Angular distribution theory for particle-capture-γ reactions

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Legendre polynomial coefficients are derived and compared with the results of other authors for the angular distributions encountered in gamma reactions initiated by the capture of vector or tensor polarized or unpolarized particles of arbitrary spin. Attention is given to phase conventions and the consequences of using different angular momentum coupling schemes. Results are given for both the channel spin and the total-j representations. Correction factors for use with other author's results are included. Some potentially useful sum rules are deduced and illustrated.

[NUCLEAR REACTIONS Rederived expressions for γ ray angular distributions resulting from capture of vector or tensor polarized and unpolarized particles of arbitrary spin. Compared with previously published results.]

INTRODUCTION

The 1954 work of Huby[1] and the less well known earlier observation of Coester[2] focused attention on the time reversal properties of S matrix elements. Since then several authors have included these considerations in descriptions of photon reactions. Devons and Goldfarb,[3] Ferguson,[4] Rybicki, Tamura, and Satchler[5] and Debenham and Satchler[6] treat particle-γ angular correlations. The latter two references are extensions of the work of Rose and Brink[7] who focused on the important question of phase conventions, e.g., signs of multipolarity mixing ratios for γ ray angular distributions. Baldin, Goldanskii, and Rosental[8] and Welton[9] examine two-body reactions where either the initial or the final state or both involve a photon. Baldin *et al.*[8] give results for the unpolarized angular distribution and for the outgoing particle (vector) polarization distribution. Welton[9] gives completely general results for all possible outgoing tensor moments in terms of all possible incoming tensor moments. Perhaps the less general results of Baldin *et al.* are easier to understand than those of Welton. Whatever the reason, the Baldin *et al* results are more popular as is evidenced by the fact that the review by Firk[10] and the recent works of Kabachnik and Razuvaev,[11] of Ulbricht *et al.*,[12] and of Laszewski and Holt[13] all use the Baldin *et al.* results. Laszewski and Holt[13] present tables of numerical coefficients of an associated Legendre polynomial expansion of the polarization distribution by evaluating the Baldin *et al.* equations. These tables supplement the tables of Carr and Baglin[14] that give the coefficients of the standard Legendre polynomial expansion of the unpolarized angular distribution.

Since tables are easier to use than the original equations, their use can be expected to increase. In a field that has been beset with errors and results almost defying comparison, e.g., because of incompletely specified information (as noted in Ref. 7), we felt it worthwhile to attempt a rederivation of the results of Refs. 8 and 9 in an effort to establish some confidence in results already in use or to correct existing results before their use continues. Since existing references rarely compare their results or give much attention to phase questions and angular momentum coupling order considerations, our second objective is to improve this situation.

NOTATIONS AND CHOICE OF AXES

The well-known differential cross section expression for a polarized incident spin $\frac{1}{2}$ beam is

$$\sigma(\theta,\phi)=\sigma_u(\theta)[1+\vec{p}\cdot\vec{A}(\theta)], \quad (1)$$

where $\vec{p}$ is the polarization, $\vec{A}(\theta)$ the analyzing power vector, and $\sigma_u(\theta)$ the unpolarized cross section.

We choose the following right-handed set of axes:

$$y \text{ along } \vec{k}_{in}\times\vec{k}_{out} \text{ and } z \text{ along } \vec{k}_{in}. \quad (2)$$

This choice of axes allows Eq. (1) to be rewritten as

$$\sigma(\theta,\phi)=\sigma_u(\theta)[1+p_yA_y(\theta)]. \quad (3)$$

If the polarization of an arbitrary spin beam is described by tensor moments t_{kq}, the analog of Eq. (1) becomes

$$\sigma(\theta,\phi)=\sigma_u(\theta)\sum_{kq} t_{kq}T^*_{kq}(\theta)\,, \qquad (4)$$

where the $T^*_{kq}[=(-1)^q T_{k-q}$ like spherical harmonics$]$ are the analyzing (power) tensors. Parity conservation and Eq. (2) imply $T_{10}=0$, T_{11} imaginary and T_{2q} real, so for a polarized beam of unit spin

$$\sigma(\theta,\phi)=\sigma_u(\theta)[1+\tfrac{3}{2}p_y A_y(\theta)+t_{20}T_{20}(\theta)+2\,\mathrm{Re}t_{21}T_{21}(\theta)+2\,\mathrm{Re}t_{22}T_{22}(\theta)]\,. \qquad (5)$$

We consider an x capture-γ reaction whose angular momenta are specified by the form

$$a(x,L)c\,, \qquad (6)$$

where a is the spin of the target, x the spin of the projectile which carries orbital angular momentum l, b the spin of the gamma emitting (compound) state, L the multipolarity and p the mode (1 = electric, 0 = magnetic) of the gamma ray, and c is the spin of residual nuclear state after the gamma emission.

We adopt the notation $\hat{x}\equiv(2x+1)^{1/2}$ and define a set of Legendre polynomial coefficients by the equation

$$\sigma_u(\theta)=(\tfrac{1}{2}\lambda\!\!\!^{-})^2\hat{x}^{-2}\hat{a}^{-2}\sum_k a_k P_k(\cos\theta)\,, \qquad (7)$$

where $\lambda\!\!\!^{-}$ is the usual reduced wavelength of the incident beam. Clearly

$$\int\sigma_u(\theta)d\Omega=(\tfrac{1}{2}\lambda\!\!\!^{-})^2\hat{x}^{-2}\hat{a}^{-2}4\pi a_0\,. \qquad (8)$$

An alternative form of Eq. (7) is

$$\sigma_u(\theta)=A_0\left(1+\sum_{k\neq 0}\bar{a}_k P_k(\cos\theta)\right), \qquad (9)$$

where

$$\bar{a}_k=a_k/a_0\,,\qquad A_0=(\tfrac{1}{2}\lambda\!\!\!^{-})^2\hat{x}^{-2}\hat{a}^{-2}a_0\,. \qquad (10)$$

For a beam of arbitrary spin polarized with moments of rank ≤ 2 we define the associated Legendre polynomial coefficients b_k, c_k, d_k, and e_k by the equation,

$$\sigma(\theta,\phi)=(\tfrac{1}{2}\lambda\!\!\!^{-})^2\hat{x}^{-2}\hat{a}^{-2}\sum_k(a_kP_k+b_kP^1_kp_y+c_kP_kt_{20}+d_kP^1_k\mathrm{Re}t_{21}+e_kP^2_k\mathrm{Re}t_{22})\,. \qquad (11)$$

The analyzing tensors can be expressed in terms of the coefficients by comparing Eqs. (5) and (11), e.g., $T_{20}(\theta)=\sum_k c_kP_k/\sum_k a_kP_k$. The associated Legendre polynomial P^M_L used here is the usual one, which is sometimes referred to as being unnormalized and is related to the normalized associated Legendre polynomial $\bar{P}^M_L$ by the following equation

$$\bar{P}^M_L\equiv(2\pi)^{1/2}e^{-iM\phi}Y_{LM}(\theta\phi)\overset{M\geq 0}{=}(-1)^M 2^{-1/2}\hat{L}[(L-M)!/(L+M)!]^{1/2}P^M_L\,. \qquad (12)$$

This equation is as given on page 219 of Baldin *et al.*[8] Unfortunately Laszewski and Holt[13] when quoting other results of Ref. 8, give this equation with the factor $(-1)^M$ omitted. Thus for $M=1$ (the polarization distribution) the use of the Laszewski and Holt equation for converting the Baldin *et al.* results from $\bar{P}^1_L$ to P^1_L coefficients would introduce a change of sign. We return to this point later.

Parity conservation implies that

$$\pi_x\pi_a(-1)^l=\pi_c(-1)^{L+p+1}\,, \qquad (13)$$

where the π's are the intrinsic parities of the particles having the angular momenta labeled by the subscripts. Multiplying this equation by its primed version leads to another statement of parity conservation

$$(-1)^{l+l'}=(-1)^{L+p+L'+p'}\,. \qquad (14)$$

References 8 and 9 follow the above convention of p values but some authors, e.g., Ref. 7, interchange the electric and magnetic values, which leaves Eq. (14) unchanged but changes the sign of the right side of Eq. (13). The derivation of the coefficient of a Legendre polynomial of order k reveals a restriction on k such that $(-1)^k$ is equal to Eq. (14) (this would not hold if we were calculating the circular polarization of the γ ray). This parity restriction is included in all the expressions for a_k, b_k, ..., e_k by defining and using the empty bracket symbol

$$[\;]\equiv\tfrac{1}{2}[1+(-1)^{L+p+L'+p'+k}]\,, \qquad (15)$$

which clearly vanishes for nonallowed k and equals unity for allowed k values. For a given pair of multipoles Lp and $L'p'$, k will be only even or only odd.

CHANNEL SPIN REPRESENTATION

Here we adopt the following coupling order:

$$\vec{x}+\vec{a}=\vec{s}\,, \qquad (16a)$$

$$\vec{l}+\vec{s}=\vec{b}\,, \qquad (16b)$$

$$\vec{L}+\vec{c}=\vec{b}\,. \qquad (16c)$$

One other equation which is invariably followed in constructing the vector potential is

$$\vec{l}_\gamma+\vec{1}=\vec{L}\,, \qquad (16d)$$

where $\vec{1}_\gamma$ is the (unobservable) photon "orbital" angular momentum and the unit vector is its "spin."

Having announced by Eq. (16) the coupling order we feel justified in abbreviating the (reduced) transition matrix elements and write

$$R \equiv \langle pL(c)b\pi \,||R||\, l(xa)s\, b\pi\rangle ,$$

and

$$R' \equiv \langle p'L'(c)b'\pi' \,||R||\, l'(xa)s'b'\pi'\rangle .$$

Also for brevity we shall use the channel label t to represent the set of quantum numbers $\{pLbls\}$.

We cannot overemphasize how important it is to know the complete coupling scheme which leads to a particular set of equations for the physical observables in terms of, say, collision matrix elements. Another author using another coupling scheme may deduce a different, but equally valid, set of equations containing some unknown collision matrix elements to be determined by fitting data. The two sets of collision matrix elements can be related by simple phase factors which although different for different channels can easily be calculated *if* the coupling schemes are known. Admittedly it is sometimes far from a simple task to discover the complete coupling scheme. The author may not give all the equations or worse he may give some of them incorrectly. Reference 8 on page 178 gives four equations, where one equation (in our notation) is the same as Eq. (16a), but the other three equations all have the order of terms switched on the left side compared to Eqs. (16b)–(16d). One might well conclude that a very different coupling scheme was employed in Ref. 8, but upon much closer inspection of the derivation it appears that the coupling scheme actually used was in fact that of Eqs. (16). Fortunately for comparison purposes Ref. 9 also chooses the Eq. (16) coupling order. On the other hand, Refs. 3 and 4 use the opposite order for Eqs. (16b) and (16c). Using Eq. (16b) with the opposite order will introduce into all Legendre polynomial coefficient sums a factor $(-1)^{l-l'+s-s'+b-b'}$ and Eq. (16c) used in opposite order will introduce a factor $(-1)^{L-L'+b-b'}$. The combination would reduce to a factor $(-1)^{s-s'+p+p'}$. For the unpolarized beam (a_k) case, $s=s'$ and the factor would simplify to $(-1)^{p+p'}$ as is easily verified by comparing final expressions in Ref. 3 or 4 with ours.

Phase factor differences in two treatments may also arise from another source, related to the choice of phase of the vector potential (serving as the photon wave function). Let the vectors $\vec{A}_{LM}(e)$ and $\vec{A}_{LM}(m)$ designate, respectively, the electric and magnetic multipole components of the transverse vector potential [e.g., Eq. (3.6) of Ref. 7]. The vector wave equation requires that the curl of one multipole be proportional to the other multiplied by the wave number κ but leaves a sign undetermined

$$\nabla \times \vec{A}_{LM}(e \text{ or } m) = \pm\kappa \vec{A}_{LM}(m \text{ or } e) . \tag{17}$$

References 7, 15, and 16 define $\vec{A}_{LM}(e)$ and $\vec{A}_{LM}(m)$ such that the + sign applies in Eq. (17). Devons and Goldfarb[3] evaluate the Racah parameter of radiation for mixed multipoles and their result [their Eq. (14.14)] reads

$$C_{k0}(LL') = \pm(4\pi)^{-1}\hat{L}\hat{L}'(-1)^{L'-1}(L1, L'-1\,|k0)[\;] , \tag{18}$$

where the last bracket is defined by Eq. (15). The $\pm$ choice in this equation is the same choice as in Eq. (17). Reference 4 adopts the + sign in Eq. (18), as we shall, which means that $E2/M1$ γ mixing ratios will have the opposite sign to those of anyone who, in effect, chose the − sign in Eq. (18), such as, e.g., Biedenharn and Rose.[17] The $\vec{A}_{LM}(m)$ or magnetic multipole expressions given by Eq. (3.6) of Ref. 7, Eq. (3.36a) of Ref. 16, Eq. (181) of Ref. 9, and Eq. (33.5) of Ref. 8 all agree. For the electric multipole Refs. 7 and 16 agree but are opposite in sign to Refs. 8 and 9, allowing for correction of obvious typographical errors in each of the latter (i.e., deleting $\sqrt{2}$ in Eq. (182) of Ref. 9 and interchanging $\sqrt{g}$ and $\sqrt{g+1}$ in Ref. 8). In an effort to combine the electric and magnetic multipole equations into one equation, Ref. 8 in Eq. (33.6), and Ref. 9, in a corrected version of its Eq. (184) [with $2^{-1/2}$ replaced by $(-1)^{p+1}2^{1/2}$, to be consistent with Eqs. (181) and (182)], present the following factor

$$(l_\gamma\,|\,p) = -(-1)^p\sqrt{2}(L-1, 11\,|l_\gamma 0)\tfrac{1}{2}(1+(-1)^{p+L-l_\gamma}). \tag{19}$$

This factor, which is exactly unity for the magnetic ($p=0$) case, corresponds to choosing the − sign in Eqs. (17) and (18). For $p=1$, Ref. 16 and indirectly Refs. 7 and 15 use a sign opposite to that of Eq. (19), or in other words do not include the $(-1)^p$, which implies a choice of the + sign in Eqs. (17) and (18). We would thus expect Refs. 8 and 9 to have a missing factor $(-1)^{p+p'}$ in their final cross sections,[18] which suggests that (intentionally or not) both authors dropped the factor $(-1)^p$ and thereby adopted the + sign in Eqs. (17) and (18). In summary, in spite of indications of opposite intentions the use of the + sign in Eqs. (17) and (18) now seems to be standard practice.

Another point concerning these sign choices is that even if two authors choose the same, say +, sign in Eqs. (17) and (18) they could do so in different fashions. For example, when Refs. 3 and 4 use the + sign in Eqs. (17) and (18) they actually introduce an additional minus sign in *both* $\vec{\mathrm{A}}_{LM}(e)$ and $\vec{\mathrm{A}}_{LM}(m)$ or equivalently eliminate the factor $-(-1)^p$ from Eq. (19). This choice, which amounts to multiplying the photon wave function by (−1), has absolutely no effect on the equations for physical observables as one would expect. For this sign choice to be important, one would need a model capable of calculating particle and photon collision matrix elements, which were related by a version of Eq. (19).

Employing the statistical-tensor–efficiency-tensor approach of Refs. 3 and 4 the following expressions for the Legendre coefficients in the channel spin representation were derived

$$a_k=\sum_{tt'}(-1)^{s-c+1}[\ ]\hat{l}\hat{l}'\hat{L}\hat{L}'\hat{b}^2\hat{b}'^2(l0,l'0\,|k0)W(lbl'b';sk)(L1,L'-1\,|k0)W(LbL'b';ck)\,\mathrm{Re}RR'^*\,, \tag{20}$$

e.g., $a_0=\sum_t\hat{b}^2|R|^2$. The sum over t and t' means over $pp'LL'bb'll's$ and s' (but here $s'=s$):

$$b_k=\frac{3\sqrt{x}\,\hat{x}\hat{k}}{[(x+1)k(k+1)]^{1/2}}\sum_{tt'}[\ ]\hat{s}\hat{s}'\hat{l}\hat{l}'\hat{L}\hat{L}'\hat{b}^2\hat{b}'^2(-1)^{a-x+c-b-s+l}(l0,l'0\,|k0)$$
$$\times W(xsxs';a1)(L1,L'-1\,|k0)W(LbL'b';ck)X(lsb;l's'b';k1k)\,\mathrm{Re}(iRR'^*). \tag{21}$$

The other three coefficients involve several common terms, so we define

$$F(k,k_l)\equiv\hat{2}\hat{x}\hat{k}_l\sum_{tt'}[\ ]\hat{s}\hat{s}'\hat{l}\hat{l}'\hat{L}\hat{L}'\hat{b}^2\hat{b}'^2(-1)^{a-x+c-b-s+l+1}(l0,l'0\,|k_l0)W(xsxs';a2)$$
$$\times(L1,L'-1\,|k0)W(LbL'b';ck)X(lsb;l's'b';k_l2k)[\ ]\mathrm{Re}RR'^*\,. \tag{22}$$

In terms of this F we find

$$c_k=\sum_{k_l}(k_l0,20\,|k0)F(k,k_l)\,, \tag{23}$$

$$d_k=\frac{-2}{[k(k+1)]^{1/2}}\sum_{k_l}(k_l0,21\,|k1)F(k,k_l)\,, \tag{24}$$

and

$$e_k=\frac{2}{[(k-1)k(k+1)(k+2)]^{1/2}}\sum_{k_l}(k_l0,22\,|k2)F(k,k_l)\,. \tag{25}$$

Parity conservation implies $(-1)^{k_l}=(-1)^k$. Therefore k_l in the above sums equals k and $k\pm2$ only.

Baldin *et al.*[8] give an expression for the differential polarization $dP/d\Omega$, which can be related to the above coefficient b_k. Equation (33.11) of Ref. 8 states in our notation

$$\frac{dP}{d\Omega}=(\tfrac{1}{2}\bar{\lambda}_\gamma)^2\frac{[x(x+1)]^{1/2}\hat{x}}{\sqrt{2}\,\hat{c}^2}\sum_{tt'k}\mathrm{Re}(iR^*R')b(ktt')\overline{P}{}^1_k\,, \tag{26}$$

where the symbol $b(ktt')$ introduced by Laszewski and Holt[13] represents

$$b(ktt')=(-1)^{a-x+c+b'+s'+l'-1+p+p'}\hat{s}\hat{s}'\hat{l}\hat{l}'\hat{L}\hat{L}'\hat{b}^2\hat{b}'^2$$
$$\times(l0,l'0\,|k0)(L-1,L'1\,|k0)W(xsxs';a1)$$
$$\times W(LbL'b';ck)X(bls;b'l's';kk1)[\ ]. \tag{27}$$

If we write

$$\frac{dP}{d\Omega}=\sigma_{u|\gamma x}\langle S_y\rangle_{\gamma x}$$
$$=x\sigma_{u|\gamma x}A_{y|x\gamma}$$
$$=x\left(\frac{\bar{\lambda}_\gamma}{\bar{\lambda}_x}\right)^2\frac{\hat{x}^2\hat{a}^2}{2\hat{c}^2}\sigma_{u|x\gamma}A_{y|x\gamma}$$
$$=(\tfrac{1}{2}\bar{\lambda}_\gamma)^2\frac{x+1}{6\hat{c}^2}\sum_k b_kP^1_k\,, \tag{28}$$

we deduce upon comparing Eqs. (26) and (28) and using the Baldin *et al.* definition of $\overline{P}{}^1_L$ [our Eq. (12)],

$$b_k=\frac{3\sqrt{x}\,\hat{x}\hat{k}}{[(x+1)k(k+1)]^{1/2}}\sum_{t,t'}\mathrm{Re}(iRR'^*)b(ktt')\ . \tag{29}$$

Equation (29) (the Baldin *et al.* result) differs from Eq. (21) (our result) by the phase factor

$$f=(-1)^{b-b'+s-s'+l-l'+1}$$
$$=(-1)^{b+l+s+b'+l'+s'+k+k+1}\,, \tag{30}$$

which is precisely the phase factor that results upon interchanging any two rows (or columns) of the 9-j symbol. Inclusion of this factor in Eq. (27) would bring Eqs. (29) and (21) into agreement. We have compared our results to those of Welton[9] and find a difference of a factor $(-1)^k$. A preliminary result of a comparison of Welton's starting equation [expressed in terms of magnetic sums, i.e., a corrected version of his Eq. (200)] with his final equation (the result of much Racah algebra) confirms (in the special test case) the $(-1)^k$ error in his final result and thus lends plausibility to our results. In summary we believe the results of Baldin *et al.* for a_k are correct (thus the tables of

Carr and Baglin[14] are correct) but for b_k are in error by the factor f of Eq. (30). The results of Laszewski and Holt for b_k (they considered only the case where $x=\frac{1}{2}$) are in error by the factor $-f$, where the minus sign takes into account their version of Eq. (12).

j-j COUPLING REPRESENTATION

As an alternative to the channel spin coupling to Eqs. (16a) and (16b) we can couple as follows

$$\vec{l}+\vec{s}=\vec{j}\,, \tag{31a}$$

$$\vec{j}+\vec{a}=\vec{b}. \tag{31b}$$

The photon coupling is as before in Eqs. (16c) and (16d). Now letting

$$R\equiv\langle pL(c)b\pi\|\,R\,\|(lx)jab\pi\rangle$$

and $t=\{pLblj\}$ we can either convert our channel spin results using

$$|l(xa)sb\rangle=\sum_j \hat{j}\hat{s}\,W(lxba;js)\,|(lx)jab\rangle \tag{32}$$

or start from fundamentals as we did in the channel spin case. We find

$$a_k=\sum_{tt'}(-1)^{a-c+1-x+k+j-j'}\hat{j}\hat{j}'\hat{l}\hat{l}'\hat{L}\hat{L}'\hat{b}^2\hat{b}'^2(l0,l'0\,|k0)W(ljl'j';xk)[\ ]$$
$$\times(L1,L'-1\,|k0)W(jbj'b';ak)W(LbL'b';ck)\,\mathrm{Re}(RR'^*)\,. \tag{33}$$

Again, of course, $a_0=\sum_t \hat{b}^2|R|^2$.

$$b_k=\frac{3\sqrt{x}\,\hat{x}\hat{k}}{[(x+1)k(k+1)]^{1/2}}\sum_{tt'}\hat{j}\hat{j}'\hat{l}\hat{l}'\hat{L}\hat{L}'\hat{b}^2\hat{b}'^2(-1)^{a-c+1+l'-j'}[\ ]$$
$$\times(l0,l'0\,|k0)W(jbj'b';ak)(L1,L'-1\,|k0)W(LbL'b';ck)X(lxj;l'xj';k1k)\,\mathrm{Re}(iRR'^*)\,. \tag{34}$$

Defining

$$G(k,k_l)\equiv\hat{2}\hat{x}\hat{k}_l\sum_{tt'}\hat{j}\hat{j}'\hat{l}\hat{l}'\hat{L}\hat{L}'\hat{b}^2\hat{b}'^2(-1)^{a-c+1+l'-j'}[\ ](l0,l'0\,|k_l0)$$
$$\times W(jbj'b';ak)(L1,L'-1\,|k0)W(LbL'b';ck)X(lxj;l'xj';k_l2k)\,\mathrm{Re}(RR'^*)\,, \tag{35}$$

we obtain

$$c_k=\sum_{k_l}(k_l0,20\,|k0)G(k,k_l)\,, \tag{36}$$

$$d_k=\frac{-2}{[k(k+1)]^{1/2}}\sum_{k_l}(k_l0,21\,|k1)G(k,k_l)\,, \tag{37}$$

and

$$e_k=\frac{2}{[(k-1)k(k+1)(k+2)]^{1/2}}\sum_{k_l}(k_l0,22\,|k2)G(k,k_l)\,. \tag{38}$$

For $x=\frac{1}{2}$ these expressions for a_k and b_k agree with results of Glavish.[19] This agreement of the j-j coupling results [Eqs. (33) and (34)] lends support to the correctness of our channel spin results [Eqs. (20) and (21)], since the former are easily obtained from the latter by using Eq. (32).

RESTRICTED SUM RULES

Let us consider a very special case where R and R' are independent of b and where $l=L$, $l'=L'$, and $s=s'=c$. For this case it is easy to see that a_k is proportional to $(l0,l'0\,|k0)(l1,l'-1\,|k0)$ and therefore that $\sum_{k\,\mathrm{even}}a_k=\sum_{k\,\mathrm{odd}}a_k=0$. Similarly b_k is found to involve

$$\sum_{bb'}\hat{b}^2\hat{b}'^2X(Lcb;L'cb';k0k)X(Lcb;L'cb';k1k)\,,$$

which vanishes. Likewise, for c_k, d_k, and e_k,

$$b_k=c_k=d_k=e_k=0\,.$$

We note that the unmodified results of Welton would satisfy the equation $b_k=0$, but the unmodified results of Baldin *et al.* would not.

An example will illustrate the usefulness of these results for checking coefficients of RR' combinations which fail to satisfy the conditions of this highly special case. We present the results for a_k and b_k for the case $\vec{\frac{1}{2}}+1\to(E1\text{ or }E2)+\frac{1}{2}$, where it will suffice to ignore one of the possible s values ($s=\frac{3}{2}$) and thus have $s=s'=c$. For an s-wave final state this ought to be a good approximation, but that is not our concern here. Thus

$$a_0=2R_{1\,1/2}{}^2+4R_{1\,3/2}{}^2+4R_{2\,3/2}{}^2+6R_{2\,5/2}{}^2\,,$$

where $R=R_{Lb}e^{i\phi_{Lb}}$

$$a_2=-2R_{1\,3/2}{}^2+2R_{2\,3/2}{}^2+3.42R_{2\,5/2}{}^2$$
$$-4R_{1\,3/2}R_{1\,1/2}C+1.72R_{2\,5/2}R_{2\,3/2}C\,,$$

$$a_4=-3.42\,R_{2\,5/2}{}^2-13.72R_{2\,5/2}R_{2\,3/2}C\,,$$

where each C (all different) is defined such that

$RR'C \equiv RR'\cos(\phi - \phi')$, and

$$a_1 = 6.92R_{2\,3/2}R_{1\,1/2}C + 1.38R_{2\,3/2}R_{1\,3/2}C + 12.48R_{2\,5/2}R_{1\,3/2}C\,,$$

$$a_3 = -6.92R_{2\,5/2}R_{1\,1/2}C - 8.32R_{2\,3/2}R_{1\,3/2}C - 5.54R_{2\,5/2}R_{1\,3/2}C\,,$$

$$b_1 = -1.16R_{2\,3/2}R_{1\,1/2}S - 0.92R_{2\,3/2}R_{1\,3/2}S + 2.08R_{2\,5/2}R_{1\,3/2}S,$$

$$b_2 = -0.66R_{1\,3/2}R_{1\,1/2}S + 0.47R_{2\,5/2}R_{2\,3/2}S\,,$$

$$b_3 = -0.77R_{2\,5/2}R_{1\,1/2}S + 0.92R_{2\,3/2}R_{1\,3/2}S - 0.15R_{2\,5/2}R_{1\,3/2}S\,,$$

$$b_4 = -1.14R_{2\,5/2}R_{2\,3/2}S\,.$$

Now let us imagine the R_{Lb} to be independent of b, i.e., $R_{Lb} = R_L$ only. Then since $l = L$ and $l' = L'$ and $s = s' = c$, the special case conditions are satisfied and the special results can be applied. Notice how for k odd the $b_k = 0$ result serves as a very useful check on the numerical coefficients, e.g., for b_1 we find $-1.16 - 0.92 + 2.08 = 0$. For k even, the S factors in b_k vanish and thus $b_k = 0$ is not helpful here. For both even and odd k the $\sum a_k = 0$ result is a useful check. In fact it is easy to see that for even k, $\sum a_k = 0$ for each L value separately.

Finally, we note once again that the published "Tables of Differential Polarization Coefficients"[13] must be multiplied by the phase factor given in Eq. (30). In the notation of the tables of Ref. 13, this additional phase is

$$(-1)^{(L+J+S)_{\text{first}} - (L+J+S)_{\text{second}} + 1}\,.$$

Furthermore, if the results are to be written in terms of $P_\nu^1(x)$ rather than $\overline{P}_\nu^1(x)$, the equation which relates the two, given on page 309 of Ref. 13, must be multiplied by a minus sign.

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[18] More precisely, one cannot really tell whether or not Welton dropped the $(-1)^{p+p'}$, since his final result is inconsistent with his starting point by a factor $(-1)^{L_1+L_2}$ if the $(-1)^{p+p'}$ was included at the start and by a factor $(-1)^{k+q}$ if not [the rank of the γ polarization q is zero in our work, so the latter factor reduces to $(-1)^k$].

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