S-wave pion-nucleus dynamics in the $\sigma + \omega$ model

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The $\sigma + \omega$ model of nuclear forces has many desirable properties, including approximate chiral symmetry (partial conservation of axial vector current) and renormalizability, and it gives a good account of nuclear structure. It is therefore a useful framework within which to explore the modification of pion-nucleon dynamics in a nuclear medium, a problem which resists the usual soft-pion current-algebra *cum* analyticity techniques owing to the presence of anomalous thresholds (nuclear structure effects). It is found that through order G^4 (for pion elastic scattering) and G^3 (for pion absorption or emission) the chiral invariance ensures sufficient cancellations between otherwise large terms so as to produce little renormalization of free-particle dynamics. Moreover, the pseudoscalar-coupled $\sigma + \omega$ model is equivalent in pion emission or absorption, through $O(G^3)$, to a pseudovector-coupled phenomenological model (with no π-σ interaction, but with a form of partially conserved axial vector current). Thus, the question of whether one can distinguish experimentally between pseudoscalar and pseudovector πNN coupling is probably meaningless.

[NUCLEAR REACTIONS $A(\pi,\pi)A$, $A(p,\pi)B$, $A(\pi,NN)C$. $E_\pi < 50$ MeV. Wick-formalism treatment of field-theoretic σ-model Lagrangian with ωN coupling. Consequences of broken chiral symmetry for finite-mass pions in nuclei. Nonlinear effects.]

I. INTRODUCTION

The σ model[1] gives a good account of pion-nucleon dynamics at low energies, and especially the small π-N scattering lengths. It is useful to consider how this comes about, from two different viewpoints: First, the large, repulsive, s-wave nucleon-antinucleon pair term [Fig. 1(a)] predicted by the pseudoscalar coupling in Born approximation is almost identically canceled by the large, attractive, σ-exchange term [Fig. 1(b)] to order G^2 in perturbation theory. (The higher-order terms contain similar cancellations, so the full scattering amplitude remains small.) The second viewpoint is to note that the σ model, by construction,[1] predicts a partially-conserved axial-vector current (PCAC)—this dynamical symmetry, together with some plausible assumptions about the analytic structure and kinematic properties of the πN amplitude, allows the proof of a low-energy theorem (analogous to that for Compton scattering[2,3]) relating πN scattering lengths to the pion decay constant.[4,5]

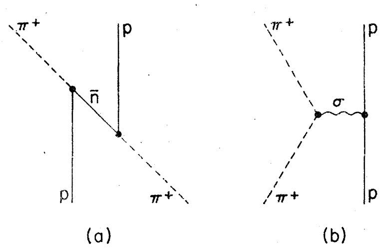

FIG. 1. Time-ordered diagrams representing the dominant contributions to s-wave π-nucleon scattering. (a) Standard PS Born term. (b) σ exchange.

From the first point of view we see that something quite interesting might happen to the πN scattering if the nucleon and pion were no longer isolated, but could interact with the other nucleons in a finite nucleus. In the (on-shell) elastic scattering, we would have to worry about the fact that an $N\bar{N}$ pair might require substantially less energy to create in a nucleus, than in free space—this effect would greatly increase Fig. 1(a) while leaving Fig. 1(b) unchanged, thereby spoiling the cancellation and giving rise to a large (s-wave) repulsive π-nucleus interaction. Such an effect contradicts the low-energy elastic scattering data,[6] and so can be presumed not to take place.

A closely related problem was recently pointed out[7] by Bertsch and Riska: If the process $A(\pi^-,NN)B$ may be thought of as proceeding via the effective two-body amplitude of Fig. 2(a) (neglecting, for the moment, charge-exchange terms), *and* if the πN rescattering may be decomposed as in Fig. 1, into the separate pieces 2(b) and 2(c), we may expect (because the intermediate pion is far from the mass shell) that the σ-exchange term Fig. 2(c) will be substantially diminished relative to the pair term 2(b) by the σ-meson propagator $(1+q_\pi{}^2/m_\sigma{}^2)^{-1} \simeq 0.8$, once again spoiling the cancellation, and (naively) increasing the amplitude fourfold.[7]

From the second point of view (PCAC + pion pole dominance), we can say nothing about what happens in a nucleus, since the presence of anomalous thresholds prevents the application of the pole dominance idea.[5] Put another way, for the πN case

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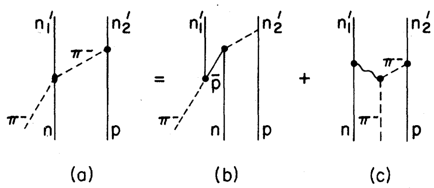

FIG. 2. Decomposition of the s-wave rescattering part of the pion absorption amplitude into basic processes.

it is plausible that with good accuracy we may write $(-m_\pi^2 < q^2 < m_\pi^2)$

$$(q^2 - m_\pi^2)^{-1}K(-q^2) \simeq (q^2 - m_\pi^2)K(-m_\pi^2)\ , \tag{1}$$

where $K(-q^2)$ is the (pseudoscalar) nucleon structure factor, with a "size" $\sim M^{-1} \sim 0.2$ fm (the nucleon "core" radius) much smaller than $m_\pi^{-1} = 1.4$ fm. In the nuclear case, however, the corresponding structure factor would be characterized by a length R (the nuclear radius) at least as large as m_π^{-1}, even for light nuclei, and generally much larger. About all we can do with current algebra, then, is to express the hope that somehow the dynamical symmetry [chiral SU(2)× SU(2)] embodied in PCAC will lead to cancellations which will compensate the large effects noted above. If we wish to study nuclear problems, therefore, we must resort to a specific model.

A variant of the σ model, in which a neutral vector meson (ω) coupled to the (conserved) baryon current has been introduced, has had considerable success in accounting for a number of important nuclear properties.[8-13] Thus it seems worrisome that the $\sigma+\omega$ model should have trouble with pion-nucleus dynamics, especially since it does well for the πN system (where it is essentially the σ model). The object of this paper is to show that a consistent treatment of π-nucleus elastic scattering and absorption in the $\sigma+\omega$ model eliminates the troubles alluded to previously by bringing in additional large corrections which precisely cancel the offending terms.

II. THE $\sigma+\omega$ MODEL

The Lagrangian of the $\sigma+\omega$ model is

$$\begin{aligned}\mathcal{L} = &-\bar{N}[-i\gamma^\mu\partial_\mu + M + G_\omega^V\gamma^\mu\omega_\mu + G(\sigma + i\gamma^5\vec{\tau}\cdot\vec{\pi})]N + \tfrac{1}{2}g^{\mu\nu}(\partial_\mu\vec{\pi}\cdot\partial_\nu\vec{\pi} + \partial_\mu\sigma\partial_\nu\sigma) + \tfrac{1}{4}F^{\mu\nu}F_{\mu\nu} - \tfrac{1}{2}m_\omega^2\omega^\mu\omega_\mu \\ &- \tfrac{1}{2}m^2\vec{\pi}\cdot\vec{\pi} - \frac{1}{2}\left(m^2 + \frac{8\lambda M^2}{G^2}\right)\sigma^2 - \lambda\left[(\vec{\pi}\cdot\vec{\pi}+\sigma^2)^2 + \frac{4M}{G}\sigma(\vec{\pi}\cdot\vec{\pi}+\sigma^2)\right]\ .\end{aligned} \tag{2}$$

The inclusion of the vector field ω^μ alters neither the conservation of the vector current

$$\partial_\mu\vec{V}^\mu = 0\ , \tag{3}$$

nor the partial conservation of the axial current

$$\partial_\mu\vec{A}^\mu = -\frac{2Mm^2}{G}\,g_A\vec{\pi}\ , \tag{4}$$

nor the renormalizability[14] of the theory.

The average nuclear dynamics (in the Hartree-Fock sense) is given by the effective one-body Dirac Hamiltonian

$$H_{\rm eff} = -i\vec{\alpha}\cdot\nabla + \beta M + \beta U + V\ \ (\vec{\alpha} = \gamma^0\vec{\gamma},\ \beta = \gamma^0)\ , \tag{5}$$

where

$$U(\vec{r}) = G\langle \text{g.s.}|\sigma(\vec{r})|\text{ g.s.}\rangle\ ,\ V(\vec{r}) = G_\omega\langle \text{g.s.}|\omega^0(\vec{r})|\text{g.s.}\rangle\ , \tag{6}$$

and the space components of the ω field, as well as of the pion field, average to zero for reasons of symmetry. As pointed out elsewhere,[13] we may regard the ρ meson as of secondary importance in determining average nuclear properties, and so omit it from the present discussion.

Let us now calculate the diagrams, Figs. 1(a) and 1(b), in canonical fashion, taking the spinors to be solutions of the Dirac equation with $H_{\rm eff}$ from Eq. (5), rather than of the free Dirac equation: We then find the effective π-A potentials, to order G^4, to be

$$v_{\rm eff} \simeq (2k^0)^{-1}\frac{G^2}{M}\,\frac{V(r)}{M}\,\langle \text{g.s.}|N^\dagger(\vec{r})N(\vec{r})|\text{g.s.}\rangle\ . \tag{7}$$

Equation (7) represents a repulsive potential (some 300 MeV strong at the nuclear center) which behaves like $\rho^2(r)$. Clearly, such a thing is not observed—what has gone wrong? The source of the difficulty can only be that in keeping certain $O(G^4)$ terms leading to Eq. (7) (which corresponds to the covariant diagrams of Fig. 3), we have neglected other diagrams of the same order and density dependence.

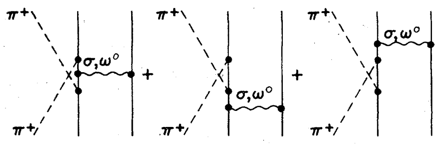

FIG. 3. Covariant diagrams representing part of the $O(G^4)$ effects on the *PS* Born term.

III. ORDER-G^4 CORRECTIONS TO ELASTIC SCATTERING

In order to identify the missing terms, we write the elastic π^+-nucleus scattering amplitude in the Wick[15] formalism

$$T_{\vec{k}'\vec{k}}=\langle A|\mathcal{J}^\dagger_{\vec{k}'}|\Psi_i^{(+)}\rangle\,, \tag{8}$$

where

$$\mathcal{J}_{\vec{k}'}=[H,a^\dagger_{\vec{k}'}]-k^{0\prime}a^\dagger_{\vec{k}'} \tag{9}$$

is the source term in the pion field equation, $|A\rangle$ is the target wave function (containing no asymptotic pions), and $a^\dagger_{\vec{k}'}$ is the creation operator for a positive pion of momentum $\vec{k}'$. The initial-state wave function is

$$|\Psi_i^{(+)}\rangle=a^\dagger_{\vec{k}}|A\rangle+(E_A+k^0+i\eta-H)^{-1}\mathcal{J}_{\vec{k}}|A\rangle\,. \tag{10}$$

The portion of the Hamiltonian containing pion interactions is

$$H'=\int d^3x\, iGN^\dagger(\vec{x})\gamma^0\gamma^5\vec{\tau}\cdot\vec{\pi}(\vec{x})N(\vec{x})+\lambda[\vec{\pi}\cdot\vec{\pi}+\sigma^2]^2 + \frac{4M\lambda}{G}\sigma(\vec{x})(\vec{\pi}\cdot\vec{\pi}+\sigma^2)\,, \tag{11}$$

and using Eq. (11) in the expression (9) for the source term $\mathcal{J}_{\vec{k}}$ we find

$$\mathcal{J}_{\vec{k}}=\int\frac{d^3x\,e^{i\vec{k}\cdot\vec{x}}}{(2\pi)^{3/2}(2k^0)^{1/2}}\Big\{iGN^\dagger(\vec{x})\gamma^0\gamma^5\tau_+N(\vec{x}) + \frac{8M\lambda}{G}\sigma(\vec{x})\pi_+(\vec{x}) + 4\lambda(\vec{\pi}\cdot\vec{\pi}+\sigma^2)\pi_+(\vec{x})\Big\}\,. \tag{12}$$

Putting Eq. (12) into Eq. (8) and performing the requisite commutations, as well as using the relation

$$a_{\vec{k}}|A\rangle=(E_A-k^0-H)^{-1}\mathcal{J}^\dagger_{\vec{k}}|A\rangle\,, \tag{13}$$

we find for the on-shell elastic amplitude the exact expression

$$T_{\vec{k}'\vec{k}}=\int\frac{d^3x\,e^{i(\vec{k}-\vec{k}')\cdot\vec{x}}}{(2\pi)^3\,2k^0}\left\langle A\left|\left\{\frac{8M\lambda}{G}\sigma(\vec{x})+4\lambda[\sigma^2(\vec{x})+\vec{\pi}(\vec{x})\cdot\vec{\pi}(\vec{x})]+8\lambda\pi_+(\vec{x})\pi_-(\vec{x})\right\}\right|A\right\rangle + \langle A|\mathcal{J}_{\vec{k}}(E_A-k^0-H)^{-1}\mathcal{J}^\dagger_{\vec{k}'}+\mathcal{J}^\dagger_{\vec{k}'}(E_A+k^0+i\eta-H)^{-1}\mathcal{J}_{\vec{k}}|A\rangle\,. \tag{14}$$

There are several ways to approach the reduction of Eq. (14): (a) We might attempt to eliminate the antinucleon states by means of a suitable Foldy-Wouthuysen transformation. Unfortunately, the usual FW transformation[16] does not preserve the chiral invariance which characterizes the $\sigma+\omega$ model and leads to PCAC, and we have so far failed to find one which *does* maintain PCAC. (b) A second approach is to insert various sorts of intermediate states between the pion field operators appearing quadratically in the first term of (14), and between the source operators in the second term. (Naturally, this expansion would be truncated at some point.) In this way we should be able to obtain coupled Low equations for elastic and inelastic π-A scattering, including crossing, analogous to those derived by, e.g., Miller.[17] Here we shall stop with the Born terms appropriate to π-nucleus elastic scattering in s waves, to order G^2 and G^4, since our aim is to show that the large $O(G^4)$ Born term Eq. (7) is appropriately canceled.

From Eq. (14) and Eq. (12) we can easily find the terms in $T_{\vec{k}'\vec{k}}$ which are of orders G^2 and G^4: To (overt) order G^2 we have

$$T^{(2)}_{\vec{k}'\vec{k}}=\int\frac{d^3x\,e^{i(\vec{k}-\vec{k}')\cdot\vec{x}}}{(2\pi)^3\,2k^0}\left\langle A\left|\frac{8M\lambda}{G}\sigma(\vec{x})\right|A\right\rangle + \frac{(\sqrt{2}\,iG)^2}{(2\pi)^3 2k^0}\int d^3x\int d^3x'\,e^{i(\vec{k}\cdot\vec{x}-\vec{k}\cdot\vec{x}')}\{\langle A|\bar{N}_p(\vec{x})\gamma^5N_n(\vec{x})(E_A-k^0H)^{-1}\bar{N}_n(\vec{x}')\gamma^5N_p(\vec{x}')|A\rangle + \langle A|\bar{N}_n(\vec{x}')\gamma^5N_p(\vec{x}')(E_A+k^0+i\eta-H)^{-1}\bar{N}_p(\vec{x})\gamma^5N_n(\vec{x})|A\rangle\}\,, \tag{15}$$

whereas the terms which are (overtly) of order G^4 are

$$T^{(4)}_{\vec{k}'\vec{k}}=\int d^3x\int d^3x'\,\frac{e^{i(\vec{k}\cdot\vec{x}-\vec{k}'\cdot\vec{x}')}}{(2\pi)^3\,2k^0}\,\{\langle A|\,4\lambda[\sigma^2(\vec{x})+\vec{\pi}(\vec{x})\cdot\vec{\pi}(\vec{x})+2\pi_+(\vec{x})\pi_-(\vec{x})]\,|A\rangle\delta(\vec{x}-\vec{x}')\}$$
$$+(iG\sqrt{2})\,\frac{8M\lambda}{G}\langle A|[\overline{N}_p(\vec{x})\gamma^5N_n(\vec{x})(E_A-k^0-H)^{-1}\sigma(\vec{x}')\pi_-(\vec{x}')+\sigma(\vec{x})\pi_+(\vec{x})(E_A-k^0-H)^{-1}\overline{N}_n(\vec{x}')\gamma^5N_p(\vec{x}')$$
$$+\overline{N}_n(\vec{x}')\gamma^5N_p(\vec{x}')(E_A+k^0+i\eta-H)^{-1}\sigma(\vec{x})\pi_+(\vec{x})+\sigma(\vec{x}')\pi_-(\vec{x}')$$
$$\times(E_A+k^0+i\eta-H)^{-1}\overline{N}_p(\vec{x})\gamma^5N_n(\vec{x})]\,|A\rangle$$
$$+\left(\frac{8M\lambda}{G}\right)^2\langle A|\,\sigma(\vec{x})\pi_+(\vec{x})[(E_A-k^0+i\eta-H)^{-1}+(E_A-k^0-H)^{-1}]\,\sigma(\vec{x}')\pi_-(\vec{x}')+\text{H.c.}|A\rangle\,. \tag{16}$$

The $O(G^4)$ portion of the last part of (16) arises from diagrams such as Fig. 4. Now, in fact, the claim that Eq. (15) is of order G^2, and Eq. (16) of order G^4 is misleading: The interacting fields actually contain all orders of the coupling constants, as do the wave functions $|A\rangle$ (since they are eigenstates of H). The best we can do, therefore, is to say Eq. (15) is *at least* of order G^2, and Eq. (16), at least of order G^4.

The leading contributions to Eq. (15) correspond to the diagrams in Figs. 1(a) and 1(b) which we write as

$$T^{(2)}_{\vec{k}'\vec{k}}=-\frac{G^2}{2Mk^0}\,\frac{m_\sigma{}^2-m^2}{m_\sigma{}^2+(\vec{k}-\vec{k}')^2}\int\frac{d^3x}{(2\pi)^3}\,e^{i(\vec{k}-\vec{k}')\cdot\vec{x}}\langle A|\,N^\dagger(\vec{x})\gamma^0N(\vec{x})|\,A\rangle$$
$$+\frac{G^2}{2Mk^0}\int\frac{d^3x}{(2\pi)^3}\,e^{i(\vec{k}-\vec{k}')\cdot\vec{x}}\,\langle A|\,N^\dagger(\vec{x})N(\vec{x})|\,A\rangle\,. \tag{17}$$

Equation (17) follows from Eq. (15) upon substitution for $\sigma(\vec{x})$, the lowest-order (in G) contribution to it

$$\sigma_0(\vec{x})=-\frac{G}{4\pi}\int d^3x'\,\frac{e^{-m_\sigma|\vec{x}-\vec{x}'|}}{|\vec{x}-\vec{x}'|}\,N^\dagger(\vec{x}')\gamma^0N(\vec{x}')\,, \tag{18}$$

and upon taking as the most important class of intermediate states in the Green's functions $(Z-H)^{-1}$, the states containing a nucleon-antinucleon pair in addition to the A nucleons in the state $|A\rangle$. (Since such $N\overline{N}$ pair excitations have energy ~ $2M$ greater than the state $|A\rangle$, we may take both Green's functions to be constant, neglecting k^0 and the internal excitation energies.) Since, to this order,

$$\langle A|\,N^\dagger(\vec{x})\gamma^0N(\vec{x})|\,A\rangle\simeq\langle A|\,N^\dagger(\vec{x})N(\vec{x})|\,A\rangle\,, \tag{19}$$

we see that at low pion energies there is almost complete cancellation between the two terms of Eq. (17), exactly as for free nucleons.

There are three $O(G^4)$ contributions from Eq. (15), two extremely important, the other somewhat smaller. Suppose, for a moment, that the potentials in (5) were constant: Then we would have

$$\langle N^\dagger\gamma^0N\rangle\simeq\left(\frac{1+U/M}{1-V/M}\right)\langle N^\dagger N\rangle\simeq[\,1+(U+V)/M]\langle N^\dagger N\rangle\,. \tag{20}$$

Although it is correct to say that $(U+V)/M$ is $O(G^2)$, it is a relatively small correction, of order 0.1. On the other hand, the energy required to make an $N\overline{N}$ pair would be

$$2M+U-V\simeq M\,, \tag{21}$$

which amounts to a factor of 2 correction to the repulsive term of (17). In fact, a more careful treatment, using the effective Hamiltonian (5), leads to

$$\hat{T}^{(4)}_{\vec{k}'\vec{k}}=\frac{G^2G_\omega{}^2}{2M^2k^0}\int\frac{d^3x}{(2\pi)^3}\int d^3x'\,e^{i(\vec{k}-\vec{k}')\cdot\vec{x}}\,\frac{e^{-m_\omega|\vec{x}-\vec{x}'|}}{4\pi|\vec{x}-\vec{x}'|}\langle A|\,N^\dagger(\vec{x})N^\dagger(\vec{x}')N(\vec{x}')N(\vec{x})|A\rangle\,. \tag{22}$$

Equation (22) is just Eq. (7) corrected to eliminate such self-diagrams as those in Fig. 5, which are in principle already included in the vertex and propagator renormalizations. [Equation (20) should be regarded as similarly corrected.]

When we look for additional $O(G^4)$ contributions from Eq. (15), we find only two further classes: The first comes from expanding states such as $N(\vec{x})|A\rangle$ in powers of the interaction:

$$N(\vec{x})|\,A\rangle=N^{(0)}(\vec{x})|\,A\rangle+\int\frac{d^3q}{(2\pi)^{3/2}}\,e^{i\vec{q}\cdot\vec{x}}(H-E_A-\epsilon_q)^{-1}J_{\vec{q}}|\,A\rangle\,. \tag{23}$$

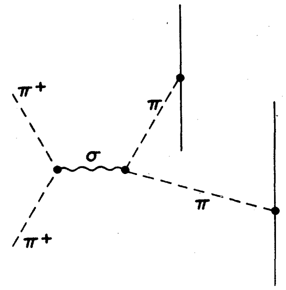

FIG. 4. Elastic scattering of the external pion from the nuclear pion field, mediated by σ exchange.

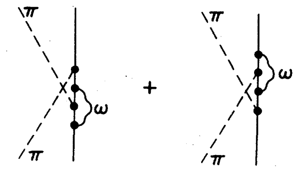

FIG. 5. Typical self-diagrams which should be omitted since they are implicitly contained in the renormalized masses and coupling constants.

The "current" $J_q^{\mp}$ contains π, σ, and ω fields, and the new term will give rise to diagrams such as Fig. 6, in which the pion is absorbed (or emitted) by one nucleon, which at some point exchanges an ω or a neutral pion with a second nucleon, which then emits (or absorbs) the pion. For charged pions this can only occur on an np pair. The reason that only the ω or π^0 exchange is important in Fig. 6 is that in order to have a large matrix element for two mesons on one nucleon, the product of two even or two odd operators is necessary, giving $\vec{\gamma}\gamma^5 \equiv \gamma^0\vec{\sigma}$ for the case of ω exchange (the external pion introduces a γ^5 on each line). Because of the antinucleon intermediate states which dominate Fig. 6, none of the four amplitudes can be considered part of the model wave function $|A\rangle$, by contrast with the *time* component of ω exchange which, being even, is included. Thus no double counting is involved here. As it happens, the s-wave parts of the graphs in Fig. 6 sum to zero both for ω and π^0 exchange. There is no important σ-exchange amplitude because the σNN coupling is an even operator. Finally, although the $\sigma+\omega$ model contains no *isoscalar*, pseudoscalar meson (for which the amplitude corresponding to Fig. 6 would not vanish), the η meson *is* found in nature and one might reasonably inquire why it should be neglected. The answer is that the ηNN coupling constant is badly known, but small,[18] and so η exchange can be neglected.

The only significant $O(G^4)$ correction arising from Eq. (15) [other than (22)] is a correction to the nuclear σ field. That is, the (time-independent) equation of motion for the σ meson is [from Eq. (2)]

$$m_\sigma^2\sigma(\vec{x})\left\{1+\frac{3}{2}\,\frac{G\sigma(\vec{x})}{M}+\frac{1}{2}\left[\frac{G\sigma(\vec{x})}{M}\right]^2\right\}-\nabla^2\sigma(\vec{x}) = -G\bar{N}(\vec{x})N(\vec{x})\,. \quad (24)$$

Thus, if we solve by successive approximation (and make sure to normal-order all nucleon operators at the end, consistent with our "tree" approximation) we have

$$\sigma(\vec{x})=\sigma_0(\vec{x})+\sigma_1(\vec{x})+\cdots\,,$$

where $\sigma_0(\vec{x})$ satisfies Eq. (18), and

$$(m_\sigma^2-\nabla^2)\sigma_1(\vec{x}) = -\tfrac{3}{2}m_\sigma^2\frac{G}{M}\,\sigma_0^2(\vec{x})\,, \quad (25)$$

or

$$\left\langle A\left|\frac{8M\lambda}{G}\,\sigma_1(\vec{x})\right|A\right\rangle = -\tfrac{3}{2}m_\sigma^4\frac{G^4}{M^2}\int d^3x'\int d^3x''\,D(\vec{x},\vec{x}',\vec{x}'')\,\langle A|N^\dagger(\vec{x}')N^\dagger(\vec{x}'')N(\vec{x}'')N(\vec{x}')|A\rangle\,, \quad (26)$$

where

$$D(\vec{x},\vec{y},\vec{z}) = \int d^3r\,\frac{\exp[-m_\sigma(|\vec{x}-\vec{r}|+|\vec{y}-\vec{r}|+|\vec{z}-\vec{r}|)]}{(4\pi)^3|\vec{x}-\vec{r}|\,|\vec{y}-\vec{r}|\,|\vec{z}-\vec{r}|}\,. \quad (27)$$

The σ_1 field, Eq. (25), corresponds to the process of Fig. 7.

We are now in a position to calculate the overtly G^4 terms from Eq. (16). Certain self-terms, such as Fig. 8(a), should be omitted because they are vertex or propagator renormalizations, whereas others [Fig. 8(b)] are known (small) corrections to the πN scattering amplitudes. The remaining contributions from the terms in (16) linear in $\sigma(\vec{x})\pi_\pm(\vec{x})$ are small, as we shall see in Sec. IV below [see Eqs. (42)–(44)].

The contribution from the term in (16) corresponding to Fig. 4 [this is obtained by introducing the one-σ intermediate states in the Green's function between $\sigma(\vec{x})$ and $\sigma(\vec{x}')$, and neglecting the excitation energies by comparison with m_σ] precisely cancels that from all the charged meson terms in the first bracket, under the approximation that

$$\delta(\vec{x}-\vec{x}') - \frac{m_\sigma^2 e^{-m_\sigma|\vec{x}-\vec{x}'|}}{4\pi|\vec{x}-\vec{x}'|}\simeq 0\,. \quad (28)$$

This cancellation is more general than it might

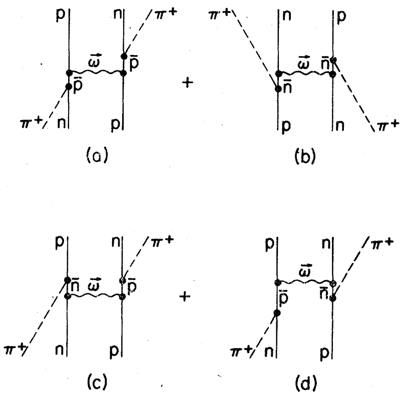

FIG. 6. Elastic pion scattering from a neutron-proton pair, mediated by exchange of a spacelike ω meson.

seem at first: The terms quadratic in charged-pion fields correspond to the time-ordered diagrams Figs. 9(a) and 9(b) (with time increasing upward), in which the blobs are s-wave $\pi\pi$ elastic amplitudes (including σ exchanges, loops, etc.). In a $T=0$ nucleus, both terms will appear with equal weight, thus, only the vanishing linear combination of $\pi\pi$ amplitudes[4]

$$\tfrac{7}{12}t_2+\tfrac{1}{6}t_0=0 \tag{29}$$

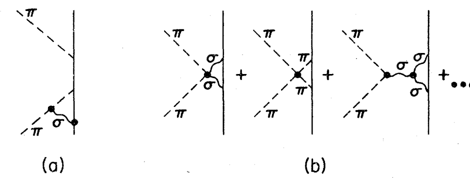

FIG. 8. (a) Typical $O(G^4)$ term which should be omitted because it represents a contribution to the π-nucleon PS structure function. (b) Terms representing known, small corrections to the π-nucleon scattering amplitude.

appears, so we expect the cancellation to persist in higher orders. We are left with the $\pi^0\pi^0$ term which, after summing all contributions, is

$$T^{(\pi\pi)}_{\vec{k}'\vec{k}}\simeq\frac{G^2m^2}{4M^2k^0(2\pi)^3}\int d^3x\, e^{i(\vec{k}-\vec{k}')\cdot\vec{x}}\langle A|[\pi^0(\vec{x})]^2|A\rangle . \tag{30}$$

To obtain (30) we have assumed pure $T=2$, s wave for the $\pi^+\pi^0$ elastic amplitude, and have used Weinberg's theoretical value[4] of t_2 (but with $g_A=1$ for consistency).

Combining all $O(G^4)$ terms including (20), (22), (26), and (30), we obtain

$$\begin{aligned}T^{(4)}_{\vec{k}'\vec{k}}\simeq\int d^3x\int d^3x'\,(2\pi)^{-3}(2k^0)^{-1}\Big\{&\langle A|N^\dagger(\vec{x})N^\dagger(\vec{x}')N(\vec{x}')N(\vec{x})|A\rangle\\&\times\frac{G^4}{M^2}\left[\frac{e^{-m_\sigma|\vec{x}-\vec{x}'|}}{4\pi|\vec{x}-\vec{x}'|}e^{i(\vec{k}-\vec{k}')\cdot\vec{x}}\right.\\&+\frac{m_\sigma}{16\pi}e^{-m_\sigma|\vec{x}-\vec{x}'|}e^{i(\vec{k}-\vec{k}')\cdot(\vec{x}-\vec{x}')/2}(1-m^2/m_\sigma^2)\\&\left.-\tfrac{3}{2}m_\sigma^4\left(1-\frac{m^2}{m_\sigma^2}\right)^2\int d^3x''\,D(\vec{x},\vec{x},\vec{x}'')e^{i(\vec{k}-\vec{k}')\cdot\vec{x}''}\right]\\&-\frac{G^4m^3}{192\pi M^2}e^{-m|\vec{x}-\vec{x}'|}e^{i(\vec{k}-\vec{k}')\cdot(\vec{x}-\vec{x}')/2}\langle A|N^\dagger(\vec{x})N^\dagger(\vec{x}')\vec{\sigma}\cdot\vec{\sigma}\tau_3\tau_3'N(\vec{x}')N(\vec{x})|A\rangle\Big\}.\end{aligned} \tag{31}$$

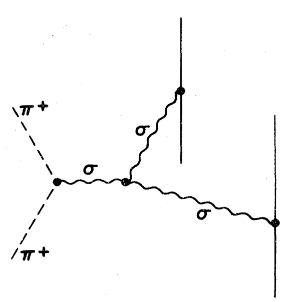

FIG. 7. Elastic scattering of the external pion from the nuclear σ field, mediated by σ. This process may also be regarded as a nonlinear effect of the dynamical equation of the σ.

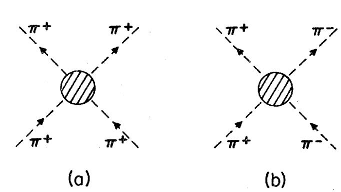

FIG. 9. Time-ordered diagrams representing elastic scattering of the external (positive) pion from the charged nuclear pion field.

By neglecting the range of the propagators, we obtain the approximate, equivalent (repulsive) local potential

$$v_{\rm eff} \simeq (2k^0)^{-1}[U(r)/M]^2 \times\left(\tfrac{5}{2}m^2\int_{r_c m_\sigma}^{\infty} dx\, x^2\frac{e^{-x}}{2}+\frac{m_\sigma^4}{32M^2}\right). \qquad (32)$$

This potential has a central depth of ~ 31–42 MeV, depending on how we handle short-range correlations, which is about what is needed in pion-nucleus low-energy scattering.[6] Note that the first term in the parentheses in (32) comes from the inexact cancellations in the bracketed terms in (31)(with a correction for an NN hard core), whereas the second term was from the $T=2$ part of $\pi\pi$ scattering, which is singled out for a nucleus with $N\simeq Z$.

IV. PION ABSORPTION NEAR THRESHOLD–EFFECTS OF ORDER G AND G^3

We consider here the process $\pi^- + A \to$ anything (where "anything" means a state with no explicit pions) very near the threshold. The amplitude for this process is

$$T_{fi} = \langle B^{(-)}|\mathcal{J}_{\vec{k}}^{\dagger}|A\rangle, \qquad (33)$$

where $|B^{(-)}\rangle$ is the final continuum nuclear wave function (with incoming-wave boundary conditions, as the superscript reminds us), and $\mathcal{J}_{\vec{k}}^{\dagger}$ is the current

$$\mathcal{J}_{\vec{k}}^{\dagger} = \int \frac{d^3x}{(2\pi)^{3/2}(2k^0)^{1/2}}\, e^{i\vec{k}\cdot\vec{x}}\Big[iG\sqrt{2}\,N_n^{\dagger}(\vec{x})\gamma^0\gamma^5 N_p(\vec{x}) + 4\lambda(\sigma^2+\vec{\pi}\cdot\vec{\pi})\pi_-(\vec{x}) + \frac{8M\lambda}{G}\sigma(\vec{x})\pi_-(\vec{x})\Big]. \qquad (34)$$

If we were speaking of a pionic atom in its ground state, the pion plane wave would be replaced by an appropriate pionic atom wave function, e.g.,

$$\frac{e^{i\vec{k}\cdot\vec{r}}}{(2\pi)^{3/2}} \to (Z\alpha m)^{3/2}\pi^{-1/2}e^{-Z\alpha m r}, \qquad (35)$$

and we would speak of a capture rate, rather than a T matrix. In either case, we shall assume that the variation of the "free" pion wave function over the nuclear region is slow.

Performing the same hierarchical ordering as in the previous section, we find that the $\lambda(\sigma^2+\vec{\pi}^2)\pi_-$ term is $O(G^5)$ and so can be dropped, whereas the $N_n^{\dagger}\gamma^0\gamma^5N_p$ term is overtly of order G, but in fact contains an important (dominant) order-G^3 part due to interactions. Finally, the $\sigma(\vec{x})\pi_-(\vec{x})$ term is of order G^3 *ab initio*. Thus,

$$T_{fi} \simeq \int \frac{d^3x}{(2\pi)^{3/2}(2k^0)^{1/2}}\Big[\langle B^{(-)}|iG\sqrt{2}N_n^{\dagger}(\vec{x})\gamma^0\gamma^5N_p(\vec{x})|A\rangle + (m_\sigma^2-m^2)\frac{G}{M}\langle B^{(-)}|\sigma(\vec{x})\pi_-(\vec{x})|A\rangle\Big]. \qquad (36)$$

Now to evaluate the first term in (36) we use a trick: We may write

$$\langle B^{(-)}|N_n^{\dagger}(\vec{x})\gamma^0\gamma^5N_p(\vec{x})|A\rangle \equiv \tfrac{1}{2}\sum_C \frac{\langle B|[N_n^{\dagger}(\vec{x}),H]|C\rangle\langle C|\gamma^0\gamma^5N_p(\vec{x})|A\rangle}{(E_C-E_B)} + \tfrac{1}{2}\sum_C \frac{\langle B|N_n^{\dagger}(\vec{x})\gamma^0\gamma^5|C\rangle\langle C|[H,N_p(\vec{x})]|A\rangle}{(E_C-E_A)}. \qquad (37)$$

The state $|C\rangle$ has one fewer nucleon than either $|A\rangle$ or $|B^{(-)}\rangle$, so $E_C-E_B\simeq E_C-E_A\simeq -M$. Thus, performing the indicated commutations (with the *full* Hamiltonian) and using closure, we find

$$\langle B^{(-)}|N_n^{\dagger}(\vec{x})\gamma^0\gamma^5N_p(\vec{x})|A\rangle \simeq -\frac{i}{2M}\nabla\cdot\langle B^{(-)}|N_n^{\dagger}(\vec{x})\gamma^0\vec{\sigma}N_p(\vec{x})|A\rangle - \frac{iG\sqrt{2}}{2M}\langle B^{(-)}|N^{\dagger}(\vec{x})N(\vec{x})\pi_-(\vec{x})|A\rangle + \frac{G_\omega}{M}\langle B^{(-)}|N_n^{\dagger}(\vec{x})\gamma^0\gamma^5N_p(\vec{x})\omega^0(\vec{x})|A\rangle, \qquad (38)$$

which we approximate, to order G^2, as

$$\langle B^{(-)}|N_n^{\dagger}(\vec{x})\gamma^0\gamma^5N_p(\vec{x})|A\rangle \simeq -\frac{i}{2M}\langle B^{(-)}|(1+G_\omega\omega^0(\vec{x})/M)\nabla\cdot N_n^{\dagger}(\vec{x})\gamma^0\vec{\sigma}N_p(\vec{x})|A\rangle - \frac{iG\sqrt{2}}{2M}\langle B^{(-)}|N^{\dagger}(\vec{x})N(\vec{x})\pi_-(\vec{x})|A\rangle. \qquad (39)$$

Putting Eq. (39) into Eq. (36), and assuming we can neglect the variation of the pion wave function over the nuclear volume (this is a correction of order $mRZ\alpha$ for the pionic atom case, and of order kR for capture from a continuum state), we find that the divergence term from (39) drops out, leaving [to $O(G^3)$]

$$T_{fi}\simeq \int d^3x \int d^3x' \frac{G\sqrt{2}}{(2\pi)^{3/2}\sqrt{2k^0}} \left\langle B^{(-)}\middle| N_n^\dagger(\vec{x})\vec{\sigma}N_p(\vec{x})\cdot\nabla\left[-\frac{G_\omega^2}{2M^2}\frac{e^{-m_\omega|\vec{x}-\vec{x}'|}}{4\pi|\vec{x}-\vec{x}'|}+\frac{G^2}{2M^2}\frac{e^{-m|\vec{x}-\vec{x}'|}}{4\pi|\vec{x}-\vec{x}'|} -\frac{G^2}{2M^2}\left(\frac{e^{-m|\vec{x}-\vec{x}'|}}{4\pi|\vec{x}-\vec{x}'|}-\frac{e^{-m_\sigma|\vec{x}-\vec{x}'|}}{4\pi|\vec{x}-\vec{x}'|}\right)\right]N^\dagger(\vec{x}')N(\vec{x}')\middle|A\right\rangle . \tag{40}$$

In Eq. (40), normal ordering between the nucleon operators is implied (since the self-terms are already implicitly included in the renormalized values of the coupling constants), but the equation is written in (density) × (density) form for clarity. The terms within the brackets in Eq. (40) are obtained as follows: The fields $\omega^0(\vec{x})$ and $\sigma(\vec{x})$ are written in terms of nucleon coordinates

$$\omega^0(\vec{x})=G_\omega\int d^3x' \frac{e^{-m_\omega|\vec{x}-\vec{x}'|}}{4\pi|\vec{x}-\vec{x}'|}N^\dagger(\vec{x}')N(\vec{x}') , \tag{41}$$

and $\sigma(\vec{x})$ as in Eq. (18); PCAC is used to write

$$\pi_-(\vec{x})\simeq -\frac{G}{2Mm^2}\nabla\cdot\vec{A}_-(\vec{x}) \tag{42}$$

[the $A_-^0(\vec{x})$ term is dropped, as being $O(m/2M)$], and the axial current is expressed in terms of nucleon coordinates alone by using the expression[19] (good to order $G^0\equiv 1$)

$$\vec{A}_-(\vec{x})=\sqrt{2}N_n^\dagger(\vec{x})\vec{\sigma}N_p(\vec{x})+\frac{1}{m^2}\nabla[\nabla\cdot\vec{A}_-(\vec{x})] , \tag{43}$$

so that

$$\nabla\cdot\vec{A}_-(\vec{x})=m^2\sqrt{2}\int d^3x' \frac{e^{-m|\vec{x}-\vec{x}'|}}{4\pi|\vec{x}-\vec{x}'|}\times\nabla'\cdot N_n^\dagger(\vec{x}')\vec{\sigma}N_p(\vec{x}') . \tag{44}$$

[The procedure leading to Eqs. (42)–(44) was used to obtain Eq. (31), as well as to neglect the terms in (16) linear in $\sigma\pi$.] Finally, we have used the result

$$(m_\sigma^2-m^2)\int d^3x'' \frac{e^{-m_\sigma|\vec{x}''-\vec{x}'|}}{4\pi|\vec{x}''-\vec{x}'|}\frac{e^{-m|\vec{x}''-\vec{x}|}}{4\pi|\vec{x}''-\vec{x}|} = \frac{e^{-m|\vec{x}-\vec{x}'|}}{4\pi|\vec{x}-\vec{x}'|}-\frac{e^{-m_\sigma|\vec{x}-\vec{x}'|}}{4\pi|\vec{x}-\vec{x}'|} . \tag{45}$$

The expression (42) is used together with (44) to rewrite the last term of (39), giving the second term in brackets in (40); the first term comes from the $\nabla\omega^0(\vec{x})$ term of (39) using (41), and the last term comes from the $\sigma(\vec{x})\pi_-(\vec{x})$ term of (36), using (42), (44), and (45). We see that there is exact cancellation between the second term and the first part of the third term (this is good to $m/2M$) corresponding to the almost perfect cancellation of Figs. 1(a) and 1(b) in free πN scattering. As Bertsch and Riska[7] have pointed out, the finite mass of the σ meson leads to a nonvanishing contribution, which we identify as the last term of Eq. (40), and which would dominate the small (noncanceled) s-wave remainder proportional to

$$\frac{G^2}{2M^2}\frac{e^{-m|\vec{x}-\vec{x}'|}}{4\pi|\vec{x}-\vec{x}'|}\begin{cases}\dfrac{m}{2M}\ \text{(deuterium target)},\\[2ex] \left(\dfrac{m}{2M}\right)^2\text{(large nucleus)}.\end{cases} \tag{46}$$

However, there is an ω contribution from the πNN vertex renormalization which almost cancels the σ contribution. Clearly, the criterion for which term will dominate is the relative size of the factors

$$\frac{G^2}{2M^2m}\left(\frac{m}{2M}\right)^\nu \ \text{("free" pion-nucleon rescattering)} \tag{47}$$

and

$$\frac{G^2}{2M^2m}\left(\frac{m}{m_\sigma}-\frac{G_\omega^2 m}{G^2 m_\omega}\right)\ (\sigma+\omega\ \text{effects}). \tag{48}$$

We see that if there had been no ω contribution, the relative size of σ and π terms would have been 2.6 for deuterium ($\nu=1$) and 35 for a large nucleus ($\nu=2$). With the ω correction, the ratio of (48) to (47) is either 0.4 ($\nu=1$) or 5 ($\nu=2$). Perhaps a word is in order here as to why the power ν in Eq. (47) is different for the $A=2$ system and heavier nuclei. It is simply that in π^-N s-wave scattering, the recoil correction is $\sim(1-m/2M)^{-1}$ for neutrons, $(1+m/2M)^{-1}$ for protons. With a deuterium target, only the π^-n rescattering term is present, whereas for a large nucleus we get the average of π^-n and π^-p terms. More generally, the combination of πN scattering lengths which enters, $2a_{3/2}+a_{1/2}$, is zero within experimental error.

What we have learned from the above exercise is why the usual impulse approximation works on $\pi^-d\to 2n$, whereas with heavier targets, it is the average σ and ω fields which prevail. However, they tend to cancel, and so lead to reasonable absorption rates (we must keep in mind that the NN short-range correlations present in realistic nuclear wave functions will tend to slightly reduce the $\sigma+\omega$ term).

V. THE PSEUDOSCALAR-PSEUDOVECTOR (PS-PV) EQUIVALENCE THEOREM

In a previous paper,[20] I noted that pion emission or absorption by a nucleon described by the Hamiltonian (5) would have the effective vertex

$$H^{eff}_{\pi NN} = -\frac{G}{2M}\int d^3x\, N^\dagger(\vec{x})\gamma^0\vec{\sigma}\tau N(\vec{x})\cdot\nabla[\chi_\pi(\vec{x})f(\vec{x})] \tag{49}$$

as a consequence of orthogonality of initial and final nuclear states, where the form of $f(x)$ depends on whether $\gamma^5\pi$ or $\gamma^\mu\gamma^5\partial_\mu\pi$ coupling is assumed, with

$$f(x)=\begin{cases}(1-V(x)/M)^{-1}, & \gamma^5\\ (1+U(x)/M)(1-V(x)/M)^{-1}, & \gamma^\mu\gamma^5\,.\end{cases} \tag{50}$$

(A closely related result was also found by Friar.[21]) Although for threshold (p,π^+), e.g., there is still a substantial difference (perhaps a factor of 3 in rate) between the two forms of $f(x)$, the prediction of (49) was a great improvement over earlier attempts[22-24] to apply relativistic nucleon wave functions to this process.

However, the result we have found in Eq. (40) is that in the $\sigma+\omega$ model, although we have pseudoscalar πNN coupling, the constraint of approximate chiral symmetry (PCAC) is such as to restore completely the *PS-PV* equivalence. As we see from Eq. (40),

$$T_{fi}\simeq-\int\frac{d^3x\,G\sqrt{2}}{(2\pi)^{3/2}(2k^0)^{1/2}}\langle f|N_n^\dagger(\vec{x})\vec{\sigma}N_p(\vec{x})|i\rangle \times\nabla[U(x)+V(x)/M]\,, \tag{51}$$

which is just what we get from (45) with the *PV* form of $f(x)$. This is a result I had expected to be true, but have been unable to demonstrate until now.

VI. CONCLUSIONS

In a sense, it should not have surprised us that terms independent of pion mass should cancel completely in the $\sigma+\omega$ description of π-nucleus elastic scattering: In the exact chiral limit ($m_\pi\to 0$) the scattering length is proportional to[4] $(G^2/8\pi)\, m_\pi/M^2$, which vanishes as $m_\pi\to 0$. Even with nonzero pion mass, the amplitude to scatter from an elementary $T=0$ target vanishes.[4] Inspecting the effective π-A potential Eq. (32), we see that the dominant term ($\propto m_\pi^2$) would vanish in the chiral limit, but that the term arising from π-π elastic scattering would appear not to. The reason for this is that near $m_\pi=140$ MeV the $\pi\pi$ amplitude times the nuclear π^0 source is approximately independent of m_π. As $m_\pi\to 0$, this term becomes proportional to $m_\pi A$ and so vanishes. It is worth noting that in the absence of Pauli correlations there could be no nonvanishing matrix element of $[\pi^0(x)]^2$, to $O(G^2)$, so this effect is due to the compositeness of the nucleus. The fact that for a $T=0$ target there is a nonvanishing scattering length for physical pions is also a consequence of the compositeness of the nucleus. It is interesting that the net violation of the Weinberg theorem[4] (which may be attributed to failure of the impulse approximation) is very small.

In retrospect, the complete cancellation of the effects involving ω exchange (from π-nucleus scattering amplitudes) should not have surprised us either, since the ω has nothing to do with chirality, i.e., the chiral invariance, upon which the major cancellations (provable by soft-pion techniques) depended, involves only the π, σ, and N parts of the Lagrangian.

What *should*, perhaps, surprise us is that the terms which survive as the pion mass is increased from zero, are of order $(m/2M)^2$ or $(m/m_\sigma)^2$, rather than of order mR or $(mR)^2$. Somehow, the presence of anomalous thresholds does not introduce another (and much smaller) energy scale from that appropriate to πN scattering. In a way this is quite similar to what happens when we consider corrections (due to retardation or higher multipoles) to the integrated nuclear photoabsorption cross section in the dipole approximation[25]—rather than being of order kR, the corrections are of order kM^{-1}. Since soft pions and soft photons are rather similar, it may be possible to find a connection between these phenomena which explains the relative paucity of nuclear structure effects.

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