Relation between the interacting boson approximation and the quasiparticle formalism

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A direct relationship is demonstrated between the interacting boson approximation and the conventional quasiparticle formalism. Numerical calculations are carried out for the Sn isotopes using a Hamiltonian derived from a realistic nucleon-nucleon interaction.

[NUCLEAR STRUCTURE quasiparticle formalism, interacting boson approximation.]

The interacting boson approximation (IBA) provides a very powerful and flexible formalism for the study of collective states in nuclei, whether they be deformed, vibrational, or in between.[1] Coupled with group theoretical techniques, and with very few free parameters, it has been shown to describe very well both energy level systematics and electromagnetic transition rates. Furthermore, a microscopic interpretation of the s and d bosons in terms of the shell model has been given by Arima, Iachello, Otsuka, and Talmi[2-4]; the agreement between the shell model and this version of the IBA is particularly good in the limit of good seniority.

In this paper, we wish to discuss the relation between the IBA and calculations done in the usual quasiparticle formalism. Let us write the ground state of a spherical nucleus in which only one kind of particle (proton or neutron) is active as[5]

$$| \text{BCS}\rangle = \prod_{\alpha>0} (u_\alpha + v_\alpha a_\alpha^\dagger a_{\bar{\alpha}}^\dagger)|0\rangle , \tag{1}$$

where the $a_\alpha^\dagger$ are particle creation operators and the coefficients v_α and u_α are obtained by solving the generalized Bardeen-Cooper-Schrieffer (BCS) gap equations. The first excited 2^+ state can then be written as

$$\begin{aligned} |\text{TDA}; 2^+\rangle &= \sum_{\alpha\geq\beta} X_{\alpha\beta}\{b_\alpha^\dagger b_\beta^\dagger\}_{2M}|\text{BCS}\rangle \\ &\equiv q_{2M}^\dagger |\text{BCS}\rangle , \end{aligned} \tag{2}$$

where the coefficients $X_{\alpha\beta}$ are obtained by a two quasiparticle Tamm-Dancoff approximation (TDA) calculation and the quasiparticle operators are as usual

$$b_\alpha^\dagger = u_\alpha a_\alpha^\dagger - v_\alpha a_{\bar{\alpha}} .$$

Such a formalism can give an excellent description of the Sn isotopes for example.[6,7]

We can relate this formalism to the IBA by noting that number projection from the above states gives

$$\hat{P}(2n)|\text{BCS}\rangle = N_0\left(\sum_{\alpha>0} \frac{v_\alpha}{u_\alpha} a_\alpha^\dagger a_{\bar{\alpha}}^\dagger\right)^n |0\rangle , \tag{3}$$

$$\hat{P}(2n)|\text{TDA}; 2^+M\rangle = N_2\left(\sum_{\alpha>\beta} \frac{X_{\alpha\beta}}{u_\alpha u_\beta}\{a_\alpha^\dagger a_\beta^\dagger\}_{2M}\right) \times \left(\sum_{\gamma>0}\frac{v_\gamma}{u_\gamma} a_\gamma^\dagger a_{\bar{\gamma}}^\dagger\right)^{n-1}|0\rangle . \tag{4}$$

It is then natural to associate the s and d bosons with the two-nucleon clusters that occur in the above expressions,

$$s^\dagger \longrightarrow S^\dagger \equiv N_S \sum_{\alpha>0}\frac{v_\alpha}{u_\alpha} a_\alpha^\dagger a_{\bar{\alpha}}^\dagger , \tag{5}$$

$$d_M^\dagger \longrightarrow D_M^\dagger \equiv N_D \sum_{\alpha\geq\beta} \frac{X_{\alpha\beta}}{u_\alpha u_\beta}\{a_\alpha^\dagger a_\beta^\dagger\}_{2M} . \tag{6}$$

One can then construct the corresponding boson space Hamiltonian truncated to fourth order:

$$\begin{aligned} H^B = {} & E_s s^\dagger s + E_d \sum_M d_M^\dagger d_M + \tfrac{1}{4} U_0 s^\dagger s^\dagger s s \\ & + U_1 \sum_M d_M^\dagger s^\dagger s d_M + \tfrac{1}{4}\sum_{JM} U_2(J)\{d^\dagger d^\dagger\}_{JM}\{dd\}_{JM} \\ & + \tfrac{1}{2}U_3\sum_M (\{d^\dagger d^\dagger\}_{2M} s d_M + d_M^\dagger s^\dagger \{dd\}_{2M}) \\ & + \tfrac{1}{4}U_4(\{d^\dagger d^\dagger\}_0 s s + s^\dagger s^\dagger\{dd\}_0) . \end{aligned} \tag{7}$$

The coefficients U_i are determined by the requirement that the matrix elements of H^B taken between normalized one and two boson states be equal to the matrix elements of H^F taken between the corresponding, orthonormalized,[2] two and four fermion cluster states.[8,9] Here H^F is the usual fermion space Hamiltonian of the shell model:

$$H^F = \sum_\alpha \epsilon_\alpha a_\alpha^\dagger a_\alpha + \tfrac{1}{4}\sum_{\alpha\beta\gamma\sigma} V_{\alpha\beta\gamma\sigma} a_\alpha^\dagger a_\beta^\dagger a_\gamma a_\sigma . \tag{8}$$

In general, the boson Hamiltonian should contain

TABLE I. The ground state energy and the excitation energy of the first 2^+ state with a semirealistic interaction and the relevant coefficients of H^B. For comparison the other coefficients of H^B for ^{120}Sn are $U_2(0)=0.348$, $U_2(2)=-0.804$, $U_2(4)=-0.208$, $U_3=-0.0786$, and $U_4=-0.350$.

	$E(0^+)$			$E^*(2^+)$					
N	BCS; PROJ	BCS	IBA	TDA	IBA	E_s	E_d	U_0	U_1
4		−3.81	−3.75	1.31	1.23	−1.799	−0.549	−0.293	−0.165
6		−5.83	−5.85	1.36	1.22	−1.782	−0.538	−0.337	−0.182
8	−7.90	−7.90	−8.18	1.41	1.23	−1.762	−0.513	−0.376	−0.194
10	−10.01	−9.98	−10.74	1.47	1.29	−1.746	−0.462	−0.401	−0.201
12	−12.08	−12.05	−13.49	1.51	1.38	−1.739	−0.367	−0.407	−0.202
14	−14.12	−14.14	−16.36	1.50	1.55	−1.740	−0.224	−0.398	−0.193
16	−16.17	−16.28	−19.21	1.45	1.76	−1.745	−0.050	−0.375	−0.178
18	−18.33	−18.52	−22.05	1.37	1.97	−1.750	0.130	−0.350	−0.164
20	−20.65	−20.89	−24.86	1.30	2.14	−1.752	0.295	−0.326	−0.153
22	−23.15	−23.42	−27.58	1.24	2.23	−1.750	0.443	−0.303	−0.148
24	−25.87	−26.14	−30.30	1.20	2.28	−1.746	0.583	−0.283	−0.146
26	−28.81	−29.05	−32.99	1.18	2.30	−1.740	0.716	−0.266	−0.146

higher order terms with six, eight, ten,... operators; however, the calculation of the required coefficients U_i becomes prohibitively difficult. One must then hope that the truncation is all right and proceed accordingly. In fact, this truncation is essentially exact in the case of single j-shell calculations in the seniority scheme, and appear to be very good also in more general calculations with pairing plus quadrupole forces.[4]

The states $(n!)^{-1/2}(s^\dagger)^n|0\rangle$ and $((n-1)!)^{-1/2}d_M^\dagger \times (s^\dagger)^{n-1}|0\rangle$ evidently correspond to the projected BCS and TDA states. One can also proceed to construct more complicated states containing two, three, or more d bosons. In general, the Hamiltonian H^B must be diagonalized in a space of many such states. With the identification of the s and d boson proposed in Eqs. (5) and (6), we have established an equivalence between the number conserving IBA and the quasiboson methods[8, 9] which use the operator $q_M^\dagger$ [Eq. (2)] to define the boson space.

To study how well this approach works, we have applied it to the Sn isotopes. The fermion space includes the five neutron orbitals $d_{5/2}$, $g_{7/2}$, $s_{1/2}$, $d_{3/2}$, and $h_{11/2}$, and the Hamiltonian H^F is derived from a realistic nucleon-nucleon potential. Details of the single particle energies and the Hamiltonian are given in Ref. 10. The results of the BCS and TDA calculations together with the ground state energies calculated in the BCS formalism, but including number projection,[11] are given in Table I. The IBA wave functions which correspond to the projected BCS and TDA states, and which are expected to describe quite accurately the ground state and the first excited 2^+ state of the Sn isotopes without significant mixing of more complicated configurations, are

$$|0^+\rangle = \frac{1}{\sqrt{n!}}(s^\dagger)^n|0\rangle ,$$

$$|2^+\rangle = \frac{1}{[(n-1)!]^{1/2}}\, d_M^\dagger (s^\dagger)^{n-1}|0\rangle .$$

The energies of these states are also given in Table I. For the sake of completeness, we also give in Table II the results obtained with a pure pairing force and the parameters used in Ref. 12 compared to the exact calculations which are possible in this case.[13, 14]

It is clear that, although the IBA works very well for the lighter isotopes, the binding energy of the ground state and the excitation energy of the first 2^+ state are seriously overestimated in the heavier isotopes. It is interesting to note that the structure of the s boson remains roughly independent of neutron number, even though the occupation amplitudes

TABLE II. Ground state energy of the Sn isotopes with a pairing force and the relevant coefficients of H^B.

N	E(exact)	E(BCS)	E(IBA)	E_s	U_e
4	−2.62	−2.27	−2.62	−1.53	0.87
6	−3.26	−2.78	−3.27	−1.52	0.85
8	−3.42	−2.85	−3.48	−1.50	0.84
10	−3.08	−2.45	−3.25	−1.47	0.82
12	−2.21	−1.49	−2.54	−1.42	0.80
14	−0.70	0.21	−1.38	−1.38	0.79
16	2.16	2.93	−0.13	−1.41	0.80
18	5.70	6.43	1.60	−1.45	0.81
20	9.83	10.52	3.95	−1.47	0.83
22	14.50	15.14	6.96	−1.49	0.85
24	19.67	20.24	10.56	−1.50	0.86
26	25.32	25.80	14.84	−1.50	0.88

v_α change, but that the structure of the d boson does change quite markedly.

One is then led to conclude that the truncation of H^B at fourth order is not a sufficiently good approximation when using a relatively large number of bosons. Relatively small empirical changes in the calculated boson interactions can very decidedly improve the IBA results, but the justification of such changes remains a problem. The quasiboson approach (which gives up particle number conservation by using $q_M^\dagger$ as the boson operator) should have an advantage over the IBA at least for vibrational nuclei away from closed shells where a small number of quasibosons is expected to be sufficient.

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