Communications

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Orthogonality constraints and proton-induced reactions

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(Received 21 March 1979)

A simple method of imposing orthogonality constraints upon the relevant bound and continuum waves involved in the calculation of (p,π^+) and other proton induced reactions is presented.

[NUCLEAR REACTIONS Orthogonality relations for optical model and bound nuclear wave functions.]

Continuum proton (neutron) wave functions which are solutions of an optical-model Hamiltonian are generally not orthogonal to the bound proton (neutron) wave functions having the same quantum numbers. This problem has received much attention recently[1,2] and is still an outstanding problem.

One may consider the calculation of such reactions as (p,π^+), (p,γ) and (p,d) among others. For concreteness let us focus on the reaction $^{12}C(p,\pi)^{13}C$. We consider the simplest case where pion emission proceeds via a single nucleon process. The proton, upon emitting a pion, becomes a neutron which is captured into the ith bound particle state of ^{13}C. If the impinging proton were not to feel a Coulomb force and if charge independence were assumed, the proton distorted wave, in principle, would have to be orthogonal to the bound wave function of the neutron. In practice this constraint is violated since the proton wave is taken to be an eigenstate of a complex energy-dependent optical-model Hamiltonian with an energy argument that lies in the continuum. The neutron bound-state wave function, however, may be considered to be an eigenstate of a generalized (real) effective Hamiltonian. (One model for such energy-dependent Hamiltonians may be obtained from the projection operation reaction theory of Feshbach.[3]) Since two different Hamiltonians are used to define the continuum and bound waves, it is clear that these are not orthogonal in general.

Similar considerations apply for other reactions. For example, if we consider the direct capture process (p,γ) the proton wave which would be obtained from an optical-model calculation would not generally be orthogonal to proton bound states with the same quantum numbers.

The method we suggest below which has been outlined and applied previously to other problems[4] yields wave functions which have the correct orthogonality properties and also the correct phase shifts. Specifically, the optical proton wave function will be orthogonal to the proton bound states.

Let $|\phi_i\rangle$ represent the bound-state proton wave functions having energy ϵ_i obtained from an appropriate model Hamiltonian. (For example, $|\phi_i\rangle$ may be the bound-state eigenfunctions of a Woods-Saxon potential.) Define projection operators

$$P_i = |\phi_i\rangle\langle\phi_i| \tag{1}$$

and

$$P = \sum_i P_i . \tag{2}$$

The proton continuum wave $|\psi_{\vec{k}}^{(+)}(\lambda)\rangle$ satisfies the equation

$$H_{\rm opt}(\lambda, \epsilon_{\vec{k}})\,|\psi_{\vec{k}}^{(+)}(\lambda)\rangle = \epsilon_{\vec{k}}\,|\psi_{\vec{k}}^{(+)}(\lambda)\rangle, \tag{3}$$

where $\epsilon_{\vec{k}} = k^2/2\mu$. Here λ stands for the strengths and ranges of the phenomenological potentials used in the construction of $H_{\rm opt}(\lambda,\epsilon_{\vec{k}})$. Let us agree that λ represents the set of parameters which produce the best fit to the elastic data. It should be clear that in general

$$\langle\phi_i|\psi_{\vec{k}}^{(+)}(\lambda)\rangle \neq 0. \tag{4}$$

We now outline a procedure which may be used to obtain continuum wave functions that are orthogonal to the bound states. We introduce a new optical model Hamiltonian

$$\hat{H}(\lambda,\epsilon)=\sum_i \epsilon_i P_i + (1-P)H_{\rm opt}(\lambda,\epsilon)(1-P). \qquad (5)$$

The scattering states of $\hat{H}(\lambda,\epsilon)$ will be clearly orthogonal to the $|\phi_i\rangle$. The new scattering states $|\hat{\psi}^{(+)}_{\vec{k}}(\lambda)\rangle$ satisfy

$$\hat{H}(\lambda,\epsilon_{\vec{k}})|\hat{\psi}^{(+)}_{\vec{k}}(\lambda)\rangle=\epsilon_{\vec{k}}|\hat{\psi}^{(+)}_{\vec{k}}(\lambda)\rangle, \qquad (6)$$

or

$$(\epsilon_{\vec{k}}-H_{\rm opt}(\lambda,\epsilon_{\vec{k}}))|\hat{\psi}^{(+)}_{\vec{k}}(\lambda)\rangle = -\sum_i |\phi_i\rangle\langle\phi_i|H_{\rm opt}(\lambda,\epsilon_{\vec{k}})|\hat{\psi}^{(+)}_{\vec{k}}(\lambda)\rangle. \qquad (7)$$

Equation (7) may be solved easily to yield

$$|\hat{\psi}^{(+)}_{\vec{k}}(\lambda)\rangle=|\psi^{(+)}_{\vec{k}}(\lambda)\rangle - G^{(+)}_{\rm opt}(\lambda,\epsilon_{\vec{k}})PM^{-1}(\lambda,\epsilon_{\vec{k}})P|\psi^{(+)}_{\vec{k}}(\lambda)\rangle, \qquad (8)$$

where

$$G^{(+)}_{\rm opt}(\lambda,\epsilon)=[\epsilon-H_{\rm opt}(\lambda,\epsilon)+i\eta]^{-1} \qquad (9)$$

and the matrix M is defined such that

$$\langle\phi_i|M|\phi_j\rangle\equiv\langle\phi_i|G^{(+)}_{\rm opt}(\lambda,\epsilon)|\phi_j\rangle. \qquad (10)$$

We now note that the scattering amplitude obtained from $\hat{H}(\lambda,\epsilon_{\vec{k}})$ may no longer fit the elastic data, but for any choice of λ we have

$$\langle\phi_i|\hat{\psi}^{(+)}_{\vec{k}}(\lambda)\rangle=0. \qquad (11)$$

We are now free to readjust the parameter set λ such that the $|\hat{\psi}^{(+)}_{\vec{k}}(\lambda)\rangle$ have the correct asymptotic behavior. We may denote the new parameter set as $\hat{\lambda}$ and the new wave functions as $|\hat{\psi}^{(+)}_{\vec{k}}(\hat{\lambda})\rangle$.

In conclusion, we see that we may obtain continuum wave functions $|\hat{\psi}^{(+)}_{\vec{k}}(\hat{\lambda})\rangle$ which are solutions of $\hat{H}(\hat{\lambda},\epsilon)$ and which have the desired property,

$$\langle\phi_i|\hat{\psi}^{(+)}_{\vec{k}}(\hat{\lambda})\rangle=0. \qquad (12)$$

These techniques may be extended to apply to wave functions obtained from relativistic optical-model Hamiltonians if the appropriate relativistic models are also used to obtain the bound-state wave functions. This method also has applications to other problems such as the proper description of (γ,p), $(e,e'p)$, and stripping reactions.

This work was supported in part by the National Science Foundation and the PSC-BHE Award Program of the City University of New York.

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