Comments and Addenda

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Comment on "Statistical significance of spreading widths for doorway states"

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We reply to the criticism of our method for the analysis of doorway fine structure.

[NUCLEAR REACTIONS Analysis of doorway fine structure.]

MacDonald[1] has made criticisms of our earlier work[2] in the course of proposing a method for the analysis of doorway fine structure. While we accept MacDonald's results, we reject the implication that they discount our 1974 method. The objective that MacDonald sets himself is different from ours; his claim that our method does not attain *his* objective is true, but in no way detracts from the success of our method in attaining *our* objective.

Our objective was the analysis of actual doorway fine-structure data *taking account of the imperfections of such data*: errors on widths, misassigned levels, truncated sequences. Since these shortcomings are worse in the wings (where one has also the complication of background contributions), we used a method that upweights the central levels of a distribution. Necessarily this was not an elegant, analytical procedure, but a simple practical recipe for handling imperfect data, and extracting a best-fit value of $W = 2\pi\langle M^2\rangle/\langle D\rangle$. This was explicitly discussed below Eq. (27) of our paper.

In contrast, MacDonald's method pays no attention to shortcomings of data, especially in the wings, and proceeds as if ideal data were provided. This leads to an analysis whose spirit is essentially different from ours. To appreciate this difference more fully, let us note that W has two essentially independent roles in the line-broadening situation. First, it determines the degree of spreading of the original pure doorway state; in fact, in strong-coupling situations, W is the width of the distribution. Second, it determines the normalization of the distribution; for instance, in the wings, the strength function equals $W\gamma_D^2/2\pi(E_D - E)^2$, where γ_D^2 is the doorway width, equal to the sum of the widths of the fine-structure levels (in the absence of background strength). Given ideal data, one can immediately determine W from its normalization role by fitting the strength function in the wings. In this sense, given ideal data, determination of W is trivial, and requires no detailed analysis of the kind proposed by MacDonald. With actual data, with its imperfections in the wings and the presence of background strength, the situation is very different for two reasons: (i) The wing strength function is uncertain because of the imperfections; (ii) the presence of background strength causes a double uncertainty, viz., in the separation of background and nonbackground strength functions, and also in the determination of γ_D^2 as the total of all nonbackground widths.

It follows that, when confronted with actual data rather than ideal data, one needs a method which focuses more on the width role of W and less on its normalization role, i.e., it should upweight the central levels of the distribution and downgrade the wings. *The suitability of a method of analysis* *depends on the extent to which it downgrades the wings to a level commensurate with the imperfections of data in the wings.* There is no doubt that our method upweights the central levels more than MacDonald's; in the weak-coupling situation $W=1.57\langle D\rangle$, his evaluations of W from computer-produced distributions (without background) have a spread of only 4% of the mean value, while ours have 40%. Thus his method focuses on the normalization role of W, as revealed in the wings, while ours focuses on the width role of W, as revealed by the central levels (with their statistical fluctuations which cause the large spread).

For data as they exist at present, it is not clear which of the methods is better. MacDonald's method possibly errs on the side of insufficient downgrading of the wings, and produces overoptimistic errors in best-fit W, while ours possibly errs on the side of excessive downgrading, with consequent overpessimistic errors. Only a detailed comparative numerical study of fitting actual data with the two methods can settle the issue. Of course if there were the prospect of a new generation of much more accurate data, then our method would become less appropriate. However, as we have seen, for ideal data, all methods become unnecessary since the determination of W becomes trivial.

APPENDIX: PROOF OF ENSEMBLE THEOREM

In Ref. 2, it was proposed, without proof, that a certain theorem was valid, applying to the ensemble of all mixing patterns with the same mean level spacing and mean square mixing matrix element, viz., that if all members of the ensemble are superposed, the resulting lineshape is Lorentzian. We now give the missing proof. This uses the same assumptions of randomness of levels and matrix elements as in recent reaction theory.[3]

Let us denote a particular member of the ensemble by α; then its R function is

$$R_\alpha(E)\equiv\left(E_0-E-\sum_{i_\alpha}\frac{M_{i_\alpha}{}^2}{E_{i_\alpha}-E}\right)^{-1},$$

where E_0 is the energy of the doorway state (taken to be the same for all α), and i_α label the levels of member α. E_{i_α} are the energies, and M_{i_α} are the coupling matrix elements to the doorway.

If there are $N(\gg 1)$ members of the ensemble, then the R function of the superposed distribution is

$$R(E)\equiv N^{-1}\sum_\alpha R_\alpha(E).$$

The lineshape of the superposed distribution is $\pi^{-1}\,\mathrm{Im}\,R(E+i\epsilon)$, where $(\langle M^2\rangle/D)\gg\epsilon\gg D/N$, $\langle M^2\rangle$, D being the mean square matrix element and level spacing. In the case of strong coupling $\langle M^2\rangle\gg D^2$, we know that the individual $\mathrm{Im}\,R_\alpha(E+i\epsilon)$ are Lorentzian, so it follows that $\mathrm{Im}\,R(E+i\epsilon)$ also has this property. Thus we look at weak coupling $\langle M^2\rangle\ll D^2$, implying $\epsilon\ll D$.

Let us note that the quantity defined by

$$\mathcal{R}(E)\equiv\left[N^{-1}\sum_\alpha R_\alpha{}^{-1}(E)\right]$$

has a smooth strength function, even in weak coupling, viz.,

$$\mathcal{R}(E+i\epsilon)=(E_0-E-i\pi\langle M^2\rangle/D)^{-1}.$$

It follows that the theorem is proved if we have

$$\mathcal{R}(E+i\epsilon)-R(E+i\epsilon)=0.$$

This can be shown to be so, using the same arguments of randomness as in Ref. 3. Expanding $\mathcal{R}$ and R in orders of $M_{i_\alpha}{}^2$, the terms first order in $M_{i_\alpha}{}^2$ cancel directly. Those of second order contain

$$N^{-2}\left[\left(\sum_\alpha m_\alpha\right)^2-N\sum_\alpha m_\alpha{}^2\right],$$

where

$$m_\alpha\equiv\sum_{i_\alpha}\frac{M_{i_\alpha}{}^2}{E_{i_\alpha}-E-i\epsilon}=p_\alpha+iq_\alpha$$

with p_α, q_α real. The randomness assumption gives

$$\left(\sum_\alpha p_\alpha\right)^2=\sum_\alpha p_\alpha{}^2=\pi N\langle M^2\rangle^2/2\epsilon D,$$

$$\sum_\alpha q_\alpha=\pi N\langle M^2\rangle/D,$$

$$\sum_\alpha q_\alpha{}^2=N(\pi\langle M^2\rangle/D)^2+\pi N\langle M^2\rangle^2/2\epsilon D,$$

$$\left(\sum_\alpha p_\alpha\right)\left(\sum_{\alpha'} q_{\alpha'}\right)-\sum_\alpha p_\alpha q_\alpha=2i\left(\frac{\pi N\langle M^2\rangle^2}{2\epsilon D}\right)^{3/2}.$$

From these results, it follows that the second order term vanishes as $N\to\infty$. The vanishing of higher-order terms can be shown similarly.

[1]W. M. MacDonald, Phys. Rev. C 20, 429 (1979).
[2]A. M. Lane, J. E. Lynn, and J. Moses, Nucl. Phys. A232, 189 (1974).
[3]D. Agassi, H. A. Weidenmuller, and G. Mantazouris, Phys. Rep. 22C, No. 3 (1975).