PHYSICAL REVIEW C VOLUME 20, NUMBER 2 AUGUST 1979

Test of bound-state approximations in a three-body model of rearrangement collisions

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(Received 11 April 1979)

A common approximation in atomic, molecular, and nuclear rearrangement processes is to neglect n-body breakup contributions ($n \geq 3$) by replacing the full wave function by (sums of) product wave functions, each of which is related to a specific asymptotic arrangement channel. We test this bound-state approximation in a simple three-body model of nuclear stripping by comparing bound-state approximation calculations with exact and distorted-wave Born-approximation-type calculations. Calculated are the stripping and elastic amplitudes and cross sections for deuteron energies E_d = 1.78, 6.7, 11.2, and 15.12 MeV. The bound-state approximation is best below the breakup threshold (E_{BU} = 2.225 MeV), where it provides an excellent fit in the forward direction and a qualitative fit over the whole angular range. For E_d = 6.7 MeV the breakup cross section is still very small; however, the intermediate continuum states already play an important role and the bound-state approximation becomes quite poor. At this energy the distorted-wave Born approximation-type calculation, which is based on exact elastic wave functions and therefore accounts partly for the continuum, is still quite good. For the two higher energies both continuum and multistep effects appear to be very important, rendering both distorted-wave Born approximation and bound-state approximation poor approximations outside the forward region. The qualitative features of the elastic cross sections are explained in terms of the momentum dependence of the bound-state wave functions and two-body T matrices.

[NUCLEAR REACTIONS Quality of DWBA and bound-state approximations; Three-body methods; Model (d,p) calculations at E_d=1.7–15 MeV; Multistep effects]

I. INTRODUCTION

Collision processes of atomic,[1] molecular,[2] and nuclear[3] nature have traditionally been analyzed in terms of quasi two-body formulations, even if rearrangement processes between the colliding particles were considered. A quasi two-body formulation can be invoked by retaining only channels with two fragments, and by including only bound states of those fragments in the calculations. Such a bound-state approximation (BSA) can be implemented in a wave function formulation by replacing the full wave function by a sum of product wave functions each corresponding to an asymptotic two-body arrangement channel, or alternatively into a T-matrix formulation by keeping only the two-body contributions in the spectral representation of the Green's functions.

The BSA for transfer processes is often supplemented by a first-order approximation. Depending on the specific stage at which this approximation is introduced one can obtain plane-wave and distorted-wave (DWBA) amplitudes. The distorted-wave Born approximation[4] (DWBA) has gained great popularity in the description of direct nuclear reactions and has been very useful in explaining much, though not all, experimental data. We should caution, however, that such success does not necessarily imply that the approximations made are valid ones; use of effective potentials can compensate for many of the shortcomings of the approximations and may obscure the important features of the reaction. This is particularly clear at intermediate energies, where both the DWBA and more fundamental approaches based on multiple scattering theory are used. Only approximations made in the latter approach can be tested unambiguously.

If one wants to estimate the validity of the BSA, one can do so by computing certain effects of the continuum (i.e., n-body effects with $n \geq 3$), for example, by extending the wave function space or the spectral representation of the Green's function to include continuum components. However, lacking an exact result, one is still left uncertain about the full importance of the continuum, the role of multistep processes, and possible cancellations between either of these effects.

A much cleaner analysis is possible if one studies a system which can be solved exactly, and also allows the use of various approximations which are standard in realistic nuclear reaction

studies. The Mitra three-body model[5] of nuclear stripping reactions provides such a framework. Admittedly this model is too simplified for a realistic description of most direct reactions, since it allows only real, separable S-wave interactions between the particles. However, it does contain the essential features of rearrangement processes, and it does admit exact solutions and solutions of plane-wave and distorted-wave type. Therefore, this model has been used by several authors[6–8] to study approximations, and to study the validity of a three-body description of stripping for its own sake. Although these studies have provided insight into the quality and some overall features of the PWBA and the DWBA, they have not allowed a fully satisfactory separation of the effects of different approximations.

In the present paper we discuss new calculations with the BSA. Since these calculations go beyond the first-order Born approximation, they give insight into the role of multistep effects proceeding via intermediate two-fragment channels. There has been much interest in such multistep effects in direct nuclear reactions recently,[9] but in none of these studies were exact and approximate results compared. In the present paper we do present comparisons between the exact and the BSA calculations, which will allow us to disentangle continuum and multistep effects. Using unitarity considerations we are also able to distinguish between real and virtual (or intermediate) continuum effects.

We start this paper with a brief review of the reaction model. The scattering equations are derived from the general channel coupling array approach to n-body scattering.[10–12] They are reduced using the identity of proton and neutron in the model. In first order these equations give rise to the PWIA (plane-wave impulse approximation) for the elastic scattering amplitude, and to the PWBA for the transfer amplitude.

In Sec. III we present the results for elastic and stripping cross sections. We explain the qualitative features of the elastic cross section through an analysis of the PWIA and also discuss the unitarity properties of amplitudes and cross sections. We analyze these results and present conclusions concerning the validity of bound-state and low-order approximations in rearrangment scattering. Some comments are made on the applicability of these results for more realistic descriptions of the rearrangement processes.

II. REACTION MODEL AND T-MATRIX EQUATIONS

If one neglects the internal structure of the target nucleus, then deuteron stripping reduces basically to a three-body process. By making the further assumptions that the two-body interactions are separable, that the proton and neutron are identical particles, and that the nucleus has infinite mass, one arrives at the Mitra model of stripping.[5] This model was employed by Shanley and Aaron[6] to study the validity of the three-body approach and of various approximations (notably the DWBA) used in standard direct reaction analysis. Bouldin and Levin[8] used the same model with slightly different parameters but also included the heavy-particle exchange diagram. A somewhat different set of calculations was performed by Reiner and Jaffe,[7] who used different neutron and proton binding energies (as is the case in heavier nuclei), and also included an approximation to the Coulomb interaction between proton and target nucleus. They performed DWBA calculations using the exact elastic wave functions in the DWBA matrix elements. Similar "exact" DWBA matrix elements were calculated by Bouldin and Levin[8] and will be shown here for comparison.

The three particles in the problem are the two nucleons n and p, and the target nucleus A. The particles interact via separable S-wave potentials

$$\langle \vec{p} \,|\, V^{(k)} \,|\, \vec{p}\,' \rangle = -\lambda_k g_k(\vec{p}) g_k(\vec{p}\,') \tag{1}$$

with Yamaguchi form factors[13]

$$g_k(\vec{p}) = (p^2 + \beta_k^2)^{-1} . \tag{2}$$

The proton-neutron interaction $V^{(1)}$, which has $\beta_1 = 1.45$ fm^{-1}, will support one bound state with energy $\epsilon_1 = 2.225$ MeV, so chosen to fit the deuteron. The interactions of the nucleus A with the proton ($V^{(2)}$) and the neutron ($V^{(3)}$) are identical and have a range parameter $\beta_2 = 1.06$ fm^{-1}. The binding energy $\epsilon_2 = 3.3$ MeV. The strength parameters λ_k are fixed by β_k and the binding energy.

The main motivation for this study is to test the BSA. Since this approximation is easily implemented within the channel coupling array (CCA) equations,[14] even if more than three arrangement channels are present, it is of interest to see how the Mitra equations can be derived starting from these CCA equations. We start from the CCA equations in the form[11]

$$T_{ik} = W_{il} V_k + \sum_m W_{il} V_m G_m T_{mk}, \quad i,k = 1,2,3 \tag{3}$$

where $V_k = \sum_{i \neq k} V^{(i)}$ is a sum of two-particle interactions between the fragments in channel k, and W_{il} is the channel coupling array satisfying $\sum_l W_{il} = 1$. The channel coupling array is further restricted by the requirement of connectivity. In order to get the Mitra equations we use the Faddeev-Lovelace choice[11] for W: $W_{il} V^{(k)} \equiv W_{ik} V^{(k)} = \delta_{ik} V^{(k)}$, so that $W_{il} V_k = V^{(i)}(1 - \delta_{ik}) \equiv V^{(i)} \bar{\delta}_{ik}$. This

yields

$$T_{ik} = V^{(i)}\bar{\delta}_{ik} + \sum_m V^{(i)}\bar{\delta}_{im} G_m T_{mk}, \quad i,m,k = 1,2,3 . \tag{4}$$

These equations are now simplified by using the identity of the nucleons, i.e., by using $PT_{21}P = T_{31}$ and $PT_{11}P = T_{11}$, where P is the permutation operator for particles 2 and 3. One obtains

$$T_{11} = V^{(1)}(G_2 T_{21} + P G_2 T_{21} P) , \tag{5}$$

$$T_{21} = V^{(2)}(1 + G_1 T_{11} + P G_2 T_{21} P) . \tag{6}$$

For separable potentials these equations can be rewritten in terms of the elastic and stripping amplitudes defined by

$$A_{ij}(\vec{k}_i, \vec{k}_j, E+i\epsilon) = \int d\vec{p}_i \int d\vec{p}_j \phi_i^*(\vec{p}_i) \langle \vec{p}_i \vec{k}_i | T_{ij}(E+i\epsilon) | \vec{p}_j \vec{k}_j \rangle \phi_j(\vec{p}_j) , \tag{7}$$

where ϕ_i is the bound-state wave function of pair i. The resulting equations are on-shell equivalent to the three-body Amado equations[15] for two identical particles and a structureless nucleus. Off-shell the equations differ because the inhomogeneous term in Eq. (6) does not depend on energy.

In the bound-state approximation G_i is replaced by[14]

$$\tilde{G}_i = \int d\vec{k} \, |\vec{k}_i \phi_i\rangle (E - k_i^2/2M_i + \epsilon_i)^{-1} \langle \vec{k}_i \phi_i | . \tag{8}$$

The scattering equations (5) and (6) now acquire the simple form

$$A_{11}(\vec{k}_1, \vec{k}_0; E+i\epsilon) = -2\lambda_1 a_1 \int d\vec{k} \, g_1(\tfrac{1}{2}\vec{k}_1 - \vec{k}) \phi_2(\vec{k}_1 - \vec{k}) \frac{1}{E + \epsilon_2 - k^2/2M_2 + i\epsilon} A_{21}(\vec{k}, \vec{k}_0; E) , \tag{9a}$$

$$\begin{aligned} A_{21}(\vec{k}_2, \vec{k}_0; E+i\epsilon) = & -\lambda_2 a_2 g_2(\vec{k}_2 - \vec{k}_0) \phi_1(\vec{k}_2 - \tfrac{1}{2}\vec{k}_0) \\ & - \lambda_2 a_2 \int d\vec{k} \, g_2(\vec{k} - \vec{k}_2) \phi_1(\vec{k}_2 - \tfrac{1}{2}\vec{k}) \frac{1}{E + \epsilon_1 - k^2/2M_1 + i\epsilon} A_{11}(\vec{k}, \vec{k}_0; E) \\ & - \lambda_2 a_2 \phi_2(\vec{k}_2) \int d\vec{k} \, g_2(\vec{k}) \frac{1}{E + \epsilon_2 - k^2/2M_2 + i\epsilon} A_{21}(\vec{k}, \vec{k}_0; E) , \end{aligned} \tag{9b}$$

where $a_i = \int d\vec{q} \, \phi_i(\vec{q}) g_i(\vec{q})$. We have used units in which the masses are dimensionless. Thus the reduced masses M_1 and M_2 $(=M_3)$ equal 2 and 1, respectively. After expanding $A_{ij}(\vec{k}_i, \vec{k}_j; E)$ into partial waves, we obtain uncoupled equations for all partial waves. The last term in (9b) represents the transfer of the infinite mass particle A; it only contributes for S-wave scattering. Note that similar BSA's can be made in the Amado equations, The resulting equations differ slightly from Eqs. (9a) and (9b) because the approximation is made for the propagator $\tau_2 (\tau_2 = g_2^{-1} T^{(2)} g_2^{-1}$, where $T^{(2)}$ is the two-body T matrix for pair 2) and not for the Green's function. For the three-nucleon system it has been shown in the context of dispersion relations[16] that continuum terms are necessary so that the BSA or pole approximation to the propagator is not satisfactory. Similar conclusions have been obtained for the four-nucleon system[17] using a simple generalized Amado model.

Instead of the set Eq. (9) one can also use wave function equations, although these are numerically less convenient. The DWBA results quoted in the next section were determined[8] using exact distorted waves (i.e., the projection of the exact elastic wave functions onto the ground state) in the computation of the DWBA-matrix element of stripping. In normal reaction calculations exact elastic distorted waves are not available, and the elastic wave functions are generated by phenomenological optical potentials. In a similar fashion Shanley and Aaron[6] compute DWBA-matrix elements using Woods-Saxon-type optical potentials fitted to the exact cross sections.

III. RESULTS

In Figs. 1–4 we show the results for the elastic cross sections. The differential cross section has a very simple structure due to the absence of strong surface absorption in the model. The BSA calculations are very good in the forward direction, but become increasingly bad for higher energies and larger angles. Since the difference of the exact and BSA calculations reflects the importance of the continuum, we can conclude that the role of the continuum is very important except for the forward region.

The good agreement of BSA and exact calculations for the forward direction requires further explanation. In lowest order, the exact elastic scattering amplitude is identical to the plane-wave impulse approximation (PWIA) given by

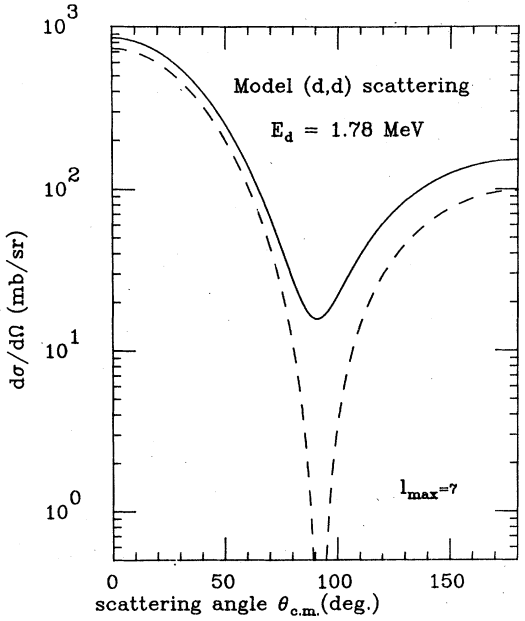

FIG. 1. Elastic differential cross section for $d+A$ scattering at 1.78 MeV. The maximum number of partial waves included in the calculation (l_{max}) is 7. ——exact; ----- BSA.

$$\langle \vec{k}'\phi_1 | t^{(2)}(E - k_p) | \phi_1 \vec{k} \rangle,$$

where k_p is the proton kinetic energy operator. Because of the separable form of the potential the two-body T matrix factorizes; hence the PWIA

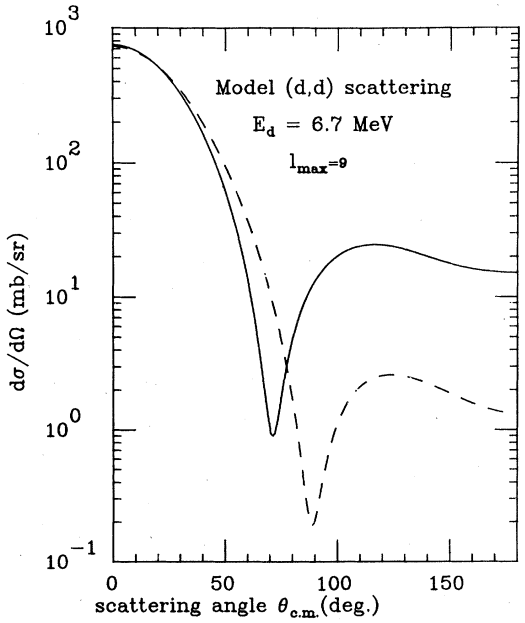

FIG. 2. Same as Fig. 1 with E_d = 6.7 MeV.

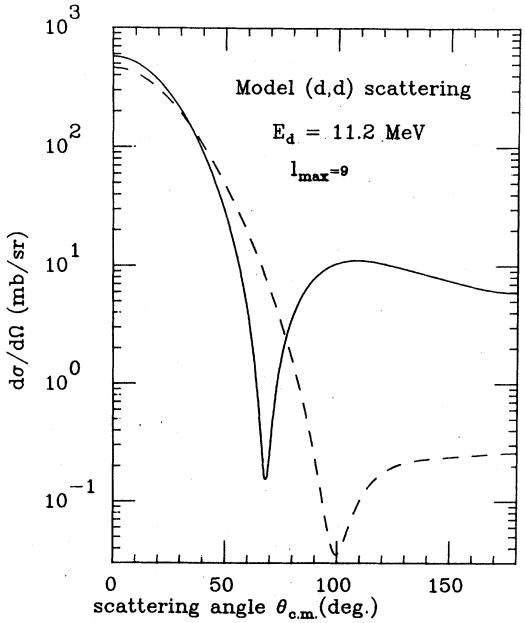

FIG. 3. Same as Fig. 1 with E_d = 11.2 MeV.

graph can be represented by the diagram in Fig. 5. The S-wave nucleon-nucleon and nucleon-nucleus wave functions both peak at zero relative momentum. This knowledge is essential for explaining the gross features of the cross sections. First we consider forward scattering, i.e., $\vec{k} = \vec{k}'$. The wave function part of the integrand in Eq. (9a) has the form

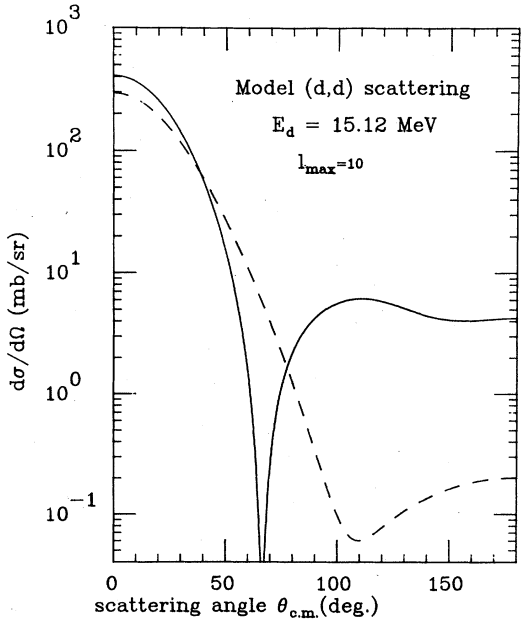

FIG. 4. Same as Fig. 1 with E_d = 15.12 MeV.

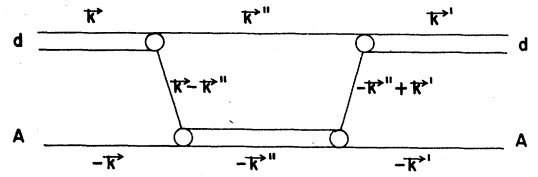

FIG. 5. Diagrammatic representation of the lowest-order elastic $d+A$ graph.

$$g_1(\vec{k}/2-\vec{k}'')\phi_2(\vec{k}-\vec{k}'')g_2(\vec{k}-\vec{k}'')\phi_1(\vec{k}/2-\vec{k}'').$$

This function peaks for $\vec{k}''=\vec{k}^{\max}\gtrsim\vec{k}/2$. Quantitatively we find that γ in $\vec{k}^{\max}=\gamma\vec{k}$ takes on the values 0.64, 0.63, 0.62, and 0.61 at the four energies (in increasing order). We expect that the distance of this $\vec{k}^{\max}$ from the on-shell value of $\vec{k}$, given by $k^{\text{on}}=[2(E+\epsilon_2)]^{1/2}$, is a good quantitative measure for the correctness of the BSA, since this approximation is exact near the bound-state pole $k^2=2(E+\epsilon_2)$. Therefore we have calculated the expression $\Delta E=E+\epsilon_2-[(k^{\max})^2/2]$. As E runs through the four bombarding energies, ΔE takes the following values: 1.39, 4.14, 6.26, and 8.09 MeV [see Fig. 6(a)]. Clearly, when we increase the energy of the deuteron, the importance of the on-shell region becomes smaller, and thus we expect the BSA to become worse. We also see that the deterioration of the BSA with increasing energy is *not* due to the necessity that bound-state scattering only can proceed via high-momentum components in the nucleon-nucleus wave function; the product of the form factors $g_2(\vec{k}''-\vec{k})g_2(\vec{k}''-\vec{k}')$ which carries the momentum dependence of the wave function also occurs in the continuum contribution. The decrease in importance of bound-state scattering for higher energies can be explained by the fact that the nucleons in the deuteron prefer to leave the deuteron with equal amounts of momentum, which means that only part of the original kinetic energy is carried away by the nucleon. The remaining energy is converted into the internal energy of the nucleon-nucleus system, which therefore is less likely to be in a bound state.

One can also approach the situation from the opposite direction. We can investigate which momentum components in the deuteron wave function emphasize bound-state scattering. We simply determine the on-shell momentum in the intermediate state $k^{\text{on}}=[2(E+\epsilon_2)]^{1/2}$, and from this the corresponding relative momentum $\vec{q}=\frac{1}{2}\vec{k}-\vec{k}^{\text{on}}$ in the deuteron wave function. Again we can assume $\vec{k}$ and $\vec{k}^{\text{on}}$ to be parallel, and we find for the four energies (in increasing order) $q=0.16$, 0.21, 0.25, and 0.28 fm^{-1} (compared with relative momenta $q=0.06$, 0.10, 0.12, and 0.13 fm^{-1} at $\vec{k}''=\vec{k}^{\max}$).

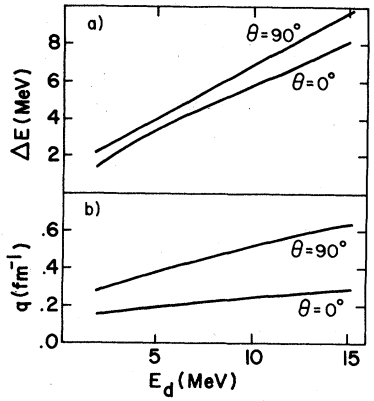

FIG. 6. ΔE and q are measures of the importance of the on-shell region (where the BSA is exact) in the integral for the PWIA graph in Fig. 5. ΔE is the difference of two kinetic energies: one corresponding to a maximalization of the integrand ($k''=k^{\max}$), and one corresponding to the on-shell region ($k''=k^{\text{on}}$). q is the momentum of the deuteron wave function corresponding to the on-energy-shell intermediate state.

This analysis also shows that bound-state scattering becomes less likely with increasing energy as higher-momentum components in the deuteron wave function are involved at higher energy.

Having explained the behavior at forward angles with increasing energy, we will now try to explain the poorness of the BSA at intermediate and backward angles, especially at higher energies. The conditions for $\vec{k}^{\max}$ are slightly different here because $\vec{k}\neq\vec{k}'$. From symmetry arguments we expect that $\vec{k}^{\max}=\gamma(\vec{k}+\vec{k}')/2$, where again $\gamma\gtrsim\frac{1}{2}$. Setting $\theta=90°$ we find that the distance to the bound-state pole $\Delta E=E+\epsilon_2-[(k^{\max})^2/2]$ is now 2.1, 5.0, 7.6, and 9.8 MeV, respectively [see Fig. 6(b)]. These values are considerably larger than the corresponding values at forward angles. We can again invert the argument and determine the relative momenta in the deuteron wave function which emphasize bound-state scattering in the intermediate state. We find $q=0.27$, 0.43, 0.55, and 0.63 fm^{-1} (compared to $q=0.15$, 0.30, 0.38, and 0.45 fm^{-1} at $\vec{k}=\vec{k}^{\max}$). Both analyses show that at $\theta=90°$ scattering via the two-body intermediate state is less likely than at forward angles. In more physical terms we can characterize the situation at 90° as follows: Only high-momentum components in the deuteron wave function lead to bound-state scattering, and since the nucleons prefer to emerge from the deuteron with equal amounts of momentum, bound-state scattering is less likely to happen than at forward angles. Most of the time the intermediate nucleon-nucleus system is expected to have a fairly large internal energy because the nucleon absorbs only part of the kinetic energy

carried by the original deuteron (even less than in the forward case), whereas the nucleon-nucleus system cannot absorb any kinetic energy at all.

The explanation for the poorness of the BSA at large angles was based on the increase in ΔE on going from $\theta = 0$ to 90°, or on the increase of the momentum transfer q—corresponding to on-shell scattering—from $\theta = 0°$ to 90°. Quantitatively we find that the increase in q is of the order of 70% for $E_d = 1.78$ MeV and more than 100% for the other energies. We thus expect that the scattering at larger angles will be better represented by the BSA at $E_d = 1.78$ MeV than at the other energies. This is indeed what is observed in the cross sections Figs. 1–4.

One may question whether the momentum-energy considerations also will apply in the usual scattering situation where there is a lot of surface absorption by the nucleus (Shanley and Aaron[6] indicate that such absorption is weak in the present reaction model). Since surface absorption is presumably related to the existence of many different reaction channels in the usual nuclear reaction process, one usually relies on effective optical potentials to describe the absorption. Because the effective potentials are manifestations of the complex many-body structure of the system, it is unclear how they should be used in a three-body coupled channel model. If one circumvents the many-body nature ($n \geq 4$) of the problem, one can introduce optical potentials in a straightforward way, but then one observes a large sensitivity to this potential if one makes a bound-state approximation.[9] Pending an exact calculation of continuum effects of more realistic scattering processes, we will have to rely on the understanding obtained for the simpler reaction model. Therefore it is reasonable to assume that in general the BSA is a fair approximation for the forward direction, but becomes poor for the energies above the breakup threshold and at larger angles. These limitations should be kept in mind in calculating multistep effects via the BSA, as is done for example in the CRC (coupled reaction channel) approach. In a previous paper we have shown that multistep effects (i.e., higher-order rearrangement effects) are unrealistically small if calculated within the BSA.[9] Another question is whether the previous momentum considerations also apply for non-S-wave transfer. In this case higher-momentum components are important in the bound-state wave function, so that the situation might be slightly more favorable for the BSA.

The stripping cross sections are shown in Figs. 7–10. Part of these results have already been discussed in a previous letter.[18] The forward peaking in this cross section can be attributed to the nature

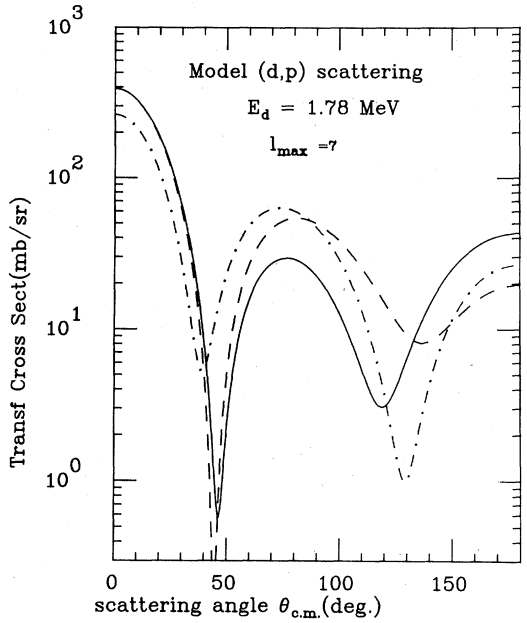

FIG. 7. Differential cross section for the stripping process $A(d,p)B$ at 1.78 MeV. ——exact; -----BSA; -·-·-DWBA.

of the first-order exchange term in Eq. (9b) which has the energy denominator $[(-\vec{k}_2 + \frac{1}{2}\vec{k}_1)^2 + \epsilon_1]^{-1}$. However, we also know that the plane-wave Born approximation, which only takes into account this pole term, strongly overestimates the forward peak.[6,8] In standard DWBA analyses of stripping

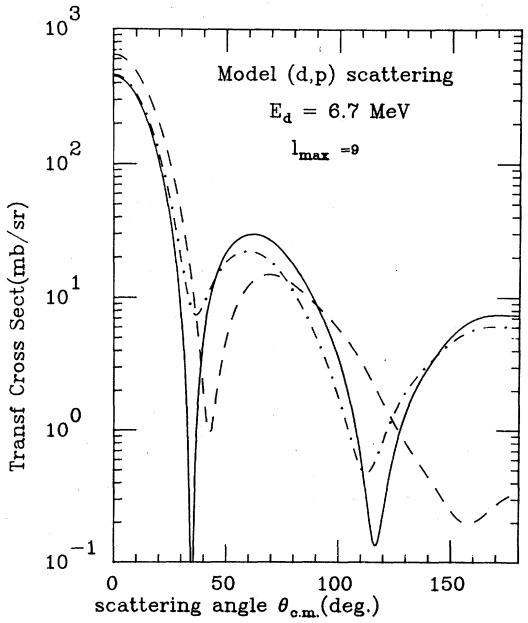

FIG. 8. Same as Fig. 7 for $E_d = 6.7$ MeV.

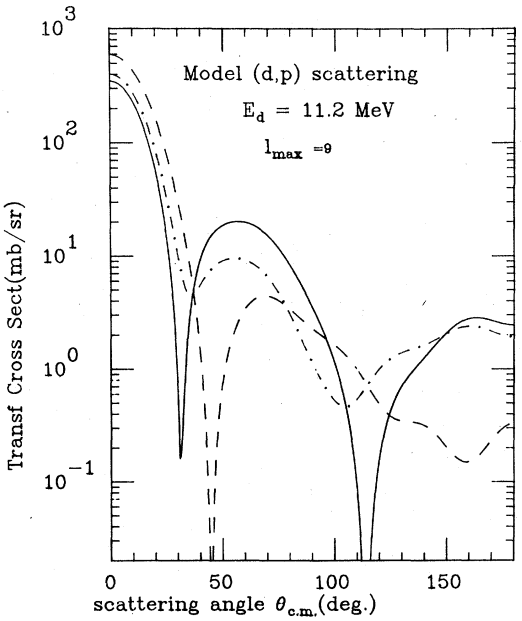

FIG. 9. Same as Fig. 7 for E_d+11.2 MeV.

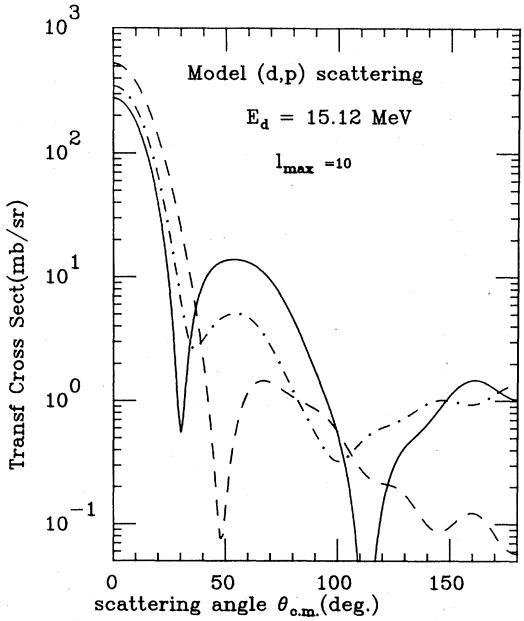

FIG. 10. Same as Fig. 7 for E_d = 15.12 MeV.

a reduction of this peak is accomplished by using distorted waves instead of plane waves. Our results at forward angles indicate that this reduction can also be accomplished by the BSA rescattering terms in Eq. (9b). Despite this improvement over the PWBA, the disagreement between BSA and exact results grows considerably with increasing energy, reaching a high of 90% at 15.12 MeV, compared to 3% at the lowest energy. The "exact" DWBA performs better at higher energies, the discrepancy at 15.12 MeV being only 25%. For comparison: The PWBA overshoots[8] the exact forward peak by a factor of 6 at this energy. The BSA and DWBA are also able to reproduce some of the qualitative features of the cross section at higher angles, especially for the lower energies. Qualitatively we can conclude that multistep effects (absent in the DWBA) and continuum effects (absent in the BSA) become increasingly important with higher energies. More quantitative conclusions are possible if we analyze the results in terms of unitarity defects arising from bound-state rearrangement and continuum rescattering.

Below the breakup threshold ($E_d < 2.225$ MeV) the scattering amplitudes satisfy two-particle unitarity exactly,[6] i.e.,

$$\mathrm{Im}A^l_{11} = -\pi\rho_1 |A^l_{11}|^2 - 2\pi\rho_2 A^l_{12} A^{*l}_{21} , \tag{10}$$

where the ρ_i are phase space factors equal to $k_i m_i \theta(E+\epsilon_i)$, and the factor 2 arises from the presence of two identical nucleon-nucleus channels. With the usual representation of the elastic amplitude by

$$A^l_{11}(E) = (2\pi i\rho_1)^{-1}(1-\eta_l e^{2i\delta l}) , \tag{11}$$

one can characterize the flux loss out of this channel by the reduction of η_l from its elastic value $\eta_l = 1$. From Eqs. (10) and (11) η_l can be related to the amplitude A_{12} by means of the transmission coefficient

$$1-\eta_l^2 = 8\pi^2\rho_1\rho_2 |A_{12}|^2 . \tag{12}$$

Equation (12) is exact below the breakup threshold; above it we use (12) to define a new parameter $\tilde{\eta}_l$, with $\eta_l < \tilde{\eta}_l$, since η_l includes the flux loss to the continuum not present in $\tilde{\eta}_l$.

In Table I we list values of η_l for $0 \leq l \leq 5$ for all the energies considered. For $E_d = 1.78$ MeV the deviation of η_l and $\tilde{\eta}_l$ merely provides a check on the accuracy of the exact calculations, and therefore allows one to determine the significance of the deviations of η_l and $\tilde{\eta}_l$ in other cases. The accuracy is better than 1% in all cases. In the BSA there is no continuum, so that (10) and (11) should hold for all energies. However, the BSA leads to a small breaking of time reversal invariance (i.e., $T_{12} \neq T_{21}$) which in turn is reflected in a difference between η_l and $\tilde{\eta}_l$. In the exact calculation the reduction of η_l from 1 represents the overall absorption (continuum and stripping), whereas the reduction of $\tilde{\eta}_l$ from 1 represents the absorption due to stripping alone. The numbers $\zeta = \sigma_{abs}/\sigma_{transf}$ listed in the same table are an overall measure of the

TABLE I. Results for absorption coefficients: $\zeta = \sigma_{abs}/\sigma_{transf}$.

	$E_d = 1.78$				$E_d = 6.7$				$E_d = 11.2$				$E_d = 15.12$			
	$\zeta = 1.00$		$\zeta^{BSA} = 1.00$		$\zeta = 1.09$		$\zeta^{BSA} = 0.97$		$\zeta = 1.32$		$\zeta^{BSA} = 0.96$		$\zeta = 1.47$		$\zeta^{BSA} = 0.97$	
l	η_l	$\tilde{\eta}_l$	η_l^{BSA}	$\tilde{\eta}_l^{BSA}$	η_l	$\tilde{\eta}_l$	η_l^{BSA}	$\tilde{\eta}_l^{BSA}$	η_l	$\tilde{\eta}_l$	η_l^{BSA}	$\tilde{\eta}_l^{BSA}$	η_l	$\tilde{\eta}_l$	η_l^{BSA}	$\tilde{\eta}_l^{BSA}$
0	0.859	0.865	0.20	0.37	0.66	0.70	0.83	0.85	0.47	0.61	1.00	1.00	0.35	0.61	0.89	0.90
1	0.995	0.999	0.97	0.96	0.97	0.97	0.80	0.76	0.93	0.94	0.59	0.52	0.88	0.92	0.40	0.28
2	0.641	0.639	0.58	0.57	0.05	0.34	0.16	$0.09i$	0.16	0.56	0.20	0.05	0.27	0.71	0.30	0.23
3	0.960	0.960	0.958	0.957	0.74	0.76	0.75	0.75	0.62	0.72	0.71	0.71	0.60	0.74	0.72	0.71
4	0.995	0.995	0.995	0.995	0.935	0.940	0.939	0.939	0.88	0.91	0.909	0.909	0.860	0.901	0.902	0.901
5	0.999	0.999	0.999	0.999	0.984	0.985	0.985	0.985	0.963	0.972	0.972	0.972	0.952	0.966	0.967	0.967

importance of continuum effects for the exact calculations, whereas they are an overall measure of the breaking of time-reversal invariance in the BSA.

With the exception of a few incidental cases the breaking of time-reversal invariance in the BSA as reflected in the difference between η_l and $\tilde{\eta}_l$ is quite small. Most absorption is found in the $l=0$ and $l=2$ waves. The absorption for $l=1$ in the BSA at higher energies is much larger than in the exact case despite the absence of the continuum.

From these results we draw the following conclusions. First, and not unexpectedly, the continuum effects become increasingly important with higher energies. For energies close to breakup (i.e., 6.7 MeV) the contribution of real breakup (proportional to $\eta_l - \tilde{\eta}_l$) is quite small. Intermediate breakup effects, however, are already quite important as the BSA is not very good at 6.7 MeV. For 11.2 and 15.12 MeV the contribution of the real continuum is quite large. This means for example that a dispersion calculation based on two-body unitarity [Eq. (10)] would still be successful for $E_d = 6.7$, but would probably be poor for higher energies. The continuum effects being so important at higher energies, we do not expect that the BSA is a good approximation at those energies. This is confirmed by Figs. 9 and 10. We still can get a lot of absorption in the BSA (cf. $l=1$); apparently the absence of the continuum in the BSA is made up partly by a stronger effective bound-state coupling so that the loss of flux to the p-nucleus channel is considerable. As the DWBA calculation provides better fits for higher energies than the BSA, we conclude that continuum effects are more important than the BSA-rescattering corrections at these energies.

Although an optical potential may reproduce part of the absorption of the continuum we see that even with the best (i.e., exact) choice for the distorted wave one does not obtain very good agreement to the (exact) cross section at higher angles and energies. This should caution experimentalists in trusting their DWBA beyond the forward scattering region.

IV. SUMMARY

We have shown within a three-body reaction model that a large portion of the coupling of different arrangement channels occurs via the continuum through both virtual and real breakup, especially at higher energies and larger angles. Thus if one wants to understand multistep corrections for a large angular range one should not resort to the bound-state approximation (BSA). On the other hand, if one wants to use a three-body

model in order to include the continuum, then the introduction of phenomenological optical potentials leads to various problems. In particular, one has to account for the fact that the coupling effects are large, so that a considerable renormalization of the phenomenological optical potential seems necessary.[19] In practice this requires performing a new search for optical model parameters in a strong coupling theory wherein the continuum provides the strong coupling between different arrangement channels. The agreement of the BSA with exact results in the forward direction is gratifying because we know that the PWBA is a very poor approximation at forward angles. Clearly the rescattering terms in the BSA give a significant and realistic reduction of the forward peak present in the PWBA. As the forward region provides important nuclear structure information (spectroscopic factors) the BSA appears to be a useful tool in this region.

This work was supported in part by the National Science and Engineering Research Council of Canada and the U. S. Department of Energy.

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