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$\Delta T = 1$, $0^+ \to 0^+$ beta decay of ^{28}Mg

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The isospin-forbidden beta-decay branching ratio of the 0^+, $T = 2$ ^{28}Mg ground state to the 972-keV 0^+, $T = 1$ level of ^{28}Al is measured to be $(0.31 \pm 0.04)\%$ corresponding to $ft = (9.20 \pm 1.1) \times 10^7$s, $\log ft = 7.96 \pm 0.06$. A charge-dependent matrix element between the 0^+, $T = 2$ analog state at 5992 keV in ^{28}Al and the anti-analog level at 972 keV of 20.6 ± 1.6 keV is deduced from this branching ratio. This is in good agreement with a previous result.

[RADIOACTIVITY ^{28}Mg [from ^{26}Mg(t,p)]; measured β-γ coin; deduced branching ratio to ^{28}Al(0.972), $\log ft$, isospin mixing matrix element. Magnetic spectrometer, NaI(Tl) detector.]

I. INTRODUCTION

The charge-dependent interaction V_{CD} between two nuclear states has a sensitivity to some features of nuclear structure not possessed by other parts of the interaction. For instance, the sensitivity to the nucleon radial wave functions (in particular, the difference between the proton and neutron wave functions) has been demonstrated by Kahana[1] in the case of Coulomb displacement energies of analog states and by Lawson[2] in the case of isospin mixing in ^{8}Be and ^{12}C. This sensitivity is not possessed by other commonly measured quantities. It has been repeatedly stressed (see, e.g., the review article of Blin-Stoyle[3]) that the charge dependent matrix element $\langle f | V_{CD} | i \rangle$ is quite sensitive to the *total* wave functions of the initial and final states and meaningful estimates of this quantity will come only from realistic evaluations of these wave functions. In order to use measured values of $\langle f | V_{CD} | i \rangle$ as tests of nuclear models it is desirable to separate as far as possible the sensitivity to the radial and nonradial parts of the wave functions. Fortunately, the difference between the neutron and proton radial wave functions can be easily varied simply by choosing levels with different binding energies; the more tightly a level is bound, the less will be this difference.

Isospin-forbidden allowed β decay provides us with a rather large set of $\langle f | V_{CD} | i \rangle$ which are not only between relatively tightly bound levels but are determined experimentally in a straightforward and relatively rigorous manner. The latter point is stressed in numerous review articles, e.g., Ref. 3. A recent survey article[4] lists 33 isospin-forbidden Fermi β decays yielding values of $\langle f | V_{CD} | i \rangle$ for $A = 20$ to 235. From the discussion in this survey article and that of Blin-Stoyle it would appear that these measurements have been underexploited and that theoretical efforts to understand them using the latest generation of nuclear models could be quite fruitful and interesting.

One recent measurement of an isospin-forbidden allowed β decay was that of Dickey, Bussoletti, and Adelberger[5] for the decay of ^{28}Mg to the 972-keV level of ^{28}Al. As shown in Fig. 1, the decay is from a 0^+, $T=2$ state to a 0^+, $T=1$ state and, as discussed by these authors, is of particular interest because the ^{28}Al 972-keV state is probably to a good approximation,[6] the anti-analog state, $|\bar{A}\rangle$, of the ^{28}Mg ground state; the analog state, $|A\rangle$, lying at 5992 keV in ^{28}Al.[7]

Because this particular case is of unusual interest and because the measurement is difficult we have repeated it using a different experimental procedure from that of Dickey *et al.*[5]

II. EXPERIMENTAL METHODS AND RESULTS

The two general ways that one might consider for determining the $0^+ \to 0^+$ β-ray branching of ^{28}Mg are to measure the intensities of either the β rays or the relevant γ rays. In an earlier decay scheme study[8] it was suggested that the intensity of the 941-keV γ ray ($972 \to 31$) should show a slight excess over the intensity of the 401-keV γ ray ($1373 \to 972$) if there was direct ^{28}Mg β-ray feeding of the 972-keV level. (The γ-ray feeding of the 972-keV level from sources other than the 1373-keV state is negligible.) However, the β-branching ratio found by Dickey *et al.*[5] is 0.21%, and at this level of accuracy the determination of relative detector efficiencies for the 401- and 941-keV γ rays is not practical.

If the β rays are to be measured then, as may be seen in Fig. 1, the sought-for $0^+ \to 0^+$ β-ray branch to the 972-keV level of ^{28}Al has an expected end-point energy of 860 keV, well above the prin-

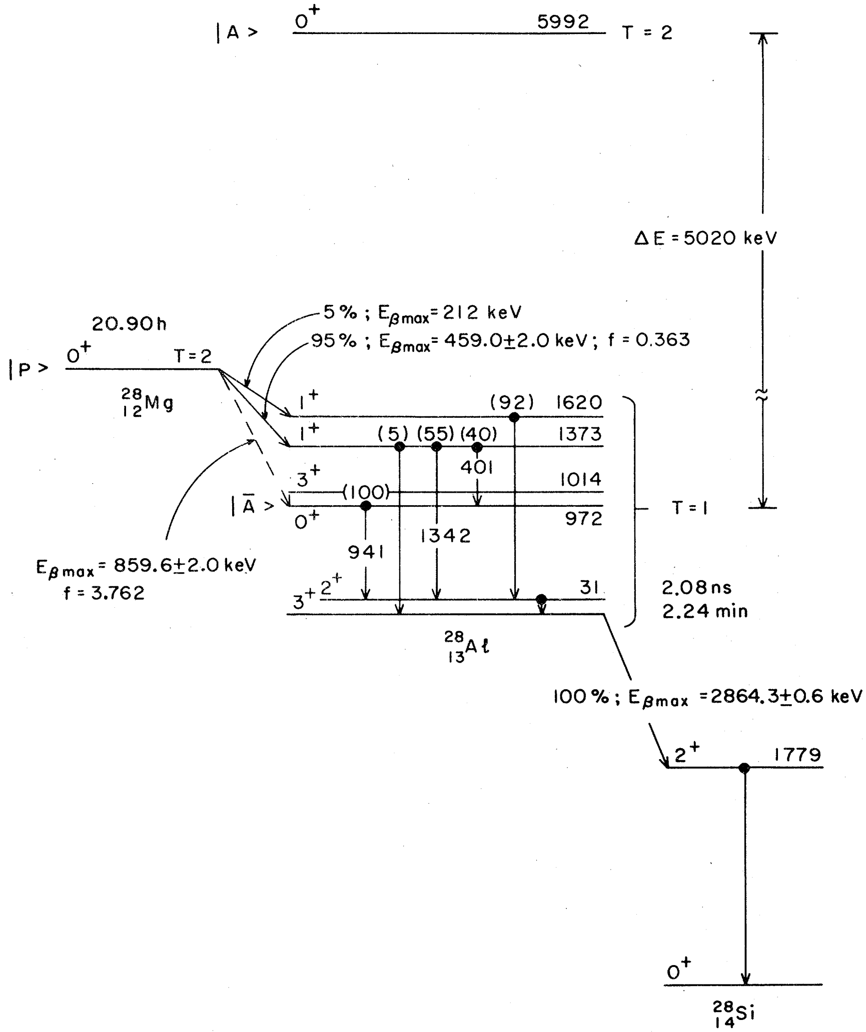

FIG. 1. Partial level schemes illustrating the β decay of ^{28}Mg and ^{28}Al (after Ref. 5). The experimental data are from Refs. 7 and 8. Level energies are in keV, γ-ray branching ratios are in percent and are enclosed in parentheses. The isospin-forbidden $0^+ \rightarrow 0^+$, $\Delta T = 1$ transition of interest is indicated by a dashed arrow.

cipal branch having an end point of 459 keV. The difficulty in a direct measurement of this weak branch is the background due to the presence of the daughter activity, 2.2-min ^{28}Al, which decays with a 100% β-ray branch of end-point energy 2864 keV to the 1779-keV first-excited state of ^{28}Si. It was concluded that the direct measurement would require a technique that would have to discriminate strongly against the ^{28}Al decay. The feature that makes such discrimination possible is that each β ray to the 972-keV state must be in coincidence with a 31-keV γ ray owing to the 100% $972 \rightarrow 31 \rightarrow 0$ γ-γ cascade.[9] The requirement then is to measure β rays in coincidence with 31-keV γ rays using a detector with a much higher efficiency for 31-keV γ rays than for those of 1779 keV. Since the principal 459-keV β-ray branch of ^{28}Mg is also largely (95%) in coincidence with the 31-keV transition, a measure of the relative coincidence intensities of the 459- and 860-keV β-ray components will lead to the desired $0^+ \rightarrow 0^+$ β-ray branching ratio.

The device used for these measurements was an iron-free intermediate-image β-ray spectrometer. Its application to a somewhat different type of coincidence measurement has been described previously.[10] In the present work β rays were detected by means of a 2-mm thick × 2-cm diameter anthracene crystal in the focal plane on the end of a light pipe leading to an RCA 6342 photomultiplier tube (see Fig. 1 of Ref. 10). This same detecting system was used for a recent measurement[11] of the β-ray spectrum of ^{20}F. Although the shape of the pulse-height spectrum from the anthracene crystal varies considerably with the energy of the focused β rays (see for example Fig. 2 of Ref. 11), the efficiency when using a low bias was >97% over most of the range of energies involved in these experiments and could be accurately determined. The spectrometer was adjusted for the maximum β-ray transmission (8% of 4π).

For detecting γ rays, a 1-mm thick by 3.8-cm diameter NaI(Tl) crystal was located close to the source on the end of a light pipe leading to an RCA 8575 photomultiplier tube. In order to avoid the coincidence detection of β rays that might enter the NaI(Tl) crystal and then scatter back into the acceptance solid angle of the spectrometer a Be absorber 5 cm in diameter and 1 cm thick was placed between the source and the crystal. The Be also absorbs about 34% of the 31-keV γ rays. With this geometry the absolute photopeak efficiency for detecting the 31-keV γ rays was measured to be ~11%. It was found that the gain of the 14-stage RCA 8575 tube is more sensitive to magnetic fields than, for example, the 10-stage RCA 6342 tube. However, with an arrangement of 4 concentric cylindrical iron shields over the RCA 8575 tube the gain shift due to changes in the field over the complete range of up to 2.9 MeV in focused β-ray energy was measured to be completely negligible.

Figure 2 shows the low-energy region of the NaI(Tl) pulse-height spectrum due to the γ rays from the ^{28}Mg- ^{28}Al source. The two pulse-height windows used for the measurements are indicated. The efficiency in the lower window, centered at 31 keV, for the 1779-keV γ rays of ^{28}Al, was measured to be a factor of 106 less than that of the 31-keV γ rays. This is considerably less than the factor of ~500 estimated by using the calculated absolute efficiencies for this size NaI(Tl) crystal from the tables of Vegors *et al.*[12] together with an estimate of the fraction of the total spectrum lying within the 31-keV window. The difference reflects the importance of Compton scattering of 1779-keV γ rays from material surrounding the detector with the subsequent detection of Compton electrons or scattered γ rays.

It was assumed that the composition of the γ-ray events in the upper γ-ray window, centered at about 70 keV, was the same as that of the background for the 31-keV full-energy peak in the 31-keV window. Then, subtraction of the β-ray spectrum in coincidence with the 70-keV window, suitably normalized, from that in coincidence with the 31-keV window will yield the β-ray spectrum in coincidence with 31-keV γ rays alone. This assumption and the uncertainties associated with it are discussed further in Sec. III.

The counting electronics consisted of a standard fast-slow system having a resolving time $2\tau \simeq 57.8 \pm 1.2$ ns (measured value) for the 31-keV γ rays in coincidence with β rays. Because of the low γ-ray energy and the losses in the light pipe, this was as good as could be obtained. Since the time resolution was determined mostly by the γ-ray detector, the coincidence efficiency, as determined in separate tests, showed <1% variation over the entire range of focused β-ray energies.

As described previously[8] sources of ^{28}Mg were made in the Brookhaven National Laboratory 3.5-MeV Van de Graaff using the ^{26}Mg$(t,p)^{28}$Mg reaction at $E_t = 3.4$ MeV. Samples were deposited on a 0.076-mm-thick Ni foil in a spot of ~6 mm diameter. The source strength was adjusted so that the initial counting rates were ~14 000/s in the 31-keV γ-ray window of the NaI(Tl) detector and ~500/s at the peak of the 459-keV spectrum in the β-ray detector. From previous observations of the β- and γ-ray spectra emitted by sources prepared with standard procedures from the ^{26}Mg ingot used as a target in the ^{26}Mg$(t,p)^{28}$Mg reaction, it was known that any contaminant activities have negligible influence on the present measurement.

Four runs on the ^{28}Mg β-γ coincidence spectrum were made over a period of about one year. Although the separate results for the $0^+ \rightarrow 0^+$ branch-

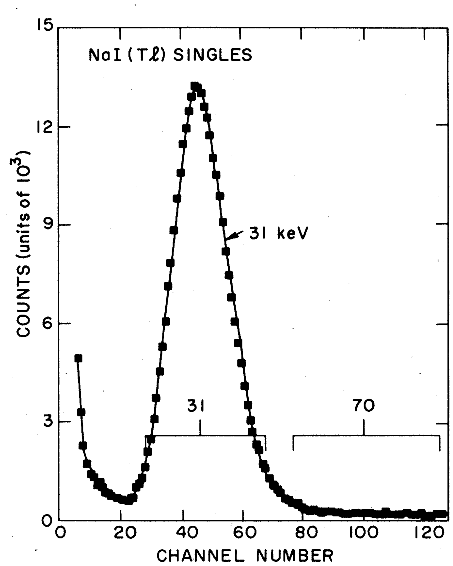

FIG. 2. Pulse-height spectrum from the 1-mm thick by 3.8-cm diameter NaI(Tl) detector due to the γ rays from a ^{28}Mg-^{28}Al source. Channels have been added in groups of four for ease of display. Brackets indicate the windows centered at 31 and 70 keV used for coincidence measurements.

ing ratio were in good agreement, the gradual improvements in technique as well as in accuracy were such that the result of the final run is the only one quoted and illustrated. All data were corrected for the 20.9-h decay of ^{28}Mg and corrections for the variation of β, γ, and coincidence efficiency with magnetic field setting were applied where appropriate. For the coincidence data the calculated random contribution was subtracted from each point using the measured resolving time and the singles rates including corrections for scaler dead time.

Figure 3(a) shows the β-ray singles spectrum which displays the dominant ^{28}Mg component of end-point energy 459 keV and the ^{28}Al β rays of end point 2864 keV. In Fig. 3(b) the yields of β rays, including all corrections, are shown in coincidence with the γ-ray windows centered at 31 and 70 keV as shown in Fig. 2. Above a focused energy of 860 keV both yields are due to the ^{28}Al β-γ coincidences and more or less follow the singles shape (see Sec. III for further discussion). The difference between these β-γ coincidence yields (suitably normalized as discussed below) in the region between 459 and 860 keV quite clearly indicates the presence of the $0^+ \rightarrow 0^+$ β-ray branch of ^{28}Mg. Above 459 keV the data in Fig. 3(b) were obtained in 5 passes over the spectrum with two sources totaling 2 h per point.

III. ANALYSIS

The first step in the analysis was to form a β spectrum in coincidence with 31-keV γ rays only. This was done by normalizing the two β-coincidence spectra in the region $E_\beta > 860$ keV and then subtracting the normalized β spectrum in coincidence with the 70-keV window from that in coincidence with the 31-keV window. The normalized coincidence spectra are shown in Fig. 3(b). The upper five points all are for $E_\beta > 860$ keV and were included in the fit to determine the normalization which was found to be

$$\frac{I(\beta; 31\text{ keV})}{I(\beta; 70\text{ keV})} = 0.86 \pm 0.02 \;\; (\chi^2/\nu = 2.25)\,. \qquad (1)$$

This value is in good agreement with that expected from the relative area of the background in the 31-keV window to the area in the 70-keV window (see Fig. 2). However, the large normalized χ^2 reflects the tendency, as can be seen in Fig. 3(b), for the ratio of Eq. (1) to vary inversely with β energy. As can also be seen in Fig. 3(b), the ratios of both coincidence yields to the β-singles also have this tendency. Utilizing a fifth source preparation, various tests were performed to understand this distortion of the coincidence spectra, and it was

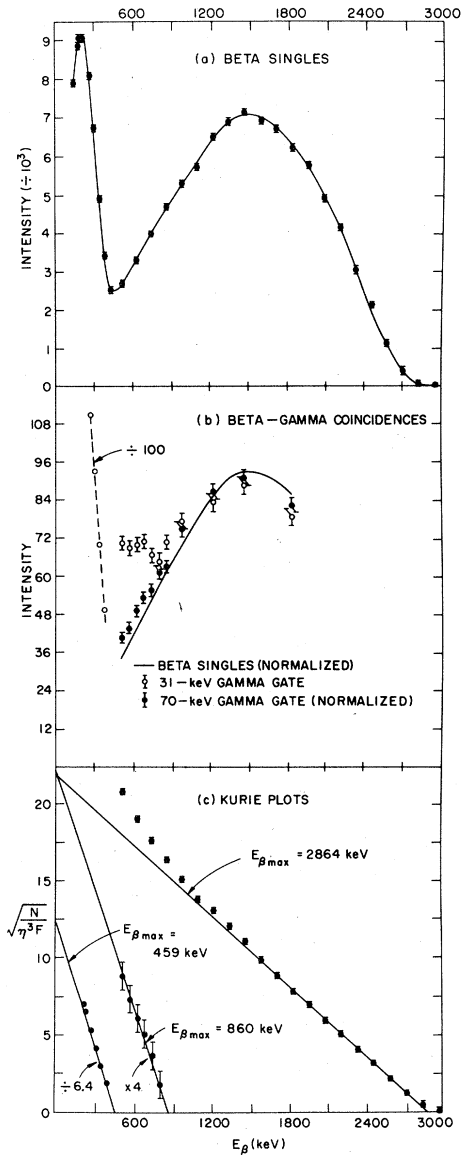

FIG. 3. ^{28}Mg β-decay spectra. (a) The singles spectrum dominated by the ^{28}Mg decay to the ^{28}Al 1373-keV level ($E_{\beta\,\max} = 459$ keV) and ^{28}Al decay to the ^{28}Si 1779-keV level ($E_{\beta\,\max} = 2864$ keV). (b) β spectra in coincidence with γ rays. The open circles indicate β's in coincidence with a γ-ray gate centered on the ^{28}Al 31-keV transition while the closed circles correspond to a gate centered at a γ-ray energy of 70 keV. For comparison, the solid curve shows the shape of the β singles. The three sets of data are normalized as explained in the text. (c) Kurie plots for the main ^{28}Mg branch ($E_{\beta\,\max} = 459$ keV), the ^{28}Al decay ($E_{\beta\,\max} = 2864$ keV) and the ^{28}Mg branch to the 972-keV level ($E_{\beta\,\max} = 860$ keV) which is the subject of study. The two ^{28}Mg Kurie plots are for the coincidence data of (b) and were obtained from the difference of the indicated β spectra in coincidence with the 70- and 31-keV windows. Note—The scale dividing factor for the 459-keV Kurie plot should be 12.8 instead of 6.4.

concluded that the cause is backscattering of ^{28}Al β rays from the 1-cm Be absorber into the magnetic spectrometer with detection of the associated bremsstrahlung in the NaI(Tl) crystal. This effect distorts the ^{28}Al coincidence β spectra by adding excess counts at lower β energies and was estimated to be somewhat worse for the 31-keV gate than for the 70-keV gate. The effect on the final $0^+ \rightarrow 0^+$ branching ratio was estimated to be -5 ± 10)% and is the largest single source of uncertainty.

In Fig. 3(c) are shown Kurie plots[13] for the main ^{28}Mg and ^{28}Al decays and for the ^{28}Mg $0^+ \rightarrow 0^+$ decay. These were formed from the Fermi function, F,[13] the β-momentum, η, and N, the number of counts per unit momentum interval, as shown. The Kurie plot for the main ^{28}Mg decay to the 1373-keV level of ^{28}Al was formed from the coincidence data since it is difficult to separate accurately the ^{28}Mg and ^{28}Al contributions to the singles spectrum of Fig. 3(a) for $E_\beta < 460$ keV.

Both the ^{28}Mg $0^+ \rightarrow 1^+$ and ^{28}Al Kurie plots show source thickness and backscattering effects, i.e., increasing deviation from linearity with decreasing E_β. A similar effect was estimated for the $0^+ \rightarrow 0^+$ Kurie plot by interpolating the deviation from the expected linear shape logarithmically between $E_{\beta\,\max} = 459$ and 2864 keV with an uncertainty assigned to encompass both values. This uncertainty contributed 4.3% to the final branching ratio. A further contribution to this final uncertainty was due to that in the end-point positions arising from uncertainty in the source thickness and in the energy calibration. This contribution was 2.1%.

If $A_{\beta\,\max} E_{\beta\,\max}$ is the zero-energy intercept in the Kurie plot, i.e.,

$$(N/\eta^3 F)^{1/2} = A_{\beta\max}(E_{\beta\max} - E_\beta)\,, \qquad (2)$$

then

$$R = \frac{I_c(0^+ \rightarrow 0^+)}{I_c(0^+ \rightarrow 1^+)} = \left(\frac{A_{860}}{A_{459}}\right)^2 \frac{f_0^{(860)}}{f_0^{(459)}}\,, \qquad (3)$$

where I_c is the integrated intensity for the indicated decay mode in coincidence with 31-keV γ rays. Using values for the Fermi integrals of[14] $f_0^{(459)} = 0.3632$, $f_0^{(860)} = 3.7617$ and the $A_{\beta\,\max}$ values from the fits of Fig. 3(c), we find $R = 3.40 \times 10^{-3}$. The actual $0^+ \rightarrow 0^+$ branching ratio is obtained by multiplying R by 0.95 twice, once to correct for the 95% $0^+ \rightarrow 1^+$ β-branching ratio and once to correct for the 5% ^{28}Al $1373 \rightarrow 0$ γ branch.[8,13] The result is a branching ratio of (0.31 ± 0.037)% where the 12% uncertainty is comprised as follows:

Statistics: 3.2%
Source thickness: 4.3%
Energy uncertainty: 2.1%
Branching ratios: 1.0%
Background subtraction: 10.0%

Using $T_{1/2} = 20.90 \pm 0.03$ h[7] we have $ft = (9.20 \pm 1.1) \pm 10^7$s, $\log ft = 7.96 \pm 0.06$ for the $0^+ \rightarrow 0^+$ $\Delta T = 1$ ^{28}Mg β branch.

Following the complete discussion of Dickey *et al.*,[5] we assume two state mixing between the analog and anti-analog states with the consequence that the mixing amplitude is given by

$$\alpha = 0.5\left(\frac{6177}{ft}\right)^{1/2} = 4.10 \times 10^{-3}$$

and the matrix element $\langle f | V_{CD} | i \rangle$ responsible for the mixing is given by

$$\langle f | V_{CD} | i \rangle = \alpha \Delta E = (20.6 \pm 1.6) \text{ keV}\,,$$

where we have used $\Delta E = 5020$ keV (see Fig. 1). The properties of the ^{28}Mg $0^+ \rightarrow 0^+$ decay measured or inferred here are collected in Table I. The branching ratio is in good agreement with the value of $(2.1 \pm 1.2) \times 10^{-3}$ found by Dickey *et al.*[5] and so are the derived quantities.

IV. CONCLUSION

Of the 39 off-diagonal matrix elements $\langle f | V_{CD} | i \rangle$ from isospin-forbidden β decay which are listed by Raman *et al.*,[4] only the values for the parent nuclei ^{57}Ni, ^{64}Ga, and possibly ^{66}Ge are greater than the 20.6 keV obtained here. Thus, the ^{28}Al analog to anti-analog matrix element must be considered as a relatively large one. It is, however, small compared to the very large matrix elements reported for ^{8}Be and ^{12}C. As recently discussed by Lawson,[2] these matrix elements, which are essentially analog-anti-analog, are of the order of 150 keV so that relative to them the ^{28}Al matrix element is small. Lawson has quite convincingly put the situation in perspective by showing that the large ^{8}Be and ^{12}C matrix elements are due mainly to the difference between the proton and neutron radial wave functions. If this effect were removed the matrix elements would be not so different from that of ^{28}Al. Lawson also demonstrates the point made by Dickey *et al.*,[5] and by previous authors, that the two-body Coulomb interaction between valence protons will usually contribute con-

TABLE I. Properties of ^{28}Mg $0^+ \rightarrow 0^+$ β decay.

Branching ratio:	$(3.1 \pm 0.4) \times 10^{-3}$
$T_{1/2}$:	$(2.4 \pm 0.3) \times 10^7$ s
$\log ft$:	7.96 ± 0.06
α:	$(4.10 \pm 0.25) \times 10^{-3}$
$\langle f \lvert V_{CD} \rvert i \rangle$:	20.6 ± 1.6 keV

siderably more to $\langle f | V_{CD} | i \rangle$ than charge-dependent single-particle energy differences. Thus, it is very important to use a large basis in the calculation of $\langle f | V_{CD} | i \rangle$ since the use of a truncated model space underestimates the number of proton-proton interactions.

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