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Pion absorption in highly excited nuclear matter

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Motivated by pion productions in high-energy heavy-ion collisions, we study the p-wave pion absorption in highly excited nuclear matter. The basic absorption mechanism is the two-nucleon model with both pion and rho-meson rescattering. The excited nuclear matter is modeled by a finite temperature Fermi gas. We deduce that for a given pion kinetic energy, the mean free path for pion absorption depends weakly on the temperature of the nuclear matter, but strongly on the density. Its relevance to high-energy heavy-ion collisions is discussed.

[NUCLEAR REACTIONS intermediate energy: high-energy heavy-ion collisions, pion production and absorption, highly excited nuclear matter.]

I. INTRODUCTION

The production of pions in high-energy heavy-ion collisions is a subject being extensively studied both experimentally[1-3] and theoretically.[4-9] The energy spectrum of the produced pions in the inclusive cross sections implies that these pions are produced from a source with a very high temperature. In the collision of Ne and NaF at energy 0.8 GeV/A, Nagamiya *et al.*[1] have deduced that the temperature of the pion source is 62 MeV. The space-time structure of this source presumably can be determined from the correlation measurement of two pions.[10,11]

In the scattering of pions from ordinary nuclei, it has been known that pions are strongly absorbed in nuclear medium.[12] For example, the total absorption cross section for 130 MeV π^+ in ^{12}C in a bubble chamber experiment is ~200 mb and is about one-third of the total cross section.[13] Recently Ginocchio[14] has done detailed cascade calculations for deep inelastic pion-induced nuclear reactions. He found, using the isobar model, that the pion is absorbed mostly on the nuclear surface where the nuclear density has almost reached the central density. Are pions still strongly absorbed in a highly excited nuclear matter produced in high-energy heavy-ion collisions?

From previous studies on pion absorptions in normal nuclei, it is well established that the most important absorption mechanism involves a pair of nucleons.[12,15-17] After the absorption of a pion, each nucleon therefore receives on the average more than 70 MeV in kinetic energy, implying that the final two nucleons are well above the Fermi energy. Hence we expect that the Pauli principle does not play an important role in pion absorptions and that pion absorptions in highly excited nuclear matter are similar to that in normal unexcited nuclei. We shall show in this paper quantitatively that this is indeed so and suggest that the pions observed in high-energy heavy-ion collisions are probably from the surface of the source, as those produced inside are absorbed.

We shall take into account only the p-wave pion absorption as it is the dominant part above threshold. The study of p-wave pion absorptions in a zero temperature Fermi gas model has been reported in Ref. 16. Here we generalize it to finite temperatures.

In Sec. II we outline the p-wave pion absorption in nuclei using the Fermi gas model. In Sec. III numerical results are presented. Conclusions are drawn in Sec. IV, and detailed formulations are given in the Appendix for completeness.

II. FORMULATION

The pion absorption rate in a nucleus is given by

$$\Gamma = 2\pi \sum_f \delta(E_f - E_i - \omega) | T_{fi} |^2 , \qquad (1)$$

where ω is the energy of the absorbed pion. The matrix element T_{fi} describes the transition of the nucleus from the initial state to the final state, their energies being E_i and E_f, respectively. For pion absorption by a pair of nucleons in the Fermi gas model, Eq. (1) is written as

$$\Gamma = 2\pi \sum_{\vec{k}_1, \vec{k}_2, \vec{k}_3, \vec{k}_4} f(k_1) f(k_2)[1-f(k_3)][1-f(k_4)]\delta\left(\frac{\hbar^2}{2m}(k_3^2 + k_4^2 - k_1^2 - k_2^2) - \omega\right) \left| \int d(1)\, d(2)\, \psi^*_{\vec{k}_3\vec{k}_4}(12) T_{\vec{q}}(12) \psi_{\vec{k}_1\vec{k}_2}(12) \right|^2 , \qquad (2)$$

where

$$f(k)=1/(1+e^{(\epsilon-\mu)/T}), \tag{3}$$

with $\epsilon=\hbar^2k^2/2m$ and T the temperature of the nuclear matter. The chemical potential μ is determined from the total nucleon number, i.e., $\sum_{\vec{k}} f(k)=N/4$. In the above, $\vec{k}_1$ and $\vec{k}_2$ are the initial momenta of the two nucleons, while $\vec{k}_3$ and $\vec{k}_4$ are their final momenta; $\vec{q}$ is the momentum of the incident pion. The normalized antisymmetric pair wave function is denoted by $\psi_{\vec{k}_1\vec{k}_2}(12)$, where (1) and (2) represent all the coordinates of nucleons 1 and 2. The mass of the nucleon is m.

From conservation of momentum, the matrix element of the pion absorption operator, $T_{\vec{q}}(12)$, must have the form

$$\int d(1)d(2)\psi^*_{\vec{k}_3\vec{k}_4}(12)T_{\vec{q}}(12)\psi_{\vec{k}_1\vec{k}_2}(12)=(2\pi)^3\delta^{(3)}(\vec{K}'-\vec{K}-\vec{q})\int d\vec{r}\, d(1')\,d(2')\psi^*_{\vec{k}'}(\vec{r};1'2')T_{\vec{q}}(\vec{r},1'2')\psi_{\vec{k}}(\vec{r};1'2'), \tag{4}$$

where we have introduced the center-of-mass and relative momenta

$$\vec{K}=\vec{k}_1+\vec{k}_2,\quad \vec{k}=\tfrac{1}{2}(\vec{k}_1-\vec{k}_2) \tag{5a}$$

$$\vec{K}'=\vec{k}_3+\vec{k}_4,\quad \vec{k}'=\tfrac{1}{2}(\vec{k}_3-\vec{k}_4) \tag{5b}$$

and

$$\vec{R}=\tfrac{1}{2}(\vec{r}_1+\vec{r}_2),\quad \vec{r}=\vec{r}_1-\vec{r}_2\,. \tag{5c}$$

The notations (1′) and (2′) represent the spin and isospin coordinates of nucleons 1 and 2. The matrix element on the right-hand side of Eq. (4) has been discussed in detail in Ref. 16 for p-wave pion absorption by a pair of nucleons with pion and rho-meson rescattering. Diagrammatically it describes the processes shown in Fig. 1. The explicit form of this matrix element is given in the Appendix.

The kinematic factors in Eq. (2) can be greatly simplified if we make the following two approximations: (1) We approximate the factors $[1-f(k_3)]$ and $[1-f(k_4)]$ by unity. This is justified for not too high temperatures as k_3 and k_4 are, on the average, well above the Fermi momentum. Certainly we should be cautious in the case of very high temperature. In this case there are finite probabilities for nucleons in such high momentum states and the above kinematic factors are smaller than unity. (2) We approximate the angular part in the δ function for the energy conservation by its average value, i.e.,

$$\frac{\hbar^2}{2m}(k_3{}^2+k_4{}^2-k_1{}^2-k_2{}^2)-\omega =\frac{\hbar^2}{4m}(\vec{k}\cdot\vec{K}-\vec{q}^2)+\frac{\hbar^2}{m}(k'^2-k^2)-\omega \approx\frac{\hbar^2}{m}\left(k'^2-k^2-\frac{q^2}{4}\right)-\omega\,. \tag{6}$$

Both approximations were used in Ref. 16.

The pion absorption cross section per nucleon is then given by Eq. (A.11) in the Appendix. The only place where the effect of temperature appears is in the function $P(x)$ defined in Eq. (A.16). This function essentially gives the relative probability for a pair of nucleons to have relative momentum $x\hbar k_F$, where k_F is the Fermi momentum and has the value 1.34 fm^{-1}. For zero temperature, $T=0$, it has the familiar form

$$P(x)=24x^2(1-\tfrac{3}{2}x+\tfrac{1}{2}x^3)\,. \tag{7}$$

To determine $P(x)$ for finite temperature, we assume that the density of the excited nuclear matter is the same as that at $T=0$. From the well known Sommerfeld expansion,[18] we can then determine the chemical potential μ for the following two limits:

$$\mu\approx\epsilon_F\left[1-\frac{\pi^2}{12}\left(\frac{T}{\epsilon_F}\right)^2-\frac{7\pi^2}{960}\left(\frac{T}{\epsilon_F}\right)^4-\cdots\right] \tag{8a}$$

for $T\ll\epsilon_F$, and

$$e^{\mu/T}\approx\frac{4}{3\sqrt{\pi}}\left(\frac{\epsilon_F}{T}\right)^{3/2}\left\{1+\frac{1}{2\sqrt{2}}\frac{4}{3\sqrt{\pi}}\left(\frac{\epsilon_F}{T}\right)^{3/2}-\frac{1}{3\sqrt{3}}\left[\frac{4}{3\sqrt{\pi}}\left(\frac{\epsilon_F}{T}\right)^{3/2}\right]^2+\cdots\right\} \tag{8b}$$

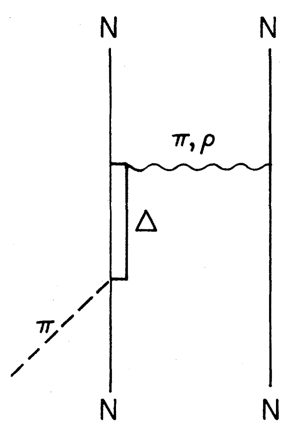

FIG. 1. Pion and rho-meson rescattering through Δ resonant intermediate state.

for $T \gg \epsilon_F$. In the above, ϵ_F is the Fermi energy and has the value ~40 MeV. For T around ϵ_F, we have to determine μ consistently from its defining equation. This procedure is more involved numerically, and we shall not consider $T \sim \epsilon_F$ in this paper.

III. RESULTS

In Fig. 2 we show the numerical results of $P(x)$ for different values of temperature. We observe that the peak of $P(x)$ moves to larger values of x as the temperature increases. At $T=0$ MeV it peaks at $x=0.5$, while at $T=60$ MeV it peaks at $x=0.9$. Also as the temperature increases, the width of $P(x)$ becomes wider and the whole function gets flatter. This change of $P(x)$ with respect to the temperature has effects on the pion absorption cross section through integrals in Eqs. (A12)–(A15).

We show in Table I the dominant integrals for different values of the pion momentum and the temperature. In order to remove the energy dependence we have multiplied these integrals by the energy factor ωD^2, where ω is the pion energy and D the propagator of the Δ resonance in Fig. 1. From the table we see that as the temperature increases, those integrals with initial relative angular momentum $L=0$ generally decrease while those with $L=1$ increase. This can be understood qualitatively in the following way: The integrals are limited to the range of r values determined by the Yukawa functions, which are defined in Eq. (A.9). For the case of pion rescattering, values of r smaller than 1.4 fm are important. As shown before, when the temperature increases from 0

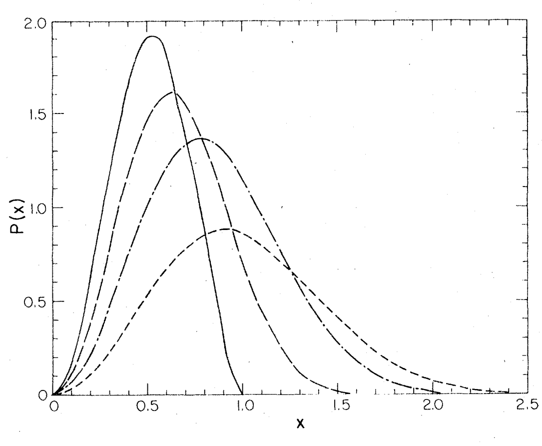

FIG. 2. Relative probability function $P(x)$ as a function of the relative momentum x in units of the Fermi momentum k_F for different temperatures. The curves are represented as: solid curve for $T=0$ MeV, long-dashed curve for $T=20$ MeV, long-dashed-dotted curve for $T=40$ MeV, and short-dashed curve for $T=60$ MeV.

MeV to 60 MeV, the maximum of the function $P(x)$ increases from $x=0.5$ to 0.9, i.e., $k_i=0.67$ fm^{-1} to 1.21 fm^{-1}. Therefore $k_i r$ increases from 0.94 to 1.68 in this temperature region. For these values of $k_i r$ the spherical Bessel functions $j_0(k_i r)$ and $j_1(k_i r)$ have different behaviors, with the former a decreasing function while the latter is an increasing function of the argument. Similar arguments apply to the case of rho-meson rescattering. These explain the features we obtained in Table I. These also account for the fact that at low temperatures the integral $\omega D^2 K_{020}$ dominates while it is $\omega D^2 \mathcal{K}_{110}$ which is dominant at high temperatures.

In Table II we show the absorption cross section per nucleon at different temperatures for different pion momenta. At a fixed temperature the energy dependence of the absorption cross section is strong, with the peak at pion momentum ~1.2 fm^{-1}. This is due to the resonance nature of the p-wave pion-nucleon interaction. As to the temperature dependence, we observe that the absorption cross section increases up to 50% from $T=0$ MeV to $T=60$ MeV.

Among the approximations we have made in evaluating the pion absorption cross section, the neglect of the Pauli blocking effect for the final nucleons may introduce errors which are temperature dependent. We have checked this effect by assuming that the final two nucleons share equally the energy of the absorbed pion, and found that this effect is negligible for all temperatures. Using the largest absorption cross sections at pion momentum 1.2 fm^{-1} in the rest frame of the nuclear matter, i.e., 140 MeV in kinetic energy, we obtain the mean free path for pion absorption, $\lambda_{abs} \approx 1/\sigma_{abs}\rho$, ranging from ~1 fm at $T=0$ to ~0.75 fm at $T=60$ MeV, if the normal nuclear matter density is used. The temperature dependence of the absorption cross section is therefore weak. This value of the pion absorption mean free path seems consistent with the conclusion in Ref. 14, but is about a factor of 2 smaller than that determined in Ref. 19.

We have also studied the dependence of λ_{abs} on the density of the nuclear matter. We have found that the following relation is approximately fulfilled:

$$\lambda_{abs} \cong \lambda_{abs}^{(0)}(\rho_0/\rho)^2 , \tag{9}$$

where ρ_0 and $\lambda_{abs}^{(0)}$ are, respectively, the normal nuclear matter density and the pion absorption mean free path in such a medium. This relation can be qualitatively understood. The fact that two nucleons are involved in the absorption gives a factor ρ^2 in the total absorption cross section. The average absorption cross section per nucleon is therefore proportional to ρ. Since λ_{abs} is given by

TABLE I. Dominant integrals in units of m_π^{-6}, with m_π the inverse pion Compton wave length, for pion momentum $q = 0.8$, 1.0, 1.2, and 1.4 fm^{-1} at different temperatures. The number −4 in the bracket is understood as 10^{-4}.

T (MeV)	q (fm^{-1})	$\omega D^2\mathcal{K}_{110}$	$\omega D^2\mathcal{K}_{011}$	$\omega D^2\mathcal{K}_{130}$	$\omega D^2\mathcal{K}_{031}$	$\omega D^2\mathcal{K}_{020}$
0	0.8	2.09(−4)	1.11(−4)	1.08(−4)	6.33(−5)	3.85(−4)
	1.0	1.98(−4)	1.67(−4)	9.46(−5)	9.49(−5)	3.15(−4)
	1.2	1.80(−4)	2.30(−4)	8.05(−5)	1.26(−4)	2.50(−4)
	1.4	1.69(−4)	2.96(−4)	6.77(−5)	1.56(−4)	1.98(−4)
20	0.8	4.06(−4)	1.22(−4)	1.45(−4)	5.07(−5)	3.22(−4)
	1.0	3.85(−4)	1.82(−4)	1.30(−4)	7.73(−5)	2.65(−4)
	1.2	3.60(−4)	2.49(−4)	1.13(−4)	1.05(−4)	2.10(−4)
	1.4	3.45(−4)	3.35(−4)	9.81(−5)	1.33(−4)	1.65(−4)
60	0.8	6.66(−4)	8.50(−5)	1.21(−4)	2.05(−5)	1.41(−4)
	1.0	6.29(−4)	1.27(−4)	1.12(−4)	3.22(−5)	1.17(−4)
	1.2	6.19(−4)	1.82(−4)	1.02(−4)	4.55(−5)	9.31(−5)
	1.4	5.63(−4)	2.29(−4)	9.09(−5)	5.94(−5)	7.37(−5)

the inverse of $\sigma_{\text{abs}}\rho$, we obtain Eq. (9). The density dependence of λ_{abs} is therefore much stronger than the case if pion absorptions involve only one nucleon, which would lead to $\lambda_{\text{abs}} \propto \rho^{-1}$.

IV. DISCUSSION AND CONCLUSIONS

In this work we have studied the dependence of p-wave pion absorption on the pion kinetic energy, the temperature, and the density of the nuclear matter. It is seen that the mean free path for pion absorption in the nuclear matter depends only weakly on the temperature of the nuclear matter, but strongly on the density and the kinetic energy of the pion. Around the (3, 3) resonance, i.e., pions with kinetic energy ~140 MeV, λ_{abs} has the largest value and is about 1 fm in nuclear matter with normal density.

TABLE II. Absorption cross section per nucleon in units of fm^2 for pion momentum $q = 0.8$, 1.0, 1.2, and 1.4 fm^{-1} at different temperatures.

T (MeV)	q (fm^{-1})	σ (fm^2)
0	0.8	3.38
	1.0	5.75
	1.2	5.79
	1.4	3.31
20	0.8	4.30
	1.0	7.37
	1.2	7.42
	1.4	4.51
60	0.8	4.69
	1.0	8.09
	1.2	8.49
	1.4	4.76

Since we have only taken into account the p-wave pion absorption, one might wonder how important are other partial waves. It is obvious that inclusions of other partial waves will reduce the mean free path for pion absorptions. Around the (3, 3) resonance region, the value $\lambda_{\text{abs}} \sim 1$ fm in normal nuclear matter density will not be much affected. However, the effect can be appreciable away from this energy region. For example, for pions with kinetic energy 35 MeV, we have estimated that $\lambda_{\text{abs}} \sim 5$ fm in normal nuclear matter for p-wave interaction only. Inclusion of s-wave pion-nucleon interaction will reduce this value to approximately half.

Our results are relevant to high-energy heavy-ion collisions. When the incident energy is high enough, pions are produced in the reaction. We do not know definitely the temperature and density of the nuclear matter during the stage pions are produced. Model calculations such as that using the classical equations of motion[20] do indicate that density of twice the normal nuclear matter density can occur in the collisions. Also the experimental energy spectra of protons are consistent with the assumption that the nuclear matter in heavy-ion collisions has a very high temperature which is 30 MeV or more.[21] If pions are produced in such hot and dense nuclear matter, then our calculations indicate that the mean free path for pion absorption is 0.25 fm or smaller. Although the pions produced on the surface of this source can leave the source and be detected by the counter, those produced in the interior of the source will have negligible probability of escaping if the size of the source is much larger than the absorption mean free path. From pion correlation experiments of

Fung *et al.*,[11] it is tentatively determined that the size of the pion source is ~3–4 fm. If this value is reliable, then we are encouraged to speculate that pions produced in the interior of the source will probably be absorbed by nucleons. These nucleons will then dissipate their energies through collisions with other nucleons or through production of new pions. In this fashion pion absorptions are probably one of the important mechanisms via which the source achieves thermal equilibrium. Such an equilibrated source has been assumed in all thermal models[4,5] for high-energy heavy-ion collisions.

As far as we know, most models for high-energy heavy-ion collisions work better for protons than for pions. From the above discussions it is probably fair to say that we should consider more seriously the effects due to true pion absorptions in high-energy heavy-ion collisions.

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APPENDIX

In this Appendix, we shall give the mathematical details of evaluating the absorption process schematically described in Fig. 1. Most of the following material can be found in Ref. 16. We include these details for completeness.

To evaluate the amplitude in Fig. 1, we use the following meson-nucleon effective interaction Lagrangians:

$$L_{\pi NN}=\frac{f}{m_\pi}\chi^\dagger\xi^\dagger\vec{\sigma}\cdot\vec{\nabla}\underline{\phi}\cdot\underline{\tau}\xi\chi\,, \tag{A.1}$$

$$L_{\rho NN}=\frac{g_\rho}{2m}(1+\kappa)\chi^\dagger\xi^\dagger(\vec{\sigma}\times\vec{\nabla})\cdot(\underline{\vec{\rho}}\cdot\underline{\tau})\xi\chi\,, \tag{A.2}$$

$$L_{\pi N\Delta}=\frac{f_\Delta}{m_\pi}\underline{\chi}^\dagger\vec{\xi}^\dagger\cdot\vec{\nabla}\underline{\phi}\xi\chi+\text{H.c.}\,, \tag{A.3}$$

$$L_{\rho N\Delta}=\frac{g_{\rho\Delta}}{2m}\underline{\chi}^\dagger\vec{\xi}^\dagger\cdot(\vec{\nabla}\times\underline{\vec{\rho}})\xi\chi+\text{H.c.} \tag{A.4}$$

Here ξ and χ are the nucleon spinor and isospinor, and $\vec{\xi}$ and $\underline{\chi}$ the Δ vector spinor and vector isospinor. The pion isovector field operator is denoted by $\underline{\phi}$ and the rho-meson vector-isovector field operator by $\underline{\vec{\rho}}$. The mass of the nucleon and pion are denoted by m and m_π, respectively. The coupling constants are taken to be $f^2/4\pi=0.081$, $f_\Delta{}^2/4\pi=0.32$, $g_\rho{}^2/4\pi=0.55$, and $\kappa=6.6$. The ρ-Δ coupling is $g_{\rho\Delta}=6\sqrt{2}/5g_\rho(1+\kappa)$ from static quark model.

The two-body π^+ absorption operator $T_{\vec{q}}(\vec{r},1'2')$ defined in Eq. (4) for pion or rho-meson rescattering is then given by

$$T_{\vec{q}}^{\pi,\rho}(\vec{r};1'2')=\frac{f_\Delta}{\sqrt{2\omega}m_\pi}\frac{1}{\omega_R-\omega-\frac{1}{2}i\Gamma_\Delta}\{e^{i\vec{q}\cdot\vec{r}/2}V_{12}^{\pi,\rho}(\vec{r})(\vec{S}_1^\dagger\cdot\vec{q})T_{1+}^\dagger+e^{-i\vec{q}\cdot\vec{r}/2}V_{21}^{\pi,\rho}(\vec{r})(\vec{S}_2^\dagger\cdot\vec{q})T_{2+}^\dagger\} \tag{A.5}$$

and

$$V_{12}^{\pi}(\vec{r})=\frac{1}{3}\frac{f_\Delta f}{4\pi}m_\pi\{Y_0(m_\pi^*r)\vec{S}_1\cdot\vec{\sigma}_2+Y_2(m_\pi^*r)S_{12}^*(\vec{r})\}\vec{T}_1\cdot\vec{\tau}_2\,, \tag{A.6}$$

$$V_{12}^{\rho}(\vec{r})=\frac{1}{3}\frac{g_\rho g_{\rho\Delta}}{4\pi m^2}m_\rho{}^3\{2Y_0(m_\rho^*r)\vec{S}_1\cdot\vec{\sigma}_2-Y_2(m_\rho^*r)S_{12}^*(\vec{r})\}\vec{T}_1\cdot\vec{\tau}_2\,, \tag{A.7}$$

$$S_{12}^*=3\vec{S}_1\cdot\hat{r}\vec{\sigma}_2\cdot\hat{r}-\vec{S}_1\cdot\vec{\sigma}_2\,, \tag{A.8}$$

$$Y_0(x)=\frac{e^{-x}}{x}\,,\quad Y_2(x)=\left(1+\frac{3}{x}+\frac{3}{x^2}\right)\frac{e^{-x}}{x}\,, \tag{A.9}$$

with S and T the transition spin and isospin operators which connect nucleons and isobars. The effective pion and rho-meson masses are defined respectively to be $m_\pi^*=(m_\pi{}^2-\omega^2/4)^{1/2}$ and $m_\rho^*=(m_\rho{}^2-\omega^2/4)^{1/2}$ in terms of the pion energy ω. The position of the isobar resonance and its width are

$$\omega_R=m_\Delta-m+\frac{q^2}{2m_\Delta}\quad\text{and}\quad\Gamma_\Delta=\frac{2}{3}\frac{f_\Delta{}^2}{4\pi}\frac{q^3}{m_\pi{}^2}\,, \tag{A.10}$$

respectively, where the mass m_Δ of Δ is 1232 MeV.

After summing over the initial and final spin and isospin states and carrying out partial wave expansions, we obtain the pion absorption cross section per nucleon

$$\sigma=\frac{4k_f^4 mq\omega}{3\pi}\left\{\sum_L^-\sum_{L'}^-\sum_l^+\left[24\mathcal{H}^{LL'l}+18\mathcal{K}^{LL'l}+\sum_{l'}^+(24\mathfrak{M}^{LL'll'}-9\sqrt{70}\mathfrak{N}^{LL'll'})\right]\right.$$
$$+\sum_L\sum_{L'}\sum_l^-\left[8\mathcal{H}^{LL'l}+22\mathcal{K}^{LL'l}+\sum_{l'}^-(16\mathfrak{M}^{LL'll'}-5\sqrt{70}\mathfrak{N}^{LL'll'})\right]$$
$$\left.+\sum_L^+\sum_{L'}^+\sum_l^+\left[18\mathcal{K}^{LL'l}-\sum_{l'}^+9\sqrt{70}\mathfrak{N}^{LL'll'}\right]\right\}, \tag{A.11}$$

where the + and − in the summations denote even and odd, respectively. In the above, we have used the following expressions:

$$\mathcal{H}^{LL'l}=[L][L'][l]\int_0^\infty dx\,P(x)x'\begin{pmatrix}L' & L & l\\ 0 & 0 & 0\end{pmatrix}^2|H^{LL'l}(k_Fx',k_Fx)|^2, \tag{A.12}$$

$$\mathcal{K}^{LL'l}=[L][L'][l]\int_0^\infty dx\,P(x)x'|K^{LL'l}(k_Fx',k_Fx)|^2\sum_C[C]\begin{pmatrix}2 & l & C\\ 0 & 0 & 0\end{pmatrix}^2\begin{pmatrix}L & L' & C\\ 0 & 0 & 0\end{pmatrix}^2, \tag{A.13}$$

$$\mathfrak{M}^{LL'll'}=(-1)^{(l-l')/2}[L][L'][l][l']\begin{pmatrix}l & l' & 2\\ 0 & 0 & 0\end{pmatrix}^2\int_0^\infty dx\,P(x)x'\,\mathrm{Re}\left\{\begin{pmatrix}L & L' & l'\\ 0 & 0 & 0\end{pmatrix}^2[H^{LL'l}(k_Fx',k_Fx)]^*K^{LL'l}(k_Fx',k_Fx)\right.$$
$$\left.+\begin{pmatrix}L & L' & l\\ 0 & 0 & 0\end{pmatrix}^2H^{LL'l}(k_Fx',k_Fx)[K^{LL'l'}(k_Fx',k_Fx)]^*\right\}, \tag{A.14}$$

$$\mathfrak{N}^{LL'll'}=(-1)^{(l-l')/2}[L][L'][l][l']\begin{pmatrix}l & l' & 2\\ 0 & 0 & 0\end{pmatrix}^2\int_0^\infty dx\,P(x)x'\,\mathrm{Re}\{K^{LL'l}(k_Fx',k_Fx)[K^{LL'l'}(k_Fx',k_Fx)]^*\}$$
$$\times\sum_C(-1)^C[C]\begin{pmatrix}2 & l & C\\ 0 & 0 & 0\end{pmatrix}\begin{pmatrix}2 & l' & C\\ 0 & 0 & 0\end{pmatrix}\begin{pmatrix}L' & L & C\\ 0 & 0 & 0\end{pmatrix}^2\begin{Bmatrix}2 & 2 & 2\\ C & l & l'\end{Bmatrix}. \tag{A.15}$$

We use the abbreviation $[L]\equiv 2L+1$. The quantity x' is given by, through energy conservation, $x'=[x^2+\omega m(1-q^2/4\omega m)/k_F^2]^{1/2}$. The function $P(x)$, which is the relative probability for a pair of nucleons with relative momentum $x\hbar k_F$, is given by

$$P(x)=24x^2\left(\frac{32\pi k_F^3}{3}\right)^{-1}\int d^3\vec{K}f\left(\left|\frac{\vec{K}}{2}+\vec{k}\right|\right)f\left(\left|\frac{\vec{K}}{2}-\vec{k}\right|\right). \tag{A.16}$$

The two functions $H^{LLl'}$ and $K^{LLl'}$ are defined, respectively, by

$$\begin{Bmatrix}H^{LL'l}(k_f,k_i)\\ K^{LL'l}(k_f,k_i)\end{Bmatrix}=\int_0^\infty dr\,r^2 j_{L'}(k_f r)j_l(\tfrac{1}{2}qr)\begin{Bmatrix}Y_0^*(r)\\ Y_2^*(r)\end{Bmatrix}j_L(k_i r), \tag{A.17}$$

with

$$Y_0^*(r)=\xi_\pi Y_0(m_\pi^*r)+2\xi_\rho Y_0(m_\rho^*r),$$
$$Y_2^*(r)=\xi_\pi Y_2(m_\pi^*r)-\xi_\rho Y_2(m_\rho^*r), \tag{A.18}$$

where

$$\xi_\pi=\frac{4}{27}f\left(\frac{f_\Delta^2}{4\pi}\right)\left(\frac{m_\pi^*}{m_\pi}\right)^3\frac{1}{D\sqrt{\omega}},$$
$$\xi_\rho=\frac{8}{75}f\left(\frac{g_\rho^2}{4\pi}\right)\left(\frac{m_\rho^{*3}}{m^2m_\pi}\right)\frac{1}{D\sqrt{\omega}}(1+\kappa)^2. \tag{A.19}$$

Hadronic form factors in the vertex can be straightforwardly included. In the monopole forms, we make the following replacements for the coupling constants:

$$f\to f\frac{\Lambda_\pi^2-m_\pi^{*2}}{\Lambda_\pi^2+\vec{Q}^2},\quad g\to g\frac{\Lambda_\rho^2-m_\rho^{*2}}{\Lambda_\rho^2+\vec{Q}^2}, \tag{A.20}$$

where $\vec{Q}$ is the momentum of the rescattered pion or rho-meson. These lead to modified expressions for Y_0 and Y_2 as shown in Ref. 16. The values of Λ_π and Λ_ρ are both taken to be 1.2 GeV in our calculations.

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