathrm{MeV}.$$

Clearly, the substitution specified by

$$G_V \to H_A,\quad G_A \to H_V,\quad G_P \to H_S \tag{22}$$

enables us to obtain from Eqs. (14), (15a), (15b), (16a), and (16b) the formulas for $\Gamma(\mu^-\,{}^{12}\mathrm{C(g.s.)} \to \nu_\mu\,{}^{12}\mathrm{B}^*(2.62))$, $P_{\mathrm{av}}(1^-)$, $A_{\mathrm{av}}(1^-)$, $P_L(1^-)$, and $A_L(1^-)$. These are

$$\Gamma(\mu^-\,{}^{12}\mathrm{C(g.s.)} \to \nu_\mu\,{}^{12}\mathrm{B}^*(2.62)) = \Gamma_0^*(3H_V^2 - 2H_V H_S + H_S^2),$$

$$\Gamma_0^* \equiv \left(\frac{GE^{(\nu)}}{\pi}\right)^2\left(1 + \frac{E^{(\nu)}}{m_\mu + M_n}\right)^{-1} C(^{12}\mathrm{C})\left(\frac{Z(^{12}\mathrm{C})}{137}\,\frac{m_\mu M_i}{m_\mu + M_i}\right)^3 = 1.268\times10^4\ \mathrm{sec}^{-1},$$

$$Z(^{12}\mathrm{C}) = 6,\quad C(^{12}\mathrm{C}) = 0.841, \tag{23}$$

$$P_{\mathrm{av}}(1^-) = \tfrac{2}{3}[1 - H_S^2/(3H_V^2 - 2H_V H_S + H_S^2)], \tag{24a}$$

$$A_{\mathrm{av}}(1^-) = 0, \tag{24b}$$

$$P_L(1^-) = \frac{2(2H_V H_A - H_A^2)}{3H_V^2 - 2H_V H_S + H_S^2}, \tag{25a}$$

$$A_L(1^-) = \frac{2(2H_V H_S - H_S^2)}{3H_V^2 - 2H_V H_S + H_S^2}. \tag{25b}$$

Unfortunately, none of $P_{\mathrm{av}}(1^-)$, $A_{\mathrm{av}}(1^-)$, $P_L(1^-)$, and $A_L(1^-)$ can be measured in the near future.

To evaluate the various form factors $F_{V,S,T,E}(q^2)$ at $q^2 = 0.681\ m_\mu^{\ 2}$, we resort to two standard hypotheses, viz:

(1) Reasonable estimates to the various form factors at low q^2 can be obtained through the invocation of the nucleon-only impulse approximation (NOIA), i.e., the standard impulse approximation without the meson-exchange corrections (MEC).

(2) Conserved-vector-current hypothesis[8] holds in the strict sense ("strong" CVC[5]).

To describe the connection between EPT and NOIA, we define

$$\mathfrak{M}_p(q^2) \equiv \left\langle \psi_{^{12}\mathrm{C}} \left| \sum_{a=1}^{A} j_0(|\vec{q}|r^{(a)}) \tau_+^{(a)} \frac{ip_z^{(a)}}{m_p} \right| \psi_{^{12}\mathrm{B}^*,\xi(0)} \right\rangle, \tag{26a}$$

$$\mathfrak{M}_{pQ}(q^2) \equiv \left\langle \psi_{^{12}\mathrm{C}} \left| \sum_{a=1}^{A} \frac{15 j_2(|\vec{q}|r^{(a)})}{(|\vec{q}|r^{(a)})^2} \tau_+^{(a)} m_\pi^2 (r^{(a)2} - 3z^{(a)2}) \frac{ip_z^{(a)}}{m_p} \right| \psi_{^{12}\mathrm{B}^*,\xi(0)} \right\rangle, \tag{26b}$$

$$\mathfrak{M}_z(q^2) \equiv \left\langle \psi_{^{12}\mathrm{C}} \left| \sum_{a=1}^{A} \frac{3 j_1(|\vec{q}|r^{(a)})}{(|\vec{q}|r^{(a)})} \tau_+^{(a)} m_p z^{(a)} \right| \psi_{^{12}\mathrm{B}^*,\xi(0)} \right\rangle, \tag{26c}$$

$$\mathfrak{M}_{r\times\sigma}(q^2) \equiv \left\langle \psi_{^{12}\mathrm{C}} \left| \sum_{a=1}^{A} \frac{3 j_1(|\vec{q}|r^{(a)})}{(|\vec{q}|r^{(a)})} \tau_+^{(a)} m_p i(\vec{r}^{(a)} \times \vec{\sigma}^{(a)})_z \right| \psi_{^{12}\mathrm{B}^*,\xi(0)} \right\rangle, \tag{26d}$$

where

$$\psi_{^{12}\mathrm{B}^*,\xi(0)} \equiv \psi_{^{12}\mathrm{B}^*,\xi(0)}(\ldots, \vec{r}^{(a)}, \sigma_z^{(a)}, \tau_z^{(a)}, \ldots),$$

$$\psi_{^{12}\mathrm{C}} \equiv \psi_{^{12}\mathrm{C}}(\ldots, \vec{r}^{(a)}, \sigma_z^{(a)}, \tau_z^{(a)}, \ldots), \quad \tau_+^{(a)} \equiv \tfrac{1}{2}(\tau_1^{(a)} + i\tau_2^{(a)}),$$

and $\xi(0)$ is the polarization four-vector describing the $^{12}\mathrm{B}^*$ state with $J_z = 0$. As already illustrated in Ref. 6, the NOIA contributions to the various form factors $F_{V,S,T,E}(q^2)$ are given as follows:

$$\sqrt{2}\, F_V(q^2) = -f_V(q^2)\left(\mathfrak{M}_p(q^2) - \frac{1}{12}\frac{q^2}{m_\pi^2}\mathfrak{M}_{pQ}(q^2)\right) - (f_V(q^2) + f_M(q^2))\frac{q^2}{4m_p^2}\mathfrak{M}_{r\times\sigma}(q^2), \tag{27a}$$

$$\sqrt{2}\, F_S(q^2) = f_V(q^2)\frac{m_\pi^2}{2m_p^2}\left(1 - \frac{m_p}{M}\right)\mathfrak{M}_z(q^2) - \tfrac{1}{4} f_V(q^2)\mathfrak{M}_{pQ}(q^2) + (f_V(q^2) + f_M(q^2))\frac{m_\pi^2}{4m_p^2}\mathfrak{M}_{r\times\sigma}(q^2), \tag{27b}$$

$$\sqrt{2}\, F_T(q^2) = 2f_V(q^2)\mathfrak{M}_z(q^2) - f_V(q^2)\frac{m_p\Delta^*}{2m_\pi^2}\mathfrak{M}_{pQ}(q^2) + (f_V(q^2) + f_M(q^2))\frac{\Delta^*}{2m_p}\mathfrak{M}_{r\times\sigma}(q^2), \tag{27c}$$

$$\sqrt{2}\, F_E(q^2) = -f_A(q^2)\mathfrak{M}_{r\times\sigma}(q^2), \tag{27d}$$

where the nucleon form factors $f_{V,M,A}(q^2)$ are defined in the same manner as in Ref. 5 or 6 [i.e., $f_V(0) = 1.00$, $f_M(0) = 3.71$, and $f_A(0) = 1.24$]. For the purpose of further discussions, we also define

$$\mathfrak{M}_z^{(0)}(q^2) \equiv \left\langle \psi_{^{12}\mathrm{C}} \left| \sum_{a=1}^{A} \frac{3 j_1(|\vec{q}|r^{(a)})}{(|\vec{q}|r^{(a)})} \frac{\tau_3^{(a)}}{2} m_p z^{(a)} \right| \psi_{^{12}\mathrm{C}^*,\xi(0)} \right\rangle, \tag{28a}$$

$$\mathfrak{M}_{r\times\sigma}^{(0)}(q^2) \equiv \left\langle \psi_{^{12}\mathrm{C}} \left| \sum_{a=1}^{A} \frac{3 j_1(|\vec{q}|r^{(a)})}{(|\vec{q}|r^{(a)})} \frac{\tau_3^{(a)}}{2} m_p i(\vec{r}^{(a)} \times \vec{\sigma}^{(a)})_z \right| \psi_{^{12}\mathrm{C}^*,\xi(0)} \right\rangle, \tag{28b}$$

$$\mathfrak{M}_{GT}(q^2) \equiv \left\langle \psi_{^{12}\mathrm{C}} \left| \sum_{a=1}^{A} j_0(|\vec{q}|r^{(a)}) \tau_+^{(a)} \sigma_z^{(a)} \right| \psi_{^{12}\mathrm{B},\xi(0)} \right\rangle, \tag{28c}$$

where $\psi_{^{12}\mathrm{C}^*,\xi(0)}$ and $\psi_{^{12}\mathrm{B},\xi(0)}$ refer, respectively, to the 17.23 MeV isospin analog state of $^{12}\mathrm{B}^*(2.62)$ and the ground state of $^{12}\mathrm{B}$ (both with $J_z = 0$).

Now, CVC in the weak sense, i.e., $\partial_\lambda V_\lambda(x) = 0$, requires

$$F_V(q^2) + \frac{q^2}{m_\pi^2} F_S(q^2) + \frac{\Delta^*}{2m_p} F_T(q^2) = 0\,. \tag{29}$$

Since $\{F_S(q^2)\}_{\mathrm{NOIA}}$ is regular at $q^2 = 0$, we obtain from Eq. (29)

$$F_V(0) = -\frac{\Delta^*}{2m_p} F_T(0)\,, \tag{30}$$

We also note from Eqs. (26a) and (26b) that $\mathfrak{M}_{pQ}(0)$ is expected to be at most the same order of magnitude as $\mathfrak{M}_p(0)$. Therefore, we obtain from Eqs. (27a)–(27d)

$$|\mathfrak{M}_{pQ}(0)| \approx |\mathfrak{M}_p(0)| \approx (\Delta^*/m_p)\,|\mathfrak{M}_z(0)| \tag{31}$$

so that

$$|F_S(0)| \lesssim (\Delta^*/2m_p)\,|F_T(0)|\,. \tag{32}$$

We proceed to observe from Eq. (32) that the $F_S(q^2)$ contribution to H_S of Eq. (21c) is smaller than that of $F_T(q^2)$ by at least a factor of $m_\mu \Delta^*/m_\pi^2 = 9\%$. To perform numerical calculations on $\mu^- \, ^{12}\mathrm{C}(\mathrm{g.s.}) \to \nu_\mu \, ^{12}\mathrm{B}^*(2.62)$, we can therefore neglect the small contribution from $F_S(q^2)$ and use a value of $F_V(q^2)$ calculated from Eq. (29) [with the $F_S(q^2)$ term neglected from the left-hand side of Eq. (29)].

To throw further light on the values of $F_{T,E}(q^2)$, we observe that, by the approximate isospin symmetry, we obtain from Eqs. (27c), (27d), (26c), (26d), (28a), and (28b)

$$F_T(q^2) \cong \sqrt{2} f_V(q^2) \mathfrak{M}_z(q^2) \cong 2 f_V(q^2) \mathfrak{M}_z^{(0)}(q^2) , \quad (33a)$$

$$F_E(q^2) \cong -\frac{1}{\sqrt{2}} f_A(q^2) \mathfrak{M}_{r\times\sigma}(q^2) \cong -f_A(q^2) \mathfrak{M}_{r\times\sigma}^{(0)}(q^2) . \quad (33b)$$

In the shell-model language, the 17.23 MeV excited state of ^{12}C may be represented as, to a sufficient approximation,

$$|^{12}\mathrm{C}^*(17.23)\rangle = \alpha \{(1s_{1/2})^{-1}(1p_{1/2})^1\}^\dagger |^{12}\mathrm{C(g.s.)}\rangle + \beta\{(2s_{1/2})^1(1p_{3/2})^{-1}\}^\dagger |^{12}\mathrm{C(g.s.)}\rangle + \gamma\{(1d_{5/2})^1(1p_{3/2})^{-1}\}^\dagger |^{12}\mathrm{C(g.s.)}\rangle , \quad (34)$$

with α, β, and γ real coefficients yet to be determined. Here, we have assumed that each particle-hole pair, as created from $|^{12}\mathrm{C(g.s.)}\rangle$, has $J^\pi = 1^-$ and $I=1$. One of our major arguments then goes as follows: If only α differs from zero, then we expect

$$\frac{F_E(0)}{F_T(0)} \cong -\frac{f_A(0)}{f_V(0)} = -1.24 . \quad (35a)$$

Similarly, $\gamma = \alpha = 0$ and $\beta \neq 0$ implies

$$\frac{F_E(0)}{F_T(0)} \cong -\frac{1}{2}\frac{f_A(0)}{f_V(0)} = -0.62 , \quad (35b)$$

while $\alpha = \beta = 0$ and $\gamma \neq 0$ implies

$$\frac{F_E(0)}{F_T(0)} \cong \frac{1}{2}\frac{f_A(0)}{f_V(0)} = 0.62 . \quad (35c)$$

Since the observed energy levels of ^{12}C suggest that the excitation energies of the three modes corresponding to the three terms of Eq. (34) are split by an amount of at least 2 MeV, it is plausible to assume that only one of the three coefficients α, β, and γ is dominant. In this case, we have

$$-1.24 \lesssim F_E(0)/F_T(0) \lesssim +0.62 , \quad (36)$$

where the lower or upper "bounds" are at most approximate. For our purpose, only the solution with gross violation of Eq. (36) shall be considered to be unlikely.

Finally, the shell-model calculation illustrated above allows us to draw the following result:

$$\frac{\mathfrak{M}_{r\times\sigma}^{(0)}(q^2)}{\mathfrak{M}_{r\times\sigma}^{(0)}(0)} = \frac{\mathfrak{M}_z^{(0)}(q^2)}{\mathfrak{M}_z^{(0)}(0)} \cong \frac{\mathfrak{M}_{\mathrm{GT}}(q^2)}{\mathfrak{M}_{\mathrm{GT}}(0)} \quad (37)$$

in the q^2 regime of our interest. A deviation of 10% from the second equality of Eq. (37) requires an unrealistic difference among the oscillator lengths of the s, p, and d shells. Since, as already given in Ref. 5,

$$F_A(q^2) \cong -f_A(q^2)\mathfrak{M}_{\mathrm{GT}}(q^2) \quad (38)$$

we obtain from Eqs. (33a), (33b), and (37)

$$\frac{F_E(q^2)}{F_E(0)} \cong \frac{F_T(q^2)}{F_T(0)} \cong \frac{F_A(q^2)}{F_A(0)} . \quad (39)$$

Numerically, we take from Ref. 5

$$\frac{F_E(0.681 m_\mu^2)}{F_E(0)} = \frac{F_T(0.681 m_\mu^2)}{F_T(0)} \cong 0.770 \pm 0.013 \quad (40)$$

with an additional theoretical uncertainty of about 5%.

To sum up our results, we can approximate Eqs. (23) and (24a) as follows:

$$\Gamma(\mu^- \, {}^{12}\mathrm{C(g.s.)} \to \nu_\mu {}^{12}\mathrm{B}^*(2.62)) = (7.52 \pm 0.03) \times 10^3 R \ \mathrm{sec}^{-1} , \quad (41a)$$

$$P_{\mathrm{av}}(1^-) = \tfrac{2}{3}(1 - P/R) \quad (41b)$$

with

$$R \equiv 2[F_V(0) + F_E(0)(E^{(\nu)}/2m_p)]^2 + [F_V(0) - F_T(0)(E^{(\nu)}/2m_p)]^2 , \quad (41c)$$

$$P \equiv [F_T(0) + F_E(0)]^2 (E^{(\nu)}/2m_p)^2 , \quad (41d)$$

$$F_V(0) = -(\Delta^*/2m_p) F_T(0) = -(8.8 \times 10^{-3}) F_T(0) . \quad (41e)$$

To determine the value of $F_V(0)$, we resort to the application of "strong" CVC to the description of the γ decay $^{12}\mathrm{C}^*(17.23) \to {}^{12}\mathrm{C(g.s.)}\gamma$. This yields

$$\langle {}^{12}\mathrm{C}^*(p^{(n)}, \xi^{(n)}) | J_\lambda(0) | {}^{12}\mathrm{C}(p^{(i)}) \rangle = -i \left(\xi_\lambda^{(n)*} F_V(q^2) + q_\lambda \frac{q \cdot \xi^{(n)*}}{m_\pi^2} F_S(q^2) - \frac{Q_\lambda}{2M} \frac{q \cdot \xi^{(n)*}}{2m_p} F_T(q^2) \right) \quad (42)$$

with $J_\lambda(x)$ the hadronic electromagnetic current. From Eq. (42), we obtain

$$\Gamma({}^{12}\mathrm{C}^*(17.23) \to {}^{12}\mathrm{C(g.s.)}\gamma) = \tfrac{4}{3}\alpha |F_V(0)|^2 . \quad (43)$$

Using the experimental value of 44 eV (Ref. 9), we find

$$|F_V(0)| = 1.62 \times 10^{-2} , \quad (44a)$$

so that, from Eq. (41e),

$$|F_T(0)| = 1.84, \quad F_V(0) F_T(0) < 0 . \quad (44b)$$

Finally, with the experimental value of Eq. (4), we can solve $F_E(0)$ from Eqs. (41a), (44a), and (44b). This yields

$$|F_E(0)| = 4.41 \text{ if } F_E(0)F_T(0) < 0 = 5.09 \text{ if } F_E(0)F_T(0) > 0 . \quad (45)$$

In view of Eq. (36), the solution with $F_E(0)F_T(0) > 0$ is unlikely. On the other hand, the solution with $F_E(0)F_T(0) < 0$ is not inconsistent with Eq. (36) if the experimental errors, as given by Eqs. (4) and (40), and the theoretical uncertainties, mainly due to the use of Eq. (39) and the neglect of the meson-exchange corrections, are properly taken into account. Using Eqs. (41b), (41c), (44a), (44b), and (45) with $F_E(0)F_T(0)<0$, we obtain

$$P_{av}(1^-)=0.58\,. \tag{46}$$

In the following section we shall direct our attention only to this solution.

APPARENT AVERAGE POLARIZATION OF ^{12}B (g.s.)

We proceed to calculate the average polarization of ^{12}B(g.s.) produced by the cascade processes of Eq. (3). As an illustrative example, we investigate in detail the case that ^{12}B*(2.62) decays *directly* to ^{12}B(g.s.). For this purpose, we introduce[10]

$$\langle{}^{12}\mathrm{B}(p^{(f)},\xi^{(f)})|J_\lambda(0)|{}^{12}\mathrm{B}^*(p^{(n)},\xi^{(n)})\rangle = i\epsilon_{\lambda\rho\eta\kappa}\xi_\rho^{(f)*}\xi_\eta^{(n)}\left(\frac{Q_\kappa}{2M}G_V(q^2)-\frac{q_\kappa}{2m_p}G_R(q^2)\right)+i\epsilon_{\rho\eta\kappa\zeta}\xi_\rho^{(f)*}\xi_\eta^{(n)}\frac{q_\lambda}{2m_p}\frac{Q_\zeta}{2M}\left(\frac{q_\lambda}{2m_p}G_S(q^2)-\frac{Q_\lambda}{2M}G_T(q^2)\right)$$
$$+i\epsilon_{\lambda\rho\eta\kappa}\xi_\rho^{(f)*}\frac{q_\eta}{2m_p}\frac{Q_\kappa}{2M}\frac{\xi^{(n)}\cdot q}{2m_p}G_K(q^2)+i\epsilon_{\lambda\rho\eta\kappa}\xi_\rho^{(n)}\frac{q_\eta}{2m_p}\frac{Q_\kappa}{2M}\frac{\xi^{(f)*}\cdot q}{2m_p}G_L(q^2) \tag{47}$$

with $q_\lambda=(p^{(f)}-p^{(n)})_\lambda$, $Q_\lambda=(p^{(f)}+p^{(n)})_\lambda$, and $M=\frac{1}{2}(M_n+M_f)$. The conservation of the hadronic electromagnetic current, i.e., $\partial_\lambda J_\lambda(x)=0$, yields

$$G_V(q^2)+(q^2/4m_p^2)G_S(q^2)=0\,. \tag{48}$$

On the other hand, we can show that, keeping the terms consistently up to the second order in $|\vec{q}|/2m_p$, the NOIA contributions to $G_K(q^2)$ and $G_L(q^2)$ be identically zero.[10] Therefore the transition amplitude for ^{12}B*(2.62) → ^{12}B(g.s.) γ is given by

$$\mathcal{T}_\gamma=(e/\sqrt{2E_\gamma})\epsilon_\lambda^*\langle{}^{12}\mathrm{B}(p^{(f)},\xi^{(f)})|J_\lambda(0)|{}^{12}\mathrm{B}^*(p^{(n)},\xi^{(n)})\rangle\,, \tag{49}$$

which, to a sufficient approximation, reduces to

$$\mathcal{T}_\gamma(\xi^{(n)},\xi^{(f)*},\epsilon^*)\cong-(e/\sqrt{2E_\gamma})(E_\gamma/2m_p)G_R(0)\vec{\epsilon}^*\cdot\vec{\xi}^{(f)*}\times\vec{\xi}^{(i)}\,, \tag{50}$$

where ϵ_λ^* is the polarization four-vector for the photon. Finally, we obtain the transition amplitude $\tilde{\mathcal{T}}$ for the two-step process of Eq. (3), viz,

$$\tilde{\mathcal{T}}=\sum_{\xi^{(n)}}\mathcal{T}_\gamma(\xi^{(n)},\xi^{(f)*},\epsilon^*)\mathcal{T}(1^-;s^{(\mu)},s^{(\nu)},\xi^{(n)*})$$
$$=(\tilde{G}/\sqrt{2})v^{(\nu)\dagger}(1-\vec{\sigma}\cdot\hat{\nu})i\vec{\epsilon}^*\times\vec{\xi}^{(f)*}\cdot[-H_A i\vec{\sigma}\times\hat{\nu}+(H_V-H_A)\vec{\sigma}+(H_S+H_A)\hat{\nu}]v^{(\mu)} \tag{51a}$$

with

$$\tilde{G}\equiv G(e/\sqrt{2E_\gamma})(E_\gamma/2m_p)G_R(0)\,. \tag{51b}$$

Making the following substitution on Eq. (12),

$$G_A\to H_V,\quad G_P\to H_S,\quad G_V\to H_A,\quad \vec{\xi}^{(f)*}\to i\vec{\epsilon}^*\times\vec{\xi}^{(f)*} \tag{52}$$

(remembering that the first term of Eq. (12) is from $\vec{\xi}^{(f)}\cdot\vec{\xi}^{(f)*}$) we obtain

$$\int\frac{d^2\Omega^{(\gamma)}}{4\pi}\int\frac{d^2\Omega^{(\nu)}}{4\pi}\sum_{\epsilon^*}\sum_{s^{(\nu)}}|\tilde{\mathcal{T}}|^2=\tfrac{2}{9}\tilde{G}^2\{2(3H_V^2-2H_VH_S+H_S^2)+i\vec{\xi}^{(f)}\times\vec{\xi}^{(f)*}\cdot\hat{n}(3H_V^2-2H_VH_S)\}\,. \tag{53}$$

Accordingly, the average polarization of ^{12}B(g.s.) produced by the two-step process of Eq. (3) is simply

$$\tilde{P}_{av}=\tfrac{1}{2}P_{av}(1^-)\,. \tag{54}$$

Intuitively, Eq. (54) means that the average polarization of the intermediate ^{12}B*(2.62) is shared equally by the two subsequent spin-one products, ^{12}B(g.s.) and γ.

We proceed to note that the coefficient $\frac{1}{2}$ of Eq. (54) can in fact be computed by a manipulation over Clebsch-Gordan coefficients. That is, we begin by assuming a 100% polarization for ^{12}B*(2.62) and then decompose the angular-momentum $|1,1\rangle$ as follows:

$$|^{12}\mathrm{B}^*;\ 1,1\rangle = \frac{1}{\sqrt{2}}\,|^{12}\mathrm{B};\ 1,1\rangle\,|\gamma;1,0\rangle - \frac{1}{\sqrt{2}}\,|^{12}\mathrm{B};1,0\rangle\,|\gamma;1,1\rangle .$$

Counting the populations $h_{\pm 1}$ and h_0, we observe that the desired coefficient is simply $h_{+1}-h_{-1}$. Generalizing this counting procedure to the multistep processes, we obtain

$$\tilde{P}_{\mathrm{av}}(1^- \to 2^\pm \to \mathrm{g.s.}) = (\tfrac{3}{5}\times 1 + \tfrac{3}{10}\times\tfrac{1}{2})P_{\mathrm{av}}(1^-) = \tfrac{3}{4}P_{\mathrm{av}}(1^-) . \quad (55)$$

The absence of the $J_z = 0$ photon mode does not hurt us since we are dealing with a statistical concept: "average" polarization. Now, experimentally, we have[7]

$$^{12}\mathrm{B}^*(2.62)\xrightarrow{\gamma}{}^{12}\mathrm{B}^*(1.67;2^-)\xrightarrow{\gamma}{}^{12}\mathrm{B}(\mathrm{g.s.}):\ (14\pm 3)\% ,$$

$$^{12}\mathrm{B}^*(2.62)\xrightarrow{\gamma}{}^{12}\mathrm{B}^*(0.95;2^+)\xrightarrow{\gamma}{}^{12}\mathrm{B}(\mathrm{g.s.}):\ (80\pm 3)\% ,$$

$$^{12}\mathrm{B}^*(2.62)\xrightarrow{\gamma}{}^{12}\mathrm{B}(\mathrm{g.s.}):\ (6\pm 1)\% .$$

(The channel $^{12}\mathrm{B}^*(2.62)\to{}^{12}\mathrm{B}^*(1.67)\to{}^{12}\mathrm{B}^*(0.95)\to{}^{12}\mathrm{B}(\mathrm{g.s.})$ can be neglected.) Weighting $P_{\mathrm{av}}(1^-)$ by the various cascade channels with their corresponding coefficients $[\tilde{P}_{\mathrm{av}}/P_{\mathrm{av}}(1^-)]$ and branching ratios, we obtain the average polarization P^*_{av} of the final $^{12}\mathrm{B}(\mathrm{g.s.})$ produced by the cascade processes $\mu^-\,{}^{12}\mathrm{C}(\mathrm{g.s.})\to\nu_\mu{}^{12}\mathrm{B}^*(2.62)\to\nu_\mu{}^{12}\mathrm{B}(\mathrm{g.s.})\gamma$:

$$P^*_{\mathrm{av}} = 0.74P_{\mathrm{av}}(1^-) = 0.43 . \quad (56)$$

Therefore, assuming that the cascade processes of Eq. (3) are the only background to produce $^{12}\mathrm{B}(\mathrm{g.s.})$, we obtain

$$P^{\mathrm{appr}}_{\mathrm{av}} = [\Gamma P_{\mathrm{av}} + \Gamma^* P^*_{\mathrm{av}}]/(\Gamma+\Gamma^*) = [\Gamma P_{\mathrm{av}} + 0.74\Gamma^* P_{\mathrm{av}}(1^-)]/(\Gamma+\Gamma^*) \quad (57)$$

with $\Gamma \equiv \Gamma(\mu^-\,{}^{12}\mathrm{C}(\mathrm{g.s.})\to\nu_\mu{}^{12}\mathrm{B}(\mathrm{g.s.}))$ and $\Gamma^* \equiv \Gamma(\mu^-\,{}^{12}\mathrm{C}(\mathrm{g.s.})\to\nu_\mu{}^{12}\mathrm{B}^*(2.62))$. Using the experimental values for Γ (6.2×10^{-3} sec^{-1}; Ref. 2), Γ^* [Eq. (4)], and $P^{\mathrm{appr}}_{\mathrm{av}}$ [Eq. (2)], we conclude with the following value of P_{av}:

$$P_{\mathrm{av}} = 0.47\pm 0.05 . \quad (58)$$

As regards the theory, we first note that the quantity specified by

$$\lambda \equiv \frac{F_P(q^2)}{F_A(q^2)}\frac{m_\mu E^{(\nu)}}{m_\pi^2} - \left(\frac{F_E(q^2)}{F_A(q^2)}+\frac{F_M(q^2)}{F_A(q^2)}\right)\frac{E^{(\nu)}}{2m_p} \quad (59)$$

can be determined unambiguously by the data on the capture rate Γ and average polarization P_{av}. Hereafter $F_E(q^2)$ is understood in the definition of Eq. (7b). Using the observed β and γ decay rates[7] of $^{12}\mathrm{B}$ and its isospin anolog $^{12}\mathrm{C}^*$, we obtain, granting validity of CVC,

$$F_M(0)/F_A(0) = 3.86\pm 0.12 , \quad (60a)$$

Meanwhile, the recent asymmetry measurements[11] yields

$$F_E(0)/F_A(0) = 3.67\pm 0.44 , \quad (60b)$$

where we have assumed the absence of second-class axial vector currents. Taking, with justification, the assumption of similar q^2 dependence for $F_{M,A,E}(q^2)$, viz.:

$$\frac{F_M(q^2)}{F_M(0)} \cong \frac{F_A(q^2)}{F_A(0)} \cong \frac{F_E(q^2)}{F_E(0)} , \quad (60c)$$

we finally obtain

$$F_P(q^2)/F_A(q^2) = -(1.08\pm 0.24), \quad q^2 = 0.740 m_\mu^2 . \quad (61)$$

On the other hand, assuming PCAC for the nucleon pseudoscalar form factor and using the Cohen-Kurath wave functions of $^{12}\mathrm{C}(\mathrm{g.s.})$ and $^{12}\mathrm{B}(\mathrm{g.s.})$ in the NOIA calculation, we obtain[12]

$$[F_P(q^2)/F_A(q^2)]_{\mathrm{NOIA}} = -1.02(1+q^2/m_\pi^2)^{-1}+\delta = -0.99, \quad (62)$$

$$q^2 = 0.740 m_\mu^2, \quad \delta = -0.28 .$$

The excellent agreement between Eqs. (61) and (62) constitutes another argument in favor of the standard picture, namely, CVC, PCAC, and the absence of second-class axial vector currents.

CONCLUSION

We have shown, in a model-independent fashion, that the "apparent" average polarization obtained by Possoz *et al.*[1] is in good agreement with the prediction of the standard picture, namely, CVC, PCAC, and the absence of second-class axial vector currents.

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