

Translationally invariant spectral function for ${}^6\text{Li}$

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A spectral energy function for the most tightly bound protons in the ${}^6\text{Li}$ ground state is calculated. A highly collective, or correlated motion wave function utilizing the translationally invariant hyperspherical harmonic formalism is used rather than an independent particle shell model wave function. A local energy approximation is made resulting in a calculated full width at half maximum of 10 MeV for the spectral function. The mean removal energy calculated is 21 MeV. These numbers generally agree with experimentally deduced values.

[NUCLEAR STRUCTURE; ${}^6\text{Li}$, K harmonics, Spectral Function, independent particle energies.]

I. INTRODUCTION

We desire to calculate the spectral energy function for a proton bound in the most tightly bound shell in the ${}^6\text{Li}$ ground state. The spectral function can be measured from high energy (e, ep)¹ or ($p, 2p$),² ($d, {}^3\text{He}$), (p, d) reactions assuming a one step impulse approximation is valid. The ($d, {}^3\text{He}$) and (p, d) reactions have small cross sections³⁻⁷ due to the absorption of particle flux that occurs before and after the reaction takes place. There is also a momentum mismatch that occurs in these reactions that also reduces these cross sections.

The spectral function can be measured experimentally^{1-3,8,9} in knock-out experiments such as ($p, 2p$), or (e, ep), see Fig. 1. One measures the spectrum of the final state particles at fixed scattering angles using a beam of definite fixed energy. Then assuming the only energy transfer occurring to take place between the incident projectile and the knocked out proton, there is a one to one correlation between the spectrum of the final state particles and the spectral energy function of the proton within the target nucleus. Assuming an impulse reaction mechanism, the cross section for (e, ep) or ($p, 2p$) reactions can be written as a product of an appropriate two body scattering cross section at a particular energy and angle, times the spectral function for the appropriate target nucleus. We now assume the incident projectile is a proton.

Thus, the spectral function can be experimentally determined^{1,11,12} (measured) from the experimental cross section as

$$S(\vec{P}, W) = \frac{d^3\sigma}{d\Omega_1 d\Omega_2 dE_1} P_0/q_2 q_1 T^2, \quad (1)$$

where

$$T = g_{LJ} \langle \frac{1}{2}(\vec{q}_2 - \vec{q}) | t_{pp} | \frac{1}{2}(\vec{P}_0 + \vec{Q}) \rangle, \quad (2)$$

$$g_{LJ} = \int \chi^{-*}(K_2 r) \chi^{-*}(K_1 r) \phi_{LJ}(r) \chi^+(K_0, ar) d\vec{r}. \quad (3)$$

Here $\vec{Q} = \vec{P} - \vec{q}_2 - \vec{q}_1$, is the recoil momentum of the residual nucleus; $\frac{1}{2}(\vec{P}_0 + \vec{Q})$ and $\frac{1}{2}(\vec{q}_2 - \vec{q})$ are the relative momenta of the protons in their common center of mass system before and after scattering. K_0 , K_1 , and K_2 are the initial and final proton momenta in the $A+1$ body system. The spectral function measured depends on the validity of the impulse approximation and on the calculation of the off-shell two body t matrix T . The factor g_{LJ} depends on knowledge of the correct in-coming and out-going distorted scattering wave functions χ^\pm , as well as the bound state proton wave function within the target nucleus. The bound state wave function is usually^{1,2,11} taken as eigenfunction for a Woods-Saxon potential fitted to an assumed binding energy. The scattering wave functions are taken as eigenfunctions^{2,13} to standard Woods-Saxon optical model potentials. One must also calculate the off-shell two nucleon t matrix in Eq. (2) to determine $S(P, W)$ experimentally. Factorization of the knock-out cross section in terms of the pp cross section does not occur¹⁰ in the presence of spin orbit nucleon forces, so these must be neglected here. Then, it has been shown^{14,15} that an accurate procedure to estimate the off-shell t matrix (squared), is to use the experimental (on-shell) cross section in the final energy prescription¹⁴ times a calculated ratio (squared) of the off-shell t matrix divided by the on-shell t matrix determined by a potential model. This ratio has been shown to be almost potential model independent, and therefore

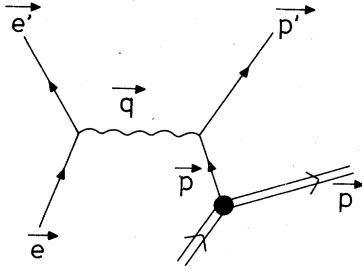


FIG. 1. The quasifree one step ($e, e' p$) reaction.

an appropriate way to describe the off-shell behavior of the required t matrix. Thus, subject to experimental errors, uncertainties in the scattering, and bound state wave functions, and the validity of the impulse approximation, one can experimentally measure the spectral function for a bound proton in a nucleus. As such, the spectral energy function is a very interesting function.

The spectral energy function tells the probability that a nucleon in a given quantum state in a nucleus has a particular bound energy. For a two body system, the spectral energy function has a δ function energy dependence. For systems of three or more interacting nucleons, the spectral function for a given nucleon is continuous smooth (non- δ) function of energy. The spectral energy function of bound protons within a nucleus is also a very interesting quantity to calculate, as it provides a separate and distinct test for two nucleon potentials appropriate for bound states. This test is separate and distinct from calculations¹⁶ of the spectrum of bound states for a nucleus.

The simple shell model predicts a spectral energy function defined as

$$S_w(E) = \int \psi^* \delta(E - H_w) \psi d\tau, \quad (4)$$

where ψ is a Slater determinant product wave function and $d\tau$ signifies integration over $3A$ spin, isospin, and space coordinates. A is the number of nucleons in the nucleus. The subscript w denotes the set of quantum numbers (n, j , parity) of the bound nucleon using normal shell model nomenclature. When calculating the spectral function the simplest shell model can be improved on.^{12, 17-19} However, the simplest shell model takes the Hamiltonian for the knocked-out proton to be a one body operator such as

$$H_{w_1}^{(1)} = \frac{-\hbar^2}{2m} \nabla_{r_1}^2 + \sum_{i=2}^A \int V(r_{1i}) \Phi_{w_i}^2(r_i) d\vec{r}_i \quad (5)$$

or

$$H_w = \frac{-\hbar^2}{2m} \nabla^2 + U_w(r), \quad (6)$$

and where

$$H_w \Phi_w = E_w \Phi_w. \quad (7)$$

Here the total Hamiltonian of the nucleus would be

$$H = \sum_i H_{w_i}, \quad (8)$$

where

$$\psi = \det |\Phi_{w_i}(r_i)| / (A!)^{1/2}, \quad (9)$$

and Φ_w are the one body wave functions or orbitals of the shell model.

This model obtains for the spectral function

$$S_w(E) = n_w \delta(E - E_w), \quad (10)$$

where n_w is the number of protons with the quantum numbers labeled by w . Thus, the spectral energy function has a δ function energy dependence in the simple shell model, just as in the two body system. This energy dependence stems from the Hamiltonian and wave function chosen for the nucleus, see Eqs. (8), (9). Experimentally, shell structure peaks are seen^{3, 11, 20-33} in the spectral energy function, but not of the zero width variety, as predicted by the simple shell model. Large widths are deduced for the deeply bound protons.

One can go beyond the simple shell model by using a better Hamiltonian, such as a Hamiltonian that includes two body forces, for example:

$$H_2 = \frac{-\hbar^2}{2m} \sum_{i=1}^A \nabla_{r_i}^2 + \sum_{i < j} V_{ij}(r_{ij}). \quad (11)$$

One can calculate the spectral energy function using this Hamiltonian and a shell model wave function. The resulting spectral energy function will not have a δ function energy dependence if A is greater than 2. This occurs because the product shell model wave function is not an eigenfunction of the operator inside the δ function of Eq. (4). The widths of the spectral energy function peaks calculated are smaller than experiment.

For ${}^{12}\text{C}$ and ${}^{16}\text{O}$, the effects of residual interactions in the independent particle shell model spectral function have been envisioned by Mougey²⁵ and studied by Gross and Lipperheide,¹² Wille, Gross, and Lipperheide,¹⁷ Wille and Lipperheide,¹⁸ and by Faessler, Kusuno, and Strobel.¹⁹ There it was difficult to calculate a spectral function energy dependence that agreed with experiment partially because the direct coupling from a residual interaction connecting continuum states to the bound states had to be neglected. One particle-two

hole and two particle-three hole configurations relative to the target wave function were included in addition to the one hole configuration. Later target correlations and core or residual nucleus correlations both were considered. Best agreement with experiment was obtained³⁴⁻³⁵ for a ^{12}C target, by adjusting the $s_{1/2}$ single particle binding energy (to 37.3 MeV) to more nearly represent the mean removal energy. All these calculations must overcome the uncertainties in the residual interaction to be used. Its parameters are generally determined by fitting excitation energies of states that do not involve deep lying holes.

For deep lying hole states in highly excited residual nuclei, the hole state is not an eigenfunction of the residual nucleus. The single particle energies of the independent particle shell for these states are perhaps not well determined from other considerations. For instance, finding the $s_{1/2}$ eigenenergy for a Woods-Saxon potential parametrized to reproduce the sd shell may not be all that appropriate a procedure to follow. Also the residual interaction deduced from low excited states may require modification for states involving deep holes, and may in fact be "large" rather than weak. We therefore seek to avoid the calculation of eigenfunctions for deep lying hole states.

Following Lipperheide,³⁵ we write the spectral function as

$$S(\vec{P}, W) = \sum_{\nu\nu'} \psi_{\nu'}^*(\vec{p}) S_{\nu\nu'}(W) \psi_{\nu}(\vec{p}), \quad (12)$$

where

$$S_{\nu\nu'}(W) = \langle \psi | a_{\nu'}^* \delta(W + H - W_A) a_{\nu} | \psi \rangle. \quad (13)$$

We are here willing to approximate the $\psi_{\nu}(\vec{p})$ as the ^6Li bound state independent particle shell model eigenfunctions in momentum space. But in evaluating $S_{\nu\nu'}(W)$, we will not make the independent particle shell model with residual interaction approximation which assumes independent particle binding energies. Instead, we use directly a translationally invariant wave function determined by solving the 6 nucleon bound state Schrödinger equation with the center of mass motion properly treated by using hyperspherical coordinates. In ^6Li we only solve for a diagonal $S_{\nu\nu'}(W)$, that is for $\nu' = \nu = 0s_{1/2}$ state with certain approximations, but we do avoid the independent particle shell model approximation. Note that single particle refers to the discrete quantum numbers of a given nucleon bound state, and is different from the independent particle implications. By independent particle, we mean an independent particle motion shell model solution to the bound

nucleon problem, possibly with residual interactions, in addition to an assumed set of single particle discrete quantum numbers. We use a translationally invariant wave function for a ^6Li target. Thus, we avoid the assumption of averaged single particle energies inherent in the shell model and calculate a spectral energy function without a δ functioned energy dependence. The δ functioned energy dependence of the spectral function is not an approximation for a deuteron, or two body target nucleus, where proper treatment of the center of mass motion requires 2.22 MeV of energy to be transferred into the deuteron to get a proton out. The δ function energy dependence of the spectral function had to be relaxed by Mougey²⁵ to approximate the experimental spectral function for deep lying hole states in ^{12}C , ^{28}Si , ^{40}Ca , and ^{58}Ni . A similar approximation was made earlier³⁶ for ^{12}C , ^7Li , and ^6Li , where the δ function dependence was replaced by a sum of Gaussians with suitably parametrized widths, strengths, and central energies.

These approximations were utilized to fit the data without doing the extremely taxing job of calculating the energy dependence of the spectral function. The spectral function has been calculated for a ^3He target by Dieperink *et al.*²⁶ There a ^3He translationally invariant ground state wave function was used that was calculated³⁷⁻³⁹ using the Faddeev three body formalism with Reid soft core potentials. This calculated spectral function

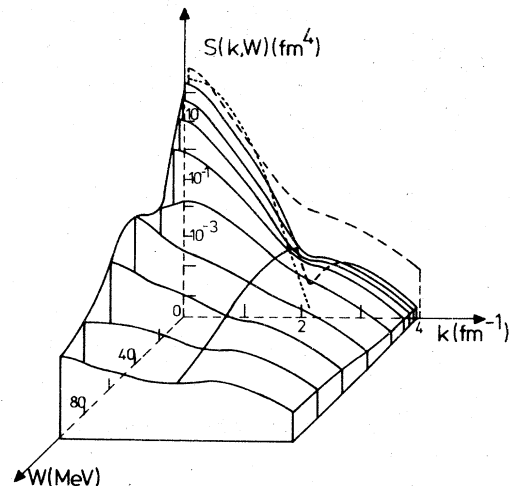


FIG. 2. The ^3He spectral function from Ref. 26 as a function of \vec{k} , the momentum of the residual two nucleon momentum and the separation energy W . The dashed curve corresponds to $d+p$ breakup of ^3He . The dotted curve is a harmonic oscillator model of that two body breakup.

is shown in Fig. 2. Note the logarithmic vertical scale. We see the separation of the energy and momentum dependence into separate factors is approximately valid. Exact spectral function calculations for heavier nuclei have not been available. Here a non- δ function spectral function for ${}^6\text{Li}$ will be calculated.

The rest of this paper is organized as follows. In the next section the details of the hyperspherical harmonic solution are spelled out. Then the calculation of the spectral function is detailed. Finally, the calculated spectral function is compared to experiment and our results are summarized.

II. THEORY AND RESULTS

We will calculate a spectral energy function for $\nu=0s_{1/2}$ protons in the ${}^6\text{Li}$ ground state ($JT=1^0$). We use a six body wave function that has previously been obtained⁴⁰ for this state via a hyperspherical expansion.⁴¹ This wave function has assumed a full s shell of 4 nucleons, and 2 nucleons in the p shell. The two nucleon potential used is a version of the Sprung and de Tourreil supersoft core potential,^{42,43} that is finite everywhere, including the origin. The version used^{42,43} predicts a 4% D state probability for the deuteron. Complete details of the hyperspherical method are available elsewhere.^{40,41,44} The ground state wave function for ${}^6\text{Li}$ can be expanded as

$$\psi = \sum_{ka} \chi_{ka}(\rho) Y_{ka}(\Omega) / \rho^{(3A-4)/2}, \quad (14)$$

where the $Y_{ka}(\Omega)$ are hyperharmonic polynomials. The hyperradius ρ is defined by

$$\rho^2 = \frac{1}{A} \sum_{i < j} (\vec{r}_i - \vec{r}_j)^2, \quad (15)$$

with Ω denoting all the rest (14) of the coordinates. The $\chi_{ka}(\rho)$ satisfy a set of coupled differential equations

$$\begin{aligned} \frac{-\hbar^2}{2m} \left(\frac{d^2}{d\rho^2} \chi_{ka} - \frac{L(L+1)}{\rho^2} \chi_{ka}(\rho) \right) + [W_{ka}^{ka}(\rho) - E] \chi_{ka}(\rho) \\ = - \sum_{k'a' \neq ka} W_{k'a'}^{ka}(\rho) \chi_{k'a'}(\rho), \quad (16) \end{aligned}$$

where $W_{k'a'}^{ka}$ is the hyperspherical angular average of the potential

$$W_{k'a'}^{ka}(\rho) = \int d\Omega Y_{k'a'}^*(\Omega) \sum_{i < j} V_{ij}(r_{ij}) Y_{ka}(\Omega). \quad (17)$$

One assigns a set of quantum numbers $n, l, j, j_\pi T_\pi$ to each nucleon, using notation paralleling the harmonic oscillator shell model, from which one can calculate

$$K = \sum_{i=1}^A (2n+l)_i, \quad (18)$$

and then L is given by

$$L = K + 3(A-2)/2 = K + 6. \quad (19)$$

The configurations considered⁴⁰ have $K=2$, the minimum value for this nucleus. The summand and subscript a , in Eq. (14) denote all the other quantum numbers of the configuration that contribute to the $J^P T=1^0$ ground state. The differential equations, Eq. (16), were solved using bound state boundary conditions and yielded a binding energy of 31.8 MeV and a root mean square radius of 2.2 fm compared to 2.53 fm experimentally. The hyperradial dependence $\chi_{ka}(\rho)$ calculated resembled

$$\chi(\rho) = N \rho^L e^{-\alpha \rho} \quad (20)$$

with a peak at ρ equalling 5 fm. In Eq. (20), α is defined as

$$\alpha^2 = |2mE/\hbar^2|. \quad (21)$$

The calculation⁴⁰ for the ground state wave function ψ has assumed the infinite sum over K , Eq. (14), is dominated by the first terms, those with $K=2$. Terms with higher K were neglected. We assume this approximate wave function is adequate for purposes of calculating the spectral energy function for $l=0$ nucleons, and in what follows drop the ka subscripts on $Y_{ka}(\Omega)$ and $\chi_{ka}(\rho)$.

Now one splits the Hamiltonian into two parts

$$H = A_\nu + B_\nu, \quad (22)$$

where $\langle \psi | A_\nu | \psi \rangle$ is the energy of the nucleon with quantum numbers specified by ν while it is in the ${}^6\text{Li}$ ground state, and $\langle \psi | B_\nu | \psi \rangle$ is the energy of the core (c) of all the rest of the nucleons, not in state ν .

A_ν and B_ν are both 6 body operators. If we could solve

$$A_\nu \phi_n = E_n \phi_n \quad (23)$$

or

$$B_\nu \phi_n = E'_n \phi'_n, \quad (24)$$

one could proceed in a traditional quantum mechanics manner.⁴⁵ That is, if we solve for the eigenfunctions and eigenvalues of A_ν or B_ν , and in what follows we discuss A_ν , preferring to discuss a bound nucleon⁴⁶ to a bound core, then we could calculate

$$E_\nu = \langle \psi | A_\nu | \psi \rangle = \sum_{nm} \langle \psi | \phi_m \rangle \langle \phi_m | A_\nu | \phi_n \rangle \langle \phi_n | \psi \rangle \quad (25)$$

or

$$E_\nu = \sum_n \langle \psi | \phi_n \rangle E_n \langle \phi_n | \psi \rangle. \quad (26)$$

Likewise, the spectral energy function would be

$$S_{\nu\nu}(W) = \langle \psi | \delta(W - A_\nu) | \psi \rangle \quad (27)$$

or

$$S_{\nu\nu}(W) = \sum_n \langle \psi | \phi_n \rangle \delta(W - E_n) \langle \phi_n | \psi \rangle. \quad (28)$$

The difficulty is that the ϕ_n or ϕ'_n are difficult eigenfunctions to obtain in a 6 body frame of coordinates. Also the overlap integrals $\langle \phi_n | \psi \rangle$ are difficult to do if one does not make an independent particle shell model approximation. Instead the utilizing Eqs. (23)–(29) we take

$$\langle \rho \Omega | A_\nu | \psi \rangle = F_\nu(\rho, \Omega) \chi(\rho) Y(\Omega), \quad (29)$$

where $F_\nu(\rho, \Omega)$ is a local function of ρ and of Ω . This leads to two similar methods for determining the spectral energy function, the approximations depending on the order of integration (ρ, Ω). In general using Eq. (29) to evaluate Eq. (27), one must solve

$$W \chi(\rho) Y(\Omega) = F_\nu(\rho, \Omega) \chi(\rho) Y(\Omega). \quad (30)$$

This provides a relationship

$$\rho = \rho_0(\Omega, W) \quad (31)$$

to be used if one integrates over $d\rho$ first, or a relation

$$\Omega = \Omega_0(\rho, W) \quad (32)$$

for 1 of the $3A - 4 = 14$ coordinates specified by Ω , if the integral $d\rho$ is done last.

First integrating over $d\rho$, Eq. (27) becomes

$$S_{\nu\nu}(W) = \int d\Omega \chi^*(\rho_0) Y^*(\Omega) \chi(\rho_0) Y(\Omega) \left/ \left(\frac{\partial F_\nu}{\partial \rho} \right) \right|_{\rho_0}. \quad (33)$$

We ask that $\chi(\rho_0(\Omega, W))$ be smooth enough to approximate Eq. (33) by

$$S_{\nu\nu}(W) \approx \frac{|\chi(\rho_a)|^2}{(\partial F / \partial \rho) |_{\rho_a(W)}}, \quad (34)$$

when ρ_a is a suitably averaged value of $\rho_0(\Omega, W)$, so that ρ_a is a function of W . A suitably averaged value would be found by considering

$$\int d\Omega Y^*(\Omega) W Y(\Omega) \chi(\rho) = \int d\Omega Y^*(\Omega) A_\nu Y(\Omega) \chi(\rho) \quad (35)$$

or

$$W - F_a(\rho) = 0, \quad (36)$$

where

$$F_a(\rho) = \int d\Omega Y^*(\Omega) A_\nu Y(\Omega) \chi(\rho) / \chi(\rho). \quad (37)$$

One has to solve Eq. (36) for ρ as a function of W . These solutions we identify as

$$\rho_a = (F_a^{-1})(W). \quad (38)$$

We note that $\chi(\rho)$, while not a constant, never changes sign so is in some sense smooth. Another way to get Eqs. (34)–(36) is to start from Eq. (27) and assume that A_ν admits to a power series expansion. Then the δ function can be written in exponential form, so Eq. (27) becomes

$$S_{\nu\nu}(W) = \frac{1}{2\pi} \int \chi(\rho) Y^*(\Omega) e^{it(W - A_\nu)} \chi(\rho) Y(\Omega) d\rho d\Omega dt. \quad (39)$$

We separate the W exponential dependence to obtain

$$S_{\nu\nu}(W) = \frac{1}{2\pi} \int d\rho \chi(\rho) \int dt e^{itW} \times \int d\Omega Y^*(\Omega) e^{-itA_\nu} Y(\Omega) \chi(\rho). \quad (40)$$

We now define

$$e^{-itF(\rho)} \chi(\rho) = \int d\Omega Y^*(\Omega) e^{-itA_\nu} Y(\Omega) \chi(\rho). \quad (41)$$

Expanding both sides of Eq. (41) in powers of t and equating coefficients, we see we must have

$$F(\rho) \cong \int d\Omega Y^*(\Omega) A_\nu Y(\Omega) \chi(\rho) / \chi(\rho). \quad (42)$$

Substituting Eq. (41) into (40) we obtain

$$S_{\nu\nu}(W) = \frac{1}{2\pi} \int d\rho \chi(\rho) \int dt e^{itW} e^{-itF(\rho)} \chi(\rho), \quad (43)$$

or

$$S_{\nu\nu}(W) = \int d\rho \chi(\rho) \delta(W - F(\rho)) \chi(\rho), \quad (44)$$

or

$$S_{\nu\nu}(W) = \frac{|\chi(\rho)|^2}{\partial F / \partial \rho |_{\rho_a(W)}}. \quad (45)$$

This assumes implicitly that $(F)^n = \int d\Omega Y^*(\Omega) \times (A_\nu)^n Y(\Omega) \chi(\rho) / \chi(\rho)$, but this is not explicitly used hereinafter. This would be true if Ψ were and eigenfunction of A_ν .

We have written $\rho = \rho_a$ in Eq. (45) because on comparing Eqs. (42)–(44) with Eqs. (36)–(37) we see the solution for the δ function in Eq. (44) are identical to those of Eqs. (36)–(38).

Let us compare Eq. (42) with Eq. (25). Equation (42) is just a hyperspherical angular average, with a $Y_{ka}(\Omega) / \chi(\rho)$ weighting factor, of $A_\nu \psi$. Thus, Eqs. (44) and (45) for the spectral function would be exact if A_ν had a local hyperspherical representation as assumed in Eq. (29), or if Ψ were

an eigenfunction of A_ν . Equation (42) is evaluated in the following way to obtain $F(\rho)$.

$Y(\Omega)$ has,⁴⁰ as a factor, a Slater determinant of single particle functions $\Upsilon^\omega(\gamma)$. The superscript ω labels the single particle quantum numbers. This Slater determinant is expanded in a cofactor expansion of A terms as

$$Y(\Omega) = \sum_{j=1}^6 \Upsilon^{\nu_0}(\vec{r}_j) C_{\nu_0 r_j}, \quad (46)$$

where the cofactor $C_{\nu_0 r_j}$ is itself an $A-1$ by $A-1$ Slater determinant. The cofactor determinant has the set of quantum numbers ν_0 omitted and also the coordinates \vec{r}_j omitted. The coordinates that the cofactor does depend on are written in cyclic order to avoid minus signs appearing in Eq. (46). The expansion Eq. (46) is sufficient for the kinetic

energy (one body part of the Hamiltonian), but the cofactors must themselves be expanded into second cofactors to handle the potential energy terms as

$$C_{\nu_0 r_j} = \sum_{i=1}^5 \Upsilon^{\nu_i}(\vec{r}_j) C_{\nu_0 \nu_i r_j r_j'}. \quad (47)$$

The $C_{\nu_0 \nu_i r_j r_j'}$ is an $A-2$ by $A-2$ Slater determinant that has the states ν_0 and ν_i missing and the coordinates $r_j r_j'$ missing. The remaining coordinates are written in a cyclic ordering.

Substituting Eqs. (46) and (47) into $Y(\Omega)$ and $Y^*(\Omega)$ in Eq. (42) results in an analysis that parallels the shell model reduction⁴⁷ or see also Ref. 40 for the reduction in hyperspherical coordinates. One finally obtains for $F(\rho)$

$$F_{\nu_0}(\rho) = \chi(\rho) \left(T_{\nu_0} + \int D(s, \rho^2) ds \frac{1}{2} \sum_{i=1}^5 \langle T^{\nu_0} T^{\nu_i} | V_{12} | T^{\nu_0} T^{\nu_i} - T^{\nu_i} T^{\nu_0} \rangle \right) \chi(\rho) / |\chi(\rho)|^2, \quad (48)$$

where

$$D(s, \rho^2) = B^2 s^{-K-9} (s/\pi)^{3/2} e^{s\rho^2} / (\pi i \rho^{2K+13}), \quad (49)$$

and the contour ds is along the imaginary s axis. B^2 is a normalization factor in the theory.⁴¹

The term $T_{\nu_0} \chi(\rho)$ corresponds to taking the kinetic energy operator, using the cofactor expansion Eq. (46) and keeping only those terms where the derivatives involve coordinates that match the coordinates of the particle with the quantum numbers ν_0 . Eventually we obtain

$$T_{\nu_0} \chi = \frac{s_1 \chi}{\rho^2} + \frac{s_2}{\rho} \frac{d\chi}{d\rho} + s_3 \frac{d^2 \chi}{d\rho^2}, \quad (50)$$

where the s are complicated sums that depend on ν_0 , and the quantum numbers of the other nucleons. Their values are given in the Appendix.

In Figs. 3-4 we compare the spectral function calculated by Eq. (44) with various experimental results. In Fig. 3 we compare our calculated spectral energy factor to that obtained⁹ from (e, ep) reaction by Nakamura *et al.* Shown as a dotted line is a Gaussian⁹ whose parameters (width, height, and central value) are fitted to best reproduce the experimental data. The solid curve, calculated from Eq. (44) under the assumption of Eq. (29), agrees somewhat better with the experimental data.

In Fig. 4 we show the differential cross section

for ${}^6\text{Li}(\rho, 2p)$ determined by Tyren *et al.*⁸ for 460 MeV incident protons. Assuming the pp cross section does not vary rapidly with the separation energy for the kinematics measured (a symmetric coplanar reaction with both protons detected at

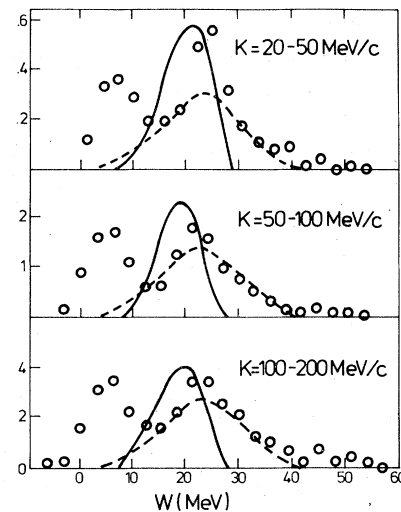


FIG. 3. Proton separation energy spectra for ${}^6\text{Li}$ from Nakamura *et al.* (Ref. 24), from 200 MeV (e, ep) data. The dashed curves are Gaussians fitted to the S shell knock-out peak. The solid curve is calculated here with arbitrary normalization.

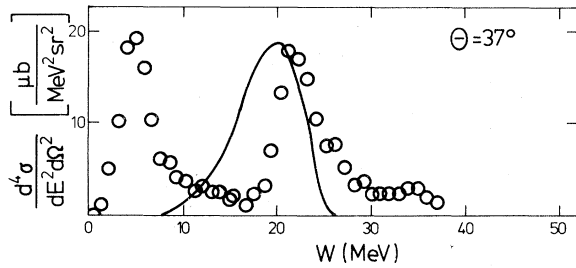


FIG. 4. The symmetric coplanar $(p, 2p)$ cross section for 460 MeV protons versus the separation energy W from Tyren *et al.* (Ref. 14). The solid curve is the spectral function calculated here normalized to the experimental data, assuming the (p, p) free scattering is approximately constant.

37° from the incident beam), this cross section should be proportional to the spectral function. We show in Fig. 4 the spectral function from Eq. (44) multiplied by an arbitrary factor to bring about best agreement to the eye of the two curves. The agreement is satisfactory.

In summary, we have calculated an approximate spectral energy function for $l=0$ nucleons in the ${}^6\text{Li}$ ground state without using the independent particle shell model assumptions. In the simplest independent particle shell model, single particle energies E_ν are input (parameters) to the model, and the spectral function energy dependence is $S_{\nu\nu}(W) = \delta(W - E_\nu)$.

Here we use a translationally invariant hyperspherical harmonic wave function and calculate the spectral function energy dependence using a local energy approximation, Eq. (42). The spectral energy function calculated has no free parameters in it, once the bound state wave function and the two nucleon interaction used to derive that wave function are specified. The spectral function calculated is in agreement with that derived from experiment. The mean removal energy is calculated for ${}^6\text{Li}$ to be 21 MeV and the full width at half maximum is about 10 MeV. The peak in the spectral function resembles a Gaussian, but only superficially.

The moderate agreement between experiment

and theoretically calculated spectral energy functions for $0s$ protons in ${}^6\text{Li}$, suggest that the shell model assumption of independent particle motion is inappropriate for deeply bound nucleons in a nucleus.

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APPENDIX

The sums in Eq. (52) come from expanding integrals of the type

$$\int_0^1 dZ (1-Z)^{M-2} Z^{l+1/2},$$

where $Z = r^2/\rho^2$ and the coordinate of the particle with quantum numbers ν_0 was \vec{r} , i.e., a function $\Upsilon^{\nu_0}(\vec{r})$. M is defined as

$$M = K + 3(A - 2)/2 - l,$$

where ν_0 specifies l . $l=0$ for the spectral function we calculate.

Then we obtain

$$S_{1/c} = S_b^{M-3} - (2l+3)S_a^{M-2} - n(2l+3)S_a^{M-1} + n(n+2)S_b^{M-1} + 2nS_b^{M-2}, \quad (\text{A1})$$

$$S_{2/c} = S_a^{M-1}(2l+3) - (2n+1)S_b^{M-1} - 2S_b^{M-2}, \quad (\text{A2})$$

$$S_{3/c} = S_b^{M-1}, \quad (\text{A3})$$

where we defined

$$n = (3A - 4)/2 = 7, \quad (\text{A4})$$

$$M = K + 3(A - 2)/2 - l = L - l, \quad (\text{A5})$$

$$S_a^m = \frac{1}{m!} \sum_{r=0}^m (-1)^r \binom{m}{r} / (2l + 2r + 3), \quad (\text{A6})$$

$$S_b^m = \frac{1}{m!} \sum_{r=0}^m (-1)^r \binom{m}{r} / (2l + 2r + 5), \quad (\text{A6})$$

$$C = \frac{-\hbar^2}{2m} \Gamma(K + 3(A - 1)/2) / \Gamma(l + 3/2). \quad (\text{A7})$$

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