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Hyperspherical harmonics method for the hypertriton and $^{9}_{\Lambda}$Be

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The hyperspherical or K-harmonics method is used to investigate the properties of $^{3}_{\Lambda}$H and $^{9}_{\Lambda}$Be (in the α-α-Λ model). A set of coupled integral equations for functions of a single variable is obtained for determining the eigenenergy. Retaining nine harmonics, the ground-state energy of $^{3}_{\Lambda}$H is calculated for two-body potentials of square-well, Gaussian, exponential, and Yukawa shape. The volume integrals of the total ΛN potential in $^{3}_{\Lambda}$H which yield the experimental energy are obtained. These are utilized to find the ΛN scattering length and effective range. For $^{9}_{\Lambda}$Be, suitable forms for the α-α and α-Λ interactions are employed to evaluate the Λ separation energy. Comparison is made with available experimental data.

[NUCLEAR STRUCTURE hyperspherical harmonics, K-harmonics, $^{3}_{\Lambda}$H, ground-state energy, $^{9}_{\Lambda}$Be, $\alpha-\alpha-\Lambda$ model.]

I. INTRODUCTION

The method of hyperspherical harmonics or K harmonics, which was introduced by Delves[1] and Smith,[2] and developed by Simonov,[3] Badalyan,[4] and Fabre[5] has recently been used to investigate the properties of a number of three-body systems in nuclear[6-8] and atomic[9-10] physics. It has also been applied to study some four-body systems like the α particle.[11-12] The basic idea of the method of K harmonics is to expand the wave function for a system of A particles (in the center-of-mass system) in terms of a complete set of orthonormal functions of $3A$-4 angular variables. The expansion coefficients are functions of a single variable that represents the length of a $3A$-3 dimensional vector. On substituting the expansion in the Schrödinger equation for the system, one gets an infinite set of coupled differential equations for the expansion coefficients. For a three-body system, the angular harmonics are functions of five angular variables. We have to calculate the matrix elements of the potential between pairs of such hyperspherical or K harmonics. The symmetry of the system under study rules out certain harmonics from appearing in the set of coupled differential equations. Further, the centrifugal barrier terms which occur in the set grow considerably for the higher harmonics. One can, therefore, truncate the set, and work with a finite set of coupled differential or corresponding one-dimensional integral equations. This can be contrasted with the Faddeev formalism[13] for a three-body problem in which one gets a set of two-dimensional integral equations, unless separable approximations for the two-body scattering amplitudes are made.

The number of equations that have to be retained in any calculations using the K harmonics method will, of course, depend on the nature of the potential used, and the results of some of the investigations have been summarized by Levinger.[6] Most of the detailed works done so far are for systems with particles of equal mass. Among the few studies on systems of three particles where the masses are not all identical, we may mention the recent work of Fang and Tomusiak,[8] who considered the ^{6}Li problem in the α-n-p model. They have developed the theory in its general form, but have made calculations by retaining only one angular harmonic.

In the present work, we have examined the $^{3}_{\Lambda}$H hypernucleus by employing the method of K harmonics. We have used different sets of two-body potentials and have retained a number of harmonics to get a convergent result for the ground-state energy of $^{3}_{\Lambda}$H. In hypernuclear physics, the study of the simple system $^{3}_{\Lambda}$H is very important, like that of the triton in nuclear physics. The Λ binding energy provides valuable information about the ΛN interaction. It is difficult to obtain direct information on such interaction, as the ΛN system is not bound and as it is troublesome to perform ΛN scattering experiments.[14] In view of the importance of the investigation on the binding energy of $^{3}_{\Lambda}$H, many calculations have earlier been made using the Faddeev theory[15-17] or the ordinary variational method.[18] However, for local two-body potentials, the K-harmonics method used by us has one advantage over the Faddeev method—the former leads to a set of coupled one-dimensional equations as explained earlier. The method of K harmonics in its general form is also free of the uncertainties involved in the choice of the trial function of the ordinary variational method.

As another application of the formalism developed in this work, we have calculated the binding energy of the hypernucleus ${}^{9}_{\Lambda}$Be in the α-α-Λ model. This model has earlier been used successfully by Bodmer and Ali[19] within the framework of the conventional variational method. One can expect the distortions of the individual α particles in ${}^{9}_{\Lambda}$Be to be small. The α-α potential used by us is that recently proposed by Buck *et al.*[20] The α-Λ potential has been taken from the work of Tang and Herndon.[21]

The method of K harmonics as applied to the hypertriton and α-α-Λ problem is discussed in Sec. II. Section III contains a discussion of our results obtained by this method.

II. METHOD OF K HARMONICS

Denoting the position vectors of the neutron, proton, and the Λ hyperon by $\vec{r}_1$, $\vec{r}_2$, and $\vec{r}_3$, respectively, and their masses by m_1, m_2 ($m_1 \approx m_2 = m$), and m_3 (m_Λ), respectively, we first introduce the coordinates $\vec{\xi}$, $\vec{\eta}$, and $\vec{R}$ defined below:

$$\vec{\xi} = \left(\frac{\mu_3}{\mu}\right)^{1/2}\left(\frac{\vec{r}_1+\vec{r}_2}{2} - \vec{r}_3\right), \tag{1}$$

$$\vec{\eta} = \left(\frac{\mu_{12}}{\mu}\right)^{1/2}(\vec{r}_1 - \vec{r}_2), \tag{2}$$

$$\vec{R} = \frac{m(\vec{r}_1+\vec{r}_2)+m_3\vec{r}_3}{M}, \tag{3}$$

where M is the total mass, $\mu_3 = 2mm_3/(2m+m_3)$, $\mu_{12} = \frac{1}{2}m$, and μ is a suitable chosen mass which we have taken to be equal to $M/3$. The interparticle separations can now be expressed as

$$r_{12} = \left(\frac{\mu}{m}\right)^{1/2}\rho(1+A\cos\lambda)^{1/2}, \tag{4}$$

$$r_{13} = \frac{a\rho}{\sqrt{2}}[1+A\cos(\lambda-2\theta)]^{1/2}, \tag{5}$$

$$r_{23} = \frac{a\rho}{\sqrt{2}}[1+A\cos(\lambda+2\theta)]^{1/2}. \tag{6}$$

Here the hyperradius $\rho = (\xi^2+\eta^2)^{1/2}$ is the length of a vector $(\vec{\xi},\vec{\eta})$ in a six-dimensional space, and

$$A\cos\lambda = \frac{(\eta^2-\xi^2)}{\rho^2},\quad A\sin\lambda = \frac{2(\vec{\xi}\cdot\vec{\eta})}{\rho}, \tag{7}$$

$$a = \left(\frac{\mu}{\mu_3}+\frac{\mu}{2m}\right)^{1/2},\quad \tan\theta = \left(\frac{2m}{\mu_3}\right)^{1/2}. \tag{8}$$

The nonrelativistic Hamiltonian for the hypertriton, after separating out the motion of the center of mass, then takes the form (using units such that $\hbar = 1$)

$$-\frac{1}{2\mu}(\nabla_\xi^2+\nabla_\eta^2)+\hat{V}. \tag{9}$$

Although three-particle forces can be included in our formalism, we ignore them and assume $\hat{V}$ to be the sum of two-particle central potentials. We will be interested in the ground state of ${}^{3}_{\Lambda}$H which is a $T=0$, $J=\frac{1}{2}$ state. The total orbital angular momentum can be taken as zero and only the triplet spin state for the np system need be considered. The space part of the wave function should remain unaltered when $\vec{\eta}$ is changed to $-\vec{\eta}$. The Schrödinger equation for the system is now

$$\left[-\frac{1}{2\mu}(\nabla_\xi^2+\nabla_\eta^2)+\hat{V}^{(12)}+\hat{V}^{(13)}+\hat{V}^{(23)}-E\right] \times\psi(\vec{\xi},\vec{\eta})\phi(1,2,3)=0, \tag{10}$$

with

$$\hat{V}^{(ij)} = V_T^{(ij)}(r_{ij})\frac{3+\vec{\sigma}^{(i)}\cdot\vec{\sigma}^{(j)}}{4} + V_S^{(ij)}(r_{ij})\frac{1-\vec{\sigma}^{(i)}\cdot\vec{\sigma}^{(j)}}{4} \tag{11}$$

and

$$\phi(1,2,3) = \frac{1}{\sqrt{6}}\{[\alpha(1)\beta(2)+\beta(1)\alpha(2)]\alpha(3) - 2\alpha(1)\alpha(2)\beta(3)\}. \tag{12}$$

Here the $\vec{\sigma}$'s are the spin operators, α and β are the spin functions, and the r_{ij}'s are given in Eqs. (4)–(6). Multiplying Eq. (10) by $\phi^\dagger(1,2,3)$ and summing over spin, we get the equation satisfied by the space part $\psi(\vec{\xi},\vec{\eta})$ of the wave function for the hypertriton:

$$\left[-\frac{1}{2\mu}(\nabla_\xi^2+\nabla_\eta^2)+V_T^{(12)}(r_{12})+\tfrac{1}{4}V_T^{(13)}(r_{13})+\tfrac{3}{4}V_S^{(13)}(r_{13}) +\tfrac{1}{4}V_T^{(23)}(r_{23})+\tfrac{3}{4}V_S^{(23)}(r_{23})-E\right]\psi(\vec{\xi},\vec{\eta})=0. \tag{13}$$

A combination of triplet and singlet nucleon-Λ potential and only the triplet np potential occur in the equation.

We now expand $\psi(\vec{\xi},\vec{\eta})$ in a complete set of orthonormal angular functions (K harmonics)[3] $U_K^\nu(\Omega_\rho)$ depending on the direction Ω_ρ of the six-dimensional vector $\vec{\rho}$,

$$\psi(\vec{\xi},\vec{\eta}) = \frac{1}{\rho^2}\sum_{K,\nu}R_K^\nu(\rho)U_K^\nu(\Omega_\rho). \tag{14}$$

For the ground state of the hypertriton with zero orbital angular momentum the quantum number K must be an even positive integer,[3] and, for a fixed value of K, ν ranges between $-K/2$ and

$+K/2$ changing in steps of 2. Some of the harmonics will, however, not appear in the expansion (14) because of the symmetry requirement $\psi(\vec{\xi}, -\vec{\eta}) = \psi(\vec{\xi}, \vec{\eta})$ mentioned earlier. After truncating the expansion and substituting in Eq. (13), one can use the standard method for getting the finite set of coupled differential equations for the partial waves $R_K^\nu(\rho)$. The corresponding equations for the case of three particles of equal mass like the triton or ^{12}C (in the α-α-α model) have been the subject of a number of investigations.[6-7,22] The partial waves are often[7,23] expanded in terms of trial functions of ρ of known form and containing adjustable parameters. The parameters are then determined from the condition of minimization of energy. This procedure, like the ordinary variational method, contains the uncertainties involved in the choice of the trial function. Fang and Tomusiak[8] have considered the α-n-p system where they have particles differing in masses. However, they have kept only the term with grand orbital zero, and have thus obtained an uncoupled differential equation. This equation has been solved numerically.

In the present work, the set of coupled differential equations for $R_K^\nu(\rho)$ is converted into a set of coupled integral equations and is solved numerically to give the ground-state energy E of the hypertriton. The size of the set is gradually enlarged by including an increasing number of K harmonics until a satisfactory convergent value for the ground-state energy is obtained. The set of integral equations is shown below:

$$\sum_{K',\nu'} \int_0^\infty I_n(x_<)K_n(x_>)M_{KK'}^{\nu\nu'}\left(\frac{x'}{\kappa}\right)x' R_{K'}^{\nu'}\left(\frac{x'}{\kappa}\right)dx' - ER_K^\nu\left(\frac{x}{\kappa}\right) = 0\,. \quad (15)$$

Here $\kappa = (-2\mu E)^{1/2}$, $n = K+2$, I_n and K_n represent modified Bessel functions of order n, $x_<$ ($x_>$) stands for the smaller (larger) of x and x', and $M_{KK'}^{\nu\nu'}$ are the matrix elements

$$M_{KK'}^{\nu\nu'}(\rho) = \int U_K^{\nu *}(\Omega_\rho)[V^{(12)}(r_{12}) + V^{(13)}(r_{13}) + V^{(23)}(r_{23})]U_{K'}^{\nu'}(\Omega_\rho)d\Omega_\rho\,, \quad (16)$$

where

$$V^{(12)}(r_{12}) = V_T^{(12)}(r_{12}) \quad (17)$$

and

$$V^{(i3)}(r_{i3}) = \tfrac{1}{4} V_T^{(i3)}(r_{i3}) + \tfrac{3}{4} V_S^{(i3)}(r_{i3}),\; i = 1, 2\,. \quad (18)$$

The modified Bessel functions are calculated numerically. As the primary object of this work is to investigate how the hyperspherical harmonics method works for a three-body hypernucleus where all the masses are not equal, we have considered four different types of simple potentials—square well, Gaussian, exponential, and Yukawa. We expect to be able to handle potentials of more complicated shape in a later work. The matrix elements can be evaluated by expanding the two-body potentials in series of Gegenbauer polynomials and integrating term by term.[4] The first few harmonics and matrix elements are presented below; others are not given because of their length:

$$U_0^0 = \frac{1}{\sqrt{\pi^3}}, \quad U_2^1 = \frac{2}{\sqrt{\pi^3}} \frac{(\eta^2 - \xi^2)}{\rho^2},$$
$$U_4^0 = (3/\pi^3)^{1/2}\{1 - 2[(\eta^2-\xi^2)^2 + 4(\vec{\xi}\cdot\vec{\eta})^2]/\rho^4\}, \quad U_4^2 = (6/\pi^3)^{1/2}[(\eta^2-\xi^2)^2 - 4(\vec{\xi}\cdot\vec{\eta})^2]/\rho^4\,. \quad (19)$$

$$M_{00}^{00}(\rho) = V_0^{(12)}(\rho') + V_0^{(13)}(\rho'') + V_0^{(23)}(\rho''),$$
$$M_{02}^{01}(\rho) = V_2^{(12)}(\rho') + \cos 2\theta[V_2^{(13)}(\rho'') + V_2^{(23)}(\rho'')],$$
$$M_{22}^{11}(\rho) = V_0^{(12)}(\rho') + V_4^{(12)}(\rho') + V_0^{(13)}(\rho'') + V_0^{(23)}(\rho'') + (\tfrac{1}{3} + \tfrac{2}{3}\cos 4\theta)[V_4^{(13)}(\rho'') + V_4^{(23)}(\rho'')],$$
$$M_{04}^{00}(\rho) = -\frac{1}{\sqrt{3}}[V_4^{(12)}(\rho') + V_4^{(13)}(\rho'') + V_4^{(23)}(\rho'')],$$
$$M_{04}^{02}(\rho) = (\tfrac{2}{3})^{1/2}\{V_4^{(12)}(\rho') + \cos 4\theta[V_4^{(13)}(\rho'') + V_4^{(23)}(\rho'')]\},$$
$$M_{24}^{10}(\rho) = -(1/\sqrt{3})\{V_2^{(12)}(\rho') + V_6^{(12)}(\rho') + \cos 2\theta[V_2^{(13)}(\rho'') + V_2^{(23)}(\rho'') + V_6^{(13)}(\rho'') + V_6^{(23)}(\rho'')]\}, \quad (20)$$
$$M_{24}^{12}(\rho) = (\tfrac{2}{3})^{1/2}\{V_2^{(12)}(\rho') + V_6^{(12)}(\rho') + \cos 2\theta[V_2^{(13)}(\rho'') + V_2^{(23)}(\rho'')] + (\tfrac{1}{4}\cos 2\theta + \tfrac{3}{4}\cos 6\theta)[V_6^{(13)}(\rho'') + V_6^{(23)}(\rho'')]\},$$
$$M_{44}^{00}(\rho) = V_0^{(12)}(\rho') + \tfrac{2}{5}V_8^{(12)}(\rho') + V_0^{(13)}(\rho'') + V_0^{(23)}(\rho'') + \tfrac{2}{5}[V_8^{(13)}(\rho'') + V_8^{(23)}(\rho'')],$$
$$M_{44}^{20}(\rho) = -(3/\sqrt{2})\{\tfrac{1}{3}V_4^{(12)}(\rho') + \tfrac{1}{5}V_8^{(12)}(\rho') + \cos 4\theta(\tfrac{1}{3}[V_4^{(13)}(\rho'') + V_4^{(23)}(\rho'')] + \tfrac{1}{5}[V_8^{(13)}(\rho'') + V_8^{(23)}(\rho'')])\},$$
$$M_{44}^{22}(\rho) = V_0^{(12)}(\rho') + \tfrac{1}{2}V_4^{(12)}(\rho') + \tfrac{7}{10}V_8^{(12)}(\rho') + V_0^{(13)}(\rho'') + V_0^{(23)}(\rho'') + \tfrac{1}{2}[V_4^{(13)}(\rho'') + V_4^{(23)}(\rho'')] + (\tfrac{1}{10} + \tfrac{3}{5}\cos 8\theta)[V_8^{(13)}(\rho'') + V_8^{(23)}(\rho'')],$$

where $\rho' = (\mu/m)^{1/2}\rho$ and $\rho'' = (a/\sqrt{2})\rho$. The functions $V_{2l}^{(ij)}$ occurring in (20) are given by

$$V_{2l}^{(12)}\left(\left(\frac{\mu}{m}\right)^{1/2}\rho\right) = \frac{2}{\pi}\int_{-1}^{+1}(1-x^2)^{1/2}C_l^{(1)}(x)V_T^{(12)}\left(\left(\frac{\mu}{m}\right)^{1/2}\rho(1+x)^{1/2}\right)dx \tag{21}$$

and

$$V_{2l}^{(i3)}\left(\frac{a}{\sqrt{2}}\rho\right) = \frac{2}{\pi}\int_{-1}^{+1}(1-x^2)^{1/2}C_l^{(1)}(x)\left[\tfrac{1}{4}V_T^{(i3)}\left(\frac{a}{\sqrt{2}}\rho(1+x)^{1/2}\right) + \tfrac{3}{4}V_S^{(i3)}\left(\frac{a}{\sqrt{2}}\rho(1+x)^{1/2}\right)\right]dx, \quad i=1,2\,, \tag{22}$$

where $C_l^{(1)}(x)$ is a Gegenbauer polynomial.

For potentials of the square well and Gaussian shape, the integrals of the above type can be analytically worked out. For exponential and Yukawa potentials, we have numerically determined the corresponding integrals.

After the evaluation of the matrix elements $M_{KK'}^{\nu\nu'}$, we have replaced the integrals occurring in Eq. (15) by summation using the Gaussian quadrature formula and converted Eq. (15) into a set of mN linear algebraic equations for $R_K^\nu(x_i/\kappa)$. Here m represents the number of harmonics retained, and N is the number of quadrature points used. We have searched for the largest κ value which makes the determinant of the coefficients of the algebraic equations zero and got the ground-state energy of ${}^3_\Lambda$H. For each type of potential, we have checked the convergence of the numerical procedure by changing the number of quadrature points. The final results with nine harmonics retained are presented in Sec. III. The potential parameters used are also described in that section. The harmonics used in the calculation are U_0^0, U_2^1, U_4^0, U_4^2, U_6^1, U_6^3, U_8^0, U_8^2, and U_8^4. The last harmonic has been found to contribute much less than one percent to the energy. Six other harmonics allied to this set, namely, U_2^{-1}, U_4^{-2}, U_6^{-3}, U_6^{-1}, U_8^{-4}, and U_8^{-2} do not appear because of the symmetry requirement mentioned earlier.

The above formalism can be easily adapted for the α-α-Λ system. Now $\vec{r}_1$ and $\vec{r}_2$ will denote the position vectors of the two α particles and m will be equal to the α-particle mass. Since α particles are spinless, the wave function must be symmetric with respect to their interchange and we have again the symmetry condition $\psi(\vec{\xi},\vec{\eta}) = \psi(\vec{\xi},-\vec{\eta})$. For the ground state, $J=\frac{1}{2}$, the orbital angular momentum is zero, and we will have to consider Eqs. (15) and (16), but with

$$V^{(12)}(r_{12}) = V^{\alpha\alpha}(r_{12}), \quad V^{(i3)}(r_{i3}) = V^{\alpha\Lambda}(r_{i3}), \quad i=1,2\,. \tag{23}$$

The α-α and α-Λ potentials used are explained in the next section. The final calculation has been done with the inclusion of nine K harmonics.

III. RESULTS AND DISCUSSION

In our work on ${}^3_\Lambda$H, we have used those np triplet potential parameters which give rise to a scattering length of 5.4 fm and a deuteron binding energy of 2.225 MeV. The parameters for the four different potential shapes are shown in the second and third columns of Table I. In each set of calculations, the form of the ΛN potential used is the same as that of the np potential. The intrinsic range for the ΛN singlet and triplet interaction has been taken to be 1.484 fm, corresponding to a mechanism of two-pion exchange. Choosing different values for the potential strength, we have found the ground-state energy of the hypertriton by the method explained in the previous section. The magnitudes V_2 of the volume integrals of the total ΛN potential[24] in ${}^3_\Lambda$H are given in the fourth column of the above mentioned table. The corresponding ground-state energies calculated by us are presented in column five. In particular, we have determined the volume integrals for each potential shape which yield the experimental[25] ground-state energy of −2.355 MeV. These V_2 values for the given hy-

TABLE I. The magnitudes (V_2) of the volume integrals of the total ΛN potential in ${}^3_\Lambda$H and the corresponding ground-state energies (E).

Potential shape	n-p triplet potential parameters Strength (MeV)	Range (fm)	V_2 (MeV fm^3)	$-E$ (MeV)
Square-well	36.2	2.02	698	2.304
			702	2.355
			719	2.612
Gaussian	72.5	1.47	714	2.280
			719	2.355
			723	2.414
Exponential	189.1	0.67	722	2.252
			729	2.355
			734	2.421
Yukawa	41.5	1.58	745	2.239
			751	2.355
			767	2.643

pertriton energy represent upper bounds, as any increase in the number of K harmonics used can only decrease the ground-state energy.

We have next utilized the ΛN potential strengths determined for $E=-2.355$ MeV to calculate the average scattering lengths (a) and effective ranges (r). The normal procedure has been used for the square-well case. For the potentials of other shape, we have employed the formulas derived by Levee and Pexton.[26] The (a,r) values (in fm) arising out of the four types of well described in Table I are $(-1.81,1.97)$, $(-1.85,2.05)$, $(-1.86,2.22)$, and $(-2.05,2.39)$, respectively.

A few years ago Londergan and Dalitz[27] used the experimental data of Alexander *et al.*[14] and Sechi-Zorn *et al.*[14] for the total Λp elastic scattering cross section (σ) and fitted them to a formula

$$\left(\frac{4\pi}{\sigma}-k^2\right)^{1/2}=-\frac{1}{a}+\frac{1}{2}rk^2\,, \tag{24}$$

where k is the Λp center-of-mass momentum. The data can be fitted well by a straight line against k^2 with the average scattering length and effective range being given by $a_{\mathrm{LD}}=-1.80$ fm and $r_{\mathrm{LD}}=3.16$ fm. Our values for the average scattering length for the ΛN interactions considered are comparable to a_{LD}, but those for the effective range are somewhat lower than r_{LD}. A more detailed comparison cannot be made because of the inadequacy of the available ΛN scattering data and the simple form of the interaction used.

The method of K harmonics has also been used by us to calculate the binding energy of ${}^{9}_{\Lambda}\mathrm{Be}$ in the α-α-Λ model. Buck *et al.*[20] have recently suggested a simple form for the α-α potential which can produce scattering data well in accord with experiment. The nuclear part of the potential is given by

$$V^{\alpha\alpha}(r)=-V_0^{(\alpha\alpha)}\exp\left(-\frac{r^2}{\beta_{\alpha\alpha}{}^2}\right), \tag{25}$$

where $V_0^{\alpha\alpha}=122.6225$ MeV and $\beta_{\alpha\alpha}=2.132$ fm. The Coulomb part is $4e^2\,\mathrm{erf}(\lambda r)/r$ with $\lambda=0.75\,\mathrm{fm}^{-1}$. The $L=0$ resonance state with this potential occurs at 92.12 keV. It is to be remembered, however, that this potential supports some redundant bound states. We have taken the potential (25) in our ${}^{9}_{\Lambda}\mathrm{Be}$ calculation. For $\alpha\Lambda$ interaction, we have used a similar Gaussian potential of Tang and Herndon[21] with $V_0^{\alpha\Lambda}=60.17$ MeV and $\beta_{\alpha\Lambda}=1.273$ fm. This potential just reproduces the α-Λ bound state at -3.04 MeV. The highest of the redundant states supported by the potential (25) occurs at a rather high negative value of -22.1 MeV. The K harmonics calculation on the α-α-Λ system yields some eigenenergies below -22.1 MeV. We have ignored them and have searched for the energy value that may correspond to the α-α resonance and α-Λ bound state. We have found only one eigenenergy between the α-Λ bound state and -22.1 MeV, and far removed from the latter. We chose this as the required energy for the α-α-Λ system. The Λ-separation energy B_Λ has been found to be 6.39 MeV. This is quite close to the experimental value of 6.71 ± 0.04 MeV,[18] in spite of the simplicity of the potential forms.

We conclude that the method of hyperspherical or K harmonics can be successfully employed to investigate the properties of many hypernuclei.

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