# Current densities in the projected Hartree-Fock approach. I. Magnetic moments

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Static magnetic dipole moments of several axially symmetric rare earth nuclei are computed for the first time in the projected Hartree-Fock approximation using the density matrix expansion effective Hamiltonian. Good agreement with experiment is found for even-even nuclei. Results for odd-$A$ nuclei show sensible discrepancies with known experimental values. These discrepancies are primarily attributed to the lack of spin polarization in the present Hartree-Fock wave functions. An estimate of spin polarization effects is also given.

[NUCLEAR STRUCTURE $^{159}$Tb, $^{154}$Sm, $^{156}$Gd, $^{165}$Ho, $^{166,167}$Er, $^{174,176}$Yb, $^{175}$Lu, $^{180}$Hf, $^{181}$Ta; calculated static $M1$ moments. Angular momentum projected Hartree-Fock approximation.]

## I. INTRODUCTION

In the past few years there has been an increasing interest in the experimental determination of electromagnetic form factors of rotational rare earth nuclei and their interpretation in terms of the mean field approximation. Charge distributions of various doubly-even and odd-mass nuclei have been accurately measured,[1] and experiments to determine current distributions are in progress.[2] It has been shown[1,3] that the predictions due to the angular momentum projected Hartree-Fock (PHF) approximation using the density matrix expansion (DME) effective Hamiltonian[4] are in excellent agreement with experimental results for longitudinal form factors of elastic and inelastic transitions to the first few levels in the ground state rotational band.

A remaining question is whether this approximation will be as successful in describing nuclear current distributions. Comparison of a schematic calculation with experimental results[5,6] on the transverse elastic form factor of $^{25}$Mg indicates that this may be the case. Hartree-Fock computations of transverse form factors are now under way and will be reported in a subsequent publication. Since experimental data on transverse form factors of rare earth nuclei are not yet available, it seems appropriate to first test the above mentioned approximation against static magnetic moments which have been extensively measured. Furthermore, this test may serve as a guide in the detailed interpretation of these form factors.

Collective gyromagnetic ratios have been successfully calculated[7] on the basis of the cranking model using pair-correlated Nilsson single particle wave functions. The cranking model yields an approximate solution of the variational equations which follow from variation of the energy after angular momentum projection[8] and, in this sense, represents one step beyond the PHF approach. As shown in Ref. 5, the PHF approximation yields an expression for $g_R$ that looks like an average of the self-consistent cranking formula. It is therefore mandatory to investigate to what extent this approximation—which avoids the ambiguities introduced by excitation energy denominators in previous[7] calculations—explains the experimental results on collective $g$ factors, before going any further in the computation of transverse form factors.

Results for doubly-even ($^{154}$Sm, $^{156}$Gd, $^{166}$Er, $^{174}$Yb, $^{176}$Yb, $^{180}$Hf) and odd-mass ($^{159}$Tb, $^{165}$Ho, $^{167}$Er, $^{175}$Lu, $^{181}$Ta) nuclei are presented and discussed in Sec. III. A detailed description of our results on $\langle J_\perp^2\rangle$ (Ref. 5)—a crucial quantity in the present approximation—can be found in Sec. II. The selection of nuclei was motivated by recent results of electron scattering experiments at the Bates Linear Accelerator Center.

## II. BRIEF SUMMARY OF THEORY AND DETAILS OF CALCULATIONS

In the PHF approach the magnetic moment of a state with angular momentum $I$ in the ground state rotational band $K$ is given, to first order[5] in $1/\langle J_\perp^2\rangle$, by the familiar[9] expression

$$\mu_I = g_R I + (g_K - g_R)\frac{K^2}{I+1} + \delta_{K,1/2}(-1)^{I+1/2}\frac{(I+\frac{1}{2})}{(I+1)\sqrt{2}}\left(\langle\phi_K|\mu_1^1|\phi_{-K}\rangle + \mathcal{O}\left(\frac{1}{\langle J_\perp^2\rangle}\right)\right). \quad (1)$$

 

The socalled collective and single particle gyromagnetic ratios are given in this approximation by

$$g_R = \langle\phi_K|(\vec{\mu}\cdot\vec{J})_\perp|\phi_K\rangle / \langle\phi_K|\vec{J}_\perp{}^2|\phi_K\rangle \quad (2)$$

and

$$g_K = \frac{1}{K}\left(\langle\phi_K|\mu_z|\phi_K\rangle - \frac{\langle\phi_K|\mu_z\vec{J}_\perp{}^2|\phi_K\rangle_L}{\langle J_\perp{}^2\rangle}\right)$$
$$\equiv g_K^0 - \delta g_K\,, \quad (3)$$

where $\phi_K$ is the intrinsic axially symmetric HF wave function, $\langle\,\rangle_L$ denotes linked diagrams, and $(\vec{A}\cdot\vec{B})_\perp = A_xB_x + A_yB_y$. As in the rotational approximation,[9] $g_R$ and $g_K$ are to this order common constants of the rotational band. A small dependence of $g_R$ and $g_K$ on angular momentum appears through higher order terms and can be neglected for the purpose of this work.

The intrinsic wave functions $\phi_K$ have been determined using Negele's DME effective interaction, as described in Ref. 3. The generalization of this effective interaction to odd-$A$ nuclei requires the addition of several current density dependent terms to the energy density. Up to now the generalization of the energy-density formalism to time reversal noninvariant systems has been restricted to the Skyrme force.[10] In the present work the pair filling approximation[11] has instead been adopted. In this approximation the mean field has reflection symmetry with respect to the plane perpendicular to the symmetry axis, and therefore conjugate orbitals are degenerate in energy. In the presence of a single nucleon the mean field would no longer be $\mathcal{R}$ invariant,[9] and the above mentioned degeneracy would be removed. Since single particle states with angular momentum components $\Omega$ and $-\Omega$ (with respect to the symmetry axis) contribute with opposite sign to $g_K$ and with same sign to $g_R$ [see Eqs. (2) and (3)], the absence of spin polarization of the core in our HF wave functions is expected to affect the $g_K$ values more strongly.

Using the one body vector operator $\vec{\mu}$, Eq. (2) reads

$$g_R = \frac{\langle\phi_K|J_\perp{}^2|\phi_K\rangle_p}{\langle J_\perp{}^2\rangle} + (g_s^p - 1)\frac{\langle\phi_K|(\vec{S}\cdot\vec{J})_\perp|\phi_K\rangle_p}{\langle J_\perp{}^2\rangle} + g_s^n\frac{\langle\phi_K|(\vec{S}\cdot\vec{J})_\perp|\phi_K\rangle_n}{\langle J_\perp{}^2\rangle}$$
$$\equiv \frac{\langle J_\perp{}^2\rangle_p}{\langle J_\perp{}^2\rangle} + \text{s.c.}\,, \quad (4)$$

where $p$ and $n$ indicate contributions from protons and neutrons, respectively, and $\langle J_\perp{}^2\rangle$ is the sum of both contributions $(\langle J_\perp{}^2\rangle = \langle J_\perp{}^2\rangle_p + \langle J_\perp{}^2\rangle_n)$. Explicit expressions of $\langle(\vec{S}\cdot\vec{J})_\perp\rangle$ are given in the Appendix for even-even and odd-$A$ nuclei, and similar expressions hold for $\langle J_\perp{}^2\rangle$ replacing $s_\pm$ by $j_\pm$. These equations are analogous to the definitions of $W$ and $\mathcal{T}$ in Ref. 7, except for the energy denominators.

The first term of Eq. (3) gives the rotational[9] model value

$$g_K^0 = g_l + (g_s - g_l)\frac{\langle K|s_z|K\rangle}{K}\,, \quad K \neq 0 \quad (5)$$

and the second term $(\delta g_K)$ gives a small projection correction to the zeroth order term $(g_K^0)$. Since we are mainly interested in the order of magnitude of this correction, it has been computed neglecting pairing effects,

$$\delta g_K = (g_s - g_l)\frac{2}{K\langle J_\perp{}^2\rangle}\Bigg\{\langle K|s_z j_y{}^2|K\rangle - \sum_h \langle K|s_z|h\rangle\langle h|j_y{}^2|K\rangle - 2\sum_{h'}\Big(\langle K|s_z j_y|h'\rangle - \sum_h \langle K|s_z|h\rangle\langle h|j_y|h'\rangle\Big)\langle h'|j_y|K\rangle\Bigg\}\,. \quad (6)$$

In Eqs. (5) and (6), $g_s$, $(g_l)$, and $|K\rangle$ are, respectively, the spin (orbital) $g$ factor and the single particle HF state assigned to the odd nucleon, $h$ and $h'$ represent occupied states. As can be seen from the calculated results in Table I, $\delta g_K$ gives a very small correction to $g_K^0$ and, consistent with our expansion, can be neglected.

Parameters describing the Hartree-Fock calculations are given in Table II. In the HF codes pairing is treated in a rather crude way.[3] Constant energy gaps $\Delta_p$, $\Delta_n$, extracted from experimental mass differences, are used to solve the pairing equations for fixed particle numbers $(Z,N)$ after each iteration. Since $g_R$ is very sensitive to the pairing of protons relative to that of neutrons, one must be cautious when using the solutions of these equations to compute $g_R$. Consequently, we have done all the calculations with and without pairing (see Appendix).

$\langle J_\perp{}^2\rangle$ of doubly-even rare earth nuclei have previously been calculated by several authors[12] using different forces and different methods. A comparison between different calculations of $\langle J_\perp{}^2\rangle$ can be found in the work of Vallieres *et al.*[12] The results presented here are in qualitative agreement with those, in spite of the fact that contributions from states with $\Omega = \pm\frac{1}{2}$ were overestimated in their former work.

In Table III we present in detail our results for $\langle J_\perp{}^2\rangle_p$ and $\langle J_\perp{}^2\rangle_n$—as well as spin corrections (s.c.)

TABLE I. Single particle gyromagnetic ratios and first order corrections, as described in the text, are given in columns three and four. Column five summarizes experimental $g_K$ values. References are as in the caption to Table IV. Column six gives the calculated values of $g_s^{\mathrm{eff}}$ in units of $g_s^{\mathrm{free}}$, while column seven summarizes the experimental data on magnetic dipole moments for states in the ground state rotational band. The last two columns contain theoretical results on magnetic moments [from Eq. (1)] using $g_R^{(P)}$ values in Table IV and $g_K$ values corresponding to $g_s^{\mathrm{free}}$ and $g_s^{\mathrm{eff}}$ (in column six).

| | $K^\pi$ | $g_K^0$ | $\delta g_K$ | $g_K^{\mathrm{exp.}}$ | $g_s^{\mathrm{eff}}/g_s^{\mathrm{free}}$ | | $\mu_I$ exp. [d] | $\mu_I^{\mathrm{th}}(g_s^{\mathrm{free}})$ | $\mu_I^{\mathrm{th}}(g_s^{\mathrm{eff}})$ |
|---|---|---|---|---|---|---|---|---|---|
| | | | | 1.830(35) [a] | | $I=\frac{3}{2}$ | 2.014(4) | 2.39 | 1.89 |
| $^{159}_{65}$Tb | $\frac{3}{2}^+$ | 2.39 | 0.010 | 1.788(43) [b] | ~0.67 | $I=\frac{5}{2}$ | 1.62(9) | | |
| | | | | 1.88(5) [c] | | | or 2.32(13) | 2.29 | 1.93 |
| | | | | 1.35(2) [a] | | $I=\frac{7}{2}$ | 4.173(27) | 4.64 | 4.07 |
| $^{165}_{67}$Ho | $\frac{7}{2}^-$ | 1.58 | 0.003 | 1.329(27) [b] | ~0.69 | | | | |
| | | | | 1.37(1) [c] | | $I=\frac{9}{2}$ | 4.13(7) | 4.52 | 4.05 |
| | | | | −0.259(3) [a] | | $I=\frac{7}{2}$ | −0.566 5(24) | −0.92 | −0.45 |
| $^{167}_{68}$Er | $\frac{7}{2}^+$ | −0.43 | 0.005 | −0.249(9) [b] | ~0.60 | | | | |
| | | | | −0.259(3) [c] | | $I=\frac{9}{2}$ | | −0.22 | 0.16 |
| | | | | 0.729(4) [a] | | $I=\frac{7}{2}$ | 2.237 99(6) | 1.29 | 2.24 |
| $^{175}_{71}$Lu | $\frac{7}{2}^+$ | 0.37 | 0.006 | 0.716(6) [b] | ~0.54 | $I=\frac{9}{2}$ | 2.01(15)<br>1.52(51) | 1.65 | 2.43 |
| | | | | 0.726(3) [c] | | $I=\frac{11}{2}$ | 2.0(7) | 2.01 | 2.67 |
| | | | | 0.780(4) [a] | | $I=\frac{7}{2}$ | 2.371(1)<br>2.370(1)<br>2.37(2) | 1.3 | 2.39 |
| $^{181}_{73}$Ta | $\frac{7}{2}^+$ | 0.37 | 0.004 | 0.771(4) [b]<br>0.78(1) [c] | ~0.48 | | | | |
| | | | | | | $I=\frac{9}{2}$ | 1.22(18)<br>1.98(63) | 1.65 | 2.55 |

[a] Reference 15. [b] Reference 16. [c] Reference 14. [d] Reference 17.

to $g_R$ in Eq. (4)—with and without pairing, corresponding to Eqs. (A1) (given within parentheses) and (A2). As these two equations are related through closure, they give different results when using a truncated basis, and Eq. (A2) is more correct in this case. However, as shown in Table III, for these large basis ($N=13$) calculations the difference in the results is quite insignificant. This can easily be understood from the fact that the ratio [see Eq. (A3)] between $\Delta N=2$ and $\Delta N=0$ matrix elements of $l_\pm$ is proportional to $\beta^-/\beta^+ \sim 10^{-1}$, and contributions from higher shells ($N>13$) are small. A more drastic change in the value of $\langle J_\perp^2\rangle$ is found between results corresponding to pairing (P) and no pairing (NP). Since $\langle J_\perp^2\rangle((\vec{S}\cdot\vec{J})_\perp)$ is proportional to the simultaneous probability of finding an unoccupied state $B$ and an occupied state $A$ connected by $j_\pm$, the value of $\langle J_\perp^2\rangle((\vec{S}\cdot\vec{J})_\perp)$ for a given deformation tends to decrease with pairing. In our present calculations with $\Delta=$const, this effect is particularly strong when the level spacing around the Fermi level is much smaller than the gap $\Delta$. This is apparent from the negative terms $u_Av_Au_Bv_B$ in Eqs. (A1) and (A2) which increase from 0 to $|\langle A|j_\pm|B\rangle|^2$ as $|\Delta/(e_A-\lambda)|^2$ goes from zero to $|\Delta/(e_A-\lambda)|^2 \gg 1$,

$$u_Av_Au_Bv_B = \tfrac{1}{4}\{[1+(e_A-\lambda)^2/\Delta^2][1+(e_B-\lambda)^2/\Delta^2]\}^{-1/2}. \tag{7}$$

If these terms are neglected, as in the work of Flocard *et al.*,[12] one gets increasingly high values of $\langle J_\perp^2\rangle$ with pairing. As an example, the omission of these terms in our calculation yields the results $\langle J_\perp^2\rangle = 225.7$ for $^{166}$Er and $\langle J_\perp^2\rangle = 177.5$ for $^{180}$Hf, which are, respectively, 59% and 38% larger than those in Table III. The decrease of $\langle J_\perp^2\rangle$ with pairing is consistent[13] with the corresponding decrease of the moment of inertia.[7]

As can be seen from Eq. (4), if pairing for protons is stronger than that for neutrons $g_R$ will decrease with pairing. The fact that the level density around the Fermi level is usually larger for neutrons than for protons gives rise in the cases $\Delta_p \approx \Delta_n$ to an effective pairing for neutrons larger

TABLE II. Gap parameters, charge radius, and mass quadrupole moments for neutrons and protons as defined in Ref. 3. Upper and lower entries for each nucleus denote theoretical predictions for pairing and no pairing solutions.

| | $\Delta_p$ (MeV) | $\Delta_n$ (MeV) | $R_c$ (fm) | $Q_n$ (fm$^2$) | $Q_p$ (fm$^2$) |
|---|---|---|---|---|---|
| $^{154}$Sm | 0.855 | 1.069 | 5.112 | 925.4 | 644.3 |
| | | | 5.108 | 938.8 | 647.7 |
| $^{156}$Gd | 0.963 | 1.07 | 5.141 | 895.5 | 647.4 |
| | | | 5.132 | 864.9 | 643.1 |
| $^{166}$Er | 0.915 | 0.903 | 5.257 | 1103.0 | 775.3 |
| | | | 5.251 | 1086.0 | 771.3 |
| $^{174}$Yb | 0.884 | 0.68 | 5.324 | 1192.0 | 810.9 |
| | | | 5.324 | 1201.0 | 808.8 |
| $^{176}$Yb | 1.18 | 0.59 | 5.316 | 1136.0 | 763.3 |
| | | | 5.315 | 1135.0 | 771.3 |
| $^{180}$Hf | 1.02 | 0.85 | 5.363 | 1110.0 | 738.0 |
| | | | 5.355 | 1105.0 | 719.3 |
| $^{159}$Tb | 0.936 | 0.78 | 5.177 | 988.7 | 701.6 |
| | | | 5.176 | 990.7 | 716.4 |
| $^{165}$Ho | 0.88 | 0.8 | 5.241 | 1108.0 | 766.4 |
| | | | 5.243 | 1089.0 | 760.8 |
| $^{167}$Er | 0.756 | 0.84 | 5.263 | 1133.0 | 784.2 |
| | | | 5.258 | 1149.0 | 779.7 |
| $^{175}$Lu | 0.912 | 0.57 | 5.342 | 1202.0 | 832.7 |
| | | | 5.357 | 1208.0 | 848.0 |
| $^{181}$Ta | 0.806 | 0.62 | 5.369 | 1078.0 | 715.7 |
| | | | 5.359 | 1077.0 | 665.8 |

than that for protons, causing an increase in $g_R$. This is particularly the case in $^{154}$Sm, $^{156}$Gd, and $^{166}$Er (see Table IV), where both $\langle J_\perp^2\rangle_p/\langle J_\perp^2\rangle$ and the spin correction (s.c.) increase. In the case of odd-$A$ nuclei, generally, the contribution to $\langle J_\perp^2\rangle((\vec{S}\cdot\vec{J})_\perp)$ from the even group of nucleons decreases more with pairing than that from the odd group [see Eqs. (A1) and (A1′) and Table III], and therefore one has $g_R^{(P)}$ (odd $Z$) $> g_R^{(P)}$ (odd $N$), as experimentally observed.

## III. DISCUSSION OF RESULTS AND COMPARISON WITH EXPERIMENT

Results for collective and single particle gyromagnetic ratios are shown in Tables IV and I, respectively. Experimental $g_R$ values for even-even nuclei are from magnetic moments of first $2^+$ states in the compilation by Grodzins.[14] For a more extensive tabulation see Refs. 7, 17, and references therein.

For odd-$A$ nuclei with $K>\frac{1}{2}$, experimental $g_R$ and $g_K$ values have been extracted from magnetic moments of ground states and $M1$ transition probabilities.[14–16] It should be pointed out that again, to first order in $1/\langle J_\perp^2\rangle$, $M1$ transition probabilities in the PHF approach are given by the rotational[9] model expression,

$$B(M1;\, IK\to I\pm 1K)=\left(\frac{e\hbar}{2Mc}\right)^2\frac{3}{4\pi}\langle IK10\,|\,I\pm 1K\rangle^2 \times (Kg_K-Kg_R)^2,\quad K>\tfrac{1}{2},$$

with $g_R$, $g_K$ as given by Eqs. (2) and (3), respectively, and the usual analysis[9] for extracting $g_R$,

TABLE III. Contributions to $\langle J_\perp^2\rangle$ from protons and neutrons and spin correction (s.c.) to $g_R$ as described in the text. The notation (P) and (NP) stands for pairing and no pairing. Entries within parentheses correspond to results from Eqs. (A1) or (A1′).

| | $\langle J_\perp^2\rangle_p^{(P)}$ | $\langle J_\perp^2\rangle_p^{(NP)}$ | $\langle J_\perp^2\rangle_n^{(P)}$ | $\langle J_\perp^2\rangle_n^{(NP)}$ | s.c.$^{(P)}$ | s.c.$^{(NP)}$ |
|---|---|---|---|---|---|---|
| $^{154}_{62}$Sm | 58.48 | 72.12 | 73.99 | 107.5 | 0.0033 | −0.0155 |
| | (58.10) | (71.87) | (73.15) | (107.1) | (0.0033) | (−0.0156) |
| $^{156}_{64}$Gd | 52.60 | 68.78 | 67.12 | 101.6 | 0.0037 | −0.0127 |
| | (52.18) | (68.54) | (66.34) | (101.2) | (0.0037) | (−0.0128) |
| $^{166}_{68}$Er | 58.74 | 69.74 | 83.14 | 111.7 | −0.0041 | −0.0228 |
| | (58.29) | (69.46) | (82.43) | (111.3) | (−0.0043) | (−0.0229) |
| $^{174}_{70}$Yb | 59.54 | 67.05 | 78.26 | 107.3 | −0.0097 | −0.0101 |
| | (59.07) | (66.73) | (77.27) | (106.8) | (−0.0099) | (−0.0102) |
| $^{176}_{70}$Yb | 57.06 | 65.20 | 92.96 | 98.89 | −0.0199 | −0.0182 |
| | (56.30) | (64.66) | (91.71) | (97.78) | (−0.0202) | (−0.0183) |
| $^{180}_{72}$Hf | 50.08 | 56.40 | 78.11 | 98.10 | −0.0117 | −0.0188 |
| | (49.54) | (56.11) | (77.34) | (97.59) | (−0.0118) | (−0.0189) |
| $^{159}_{65}$Tb | 58.19 | 71.41 | 84.25 | 108.6 | −0.0031 | −0.0136 |
| | (57.75) | (71.13) | (83.6) | (108.2) | (−0.0033) | (−0.0137) |
| $^{165}_{67}$Ho | 67.18 | 68.33 | 86.93 | 112.0 | 0.0046 | −0.0258 |
| | (66.78) | (68.03) | (86.26) | (111.6) | (0.0047) | (−0.0260) |
| $^{167}_{68}$Er | 62.05 | 70.16 | 116.37 | 121.2 | −0.0231 | −0.0266 |
| | (61.65) | (69.87) | (115.67) | (120.7) | (−0.0232) | (−0.0266) |
| $^{175}_{71}$Lu | 59.43 | 84.63 | 95.63 | 107.7 | −0.0190 | −0.0260 |
| | (58.92) | (84.28) | (94.99) | (107.1) | (−0.0192) | (−0.0261) |
| $^{181}_{73}$Ta | 48.80 | 53.34 | 80.83 | 95.92 | −0.0157 | −0.0017 |
| | (48.39) | (53.02) | (80.20) | 95.44 | (−0.0158) | (−0.0017) |

$g_K$ remains valid. As an alternative, one could use measured magnetic moments of excited states to determine $g_R$, $g_K$. However, in most cases the large uncertainty in the experimental results (see column seven of Table I) invalidates this procedure.

For even-even nuclei, theoretical results corresponding to no pairing ($g_R^{NP}$) are in rather good agreement with experiment, as shown in Table IV and Fig. 1. It is interesting to note that, despite the different theoretical approaches, these results agree quite well with those in the work of Prior *et al.*[7] The latter are given for comparison in the last two columns of Table IV. Except in the case of $^{176}$Yb, for which $\Delta_p \simeq 2\Delta_n$, our pairing results ($g_R^{(P)}$) are larger and tend to deviate more from experiment. As discussed in the previous section, this is due to the fact that the level density around the Fermi level is larger for neutrons than for protons while $\Delta_p \approx \Delta_n$. Varying $\Delta_p$ and $\Delta_n$ within the experimental uncertainties may change $g_R^{(P)}$ as much as 10–20%. Such a procedure is rather ambiguous, and a state dependent gap treatment of pairing is preferred. However, the wide discrepancies between experimental results on $g_R$ do not justify such detailed calculations.

For odd-$A$ nuclei, theoretical $g_R$ values vary more smoothly with $A$ than experimental ones (see Table IV and Fig. 2). Although for odd-$Z$ nuclei theoretical results corresponding to pairing agree rather well with experiment, there is a factor of ~2 between theoretical and experimental values in the case of $^{167}$Er (odd $N$). In contrast to the cranking formula,[7] our theoretical approach does not seem to explain in a quantitative way the observed large differences between collective gyromagnetic ratios of odd-$Z$ and odd-$N$ nuclei. This may be due to the fact that the cranking formula for the moment of inertia[7] is more sensitive to the odd nucleon than $\langle J_\perp{}^2\rangle$, and again a better treatment of pairing would be required. However, when comparing the results of Prior *et al.*[7] to ours, one should also take into account that these results correspond to $g_s = 0.6\, g_s^{\text{free}}$, whereas ours correspond to $g_s = g_s^{\text{free}}$ as well as to different $\Delta_p/\Delta_n$ ratios. Note that pairing effects are as in even-even nuclei, except for the contributions from the odd nucleon [see Eq. (A1′)].

Our results for single particle gyromagnetic ratios are similar to those using Nilsson wave functions.[9] The projection correction $\delta g_K$ [see Table I and Eq. (3)] is less than 2% in all cases and does not account for the large deviations from experimental $g_K$ values. As discussed in the previous section, these deviations may be accounted for in terms of spin polarization of the core.[16] An estimate[9] of this effect is given in column six of Table I. The effective values of the spin $g$ factor for the odd nucleon ($g_s^{\text{eff}}$)—for which the single particle HF wave functions used to evaluate $g_K^0$ give the observed $g_K$ values—are close to but somewhat smaller than those in Ref. 9, which correspond to Nilsson wave functions.

Theoretical magnetic moments for the first states of the ground state rotational band, for odd-$A$ nuclei, corresponding to $g_K^0$ ($g_s^{\text{free}}$) and $g_K^0$ ($g_s^{\text{eff}}$), are given in columns eight and nine of Table I, respectively. In both cases theoretical values of $g_R^{(P)}$ have been used and similar results are found for $g_R^{(NP)}$. Especially for states with large $I=K$, $\mu_I$ is less sensitive to small changes in $g_R$ than in $g_K$. It can be seen that, except in the case of $^{167}$Er in which $g_R^{\text{th}} \sim 2g_R^{\text{ex}}$, $\mu_I^{\text{th}}(g_s^{\text{eff}})$ agrees to within a few percent with the more precise experimental values in column 7. It is also interesting to note that

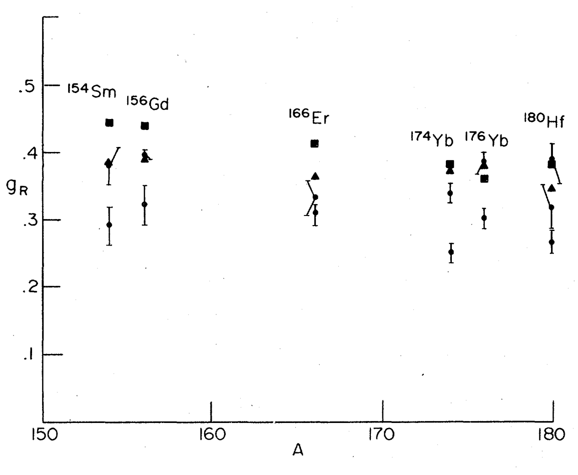


FIG. 1. Comparison between theoretical and experimental $g_R$ values in Table IV of doubly-even nuclei. Theoretical results corresponding to pairing and no pairing are represented by squares and triangles, respectively.

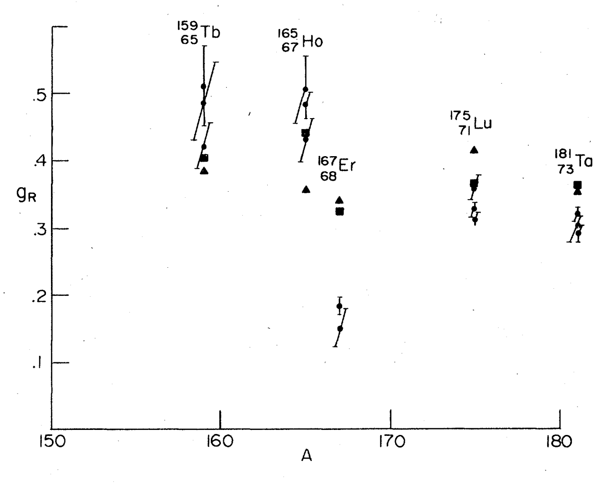


FIG. 2. Same as Fig. 1 for odd-mass nuclei.

$\mu_I^{\text{th}}$ ($g_s^{\text{free}}$) agrees better with experimental values of excited states than those of ground states. This opens the question of whether spin polarization and/or $K$-band mixing effects are different for different members of a given rotational band. It would be desirable to have more precise measurements of $\mu_I$ of excited states in order to get some further insight into this question.

## IV. CONCLUSIONS

In conclusion we may say that, within the uncertainties introduced by the simple treatment of pairing in the HF codes, the present approach gives rather satisfactory results for collective gyromagnetic ratios. Further calculations on other odd-$N$, even-$Z$ nuclei are underway to see if the disagreement found in $^{167}$Er persists in these isotopes.

The comparison of experimental $g_K$ values with single particle values $g_K^0$ indicates rather large spin polarization effects. Core contributions to $|\langle\phi_K|\mu_z|\phi_K\rangle|$ as large as 0.6 to 1.2 are needed in order to explain observed $g_K$ values. The question still remains to whether the mean field approximation when properly extended to odd-$A$ nuclei will account for these contributions. Exchange current effects—not considered in this work—are not expected to modify single particle magnetic moments by more than 10–20%.[18] The small values found for $\delta g_K$ provide a check on the convergence of our expansion in powers of $1/\langle J_\perp{}^2\rangle$.

The next interesting step in these calculations will be to analyze the $q$ dependence of the different contributions to the various multipoles that enter in transverse form factors of odd-$A$ nuclei. On the basis of the present results one would expect large core contributions at low $q$. From angular momentum considerations the highest multipoles ($M5, M7, \ldots$) are expected to be well described by single particle contributions, as seems to be confirmed by preliminary experimental results on $^{181}$Ta.[2,5] Another interesting feature will be to compare transverse electric multipoles in doubly-even and odd-mass nuclei.

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TABLE IV. Collective gyromagnetic ratios. Our results are shown in columns two (pairing) and three (no pairing). Experimental values for even-even nuclei are from the compilation by Grodzins (Ref. 14), values marked with an asterisk are from Mössbauer experiments. The last two columns contain theoretical results of Prior *et al.*, as described in Ref. 7.

| | $g_R^{(\text{P})}$ | $g_R^{(\text{NP})}$ | $g_R^{\text{exp}}$ | $g_R^{\text{I}}$ | $g_R^{\text{II}}$ |
|---|---|---|---|---|---|
| $^{154}$Sm | 0.445 | 0.386 | 0.288(29)<br>0.379(27)* | 0.42 | 0.38 |
| $^{156}$Gd | 0.443 | 0.391 | 0.296(18)<br>0.32(3)<br>0.393(7)* | 0.41 | 0.39 |
| $^{166}$Er | 0.409 | 0.361 | 0.329(27)<br>0.312(6)*<br>0.305(15)* | 0.27 | 0.36 |
| $^{174}$Yb | 0.383 | 0.374 | 0.337(7)*<br>0.247(13)<br>0.338(15)* | 0.34 | 0.32 |
| $^{176}$Yb | 0.360 | 0.379 | 0.381(16)*<br>0.299(15) | 0.35 | 0.35 |
| $^{180}$Hf | 0.379 | 0.346 | 0.263(15)<br>0.317(35)<br>0.383(35)[d] | 0.34 | 0.33 |
| $^{159}$Tb | 0.405 | 0.383 | 0.420(35)[a]<br>0.486(58)[b]<br>0.51(6)[c] | 0.49 | 0.45 |
| $^{165}$Ho | 0.440 | 0.353 | 0.429(30)[a]<br>0.504(52)[b]<br>0.48(2)[c] | 0.51 | 0.69 |
| $^{167}$Er | 0.325 | 0.340 | 0.182(10)[a]<br>0.149(30)[b]<br>0.184(10)[c] | 0.22 | 0.17 |
| $^{175}$Lu | 0.364 | 0.414 | 0.312(8)[a]<br>0.360(16)[b]<br>0.326(10)[c] | 0.33 | 0.32 |
| $^{181}$Ta | 0.361 | 0.355 | 0.293(13)[a]<br>0.320(8)[b]<br>0.30(2)[c] | 0.36 | 0.33 |

[a]Reference 15.
[b]Reference 16.
[c]Reference 14.
[d]Reference 17.

## APPENDIX A

The mean value of $(\vec{S}\cdot\vec{J})_\perp$ on a BCS state is given by

$$\langle(\vec{S}\cdot\vec{J})_\perp\rangle = \sum_{A,B}(v_A u_B - v_B u_A)^2(\langle A|s_+|B\rangle\langle A|j_+|B\rangle + \tfrac{1}{2}\langle A|s_+|\overline{B}\rangle\langle A|j_+|\overline{B}\rangle) \tag{A1}$$

$$= \sum_A n_A\langle A|s_+j_- + s_-j_+|A\rangle - \sum_{A,B}\{n_A n_B + [n_A(1-n_A)n_B(1-n_B)]^{1/2}\}(2\langle A|s_+|B\rangle\langle A|j_+|B\rangle + \langle A|s_+|\overline{B}\rangle\langle A|j_+|\overline{B}\rangle)\,, \tag{A2}$$

where the sums extend over states with third angular momentum component $\Omega>0$ and $\bar{B}$ represents the time reversed state of $B$ for $\Omega_B=\frac{1}{2}$. In Eqs. (A1) and (A2) all matrix elements are real and $n_A$, $n_B$ represent occupation probabilities, $n_A=v_A{}^2$, $v_A{}^2+u_A{}^2=1$. In the absence of pairing the same equations apply, for then $n_A=1$ or 0 for states below or above the Fermi level.

For odd-$A$ nuclei with the odd particle in orbital $K$,

$$\langle\alpha_K(\vec{S}\cdot\vec{J})_\perp\alpha_K^+\rangle=\tfrac{1}{2}\langle K|s_+j_-+s_-j_+|K\rangle+\sum_{\substack{A\neq K\\ B\neq K}}(u_Av_B-v_Au_B)^2(\langle A|s_+|B\rangle\langle A|j_+|B\rangle+\tfrac{1}{2}\langle A|s_+|\bar{B}\rangle\langle A|j_+|\bar{B}\rangle)\,. \tag{A1$'$}$$

Alternatively, the same expression (A2) can be used with the prescriptions $n_K=\frac{1}{2}$, $u_Kv_K=0$, and, in the particular case $\Omega_K=\frac{1}{2}$, the contribution to the last term in (A2) from $A=B=K$ must be excluded. The mean value of $\vec{J}_\perp{}^2$ is given by analogous equations replacing $s_\pm$ by $j_\pm$.

For completeness, the selection rules for $l_+$ operators in the deformed basis are given,

$$\langle n_r'n_z'\Lambda+1|l_+|n_rn_z\Lambda\geq 0\rangle=-\frac{1}{\sqrt{2}}\{\delta_{n_r',n_r}(\beta^+\delta_{n_z',n_z-1}[n_z(n_r+\Lambda+1)]^{1/2}+\beta^-\delta_{n_z',n_z+1}[(n_z+1)(n_r+\Lambda+1)]^{1/2})$$
$$+\delta_{n_r',n_r-1}(\beta^-\delta_{n_z',n_z-1}(n_rn_z)^{1/2}+\beta^+\delta_{n_z',n_z+1}[n_r(n_z+1)]^{1/2})\}\,, \tag{A3}$$

where $\beta^\pm=\beta_\perp/\beta_z\pm\beta_z/\beta_\perp$ with $\beta_\perp$, $\beta_z$ oscillator parameters as defined in Ref. 3. Selection rules for $j_\perp{}^2$ can easily be constructed from (A3). For the particular case $\Lambda=-1\to\Lambda=0$ that occurs in $\langle A|j_+|\bar{B}\rangle$ terms one has

$$\langle n_r'n_z'\Lambda=0|l_+|n_rn_z\Lambda=-1\rangle=-\langle n_rn_z\Lambda=1|l_+|n_r'n_z'\Lambda=0\rangle\,. \tag{A4}$$


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