Differential muon-capture rate for the reaction $\mu^- + {}^6\mathrm{Li} \to {}^3\mathrm{H} + {}^3\mathrm{H} + \nu_\mu$

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The differential muon-capture rate $d\Gamma/d\Omega({}^6\mathrm{Li} + \mu^- \to {}^3\mathrm{H} + {}^3\mathrm{H} + \nu_\mu)$ as a function of angle between the outgoing tritons is calculated by the use of the elementary particle model. The total capture rate for the same reaction but with the neutrino energy limited between 0 and 29.8 MeV is also calculated as is the rate $d\Gamma/d\nu$ under the same limitation. The reaction is also discussed as a candidate for determining a better limit on the muon-neutrino mass.

[NUCLEAR REACTIONS Muon capture ${}^6\mathrm{Li}(\mu, \nu_\mu){}^3\mathrm{H}{}^3\mathrm{H}$ calculated $d\Gamma/d\Omega$, $0° \leq \theta \leq 180°$ and, Γ, $d\Gamma/d\nu$ for $E_\nu \leq 29.8$ MeV.]

I. INTRODUCTION

The reaction $\mu^- + {}^6\mathrm{Li} \to {}^3\mathrm{H} + {}^3\mathrm{H} + \nu_\mu$ is interesting for a number of reasons. First, it is a possible candidate for determining the ν_μ mass[1] and second, it is in itself a useful test of the utility of the elementary-particle model[2] in situations involving the breakup of the nucleus. In this paper we calculate the differential muon-capture rate $(d\Gamma/d\Omega)(\mu^- + {}^6\mathrm{Li} \to {}^3\mathrm{H} + {}^3\mathrm{H} + \nu_\mu)$ as a function of the angle between the outgoing tritons. This number should be useful to experimentalists[1] who envision using the case in which the two tritons go out in approximately opposite directions, carrying off most of the available energy.

The method of calculation used here is the elementary-particle model.[2] This treatment has some advantage over the conventional impulse approximation treatment for problems of this type because it avoids the use of nuclear wave functions. Cross sections calculated by means of an impulse approximation treatment sometimes depend sensitively on the wave functions which are, in general, not well known. Also in the elementary-particle model approach the Pauli exclusion principle is incorporated directly into the matrix element.

In Sec. II of this paper we present the general form of the weak current matrix elements and discuss the form factors describing it. In Sec. III we obtain the differential muon capture rate $d\Gamma/d\Omega$ as a function of the angle between the two outgoing tritons. We also obtain a value for the total muon-capture rate Γ in the ditriton channel for neutrino energies from 0 to 29.8 MeV. In Sec. IV of this paper we discuss the results of these calculations, particularly with respect to using this reaction to obtain a limit for the muon neutrino mass.

II. THE WEAK CURRENT MATRIX ELEMENT AND FORM FACTORS

The matrix element for the muon-capture process $\mu^- + {}^6\mathrm{Li} \to {}^3\mathrm{H} + {}^3\mathrm{H} + \nu_\mu$ may be written as

$$\langle {}^3\mathrm{H}_1, {}^3\mathrm{H}_2, \nu | H_w^{(0)} | {}^6\mathrm{Li}, \mu^- \rangle = \frac{G\cos\theta}{\sqrt{2}} \bar{u}_\nu \gamma^\lambda (1-\gamma_5) u_\mu \langle {}^3\mathrm{H}_1, {}^3\mathrm{H}_2 | J_\lambda^\dagger(0) | {}^6\mathrm{Li}\rangle \quad (1)$$

to the lowest order in $G(=1.02\times 10^{-5} m_p^{-2})$, the weak coupling constant, where θ_C is the Cabbibo angle ($\cos\theta_C = 0.98$) and

$$J^\mu(0) = V^\mu(0) - A^\mu(0) \quad (2)$$

is the hadronic weak current, $V^\mu(0)$ being the vector part and $A^\mu(0)$ being the axial part respectively. Thus it is necessary to know $\langle {}^3\mathrm{H}{}^3\mathrm{H} | A_\mu^\dagger(0) | {}^6\mathrm{Li}\rangle$ and $\langle {}^3\mathrm{H}{}^3\mathrm{H} | V_\mu^\dagger(0) | {}^6\mathrm{Li}\rangle$. These have been obtained by the author in a previous paper[3] based on the observation that the reactions $\mu^- + {}^6\mathrm{Li} \to {}^3\mathrm{H} + {}^3\mathrm{H} + \nu_\mu$ and $\mu^- + d \to n + n + \nu_\mu$ are structurally very similar. Thus the current matrix[4-7] elements $\langle nn | A_\mu^\dagger(0) | d\rangle$, $\langle nn | V_\mu^\dagger(0) | d\rangle$ have precisely the same structure as $\langle {}^3\mathrm{H}{}^3\mathrm{H} | A_\mu^\dagger(0) | {}^6\mathrm{Li}\rangle$ and $\langle {}^3\mathrm{H}{}^3\mathrm{H} | V_\mu^\dagger(0) | {}^6\mathrm{Li}\rangle$ respectively.

We may therefore write[3]

$$\langle {}^3\mathrm{H}{}^3\mathrm{H} | V_\mu^\dagger(0) | {}^6\mathrm{Li}\rangle = \eta \bar{u}(p_1) \left(\frac{F_1}{M_L{}^2} \epsilon_{\mu\nu\sigma\rho} \xi^\nu Q^\rho L^\sigma + \frac{F_2}{M_L} \gamma^\nu \epsilon_{\nu\rho\sigma\mu} \xi^\rho q^\sigma \right) \gamma_5 v(p_2), \quad (3a)$$

and

$$\langle {}^3\mathrm{H}{}^3\mathrm{H}|A_\mu^\dagger(0)|{}^6\mathrm{Li}\rangle = \eta\bar{u}(p_1)\left(F_A\xi_\mu + F_P\frac{\xi\cdot Q}{M_L}q_\mu\right)\gamma_5 v(p_2), \quad (3b)$$

where $\eta=[m_T^2/(E_1E_2)]^{1/2}(2\pi)^{-1/2}(2L_0)^{-1/2}$, m_T and M_L are the triton and ^{6}Li masses respectively, L_μ is the ^{6}Li four-momentum, ξ_μ is the ^{6}Li polarization vector, E_1 and E_2 are the triton energies, $T_{1\mu}$ and $T_{2\mu}$ are the triton four-momenta, and

$$Q_\mu = T_{1\mu}+T_{2\mu}, \quad q_\mu = T_{1\mu}+T_{2\mu}-L_\mu, \quad P_\mu = T_{1\mu}-T_{2\mu}. \quad (4)$$

Equations (3a) and (3b) are completely determined by the form factors F_1, F_2 and F_A, F_P respectively. These have been obtained by the author[3] from data on the reactions[8-10] $\gamma+{}^6\mathrm{Li}\to{}^3\mathrm{H}+{}^3\mathrm{He}$, ${}^3\mathrm{H}+{}^3\mathrm{He}\to{}^6\mathrm{Li}+\gamma$, and $\pi^-+{}^6\mathrm{Li}\to{}^3\mathrm{H}+{}^3\mathrm{H}$ by using the conserved vector current hypothesis[11] and the partially conserved axial vector current hypothesis[12] respectively. They are

$$F_i(Q\cdot L, q^2, P\cdot L)\simeq F(Q\cdot L, P\cdot L)f_i(q^2), \quad i=1,2,A,P \quad (5)$$

where

$$|F(Q\cdot L,P\cdot L)|^2=\{1-0.33\exp[-9.59\times10^{-2}(q_0-20)^2]$$
$$\times\{20.84+2.01\exp[-1.589\times10^{-3}(q_0-95)^2]-0.3517q_0\exp[-1.589\times10^{-5}(q_0-16.5)^2]\}$$
$$\times[(q_0-16.5)^2+12.04]^{-1}, \quad 0\le q_0\le 75.659\ \mathrm{MeV}, \quad (6a)$$

$$|F(Q\cdot L,P\cdot L)|^2=1.15\times10^{-5}\times(105.659-q_0),\ 75.659\le q_0\le 105.659\ \mathrm{MeV}, \quad (6b)$$

and where[13] $q_0=(Q-L)\cdot L/M_L$ and all units are compatible with q_0 being expressed in MeV, and

$$f_A(q)=\frac{f_A(0)}{1-q^2/M_A^2}, \quad (7a)$$

$$M_A^2=4.95m_\pi^2, \quad (7b)$$

$$f_A(0)=0.296, \quad (7c)$$

$$|f_1(q^2)-f_2(q^2)|=\frac{|f_1(0)-f_2(0)|}{1-q^2/M_V^2},$$

$$M_V^2\simeq 4.95m_\pi^2,$$

$$|f_1(0)-f_2(0)|\simeq 4.49.$$

The derivations of Eqs. (5), (6a), (6b), (7a), (7b), and (7c) are all described in Ref. 3.

III. DIFFERENTIAL CAPTURE RATE

The matrix element squared for the process $\mu^-+{}^6\mathrm{Li}\to{}^3\mathrm{H}+{}^3\mathrm{H}+\nu_\mu$ is given by

$$|M|^2=\frac{2}{6m_\mu m_T^2}\left\{F_A^2(m_\mu+M_L)\left(\frac{E_1+E_2}{2}\right)\left[3m_\mu(m_\mu+M_L-E_1-E_2)+\frac{2m_\mu^2(m_\mu+M_L-E_1-E_2)^3}{(2m_\mu(E_1+E_2)-2m_\mu M_L-m_\pi^2)}\right.\right.$$
$$\left.\left.+\frac{m_\mu^3(m_\mu+M_L-E_1-E_2)^3}{(2m_\mu(E_1+E_2)-2m_\mu M_L-m_\pi^2)^2}\right]+(F_1-F_2)^2m_\mu(m_\mu+M_L-E_1-E_2)^3\right\}. \quad (8)$$

From this equation the differential capture rate $(d\Gamma/d\Omega)(\mu^-+{}^6\mathrm{Li}\to\nu_\mu+{}^3\mathrm{H}+{}^3\mathrm{H})$ as a function of the angle between the outgoing tritons given in Fig. 1 is obtained. The result when integrated over $d\Omega$ yields a value for Γ in agreement with Ref. 3.

We have also obtained a value for $\Gamma(\mu^-+{}^6\mathrm{Li}\to{}^3\mathrm{H}+{}^3\mathrm{H}+\nu_\mu)$ for which the value of the outgoing neutrino energy is restricted to be less than 29.8 MeV,

$$\Gamma=0.145\ \mathrm{s}^{-1}. \quad (9)$$

The significance of this number will be discussed

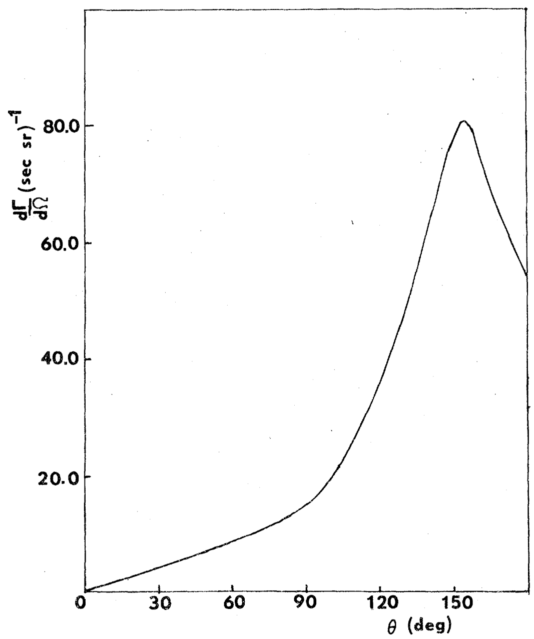

FIG. 1. Plot of the differential capture rate $d\Gamma/d\Omega$ as a function of θ, the angle between the two outgoing tritons.

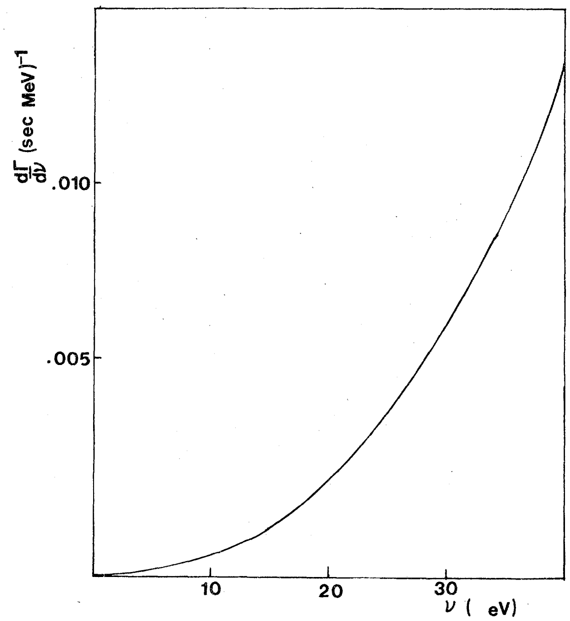

FIG. 2. Plot of the differential capture rate $d\Gamma/d\nu$ as a function ν, the neutrino energy, from 0 to 40 MeV.

in the conclusion.

Finally, in Fig. 2 we give $d\Gamma/d\nu$, the differential capture rate as a function of the outgoing neutrino energy from 0 to 40.0 MeV.

IV. CONCLUSION

A continuing interest[1] has centered on the muon-capture reaction $\mu^- + {}^6\text{Li} \rightarrow {}^3\text{H} + {}^3\text{H} + \nu_\mu$ as a possible means of improving the bounds on the mass of the muon neutrino which is currently $m_{\nu_\mu} \leq 1.6$ MeV. The standard reaction for measuring this mass is a two-body final state reaction such as pion decay, $\pi^- \rightarrow \mu^- + \nu_\mu$. However, in this reaction, the neutrino energy $\nu = 29.8$ MeV and so a mass below the 1–2 MeV range is difficult to observe.

In the reaction $\mu^- + {}^6\text{Li} \rightarrow {}^3\text{H} + {}^3\text{H} + \nu_\mu$, the three-body final state allows the selection of an outgoing configuration in which the tritons carry off most of the available energy. Ideally, if the two tritons were to go out at 180° with equal energy, the muon-neutrino energy would be totally contained in its mass (if any). If the tritons[1] carry off most of the energy, we may write

$$E_\nu \simeq m_{\nu_\mu} + \nu^2/2m_{\nu_\mu} = M_L + m_\mu - E_1 - E_2 , \qquad (10)$$

which is linear in $1/m_{\nu_\mu}$, the nuetrino mass.

In order to examine this possibility in view of the calculation presented here we look only at those cases for which the muon-neutrino energy is below that of the two-body pion decay, $\pi^- \rightarrow \mu^- + \nu_\mu$, namely 29.8 MeV.

If we confine ourselves to the case for which the two tritons go out at 180°, from Fig. 1 we see that $d\Gamma/d\Omega = 51.0(\text{s sr})^{-1}$ and is not negligible. However, this number includes all possible sharing of energy among the outgoing particles. This can be seen by considering the two cases of Fig. 3. In Fig. 3(a), the two tritons carry away essentially all of the available energy, but in Fig. 3(b) one triton and the neutrino carry away almost all of the energy. In this latter case the neutrino

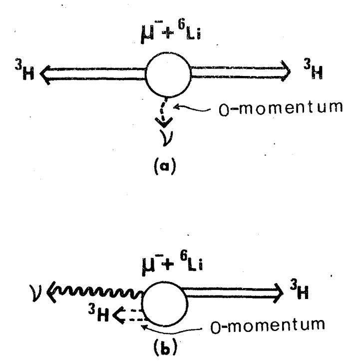

FIG. 3. Diagram of two possible configurations of the outgoing particles in the reaction $\mu^- + {}^6\text{Li} \rightarrow {}^3\text{H} + {}^3\text{H} + \nu_\mu$. In (a) the two tritons carry away all the energy. In (b) one triton and the neutrino carry away the energy. In both cases the angle between the tritons is 180°.

energy (assuming no mass) would be approximately 88.45 MeV. If we then limit the neutrino energy to a maximum of 29.8 MeV, we find that $(d\Gamma/d\Omega)(180°) = 3.24\ (\mathrm{s\ sr})^{-1}$, which is only 6.35% of the total $(d\Gamma/d\Omega)(180°)$ rate.

Contributions to the total capture rate Γ for which the neutrino energy is less than 29.8 MeV occur between the angles of 175.8° and 180°. If we look at these contributions we see that [from Eq. (9)] $\Gamma = 0.145\ \mathrm{s}^{-1}$, which is very small.

Thus we see that the contributions to Γ and $d\Gamma/d\Omega$ for which the experimental situation would be substantially better than that for two-body decays are quite small. The contributions for which the linear approximation Eq. (10) would be valid are, of course, even smaller. Of course the muon neutrino mass can be determined at all angles from the formula

$$m_\nu^2 = (M_L + m_\mu)^2 - 2(E_1 + E_2)(M_L + m_\mu) + 2m^2 + 2E_1E_2 - 2L_1L_2\cos\theta_{12}. \qquad (11)$$

The differential capture rate $(d\Gamma/d\Omega)(\mu^- + {}^6\mathrm{Li} \to \nu_\mu + {}^3\mathrm{H} + {}^3\mathrm{H})$, however, is also interesting in its own right. It is a good test of the elementary-particle model in cases of nuclear breakup. Measurement of $d\Gamma/d\Omega$ would enable a more accurate determination of the weak current form factors describing $\langle {}^3\mathrm{H}{}^3\mathrm{H} | A_\mu^\dagger | {}^6\mathrm{Li}\rangle$ and $\langle {}^3\mathrm{H}{}^3\mathrm{H} | V_\mu^\dagger | {}^6\mathrm{Li}\rangle$ respectively. These could be used in turn for calculating a number of interesting reactions, such as $\nu_\mu + {}^6\mathrm{Li} \to {}^3\mathrm{H} + {}^3\mathrm{He} + \nu_\mu$ which might be a useful test of the Weinberg-Salam theory.[14]

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[12]See M. Gell-Mann and M. Lévy, Nuovo Cimento 16, 705 (1960); Y. Nambu, Phys. Rev. Lett. 4, 380 (1960).

[13]We find that Eq. (6b) gives a better fit to the data in the range indicated, which corresponds to $\nu \le 30$ MeV, than Eq. (6a). This has almost no effect on the total capture rate.

[14]This reaction at low energies would be dominated by the axial neutral current form factor as is true in the case of the deuteron reaction $\nu_\mu + d \to n + p + \nu_\mu$. Although the final state of the reaction $\nu_\mu + {}^6\mathrm{Li} \to {}^3\mathrm{H} + {}^3\mathrm{He} + \nu_\mu$ contains isospin 0 and isospin 1 components, in the Weinberg model the weak neutral current has the form $J_\mu^{(3)} - \sin^2\theta_w J_\mu^{(\mathrm{em})}$ where $J_\mu^{(3)} = V_\mu^{(3)} - A_\mu^{(3)}$ and where the superscript (3) denotes the third component of an isovector. Thus the iospin 0 final state could be connected to the initial state only through the isoscalar part of $J_\mu^{(\mathrm{em})}$ in the Weinberg model. However, as mentioned, we expect the reaction to be dominated in this model at low to intermediate energy by the axial current part of the neutral current (see Ref. 7) $A_\mu^{(3)}$ which, being the third component of an isovector, can connect only the isospin 1 final state to the initial state. As noted in Ref. 3 the vector current contributions to the weak current matrix element are small. Experience and an examination of the data in Ref. 8 would indicate that the same would hold for the isoscalar contributions to $J_\mu^{(\mathrm{em})}$. Any striking departure from this result might suggest possible modifications of the Weinberg model.