Coulomb excitation of Yb nuclei

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Multiple Coulomb excitation measurements have been performed with ^{16}O projectiles on ^{168,170,172,174,176}Yb. Measured γ-ray yields were compared to the Winther-deBoer calculations in order to extract $B(E2)$ and $B(E3)$ values to vibrational and rotational states. The 2^+ member of the γ-vibrational band was observed in all the nuclei, the 4^+ member in three cases, and excited 0^+ states in ^{168,170,172,174}Yb. The $B(E2;0^{+\prime}\rightarrow 2^+)$ values from 0^+ states at 1228 keV in ^{170}Yb and 1154 keV in ^{168}Yb are in excess of one single-particle unit, the first such cases observed in Yb nuclei. $B(E3)$ values to octupole states were deduced for ^{172,174}Yb. Members of the ground state rotational band up to $I = 8$ were excited in each nucleus. The measured yield of the 8^+ state relative to that of the 6^+ level is compared to calculations and discussed for the five nuclei studied.

NUCLEAR REACTIONS ^{168,170,172,174,176}Yb(^{16}O, ^{16}O$'\gamma$), $E = 58$–62 MeV; measured $\sigma(E)$, I_γ. ^{168,170,172,174,176}Yb deduced levels, J, π, $B(E2)$, $B(E3)$. Enriched targets; Ge(Li) detector.

I. INTRODUCTION

Multiple Coulomb excitation measurements on deformed nuclei using heavy ions are quite important for measuring $E2$ and $E3$ reduced transition probabilities, $B(E2)$ and $B(E3)$ values, to collective states. The ytterbium isotopes provide an interesting series of nuclei to systematically study, for the five stable even-even isotopes span the range from good rigid rotors ^{174,176}Yb to the nucleus ^{168}Yb which is softer to vibrational modes. One purpose of this work was to measure transition probabilities to vibrational bands, i.e., γ, $K^\pi = 0^+$, and octupole bands. The γ-vibrational band is excited in each nucleus while $B(E2)$ values to 0^+ states are measured in three cases and $E3$ probabilities to octupole states in two nuclei. Of special interest here are the $K^\pi = 0^+$ bands. The 2^+ members can be populated in measurements involving single-step excitations, for example (d, d') reactions or Coulomb excitation experiments with α particles. However, to observe the collectivity of close-lying 0^+ states, one must resort to a heavier projectile, for example ^{16}O, to enable multiple-step excitation. The trends in the energies of 0^+ states and the associated $B(E2)$ values in the Yb nuclei are discussed here.

A second purpose of this work was to measure $B(E2)$ values to members of the ground-state rotational bands up to $I = 8$. One emphasis in this paper is on following the trend of excitation probabilities of the 8^+ state in each of the five nuclei. The extraction of absolute $B(E2)$ values from Coulomb excitation yields requires a number of sensitive corrections, and thus we seek only to observe differences in the 8^+ yields for the rather rigid rotors compared to the more neutron deficient Yb nuclei. Recent multiple Coulomb excitation experiments have been performed by Ward *et al.*[1] on ^{174,176}Yb with Kr and Xe projectiles. They observe the members of the ground-state bands up to at least $I = 18$ and report $B(E2)$ values in good agreement with the predictions of the rigid-rotor model. In this paper, we compare the properties of the lighter isotopes to the properties of these evidently good rigid rotors.

II. EXPERIMENTAL PROCEDURES

The experimental procedure for these measurements on the ytterbium nuclei was identical to that for our experiments on dysprosium nuclei, described by Oehlberg *et al.*[2] Beams of oxygen ions were accelerated to energies of 58–62 MeV by the Notre Dame FN-tandem Van de Graaff accelerator. The 2–4 mg/cm^2 metallic targets were enriched to 18, 81, 91, 96, and 96% in masses 168, 170, 172, 174, and 176, respectively. Separate measurements were performed on ^{171}Yb and ^{173}Yb targets to determine peaks from these isotopic impurities in the even-A targets.

The γ rays were detected by a 40 cm^3 Ge(Li) detector in coincidence with projectiles backscattered into a 300 mm^2 annular Si detector positioned at an average angle of 162° to the beam direction. Excitation probabilities were measured with the γ-ray detector positioned at 55° and 4.5 cm from the target, while the angular distributions of the γ rays were measured over an angular range of 33° to 90° at a distance of 7.0 cm. The observed γ-ray yields were corrected for efficiency, internal conversion, and random-coincidence effects, and then normalized to the number of scattered particles and corrected for isotopic abundance to yield excitation probabilities. In cases where sum peaks were observed, corrections were made for losses in peak area due to photopeak summing. However, corrections were not made for photopeak- Compton summing. As will be discussed in Sec. III F, we quote only ratios of excitation probabilities, P_8/P_6, in order to minimize the effects of any systematic corrections. Furthermore, we emphasize the trend of these ratios in the five nuclei studied and are therefore not concerned greatly about small 1–2% corrections which could be made uniformly to the values for all five nuclei.

The experimental excitation probabilities were then compared to values calculated with the semiclassical Winther-deBoer program; agreement between the calculated and measured values was considered indicative of a proper choice of $E2$ and $E4$ matrix elements for input to the program. The program, called SWHET,[3] integrates probabilities over the thickness of the target using stopping powers for oxygen in gold and an assumed variation of the stopping power with $Z^{-1/2}$, where Z is the atomic number. Corrections to these semiclassical calculations for quantal effects will be discussed in the next section.

Care was taken to use a projectile energy low enough to ensure minimal deviations from pure Coulomb excitation due to Coulomb-nuclear interference effects. We used ^{16}O energies in the range of 58–62 MeV which correspond to separation distances of 5.6–4.6 fm between spherical surfaces for a ^{170}Yb target, assuming that the nuclear radius is described by the formula $1.2A^{1/3}$ fm. While some experiments[4] with α particles on thin targets of rare-earth nuclei at these separation distances indicate small interference effects, for Coulomb excitation experiments[1,2,5] with heavier projectiles and thicker targets one can probably ignore interference effects at these distances in view of the uncertainties due to other corrections.

III. RESULTS AND DISCUSSION

Spectra of γ rays in coincidence with backscattered projectiles are shown in Figs. 1, 2, and 3

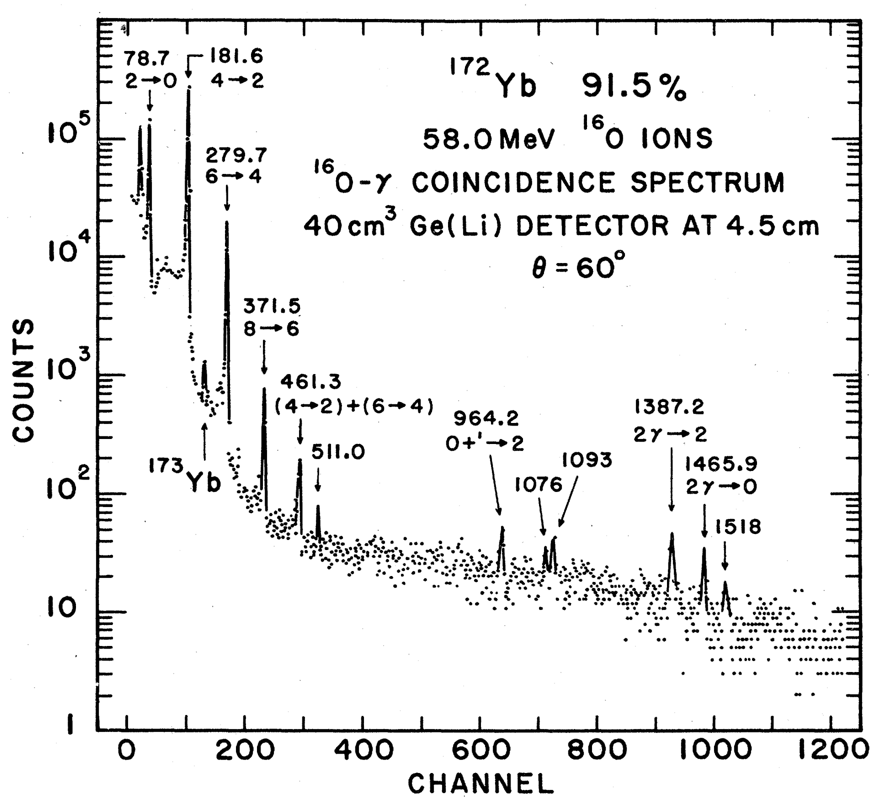

FIG. 1. Particle-γ ray coincidence spectrum for ^{172}Yb.

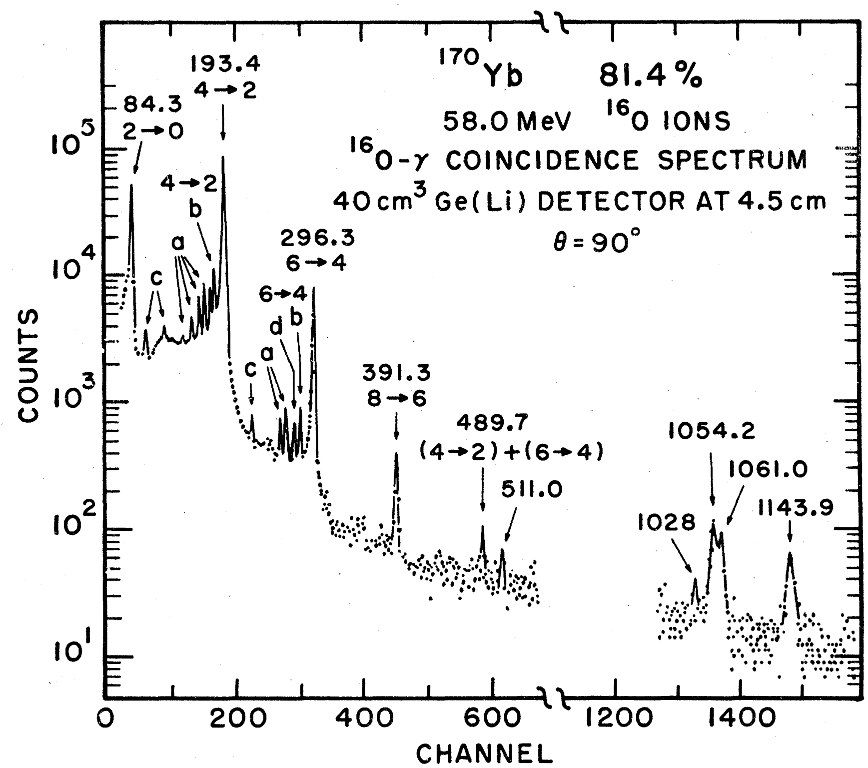

FIG. 2. Particle-γ ray coincidence spectrum for ^{170}Yb. Peaks due to other Yb isotopes are labeled as follows: (a) ^{171}Yb, (b) ^{172}Yb, (c) ^{173}Yb, (d) ^{174}Yb.

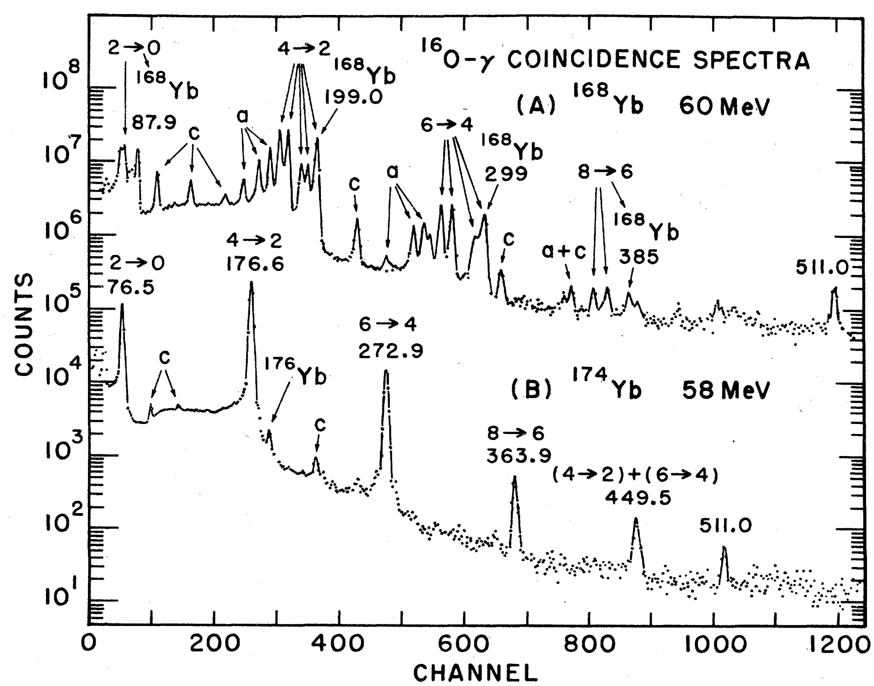

FIG. 3. Low-energy portions of particle-γ ray coincidence spectra for ^{168}Yb (top) and ^{174}Yb (bottom). In each case the Ge(Li) detector was positioned at 60° and 4.5 cm from the target. Vertical scale is arbitrary. Peaks due to other Yb isotopes are labeled as follows: (a) ^{171}Yb, (c) ^{173}Yb.

for ^{172}Yb, ^{170}Yb, and ^{168}Yb and ^{174}Yb, respectively. Adjacent channels have been summed together for these illustrations. The spectra for all the nuclei are similar in that Coulomb excitation to the 8^+ member of the ground band is observed in addition to the γ-vibrational band and possibly other excited bands. The ^{168}Yb spectrum is more complex due to the low enrichment of the target in this rare isotope.

The levels populated in the Yb nuclei are shown in Fig. 4. For the vibrational states, the γ-ray transitions actually observed in our measurements are listed in column 4 of Table I. In some cases, only the most intense of the several possible transitions from a state could be definitely identified in our spectra. It was therefore neccessary to increase the measured yields for these states to account for unobserved transitions by using branching ratios measured previously in radioactive decay experiments. These corrections are discussed for each of the five nuclei in the following sections concerning the vibrational states.

The $B(E\lambda)$ values to observed states are deduced by comparing the measured excitation probabilities to the calculated values in the Winther-deBoer program. Especially for excitation of vibrational states, one must be careful to use as input to this program realistic values of all matrix elements between the levels. Table II contains a matrix of values to demonstrate the transition elements used in our calculations. These are $E2$ matrix elements which are independent of the spin of the initial state and thus the matrix is symmetric. The rotational model is used to generate the matrix elements between members of the ground-state band, that is, the various elements are related to the $0^+ \rightarrow 2^+$ element by a ratio of Clebsch-Gordan coefficients. We assume that the intrinsic quadrupole moments of the ground and vibrational bands are equal, and therefore the matrix elements within the excited $K=0$ and $K=2$ bands are also related to the $0^+ \rightarrow 2^+$ element (denoted by the variable $-a$) by Clebsch-Gordan coefficients. It is the purpose of these measurements to extract, for example, the $B(E2;\ 0 \rightarrow 2_\gamma^+)$ value, given by b^2 in Table II, but the other matrix elements $(b_1, \ldots, b_5)$ must be included. The rotational model can be used to relate $b_1, \ldots, b_5$ to b by ratios of Clebsch-Gordan coefficients, as listed in a footnote to Table II, but these values are known to be inaccurate. One can make corrections to these simple predictions of the adiabatic rotational model by mixing the wave functions for the ground and γ bands and parametrizing the mixing in terms of Z_γ as described, for example, in Ref. 6. These mixing parameters have generally been measured previously for the nuclei studied here, enabling us to use these mixing parameters and appropriate spin-dependent factors (Ref. 6) to calculate b_i

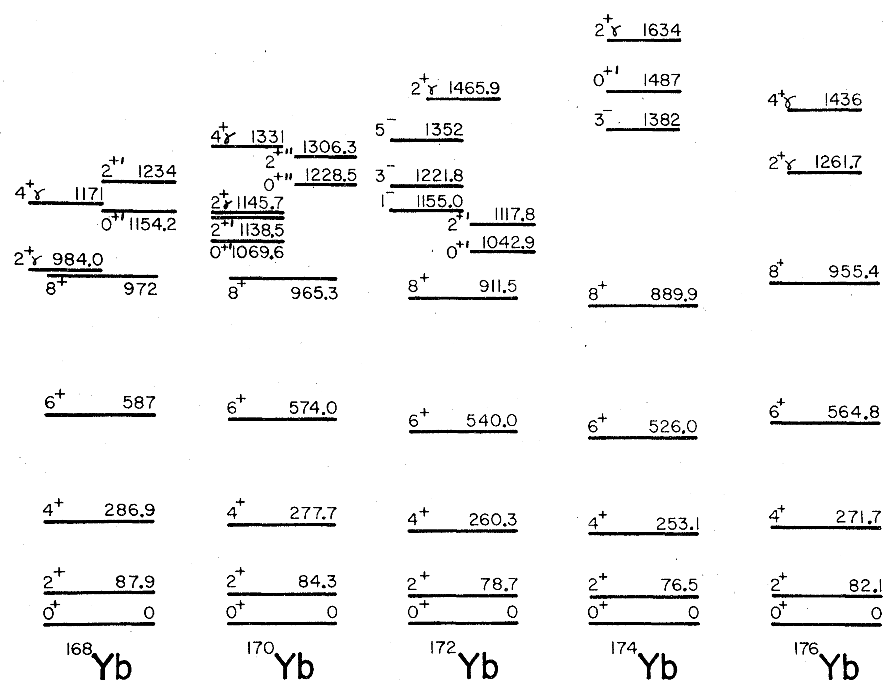

FIG. 4. Partial level schemes of Yb nuclei studied here.

TABLE I. Excitation probabilities and $B(E\lambda)$ values for vibrational states in the Yb nuclei. The single-particle-unit (SPU) is assumed to be 0.029 e^2b^2 for $B(E2)$ and 0.0125 e^2b^3 for $B(E3)$. The intrinsic quadrupole moments of the ground and vibrational bands are assumed to be equal.

A	Level energy (keV)	$I^\pi K$	Observed transitions	Excitation probability $\times 10^4$ for $I^\pi K$ state[a] exp	calc	$B(E2;0^+\to 2^+_\gamma)$ $10^{-2}\,e^2b^2$	SPU	$\frac{B(E2;2^+_\gamma\to 0^+)}{B(E2;2^+_\gamma\to 2^+)}$	Z_γ	$B(E2;0^{+\prime}\to 2^+)$ $10^{-2}\,e^2b^2$	SPU	$B(E3;0^+\to 3^-)$ $10^{-2}\,e^2b^3$	SPU
176	1262	$2^+\ 2$	$2^+2\to 0^+0$	14.8 ± 1.3	15.1	5.03 ± 0.43	1.69	1.61 ± 0.22					
			$2^+2\to 2^+0$			$7.2\ \pm 0.6$[b]		1.69 ± 0.08[b]	0.028 ± 0.008[b]				
	1436	$4^+\ 2$	$4^+2\to 2^+0$	4.4 ± 1.0	4.06								
			$4^+2\to 4^+0$										
174	1382	$3^-\ 2$	$3^-2\to 2^+0$	3.1 ± 1.1	3.04							9.3 ± 3.3	7.4
	1572	$5^-\ 2$			1.97							4.1[c]	
	1487	$0^+\ 0$	$0^+0'\to 2^+0$	0.86 ± 0.40	0.86[d] 0.86[e]					0.81 ± 0.37[d]	0.3		
	1561	$2^+\ 0$			0.26[d] 0.44[e]					$1.5\ \pm 0.7$[e]	0.5		
	1634	$2^+\ 2$	$2^+2\to 0^+0$	5.11 ± 0.70	5.07[f] 5.14[g]	5.00 ± 0.68[f]	1.72	0.65 ± 0.17	$0.013^{+0.039}_{-0.050}$				
			$2^+2\to 2^+0$			5.40 ± 0.72[g]	1.86						
	1805	$4^+\ 2$	$4^+2\to 4^+0$	1.7 ± 0.7	1.43[f] 1.58[g]								
172	1043	$0^+\ 0$	$0^+0'\to 2^+0$	2.3 ± 0.7	2.32					1.62 ± 0.49[h]	0.55		
	1118	$2^+\ 0$			0.72								
	1155	$1^-\ 1$	$1^-1\to 2^+0$	1.5 ± 0.6	1.93							4.8 ± 1.9	3.8
	1222	$3^-\ 1$			0.51							2.6[c]	
	1352	$5^-\ 1$	$5^-1\to 4^+0$	2.4 ± 0.7	1.47								
	1466	$2^+\ 2$	$2^+2\to 0^+0$	7.9 ± 1.9	7.75	4.13 ± 0.99[i]	1.44						
			$2^+2\to 2^+0$			3.73 ± 0.29[b]							
	1658	$4^+\ 2$			2.10								
170	1139	$2^+(0)$	$2^{+\prime}\to 0^+0$	12.6 ± 2.5	9.01	[3.0 ± 0.6]	1.1						
			$2^{+\prime}\to 2^+0$										
	1146	$2^+(2)$	$2^+_\gamma\to 0\ 0$	23.6 ± 4.7	24.5	$7.7\ \pm 1.5$	2.7						
			$2^+_\gamma\to 2\ 0$										
	1331	$4^+\ 2$	$(4^+2\to 4^+0)$		6.07[j]								
	1228	$0^+\ 0$	$0^+0''\to 2^+0$	11.8 ± 2.1	11.8					$5.7\ \pm 1.1$	2.0		
	1306	$2^+\ 0$	$2^+0''\to 4^+0$	2.4 ± 1.2	2.9[k]								
168	984	$2^+\ 2$	$2^+2\to 0^+0$	89.8 ± 7.3	91.9[l]	$13.2\ \pm 1.2$	4.75	0.54 ± 0.09	0.045 ± 0.029				
			$2^+2\to 2^+0$					0.57 ± 0.08[m]					
	1171	$4^+\ 2$	$4^+2\to 4^+0$	$34\ \pm 10$	30.1[l]								
	1154	$0^+\ 0$	$0^+0'\to 2^+0$	21.0 ± 7.6	21.2					$8.0\ \pm 2.9$	2.9		
	1234	$2^+\ 0$			6.2[n]								

[a] Corrected for unobserved transitions.
[b] Reference 7.
[c] Value deduced from (d,d') measurements of Ref. 8.
[d] All matrix elements calculated assuming $Z_\beta = 0$.
[e] Experimental ratios (Refs. 10 and 11) used to calculated $2_\gamma \to 2$ and $2_\gamma \to 4$ matrix elements; $Z_\beta = 0.06$ used for all others.
[f] $Z_\gamma = 0.027$ used to calculate all unobserved matrix elements.

TABLE I. (Continued)

[g] $Z_\gamma = 0.027$ used for unobserved matrix elements, except for $2_\gamma \to 4$ where value is deduced from experimental ratio of Ref. 11.
[h] The $2_\gamma^+ \to 0^{+\prime}$ and $2_\gamma^+ \to 2^{+\prime}$ matrix elements from the calculations of Ref. 15 were included, as discussed in the text and in Table III.
[i] The matrix elements from the fit I calculation of Ref. 15 were used here.
[j] Experimental ratios (Ref. 20) and $Z_\gamma = 0.053$ used to calculate the 2_γ and 4_γ matrix elements.
[k] The $E2$ matrix elements to the $2^{+\prime\prime}$ state were deduced from branching ratios (Ref. 20) and an estimate of $B(E2; 0^+ \to 2^{+\prime\prime})$ from Ref. 8.
[l] The $2_\gamma^+ \to 2^+$ matrix elements was found from our measured branching ratio; the 4_γ^+ elements were calculated assuming $Z_\gamma = 0.045$.
[m] Reference 23.
[n] The $2^{+\prime}$ matrix elements were calculated from the recent $B(E2; 0^+ \to 2^{+\prime})$ measurement (Ref. 40), assuming $Z_\beta = 0.030$.

relative to b. If a branching ratio has been measured, then we use that value to deduce the appropriate matrix element relative to b, although the predicted sign of the matrix element is still used. (If the $M1$ component in the $2_\gamma^+ \to 2^+$ transition is known, it is of course used in deducing the $E2$ matrix element; otherwise, the transition is assumed to be of pure $E2$ character.) The value of b is then varied until the calculated and experimental excitation probabilities agree. The uncertainties in $B(E2; 0^+ \to 2_\gamma^+)$ values deduced in this way reflect only the uncertainty in the measured excitation probability. Obviously our $B(E2)$ values are also dependent on the other matrix elements deduced from measured branching ratios or predictions of the rotational model. This dependence introduces an uncertainty which is very difficult to make quantitative. This is one of the perils in extracting these values from multiple Coulomb excitation data. However, most of the excitation for the 2_γ^+ state comes from only two paths, $0^+ \to 2_\gamma^+$ and $0^+ \to 2^+ \to 2_\gamma^+$. Consequently, an additional 10–15% uncertainty in the measured values would include reasonable typical uncertainties in the branching ratios used. The problems can be more severe, however, if there are conflicting branching ratio measurements or if some matrix elements deviate significantly from the mixed-wave-function rotational-model predictions. In these cases we can only quote $B(E2)$ values deduced from two different calculations using both conflicting sets of matrix elements and allow the reader to possibly draw upon future information on the conflicting branching ratios to judge which of our extracted $B(E2)$ values is appropriate. It was necessary for us to resort to such a compromise for the γ bands in ^{172,174}Yb and for the $0^{+\prime}$ band in ^{174}Yb, as discussed below.

While light-ion Coulomb excitation can be used to more directly deduce $B(E2; 0^+ \to 2^+)$ values for vibrational bands, only multiple-step excitation can populate 0^+ members of excited $K=0$ bands. In four of the Yb isotopes, 0^+ states were observed in our measurements with yields larger than for the 2^+ band members, enabling us to deduce $B(E2; 0^{+\prime} \to 2^+)$, given by c^2 in Table II. In several cases, the value of d was known previously from light-ion measurements, and d_1 and d_2 could be found relative to d from branching ratio measurements. As in the case of the γ band, known band mixing parameters Z_β were used to calculate unmeasured matrix elements, using the formalism of Ref. 6. The $4^{+\prime}$ band members were not included in the calculations because we were concerned only with extracting $B(E2)$ values for the 0^+ band heads.

A similar technique was used to analyze the

TABLE II. Reduced $E2$ matrix elements, $\langle I_i K_i \| M(E2) \| I_f K_f \rangle = (2I_i+1)^{1/2}[B(E2; I_i \rightarrow I_f)]^{1/2}$, used in Winther-deBoer calculations. The values a, b, and c are varied to give proper fits to the experimental excitation probabilities. The numerical values here are predictions of the adiabatic rotational model.

				I_2					
I_1	0	2	4	6	8	2_γ	4_γ	$0'$	$2'$
0	0	$-a$				b			d
2		$1.196a$	$-1.603a$			b_1 [a]	b_3	c	d_1
4			$1.530a$	$-2.022a$		b_2	b_4		d_2
6				$1.821a$	$-2.365a$		b_5		
8					$2.074a$				
2_γ						$-1.196a$	$1.035a$		
4_γ							$-0.612a$		
$0'$								0	$-a$
$2'$									$1.196a$

[a] The adiabatic rotational model predicts $b_1 = 1.196b$, $b_2 = 0.268b$, $b_3 = 1.035b$, $b_4 = 1.776b$, $b_5 = 0.522b$, $c = d$, $d_1 = -1.196d$, $d_2 = 1.603d$. As discussed in the text, these values are either replaced by experimental branching ratios or modified by a band-mixing factor.

observed yields for negative-parity states in ^{172,174}Yb. Only $E3$ matrix elements between the ground and negative-parity bands were used, since there is no evidence of contribution from $E1$ excitation of such low-lying collective states. The predictions of the adiabatic rotational model were used in calculating the various $E3$ matrix elements relative to $B(E3; 0^+ \rightarrow 3^-)$ because once again there is no experimental knowledge of deviations from these predictions.

The vibrational states for each nucleus will be discussed first in the following sections. The measured and calculated excitation probabilities for the observed states are listed in Table I, as are the extracted $B(E2)$ values. The results for the ground-state rotational bands are discussed separately in Sec. III F.

A. ^{176}Yb

This experiment was performed with 58 MeV ^{16}O on a target 5.8 MeV thick. In addition to γ rays from the ground band, we observe two transitions from the 2^+ member of the γ band and two from the 4^+ member (see Fig. 4). The branching ratio from the 2^+ member has been converted to a $B(E2)$ ratio and displayed in Table I, assuming that the $2_\gamma^+ \rightarrow 2^+$ transition has pure $E2$ multipolarity. The most recent published Coulomb excitation studies of ^{176}Yb were those of Ward *et al.*,[1] who used Kr and Xe ions and considered only the ground-state band, and of Sayer,[7] who used helium ions and lower energy oxygen ions and who also observed the 2_γ^+ state. Sayer's more accurate $B(E2)$ ratio[7] agrees with our measurement. Our Winther-deBoer calculations included the $I=2$ through 10 members of the ground band and the $I=2$ and 4 members of the γ band. The mixing parameter Z_γ deduced by Sayer[7] was used to calculate the unknown $E2$ matrix elements between γ and the ground bands. Comparison of our measured and calculated yields leads to $B(E2; 0 \rightarrow 2_\gamma^+) = (0.0503 \pm 0.0043)\ e^2b^2$, smaller than the value of 0.072 ± 0.006, obtained by Sayer,[7] and the approximate value of 0.070 extracted from the (d, d') work of Burke and Elbek.[8]

B. ^{174}Yb

Experiments were performed with 58 and 60 MeV ^{16}O on targets which were 9.2 and 4.4 MeV thick, respectively. A spectrum from the 58-MeV experiment is shown in Fig. 3. The excitation probabilities were obtained from averages of these two measurements which were in agreement with each other. The γ band at 1634 keV is the highest-lying one in the Yb nuclei and was not seen in the measurements of Sayer,[7] but was observed here in addition to the 4^+ member of the band. The 2_γ^+ state is not observed in the decay of ^{174}Tm,[9] but is populated in the (n, γ) experiments of Greenwood *et al.*[10] and Casten *et al.*[11] They were unable to extract the $(2_\gamma^+ \rightarrow 0)/(2_\gamma^+ \rightarrow 2)\ B(E2)$ ratio and our value of 0.65 ± 0.17 (assuming no $M1$ component in the $2_\gamma^+ \rightarrow 2^+$ transition) is the first such measurement. In the Winther-deBoer calculations, we use this ratio and $Z_\gamma = 0.027$ to calculate all other γ-band $\rightarrow$ ground-band matrix elements, leading to a $B(E2; 0 \rightarrow 2_\gamma^+)$ value of $(0.0500 \pm 0.0068)\ e^2b^2$. This value of Z_γ is calculated from the value for the branching ratio from the 4_γ^+ state observed by Greenwood *et al.*[10] and agrees, within uncertainties, with $Z_\gamma = 0.016$, the value they extract from their branching ratio for the 3^+ state. Casten *et al.*[11] quote a 4^+_γ branching ratio in agreement, within uncertainties, with that of Greenwood *et al.*[10] However, a problem exists in that both groups quote a very large $2^+_\gamma \to 4^+$ transition intensity. Comparing this intensity to that of the $2^+_\gamma \to 2^+$ transition, one finds $Z_\gamma = 0.15$, using the values of Ref. 11, and $Z_\gamma = 0.33$, using those of Ref. 10. If we had used the $(2^+_\gamma \to 4^+)/(2^+_\gamma \to 2^+)$ ratio of Casten *et al.*[11] in our Winther-deBoer calculations, we would have had to increase $B(E2; 0 \to 2^+_\gamma)$ to $(0.0540 \pm 0.0072)\ e^2b^2$ in order to explain the observed yield. Using the ratio of Greenwood *et al.*[10] would further increase this $B(E2)$ value by 7%. This, of course, demonstrates vividly the problem in extracting $B(E2)$ values from multiple excitation data if some of the matrix elements are unknown or uncertain.

Another peak seen in our γ-ray spectrum is that of 1306 keV. It is possible that this is a transition from the 1382-keV 3^- member of a $K^\pi = 2^-$ band. This state was excited in the (d, d') experiments of Burke and Elbek[8] so it is not surprising to observe this level in Coulomb excitation. Their estimated $B(E3)$ value to this 3^- state was 0.041 e^2b^3, while another 3^- level at 1846 keV was observed with an estimated $B(E3)$ of 0.051 e^2b^3. Greenwood *et al.*[10] have studied negative-parity states in the (n, γ) process and placed 3^- states of $K^\pi = 2^-$, 0^-, and 3^- at 1381.9, 1785.8, and 1851.2 keV, respectively. In our experiments we have evidence for the lowest 3^- state only. In the Winther-deBoer calculations, we have included the 3^- and 5^- members of this $K = 2$ band and used the simple rotational model to predict the unknown $E3$ matrix elements. This seems to be a good assumption in view of the work of Schmidt *et al.*,[12] who measured the multipole mixture of the transitions from the 1318-keV 2^- member of the $K = 2$ band populated in ^{174}Lu decay. From their measurements they obtain a value for $B(E3; 2^- \to 4^+)/B(E3; 2^- \to 2^+)$ equal to $0.38 \pm ^{0.79}_{0.25}$. Although the value has large uncertainties, it agrees with the rotational prediction of 0.4. As illustrated in Table I, our measured excitation probability for the 1306-keV state leads to a $B(E3; 0 \to 3^-)$ value of $(0.093 \pm 0.033)\ e^2b^3$, equivalent to 7.4 single-particle units (SPU). This is larger than the value of Burke and Elbek,[8] which was obtained in a somewhat approximate way by relating the measured (d, d') 90° differential cross section to a $B(E3)$ value. The calculations of Neergard and Vogel[13] yield a $B(E3)$ value of 0.044 e^2b^3 without Coriolis coupling included and 0.063 e^2b^3 when it is included.

Three excited $K^\pi = 0^+$ bands are seen in both the (p, t) reactions of Oothoudt and Hintz[14] and the (n, γ) measurements of Greenwood *et al.*[10] and Casten *et al.*[11] The lowest 0^+ state occurs at 1487.4 keV, and we observe a weak peak at 1411 keV which is probably a transition from this state to the 2^+_g. For Coulomb excitation measurements with ^{16}O at these energies the yield for the 0^+ state is slightly larger than that for the 2^+ member of the band, even though the latter can be excited in one step while two are required for the former. The two paths for exciting the $2^{+\prime}$ state ($0^+ \to 2^{+\prime}$ and $0^+ \to 2^+ \to 2^{+\prime}$) evidently add destructively, while the two paths for the $0^{+\prime}$ state ($0^+ \to 2^{+\prime} \to 0^{+\prime}$ and $0^+ \to 2^+ \to 0^{+\prime}$) add more constructively. The extraction of a $B(E2; 0^{+\prime} \to 2^+)$ value from the observed yield for this state in ^{174}Yb depends, of course, on the matrix elements for the $2^{+\prime}$ state. If one assumes the strict rotational model to relate all these matrix elements to the desired $0^{+\prime} \to 2^+$ element, then the data yield $B(E2; 0^{+\prime} \to 2^+) = (0.0081 \pm 0.0037)\ e^2b^2$, equivalent to (0.3 ± 0.1) SPU. However, the (n, γ) measurements[10,11] yield branching ratios from the $2^{+\prime}$ state indicative of $Z_\beta > 0.05$. Using those measured ratios[10] for the $2^{+\prime} \to 2^+$ and $2^{+\prime} \to 4^+$ elements and $Z_\beta = 0.06$ to relate the $2^{+\prime} \to 0^+$ value to the $0^{+\prime} \to 2^+$ matrix element, we extract from our data $B(E2; 0^{+\prime} \to 2^+) = (0.015 \pm 0.007)\ e^2b^2$, i.e., (0.5 ± 0.2) SPU. Although the difference in the two values is large, it appears that the $B(E2)$ for this $K^\pi = 0^+$ band is less than 1 SPU, indicating that this band is not collective. It is interesting that the (p, t) yield[14] for the 1487-keV state is quite large, consistent with the premise that this is not a collective β-vibrational state.

C. ^{172}Yb

The 1466-keV 2^+ member of the γ-vibrational band is populated; Sayer[7] observed this state in $(\alpha, \alpha'\gamma)$ measurements and quoted a $B(E2; 0 \to 2^+_\gamma)$ of $(0.0373 \pm 0.0029)\ e^2b^2$. In addition, the 964-keV peak evident in Fig. 1 corresponds to the $0^{+\prime} \to 2^+$ transition from the 1043-keV state. The 1118-keV $2^{+\prime}$ member of this band was observed by Sayer,[7] who measured $B(E2; 0 \to 2^{+\prime}) = (0.0068 \pm 0.0005)\ e^2b^2$, corresponding to 0.24 SPU. Since the ^{16}O multiple-excitation experiments populate this $0^{+\prime}$ state rather strongly, one can try to extract a $B(E2; 0^{+\prime} \to 2^+)$ value from the observed excitation probability. However, the presence of $2^+_\gamma \to 0^{+\prime}$ and $2^{+\prime}$ transitions observed in the ^{172}Tm decay measurements of Reich *et al.*[15] complicates the process since the excitation probabilities for the 2^+_γ, $0^{+\prime}$, and $2^{+\prime}$ states are interrelated. For the $2^+_\gamma \to$ ground-band matrix elements for input to the Winther-deBoer program, we used the above-quoted $B(E2)$ value of Sayer[7] and the branching ratios from Ref. 15 (which are in good agreement with those from the ^{172}Lu decay work of Sen and Zganjar[16]). The $4^+_\gamma \to$ ground-band matrix elements were calculated from the rotational model assuming a Z_γ value of 0.023, the mixing parameter obtained from the $2^+_\gamma \to 2^+/2^+_\gamma \to 0^+$ $B(E2)$ ratio.

There are, however, uncertainties involved in the choice of the $2^+_\gamma \to 0^{+\prime}$ and $2^{+\prime}$ matrix elements, since the signs of these two interband elements cannot be predicted by the rotational model and, in addition, nothing is known about possible M1 content in the $2^+_\gamma \to 2^{+\prime}$ transition. Reich *et al.*[15] have performed detailed mixing calculations involving the ground, γ, 1118- and 1476-keV (both $K=0$), and 1608-keV ($K=2$) bands. They were not able to get a consistent fit to all of the transitions between these five bands but quote the two best calculations, for which some of the results are shown in fits I and II in Table III. In fit I, the signs of the $2^+_\gamma \to 0^{+\prime}$ and $2^+_\gamma \to 2^{+\prime}$ matrix elements are positive as are the γ-band to ground-state-band matrix elements, and the magnitudes of these matrix elements agree with the measured branching ratios.[15] The only significant disagreement between this calculation and experiment is the size of the $2^{+\prime} \to 2^+$ matrix element, as demonstrated in Table III. We have used matrix elements from fit I of Reich *et al.*[15] in the Winther-deBoer program, giving the calculated excitation probabilities shown for the γ and $0^{+\prime}$ bands in column 2 of Table III. The agreement is good for the $0^{+\prime}$ state since the $B(E2;\, 0^{+\prime} \to 2^+)$ value was varied to give a good fit. Note that the calculated $P_{2\gamma}$ is 7.13×10^{-4}, somewhat lower than but within the uncertainties of the measured value of 7.9×10^{-4}. In the case of fit II (column 3 of Table III), the $2^+_\gamma \to 0^{+\prime}$ matrix element is once again positive and in agreement with experiment[15]; the $2^{+\prime} \to 2^+$ value agrees, but the $2^+_\gamma \to 2^{+\prime}$ element is negative and much smaller than experiment. The resulting excitation probability for the 2^+_γ state is reduced to 6.29×10^{-4}, farther from the measured value (but still within experimental error). Evidently a *negative* $2^+_\gamma \to 2^{+\prime}$ matrix element reduces the excitation probability of the 2^+_γ state and increases it for the $0^{+\prime}$ state. In view of the better agreement, it is tempting to conclude that the fit I calculation of Reich *et al.*[15] gives a better overall representation of the matrix elements. In this case, we would conclude that a slightly larger $B(E2;\, 0 \to 2^+_\gamma)$ value, $(0.0413 \pm 0.0099)\, e^2b^2$, is necessary to give agreement between the calculated and measured $P_{2\gamma}$ value. However, this conclusion is rather weak in view of the uncertainties involved. If the fit II results are used the $B(E2;\, 0 \to 2^+_\gamma)$ must be increased by another 13%, but the uncertainties then would still overlap the value of Sayer[7] and the value from fit I. Nevertheless, our choice for the fit I matrix elements is consistent with the conclusions of Reich *et al.*,[15] who felt that this calculation explains all of the available matrix elements except that for the $2^{+\prime} \to 2^+$ transition. This disagreement could result from an M1 component, as mentioned in Ref. 15. We have used the calculated value of Reich *et al.*[15] for this matrix element, but have checked the effect of this by performing another calculation using the experimentally determined value. We find that the $0^{+\prime}$ excitation probability is largely unaffected, whereas the $2^{+\prime}$ excitation probability is increased by 40%, still below the limit of detection in our measurement.

The result of this analysis is a rather reliable $B(E2;\, 0^{+\prime} \to 2^+)$ value. The observed excitation probability for the 2^+_γ state is best explained by using matrix elements from the fit I calculation.[15] We therefore use these same matrix elements and vary the $0^{+\prime} \to 2^+$ matrix element to explain the measured excitation probability for the $0^{+\prime}$

TABLE III. Comparison of calculated excitation probabilities for the γ and $K=0'$ bands in ^{172}Yb to the measured values. The matrix elements listed result from calculations of Reich *et al.* (Ref. 15) using two different parameter sets.

$I_i \to I_f$	Fit I	$\langle I_i \Vert M(E2) \Vert I_f\rangle$ [a] Fit II	exp [b]
$2^+_\gamma \to 0^+$	0.193	0.193	0.193
$2^+_\gamma \to 0^{+\prime}$	0.257	0.261	0.255
$2^+_\gamma \to 2^{+\prime}$	0.427	−0.103	0.430 [c]
$2^{+\prime} \to 0^+$	0.084	0.087	0.084
$2^{+\prime} \to 2^+$	−0.122	−0.150	0.161 [c]
$P_{2^+\gamma} \times 10^4$	7.13	6.29	7.9 ± 1.9
$P_{0^{+\prime}} \times 10^4$	2.04 [d]	3.53 [d]	2.3 ± 0.7
$P_{2^{+\prime}} \times 10^4$	1.13	1.33	

[a] $\langle I_i \Vert M(E2) \Vert I_f\rangle = (2I_i+1)^{1/2}[B(E2;\, I_i \to I_f)]^{1/2}$; the matrix elements listed are calculated and described in Ref. 15, the excitation probabilities listed result from our measurements and from our Winther-deBoer calculations using these matrix elements.

[b] These experimental values result from the branching ratios of Reich *et al.*, Ref. 15. Note that the sign is experimentally undetermined in each case.

[c] Experimental values assuming pure $E2$ multipolarities.

[d] Calculated assuming $B(E2;\, 0^{+\prime} \to 2^+) = 0.0144\, e^2b^2$. The best fit I agreement with the experimental excitation probabilities results from $B(E2;\, 0^+ \to 2^+{}_\gamma) = (0.203)^2\, e^2b^2 = 0.0413\, e^2b^2$ and $B(E2;\, 0^{+\prime} \to 2^+) = (0.127)^2\, e^2b^2 = 0.0162\, e^2b^2$.

state and obtain $B(E2;\ 0^{+\prime}\rightarrow 2^{+}) = (0.0162 \pm 0.0049)$ e^2b^2. In the strict rotational model, this value should equal the $B(E2;\ 0^{+}\rightarrow 2^{+\prime})$ value, (0.0068 ± 0.0005) e^2b^2 obtained by Sayer.[7] Using the simple two-band mixing formalism of Ref. 6, we find that this $(0^{+\prime}\rightarrow 2^{+})/(0^{+}\rightarrow 2^{+\prime})$ $B(E2)$ ratio yields a Z_β value of $0.036\ ^{+0.010}_{-0.016}$. The $(2^{+\prime}\rightarrow 4^{+})/(2^{+\prime}\rightarrow 0^{+})$ $B(E2)$ ratio of Reich *et al.*[15] gives a Z_β of $0.037\ ^{+0.005}_{-0.004}$. This good agreement means that, to first order, the simple two-band mixing formalism can explain these 3 transitions.

A higher energy $K=0$ band, consisting of a 0^+ member at 1042.9 keV and a 2^+ member at 1476.6 keV, was observed by Greenwood *et al.*[17] in (n,γ) measurements. There is no evidence of Coulomb excitation of this band in our spectrum, but we are able to set a limit on the intensity of a 1326.2-keV $0^{+\prime\prime}\rightarrow 2^{+}$ transition in the spectrum of Fig. 1. Our Winther-deBoer calculations indicate that $B(E2;\ 0^{+\prime\prime}\rightarrow 2^{+}) < 0.0018$ e^2b^2 (0.062 SPU). In these calculations, we have used matrix elements to the 1477-keV 2^+ state deduced in the calculations of Reich *et al.*[15] This limit demonstrates that the second excited 0^+ state in ^{172}Yb has a $B(E2)$ value which is at least 10 times smaller than that of the 1042-keV 0^+ state, and thus neither of these 0^+ states is collective.

The 1076- and 1093-keV peaks in the spectrum of Fig. 1 are interpreted as transitions from known negative-parity states. A $K^\pi = 1^-$ band has been established with $I = 1$, 2, 3, 4, and 5 members at 1155, 1198, 1222, 1331, and 1352 keV, respectively, based on the (n,γ) measurements of Greenwood *et al.*,[17] the ^{172}Tm decay work of Reich *et al.*,[15] and the deuteron-reaction work of Burke and Elbek[8] and O'Neil and Burke.[18] The energies of these states deviate greatly from the $I(I+1)$ rule but this has been explained[17,18] in terms of the Coriolis mixing of this band with the other octupole bands, especially that of $K=0$ at 1600 keV. The 1076-keV γ ray in our work corresponds to the $1^-\rightarrow 2^+$ transition; the $1^-\rightarrow 0^+$ transition is only 20% as intense as this one, according to Greenwood *et al.*,[17] and so we do not expect to find it in our spectrum. The 1093-keV γ ray corresponds to the $5^-\rightarrow 4^+$ transition, but there is no evidence in our spectrum for the 812-keV $5^-\rightarrow 6^+$ transition. The branching ratio for the 5^- state has never been measured, because the 1093-keV transition is composite in radioactive decay or neutron-capture work with the very intense transition of essentially the same energy from the 3^+ level at 1172 keV. We have increased the excitation probability of the 5^- state by 34% to correct for the unseen $5^-\rightarrow 6^+$ transition, using the pure rotational-model predictions for the ratio of $E1$ transitions from a $K=1$ band. It is interesting that we observe the 1^- and 5^- (but not the 3^-) members of this band, but this is because of the nature of multiple Coulomb excitation measurements with ^{16}O. Assuming rotational $E3$ matrix elements, we have calculated the excitation probabilities (listed in Table I) and have generally verified this population trend in the $K=1$ band. The calculated probability for the 1^- state is larger than that observed, while the calculated 5^- yield is smaller than the measured yield. However, without a knowledge of the way in which Coriolis coupling affects the $E3$ elements, this is the best agreement that can be expected. Our deduced value of (0.048 ± 0.019) e^2b^3 for the $B(E3;\ 0^+\rightarrow 3^-)$, larger than the value of 0.026 e^2b^3 estimated from the (d,d') cross section in Ref. 8, is in reasonable agreement with the calculations of Neergard and Vogel.[13] Their predictions indicate that higher-lying $K=2$ and 3 bands would have the largest $B(E3)$ values, but when Coriolis mixing is included, most of the strength is concentrated in the lowest octupole band ($K=1$) with a predicted $B(E3)$ value of 0.060 e^2b^3.

D. ^{170}Yb

A γ-ray coincidence spectrum for ^{170}Yb was shown in Fig. 2. Since the target was enriched to only 81.4%, peaks at low energies due to Coulomb excitation of the ground-state rotational bands in other Yb isotopes were observed. Corrections for contributions of these peaks to the rotational transitions in ^{170}Yb have been made. In addition, one notes a doublet at 1054–1061 keV and another peak at 1144 keV. The Ge(Li) detector was positioned at 90° for this experiment in order to minimize the tails on the peaks due to Doppler broadening. The levels of interest are shown in Fig. 4. Measurements[19-21] on the electron-capture decays of ^{170}Lu indicate the existence of four excited $K^\pi = 0^+$ bands at 1069, 1228, 1480, and 1566 keV. The absence in Fig. 2 of a 985-keV peak, corresponding to decay of the first 0^+ state ($0^{+\prime}$), indicates that the $B(E2)$ value to this state is very small (less than 0.1 SPU) and thus that it is not collective, in analogy with the 1043-keV 0^+ band in ^{172}Yb. The peak labeled 1144 keV in Fig. 2 could be assigned as the γ ray from the second excited 0^+ state at 1228 keV ($0^{+\prime\prime}$) to the 2_g^+ state or from the 1146-keV state to ground. Radioactive decay work has established 2^+ states at 1138.6 and 1145.7 keV (hereafter designated as $2^{+\prime}$ and 2_γ^+). The doublet at 1054–1061 keV in Fig. 2 is attributed to the γ rays proceeding from these two states to the 2^+ level at 84 keV. One would also expect transitions of 1139 and 1146 keV arising from de-excitations to the ground state. However, Doppler broadening makes the experimental resolution too poor to establish the various contributions to these composite peaks.

Gamma-ray angular distribution measurements were performed to resolve these difficulties. The Ge(Li) detector had been placed as close as possible (approximately 4.5 cm) to the target for normal coincidence experiments, but a source-detector distance of 6.7 cm was used for the angular distribution experiment. Spectra were taken at angles of 33°, 45°, 70°, and 90° to obtain data for the transitions in the ground-state band. The measured attenuation coefficients are significantly less than unity for the longer-lived states, but approach unity for the shorter-lived states. We have concluded that the attenuation of the γ-ray angular distribution will be negligible for the γ rays from states at 1 MeV or higher where the lifetimes are less than 5 psec. A discriminator was used to experimentally accept backscattered ions from only the first $\frac{2}{3}$ (6 MeV) of the target so that most of the recoiling Yb ions stopped in the target.

Long runs, each of approximately 18 h duration, were then recorded at 33°, 45°, and 90° in order to measure the angular distribution of the composite peak at ~1144 keV. Partial spectra are shown in Fig. 5. At 45° one sees two sets of doublets (severely Doppler broadened) resulting from deexcitation of the 2^+ states at 1139 and 1146 keV. If the γ rays at 1139 and 1146 keV were pure $2^+ \rightarrow 0$ transitions, they would have an angular distribution of a 0-2-0 sequence and would thus decrease in intensity by a factor of 33 between 45° and 90° in our geometry. However, the total intensity of the doublet decreases by only a factor of 3 between 45° and 90°, leading us to conclude that the 1144-keV peak seen at 90° is the $0^{+\prime\prime} \rightarrow 2$ transition from the state at 1228 keV.

The results of the angular distribution experiment now allow the determination of the excitation probability (see Table I) for the 0^+ member of the $K^\pi = 0^+$ band at 1228 keV. The 2^+ member of this band (labeled $2^{+\prime\prime}$) has been placed at 1306.3

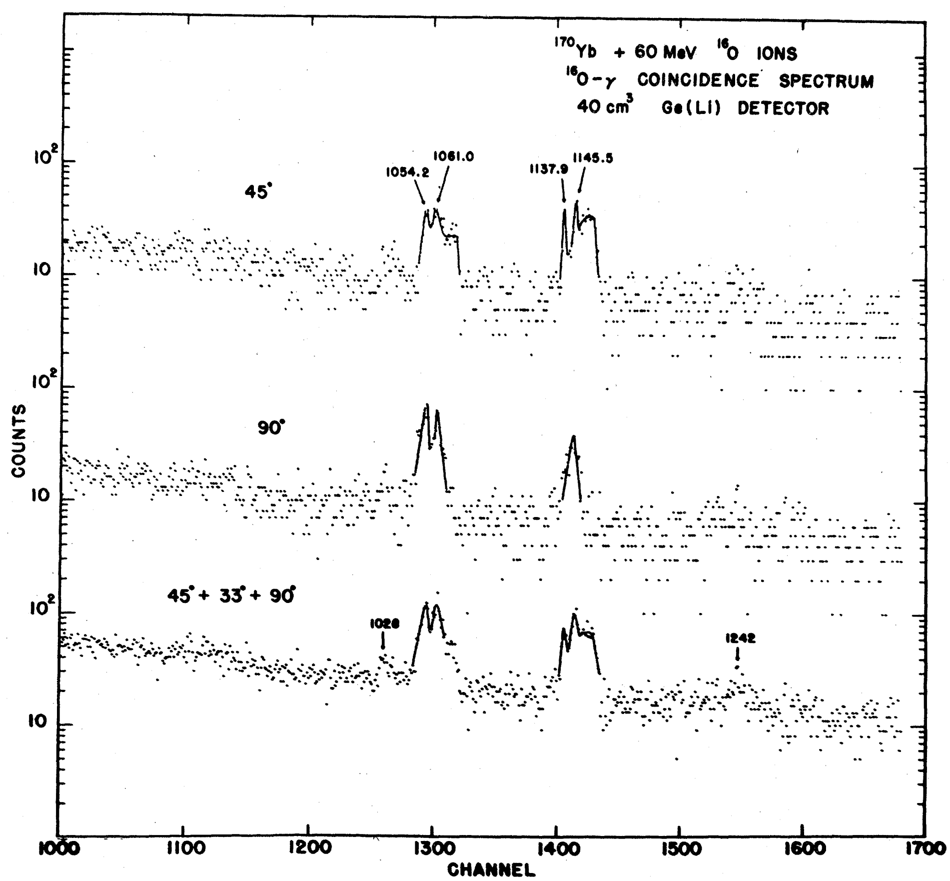

FIG. 5. High-energy portion of γ-ray coincidence spectra for ^{170}Yb, accumulated during a γ-ray angular distribution experiment.

keV in radioactive decay work[19-21] and the γ-ray peak at 1028 keV in Figs. 2 and 5 is assigned as originating from this state. The work of Camp and Bernthal[20] indicates that this $2^{+\prime\prime} \to 4$ transition is stronger than the $2^{+\prime\prime} \to 0$ or $2^{+\prime\prime} \to 2$ transitions and this is in accord with our work. A level at about 1306 keV was observed by Burke and Elbek[8] in the (d, d') reaction. Their quoted cross section for this excitation would correspond to a $B(E2)$ value of approximately 0.023 e^2b^2 (0.80 SPU), if this is indeed the $I^\pi K = 2^+0$ level. We used this $B(E2;\ 0^+ \to 2^{+\prime\prime})$ value and the branching ratios of Camp and Bernthal[20] to obtain the $E2$ matrix elements between the members of the ground band and the $2^{+\prime\prime}$ 1306-keV state. Then the $0^{+\prime\prime} \to 2^+$ matrix element was varied until a fit to the measured excitation probability was achieved. The deduced $B(E2;\ 0^{+\prime\prime} \to 2^+)$ value is (0.057 ± 0.011) e^2b^2 which corresponds to 2.0 SPU. In our work on the heavier Yb isotopes, excited 0^+ states were observed in ^{172}Yb and ^{174}Yb, but their $B(E2)$ values were much less than one single-particle unit. The large $B(E2)$ value to the second excited 0^+ state in ^{170}Yb is the first evidence for a collective $K^\pi = 0^+$ band in Yb nuclei. More discussion will be devoted to this in a later section.

Burke and Elbek[8] attributed the peak at approximately 1145 keV in the (d, d') spectrum to the excitation of a single level, the 2^+ member of the γ-vibrational band, and estimated the $B(E2)$ value as 0.094 e^2b^2 (3.2 SPU). Our measurements show that both 2^+ levels are excited and that the $B(E2)$ strength should be split between the two. This is a rather unique and peculiar situation. One of these 2^+ states is undoubtedly the bandhead for the γ vibration, while the other must be the second member of the $K = 0$ band at 1069 keV. The evidence for this latter statement comes from radioactive decay work[21]; since the 0^+ state at 1069 keV is strongly populated, one would also expect to see the 2^+ state of this band in the decay of ^{170}Lu(0^+). Within uncertainties, the branching ratios from the two states at 1139–1146 keV, as measured in our Coulomb excitation experiments, agree with those derived from decay work,[20,21] indicating that we are populating the same levels.

Tarara *et al.*[22] have recently performed (d, d') and (d, t) reactions to levels in ^{170}Yb and are able to resolve these two states. Differential cross sections for the 2^+ states at 1139 and 1146 keV were measured at 60°, 90°, and 120°. The ratio of the relative cross sections for the two states was approximately the same at each of these angles; the average of these three ratios is

$$\frac{d\sigma/d\Omega(1146)}{d\sigma/d\Omega(1139)} = 2.59\,^{+0.36}_{-0.29}\,.$$

It is reasonable to assume that the $B(E2)$ values to these two states have the same ratio. The branching ratio data of Camp and Bernthal[20] were then used to find the other $E2$ matrix elements to these 2^+ states. It is assumed, in view of the larger (d, d') yield for the 1146-keV state, that it is the γ-vibrational bandhead $(K = 2)$ and that the 1139-keV state is the second member of the $K = 0$ 1069-keV band. The 4^+ member of the γ band is not populated in ^{170}Lu decay, but is tentatively placed at (1331.3 ± 0.9) keV, according to the (d, d') and (d, t) spectra of Tarara *et al.*[22] The matrix elements to this 4^+_γ level are calculated with the rotational model using $Z_\gamma = 0.053$, the value extracted from the 3^+_γ branching ratio in decay work.[20] The $2^{+\prime} \to 4^+$ transition from the 1139-keV state is not observed either here or in the ^{170}Lu experiments, which would be an argument for assigning to this state $K = 2$ and to the 1146 keV state $K = 0$, contrary to the evidence from the (d, d') measurements. An estimate of the $2^{+\prime} \to 4$ matrix element is found by using the upper limit on the size of this transition in decay work, as discussed by Dzhelepov and Shestopalova[21]: $B(E2;\ 2^{+\prime} \to 4)/B(E2;\ 2^{+\prime} \to 0) < 0.052$. If the 1069-keV band is treated as a normal $K = 0$ band, the Winther-deBoer calculations predict a larger excitation probability for the 0^+ state than for the 2^+ state, as is the case experimentally for the 1228-keV band. We do not observe the 1069-keV 0^+ state in our measurements, so that the $0^{+\prime} \to 2$ and $2^{+\prime} \to 0^{+\prime}$ matrix elements must be reduced in the calculation in order to decrease the yield of the 0^+ state and increase that of the 2^+ member of the band. A small value, 0.0004 e^2b^2, was used for $B(E2;\ 0^{+\prime} \to 2)$, while the intraband $B(E2)$ was reduced to 5% of the value for the ground-state band. The resulting excitation probabilities are shown in Table I. The $B(E2)$ value to the 2^+ state at 1146 keV is adjusted to fit the excitation probability for that state, giving (0.077 ± 0.015) e^2b^2. If the input $B(E2)$ value to the 2^+ state at 1139 keV is assumed to be 2.59 times smaller [obtained from the (d, d') measurements], the resulting excitation probability is 33% less than the experimental value. In the case of the 1146-keV state, the area of the 1061-keV peak (see Fig. 2), corrected for γ-ray efficiency and angular distribution effects and increased using the measured branching ratios[20] to account for the $2^+_\gamma \to 0$ and $2^+_\gamma \to 4$ transitions, yielded the experimental excitation probability (Table I). The experimental probability for the 1139-keV state was more difficult to extract. We have assigned the 4^+_γ level at 1331.3 keV from (d, d') measurements[22]; the $4^+_\gamma \to 4^+$ transition would then be at 1053.9 keV, very close to the 1054.3-keV $2^{+\prime} \to 2^+$ transition from the 1139-keV level.

The 1247.0-keV $4^+_\gamma \to 2^+$ transition would be less intense than the $4^+_\gamma \to 4$ and thus more difficult to observe. The calculated excitation probability for the 4^+_γ state is 6.07×10^{-4}, accounting for 47% of the 1054-keV peak in Fig. 2. The remaining area, when corrected for efficiency, angular distribution, and other branchings,[20] gives the experimental excitation probability for the 1139-keV state listed in Table I. The 33% discrepancy between this and the calculated probability may result partially from the complex experimental analysis and from the experimentally undetermined $B(E2;\ 2^{+\prime} \to 0^{+\prime})$ value. This $B(E2;\ 2^{+\prime} \to 0^{+\prime})$ value was reduced by a factor of 22 compared to the $B(E2;\ 2^+ \to 0^+)$ value in order to decrease the calculated probability for the unobserved 1069-keV 0^+ state and increase the probability for the 1139-keV 2^+ level. Further reduction of this matrix element would alleviate the discrepancy for the $2^{+\prime}$ state, and may be reasonable in view of the mixed nature of the two 2^+ states at 1139 and 1146 keV. The fact that the 0^+ state is not Coulomb excited implies that it is not collective. The observed $B(E2)$ of 1.1 SPU to the 1139-keV 2^+ member of this band results only because it mixes heavily with the 1146-keV 2^+_γ state. Such mixing would decrease the overlap in the wave functions of the 1069- and 1139-keV states and thus decrease the $B(E2)$ between these two states.

E. ^{168}Yb

The target for this very rare isotope (0.14% in nature) was enriched to only 18% and contained nearly equal amounts of each of the even-even Yb isotopes. This is evident in Fig. 3 which displays the low energy part of the ^{168}Yb spectrum. Nevertheless, population of $K^\pi = 0^{+\prime}$ and 2^+ bands was observed, indicating that the $B(E2)$ values for these vibrational states are rather large. The 2^+ and 4^+ members of the γ band at 983.8 and 1171.2 keV were known from ^{168}Lu decay work[23] and from $(p, 2n\gamma)$ experiments,[24,25] respectively. We observe both states in our Coulomb excitation measurements. Two γ rays from the 2^+_γ state are present in our spectra and the intensity ratio for these transitions agrees quite well with the value reported by Charvet *et al.*[23] (see Table I). The Z_γ value 0.045 deduced from our $B(E2)$ ratio was used to calculate the $2^+_\gamma \to 4^+$ and all 4^+_γ matrix elements. The resulting $B(E2;\ 0 \to 2^+_\gamma)$ value is $(0.132 \pm 0.012)\ e^2b^2$, which agrees quite well with the recent value of $(0.121 \pm 0.006)\ e^2b^2$, obtained in (α, α') measurements on a highly enriched target at ORNL by Ronningen *et al.*[26] This $B(E2)$ value is 4.83 SPU and thus the 984-keV level is the most collective γ-vibrational state in the stable even Yb isotopes.

A very collective $K^\pi = 0^+$ band is also populated at 1154 keV in our experiments. Kemp and Hagemann[25] in the $(p, 2n\gamma)$ reaction observed this 0^+ state, as well as two others at 1197 and 1543 keV. The strong 1066-keV γ ray in our spectrum leads to the experimental excitation probability listed in Table I. In the SWHET calculations, matrix elements to the 2^+ member of this band were derived from the $B(E2;\ 0 \to 2^{+\prime})$ value of $(0.042 \pm 0.003)\ e^2b^2$ of Ronningen *et al.*[26] and from the rotational model assuming a Z_β of 0.030. This value of the mixing parameter is derived from the branching ratio of the 4^+ member of this band, as measured by Charvet *et al.*[23] A fit to the experimental probability of the 0^+ state yields $B(E2;\ 0^{+\prime} \to 2^+) = (0.080 \pm 0.029)\ e^2b^2$, which corresponds to 2.9 SPU. This is by far the most collective $K^\pi = 0^+$ state in the Yb nuclei studied here. Another estimate of the amount of band mixing of the excited 0^+ band with the ground-state band can be found by comparing our $B(E2;\ 0^{+\prime} \to 2^+)$ value with the $B(E2;\ 0^+ \to 2^{+\prime})$ value from (α, α') measurements,[26] as was discussed for ^{172}Yb in Sec. III C. The Z_β value from this ratio is 0.027 ± 0.016, which agrees, within large uncertainties, with the value 0.030 ± 0.005 deduced from the $B(E2)$ ratio of the $4^{+\prime}$ state.[23] A branching ratio from the 2^+ member could not be measured in Ref. 23 since the $2^{+\prime} \to 0^+$ transition was composite with another peak. The agreement in Z_β values may indicate that this band mixes strongly with the ground band and weakly with the other two 0^+ bands. This fact and the enhanced $B(E2)$ value demonstrate the similarity between this 1154-keV 0^+ band and the β-vibrational bands observed in the $N = 90$ nuclei.

F. Ground-state rotational bands

In each of the five nuclei studied, members of the ground-state rotational bands up to $I = 8$ were populated. For $A = 170$, 172, 174, 176, the γ-ray spectra in coincidence with backscattered projectiles (see Figs. 1–3), yielded accurate excitation probabilities P. The ^{168}Yb spectrum (Fig. 3) contains peaks from all of the stable Yb isotopes due to the low enrichment of this target (18%). Fairly accurate excitation probabilities could still be obtained, since the ground band for ^{168}Yb has the largest rotational constant and thus the largest rotational spacings compared to the heavier isotopes.

Within the estimated uncertainties, the measured P values for the 2^+, 4^+, and 6^+ states agree with SWHET calculations with rotational $E2$ and $E4$ matrix elements included. The only numbers we quote here are the comparisons of the ratio of the 8^+ to 6^+ experimental probabilities with the corresponding calculated ratio

$$R(\tfrac{8}{6}) = \frac{(P_8/P_6)_{\rm exp}}{(P_8/P_6)_{\rm calc}}.$$

The advantages of such a double ratio are that the experimental part depends on relative (not absolute) detector efficiency corrections and that the calculated part depends essentially on the value of $B(E2;\ 0^+ \to 2^+)$ and not on some power of this number. Likewise, systematic normalization uncertainties cancel when such a double ratio is taken and corrections for photopeak-Compton summing losses would be small. To first order, this R value is equal to $B(E2;\ 8^+ \to 6^+)_{\rm exp}/B(E2;\ 8^+ \to 6^+)_{\rm rot}$, that is, the experimental $8 \to 6$ $B(E2)$ value compared to the rotational value, where the latter is derived from the known $B(E2;\ 0^+ \to 2^+)$ through the appropriate Clebsch-Gordan coefficients. This R value thus indicates whether $E2$ transition probabilities are retarded or enhanced compared to the rotational value.

The $B(E2;\ 0^+ \to 2^+)$ values used in our SWHET calculations are listed in column 2 of Table IV. These values are essentially those that gave the best fit to the experimental 2^+, 4^+, and 6^+ excitation probabilities. Recent values from the measurements of Wollersheim *et al.*[27] and Ronnigen *et al.*[28] are listed in column 3. The $B(E4;\ 0^+ \to 4^+)$ values were calculated using the β_4 predictions of either Nilsson *et al.*[29] or Götz *et al.*,[30] who both make the assumption of the usual Fermi distribution of nuclear matter. The R values resulting when using the β_4 values of Nilsson *et al.* are listed in column 7 of Table IV. These values range from 0.81 for ^{176}Yb to 1.16 for ^{168}Yb. The uncertainties on the values for ^{170}Yb and ^{174}Yb are slightly smaller than the others, since three separate experiments were performed in the former case and two in the latter. Using the β_4 values of Götz *et al.*[30] leads to smaller $B(E4)$ and larger R values, as shown in the last column of Table IV. It is difficult to know whether to use the β_4 predictions of Nilsson *et al.*[29] or the more negative β_4 values of Götz *et al.*[30] in our SWHET calculations. The measured $B(E4)$ values[27,28] have large uncertainties and thus do not verify either calculation. The $A = 172$, 174, 176 measurements of Wollersheim *et al.*[27] agree better with the values of Nilsson *et al.*,[29] but the $A = 168$ measurement of Ronningen *et al.*[28] is in better agreement with the value of Götz *et al.*[30] Of prime interest here is the trend of the R values for the five Yb nuclei rather than the individual values. The main reason for this is the significant but somewhat uncertain quantal corrections which we must make to the calculated values, as will be discussed below. In considering the R values in columns 7 and 10 of Table IV, one notes an apparent difference in R for the light and heavy Yb nuclei. The use of the rather large experimental $B(E4)$ values[27] for $A = 172$, 174, and 176 would decrease the $R(8/6)$ value in each case below the "Nilsson" value and would thus amplify this trend of differing R values from $A = 168$ to 176. For some unknown reason, the $B(E2;\ 0^+ \to 2^+)$ values needed to obtain a best fit to the ex-

TABLE IV. Values of $R(8/6)$ for the Yb nuclei. Two sets of $B(E4)$ values were used in the calculation, one set derived from the β_4 predictions of Nilsson *et al.*, the other set from the predictions of Götz *et al*. These R values have no quantal correction included.

A	$B(E2;0^+\to 2^+)$ e^2b^2 This calc	$B(E2;0^+\to 2^+)$ e^2b^2 Recent exp [a]	$B(E4;0\to 4^+)$ e^2b^4 Recent exp [a]	β_4 Nilsson *et al.* [b]	$B(E4)$ e^2b^4 Nilsson *et al.* [b]	$R(8/6)$ This exp [c]	β_4 Gotz *et al.* [d]	$B(E4)$ e^2b^4 Gotz *et al.* [d]	$R(8/6)$ This exp [e]
168	5.68	5.81 ± 0.10	$0.036^{+0.073}_{-0.036}$	+0.02	0.077	1.16 ± 0.19	−0.01	0.027	1.21 ± 0.20
170	5.68			+0.01	0.058	0.93 ± 0.04	−0.02	0.015	0.97 ± 0.04
172	5.84	6.03 ± 0.06	$0.048^{+0.067}_{-0.046}$	−0.01	0.028	0.91 ± 0.05	−0.03	0.008	0.93 ± 0.05
174	5.74	5.95 ± 0.06	$0.044^{+0.078}_{-0.043}$	−0.02	0.015	0.85 ± 0.03	−0.04	0.002	0.87 ± 0.04
176	5.33	5.41 ± 0.08	$0.078^{+0.074}_{-0.072}$	−0.03	0.005	0.81 ± 0.06	−0.05	0	0.83 ± 0.06

[a] Wollersheim *et al.*, Ref. 27, for $A = 172, 174, 176$; Ronningen *et al.*, Ref. 28, for $A = 168$.
[b] Reference 29.
[c] The SWHET calculations in this case used $B(E4)$ values from predictions of Nilsson *et al.*, Ref. 29.
[d] Reference 30.
[e] The SWHET calculations in this case used $B(E4)$ values from predictions of Gotz *et al.*, Ref. 30.

perimental 2^+, 4^+, and 6^+ excitation probabilities are 1.5–3% smaller than the more recent precisely measured values. Increasing $B(E2;\ 0^+ \rightarrow 2^+)$ in each case would increase the SWHET calculated value and decrease R. These corrections would rather uniformly lower the R value for each nucleus, but preserve the trend of increasing R for the light isotopes. A similar argument applies to corrections due to experimental summing losses in the 8^+ probability in excess of that for the 6^+ probability. The geometrical arrangement was identical for the five isotopes studied and thus a correction for this effect would be nearly identical for the five cases, resulting in a general raising of the $R(8/6)$ values by up to 1 or 2% in each nucleus. The trend of R values for the Yb isotopes would be preserved, however.

Before one can relate the $R(8/6)$ value to an estimate of $B(E2;\ 8^+ \rightarrow 6^+)$, one must include quantum-mechanical corrections to the semiclassical Winther-deBoer program used in our calculations. Interpolation of the calculations of Alder[31] indicates that the theoretical P_8/P_6 ratio should be decreased by 8% in the case of ^{16}O excitation of Yb nuclei, resulting in an 8% increase in $R(8/6)$. (We used the same correction factor in the case of ^{16}O excitation of Dy nuclei.[2]) It would seem that this 8% correction should be applied for each Yb isotope, since very similar bombarding energies and target thicknesses were used. Such a uniform correction would raise all of the R values of Table IV, but preserve the trend of differing values. In the future, calculations need to be done using the exact quantum-mechanical code AROSA[31] (or possibly other suggested approaches[32]).

Members of the ground bands up to $I = 18$ in ^{174,176}Yb have recently been Coulomb excited with Kr and Xe beams at Berkeley.[1] The evidence there is quite convincing in favor of rotational $B(E2)$ values throughout the band. Our measurements at $I = 8$ for these two nuclei are in agreement with the Berkeley results if the 8% quantal correction is applied. The same group has performed similar measurements on ^{170,172}Yb and once again conclude that, within experimental uncertainties, the $B(E2)$ values are rotational.[33] Assuming an 8% quantal correction and the β_4 predictions of Götz *et al.*,[30] we find $R(8/6) = 0.94 \pm 0.04$ for ^{174}Yb and 1.05 ± 0.04 for ^{170}Yb. The limits of the uncertainties very nearly reach 1.0 in each case, so one might ignore the trend of increasing R values for these four nuclei and say that the $B(E2)$ values are rotational. Another consideration is that our bombarding energy was possibly high enough to cause Coulomb nuclear interference effects of the same order as some of the apparent deviations from the rotational predictions.

^{168}Yb has not been Coulomb excited with heavier projectiles but our results on this nucleus are compatible with a trend of larger R values for the lighter isotopes. Obviously, the uncertainty associated with the $R(8/6)$ measurement in ^{168}Yb is large and precludes any definite conclusions. However, there is the possibility of ^{168}Yb undergoing centrifugal stretching. The 90-neutron nuclei ^{150}Nd, ^{152}Sm, and ^{154}Gd do exhibit $B(E2)$ values[34-36] larger than rotational estimates at $I = 8$; in the case of ^{152}Sm, the $B(E2;\ 8^+ \rightarrow 6^+)$ value is enhanced by $(31 \pm 20)\%$, according to Rud *et al.*[36] Judging simply from the energy of the first 2^+ state (which is empirically related to deformation), one might conclude that ^{164}Yb ($N = 94$) would be analogous to $N = 90$ Sm and Gd nuclei. Bochev *et al.*[37] have measured lifetimes of states in Yb isotopes having $A = 160$, 162, 164, 166 in (^{40}Ar, $4n$) reactions. In ^{164}Yb they measure a $B(E2)$ value which is $(12 \pm 36)\%$ larger than the rotational estimate for $I = 8$ and $(3 \pm 34)\%$ larger for $I = 10$. At higher spins, the $B(E2)$ values are reduced, evidently as a result of a crossing with another band (backbending). In ^{166}Yb, the measured $B(E2)$ value relative to the rotational prediction is 1.006 ± 0.114 for $I = 8$ and 0.92 ± 0.44 for $I = 10$. The uncertainties are large in both cases, and thus it is difficult to conclude if there are enhanced $B(E2)$ values between states in the ground band below the backbending point. By following the trend of vibrational states in Yb nuclei (next section), one can certainly see evidence at $A = 164–168$ for nuclei which are softer against collective vibrations. Such proposed softness might allow centrifugal stretching in the ground bands, as in the case of ^{150}Nd, ^{152}Sm, and ^{154}Gd, and give rise to $B(E2)$ values slightly in excess of the rotational values before backbending occurs at $I \sim 14$. It seems important to investigate ^{168}Yb at higher spins. It does not backbend and thus provides a better case for searching for centrifugal stretching than do the backbending ^{164,166}Yb nuclei.

G. Bands based on excited states

Although many bands with $K^\pi = 2^+$ are observed in even-even deformed nuclei, throughout the rare-earth deformed region the lowest 2^+ band has the largest $B(E2)$ value, is the most collective, and can be classified as the γ-vibrational band. Such is the case in the Yb nuclei, where only the lowest-lying $K^\pi = 2^+$ band was observed in Coulomb excitation measurements. The systematic trend of the γ band is shown in Fig. 6. The 2^+_γ state is highest in energy in ^{174}Yb, but then falls steadily for the lighter isotopes. The ^{164,166}Yb nuclei are not stable and thus were not studied in this work. However, the γ-vibrational bands are strongly

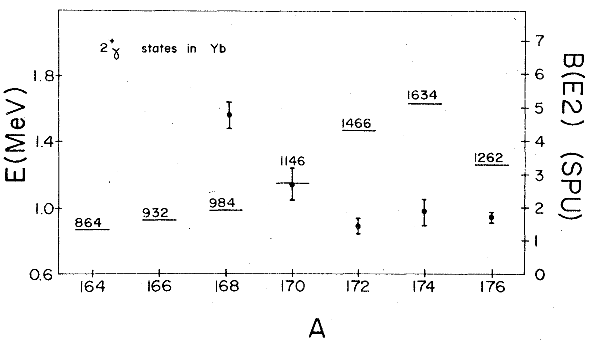

FIG. 6. Energies and $B(E2)$ values for 2^+_γ-vibrational states in Yb nuclei. The energies are written in keV on the level and our measured $B(E2;\ 0^+\rightarrow 2^+_\gamma)$ values are also given (scale on the right). The 2^+_γ placement comes from Ref. 38 for ^{166}Yb and Ref. 39 for ^{164}Yb.

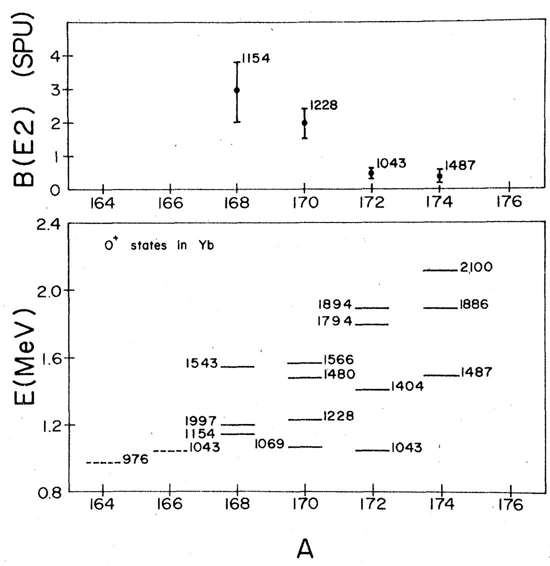

FIG. 7. Energies and $B(E2)$ values for 0^+ states in Yb nuclei. The energies are written in keV in the lower portion and our measured $B(E2;\ 0^+\rightarrow 2^+)$ values are given in the upper portion. The tentative 0^+ placements come from Ref. 14 for ^{166}Yb and Ref. 39 for ^{164}Yb.

populated in Lu decay, as observed by deBoer *et al.*[38] for $A = 166$ and Hunter *et al.*[39] for $A = 164$. The γ band in ^{164}Yb falls to 864 keV, nearly half the energy of the corresponding state in ^{174}Yb. The Gd nuclei show a similar but less pronounced trend, as the γ bandhead drops from a high of 1187 keV for $A = 158$ to 996 keV for $A = 154$. The ^{154}Gd nucleus is a transitional 90-neutron nucleus and is thought to be quite soft against vibrations since the rather low-lying γ band has a $B(E2)$ value of 5.9 SPU according to (α, α') measurements[40] and 6.9 SPU measured in (^{16}O, ^{16}O$'\gamma$) experiments.[41] In the Yb nuclei, one observes (Fig. 6) an increase in $B(E2)$ strength as the $K^\pi = 2^+$ band falls in energy. In ^{168}Yb, the γ band is at 984 keV and has a $B(E2)$ value of 4.75 SPU. Although this nucleus has a larger deformation than ^{154}Gd [the first 2^+ state in ^{168}Yb is at 87.8 keV, its $B(E2;\ 0^+\rightarrow 2^+)$ value is approximately 5.7 e^2b^2; in ^{154}Gd the corresponding state[40] is at 123 keV and has a $B(E2)$ of 3.9 e^2b^2], it is nearly as soft toward the γ-vibrational mode, judging from the energy and $B(E2)$ value for the first $I^\pi K = 2^+ 2$ state. In ^{164}Yb, the first 2^+ state is at 123.8 keV and the $B(E2;\ 0^+\rightarrow 2^+)$ value is 4.6 e^2b^2.[37] This ^{164}Yb nucleus is even less deformed than ^{168}Yb and so might be expected to have an even larger $B(E2)$ to its very low energy $K^\pi = 2^+$ band. It is possible that ^{164}Yb may be even softer against γ vibrations than ^{154}Gd.

Even when the first $K^\pi = 2^+$ band is quite high in energy in the Yb nuclei, the $B(E2)$ value is in excess of one single-particle unit (Fig. 6). Such is not the case, however, for the excited $K^\pi = 0^+$ bands, as shown in Fig. 7. Many 0^+ states are seen in the Yb nuclei in various processes. Most of the 0^+ states shown in Fig. 7 were observed in (p,t) experiments[14]; those in ^{172,174}Yb were seen predominantly in (n,γ) measurements,[10,11,17] those in ^{170}Yb in ^{170}Lu decay work,[20] those in ^{168}Yb in $(p, 2n)$ experiments,[24,25] that in ^{166}Yb by the (p,t) reaction[13] but notably not in ^{166}Lu decay,[38] and that in ^{164}Yb tentatively in ^{164}Lu decay.[39] In our Coulomb excitation measurements, we observed the 1478-keV 0^+ state in ^{174}Yb with a $B(E2;\ 0^{+\prime}\rightarrow 2^+)$ of approximately 0.4 SPU, the 1043-keV 0^+ state in ^{172}Yb with less than 1 SPU, but we saw a more collective (2.0-SPU) 1228-keV 0^+ level in ^{170}Yb and the 1154-keV 0^+ state in ^{168}Yb with an even larger $B(E2)$ of 2.9 SPU. The absence of any 0^+ bands in ^{172,174,176}Yb with enhanced $B(E2)$ values is generally interpreted as meaning that the $E2$ strength normally associated with the β-vibrational mode is distributed over many 0^+ states. The trend significantly changes in the case of ^{170}Yb, where the second excited 0^+ state has an enhanced $B(E2)$ value. In the (p,t) work of Oothoudt and Hintz,[14] the other three 0^+ states in ^{170}Yb were populated, but the 1228-keV state was not. This, of course, complements our Coulomb-excitation result in showing that the 1228-keV 0^+ state results from a superposition of several different 2-quasiparticle modes and, thus, is more collective than the other 0^+ states. An even greater degree of quadrupole collectivity is found in the lowest excited 0^+ state in ^{168}Yb. One might expect that this trend would continue for the lighter Yb nuclei. Tentatively assigned 0^+ states are found at lower energies in ^{164,166}Yb than in ^{168}Yb and it would seem reasonable that their $B(E2)$ values would be even more enhanced. The behavior of the 0^+ states in Yb nuclei resembles that found in the Gd isotopes. Two excited 0^+ states are known in ^{156,158}Gd, but none of these have $B(E2)$ values over 0.5 SPU.[40] However, in ^{154}Gd ($N = 90$), the well-known β-vibrational state occurs at 815 keV with a $B(E2; 0^{+\prime} \to 2^+)$ value of 10.4 SPU.[36,41] It may be that the same behavior occurs in a more gradual fashion in the Yb nuclei as the neutron number decreases. A significant amount of collectivity associated with the quadrupole vibration is concentrated in the second excited 0^+ state in ^{170}Yb and then in the lowest 0^+ state in ^{168}Yb. Probably neither should be called a β vibration. The lower energy 0^+ states possibly observed in ^{164,166}Yb might be more appropriately classified as β vibrations. The trend of the γ bands in these nuclei was previously mentioned, as well as the evidence that a highly collective γ vibration analogous to that in the $N = 90$ nuclei may be appearing in these light Yb nuclei. Measurements of the lifetimes of these 2^+_γ states in ^{164,166}Yb, though difficult to perform, would be important for elucidating the nature of collective vibrations in slightly deformed nuclei.

These considerations relate to the previous discussion of the possibly anomalous excitation probabilities for the 8^+ states in ^{168,170}Yb, where the question of centrifugal stretching effects in the ground-state band of ^{168}Yb was raised. $B(E2)$ values in ground bands in deformed nuclei have been rather universally found to be rotational, at least up to the backbending point. The notable exceptions are ^{150}Nd, ^{152}Sm, and ^{154}Gd, which do show enhanced $B(E2)$ values that can be explained by centrifugal stretching.[34,35,36] There is evidence that the lighter Yb nuclei, possibly including $A = 168$ but more likely $A = 166$ and 164, exhibit collective excitations that could be quite similar to the β- and γ-vibrational bands in the soft $N = 90$ nuclei. One might expect a connection between the low energy of the collective vibrational states and enhanced $B(E2)$ values in the ground-state band, as was demonstrated quantitatively in the case of ^{154}Gd.[36] Our data on the $B(E2)$ values for the 8^+ states in the light Yb isotopes are suggestive of enhancements relative to the rotational model, but are simply not sufficiently good to make a definitive conclusion. While we have established the occurrence of a rather collective 0^+ band in ^{168}Yb, more measurements with heavier projectiles and better targets are needed to find if this is accompanied by enhanced $B(E2)$ values within the ground band.

IV. CONCLUSIONS

Multiple Coulomb excitation measurements have been performed on the five stable even-A Yb isotopes using ^{16}O projectiles. We have measured $B(E2)$ values to γ-vibrational states in each nucleus, the first such measurement for three of them. Negative-parity states were populated in ^{172,174}Yb. The real advantage of using multiple Coulomb excitation for measurement of vibrational $B(E2)$ values occurs for the excited $K^\pi = 0^+$ bands. In the case of a collective 0^+ band, the interaction of it with the ground band often acts to reduce the $B(E2; 0^+ \to 2^{+\prime})$ value, but increase the $B(E2; 0^{+\prime} \to 2^+)$ value. In our ^{16}O experiments, the $0^{+\prime}$ bandhead is always populated more strongly than the $2^{+\prime}$ band member, and thus we get a measure of the $B(E2)$ to the $0^{+\prime}$ state, whereas (α, α') measurements yield the $B(E2)$ to the $2^{+\prime}$ state. In 4 of the 5 nuclei studied, we measured $B(E2)$ values to previously known 0^+ states. In ^{172,174}Yb, these $B(E2)$ values were less than one single-particle unit. In ^{170}Yb, the first excited 0^+ state at 1069 keV is remarkably noncollective. Its probable 2^+ band member (1139 keV) is populated mainly through its mixing with the 1146-keV 2^+ state. The second excited 0^+ state (1228 keV) has an enhanced $B(E2; 0^{+\prime\prime} \to 2^+)$ value of approximately 2.0 SPU. Even more of the collectivity associated with the quadrupole vibration is concentrated in the 1154-keV 0^+ state in ^{168}Yb, which has a $B(E2)$ value of approximately 3 SPU. Candidates for 0^+ states exist in the unstable ^{164,166}Yb nuclei at even lower energies, and thus it may be that the very collective β-vibrational mode may be forming in the light Yb nuclei, although somewhat more gradually than in Sm and Gd nuclei. Experiments to check the properties of these proposed states[14,39] in ^{164,166}Yb are important to test this idea. The γ-vibrational band is quite high in energy (1634 keV) in ^{174}Yb and has a $B(E2)$ value of approximately 1.7 SPU, but falls rapidly in energy until in ^{168}Yb it is found at 984 keV with a $B(E2)$ value of 4.8 SPU. Thus, the observation of collective $K^\pi = 0^+$ and 2^+ bands in ^{168,170}Yb indicates that the light Yb nuclei are becoming softer against the quadrupole vibrational modes.

Coulomb excitation of the members of the ground bands up to $I = 8$ was observed for each nucleus. The measured P_8/P_6 probability ratios were compared to Winther-deBoer calculations using rotational $E2$ and $E4$ matrix elements. The uncertainty of the corrections due to quantum-mechanical effects (and possibly Coulomb nuclear interference effects) makes it difficult to obtain absolute results for each nucleus. Nevertheless, quantal corrections should be rather uniform for the five nuclei, and thus we note the observed trend of increasing $R(8/6)$ values in moving from heavy to light Yb isotopes. Although the uncertainties on the measured value for ^{168}Yb are large, the trend suggests that centrifugal stretching in the ^{168}Yb ground band might exist. The arguments concerning the collective 0^+ and 2^+ bands indicate that the light Yb nuclei ($A = 164-168$) may resemble the $N = 90$ Nd, Sm, and Gd nuclei and thus could exhibit some degree of stretching in their ground-state bands. Clearly, more Coulomb excitation experiments with heavier projectiles are needed for ^{168}Yb.

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