Resonating-group study of the $\alpha + {}^{15}N$ system

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The resonating-group method is used to study the $\alpha + {}^{15}N$ system. The result shows that, with the use of a reasonable nucleon-nucleon interaction and a phenomenological imaginary potential, the calculation can yield a differential cross section which agrees rather well with experimental data. In addition, one learns from this study useful information concerning the cluster structures of many bound and resonance levels in the compound nucleus ^{19}F.

[NUCLEAR REACTIONS $^{15}N(\alpha,\alpha)$; calculated level structure, phase shifts, and $\sigma(\theta)$. Resonating-group method.]

I. INTRODUCTION

The resonating-group method[1] has been successfully used to study the interaction of α particles with s-shell nuclei,[2-4] ^{6}Li,[5] ^{12}C,[6] ^{16}O,[7-10] ^{18}O,[11] and ^{40}Ca.[12,13] From these studies, one obtained valuable information not only about the cluster structures of the compound nuclei involved, but also about the effects of antisymmetrization. In this investigation, we perform a similar calculation for the $\alpha + {}^{15}N$ system, the main purpose being to see in what way the presence of a single 1p hole in the target nucleus affects the characteristics of the internuclear interaction.

Triton cluster states in the compound nucleus ^{19}F have previously been investigated by a single-channel $t + {}^{16}O$ resonating-group calculation.[14] The present study is aimed to supplement that calculation by identifying also the α-cluster states. Since there is both theoretical[15] and experimental[16] evidence that at least for low-excited states the cluster configurations are relatively pure, it is our belief that in this particular nucleus, uncoupled single-channel resonating-group calculations should yield reliable results.

In the next section, we give a very brief description of the $\alpha + {}^{15}N$ resonating-group formulation. Level-structure, phase-shift, and differential-cross-section results are presented in Sec. III. Finally, in Sec. IV, concluding remarks are made.

II. FORMULATION

The formulation of a single-channel resonating-group calculation has been thoroughly described elsewhere[1] and, hence, will only be very briefly discussed here. Essentially, what one does is to derive an effective internuclear potential between the α and ^{15}N clusters by assuming reasonable cluster internal functions and by adopting the nucleon-nucleon potential given by Eqs. (117)–(119) of Ref. 1. To this effective potential one then adds a phenomenological imaginary potential $iW(R)$ to take reaction effects crudely into account. In this way, one obtains in each (J,l) state the following integrodifferential equation satisfied by the radial function $f_{Jl}(R)$ of the relative motion:

$$\left\{\frac{\hbar^2}{2\mu}\left[\frac{d^2}{dR^2}-\frac{l(l+1)}{R^2}\right]+E-V_N(R)-iW(R)-V_C(R)-\eta_{Jl}V_{so}(R)\right\}f_{Jl}(R)=\int_0^\infty [k_l^N(R,R')+k_l^C(R,R')]\,f_{Jl}(R')dR', \quad (1)$$

where the quantity η_{Jl} is given by

$$\eta_{Jl}=l \text{ for } J=l+\tfrac{1}{2},$$
$$\eta_{Jl}=-(l+1) \text{ for } J=l-\tfrac{1}{2}, \quad (2)$$

and E is the relative energy of the two clusters in the c.m. system. In Eq. (1), the functions V_N, V_C, and V_{so} represent the direct nuclear-central, the direct Coulomb, and the direct spin-orbit potentials, respectively, while the kernels k_l^N and k_l^C represent nonlocal contributions arising from the exchange character of the nucleon-nucleon potential and the antisymmetrization procedure. In deriving the intercluster spin-orbit interaction, we have adopted the simplification of omitting exchange terms by neglecting intercluster antisymmetrization. Also, two additional approximations have been made in the actual computation: (i) The

exchange-Coulomb kernel k_l^C is determined by an approximate, but self-consistent procedure[17], and (ii) the nucleon-nucleon spin-orbit range parameter λ is assumed to have a value approaching infinity [see Eq. (117) of Ref. 1]. As a result of this latter approximation, the potential $V_{so}(R)$ is characterized by only a single parameter $J_\lambda = V_\lambda \lambda^{-5/2}$ which will then be chosen as discussed below.

The α and ^{15}N clusters are considered to be described by translationally invariant antisymmetrized products of single-particle wave functions of the lowest configurations in harmonic oscillator wells. To simplify the calculation, we have assumed that the two oscillator wells are specified by a common width parameter of 0.32 fm^{-2} which is chosen to yield correctly the rms matter radius of the larger cluster, namely, ^{15}N. Quite clearly, this is not a very desirable simplification; however, a similar calculation[9] in the neighboring system $\alpha + ^{16}$O did show that the resultant uncertainties may be rather minor.

There is another undesirable feature in our calculation which should be mentioned. For a substantial saving in computational effort, we have not coupled the orbital angular momentum of the 1p hole in the target nucleus ^{15}N and the relative orbital angular momentum between the clusters. This simplification results in the appearance of a spurious rotational band which would otherwise be Pauli forbidden if the coupling had been correctly performed. Fortunately, however, this spurious band occurs deeply in the bound region and, hence, its appearance can be easily recognized and affects only in a minor way the essential findings of our calculation.

The absorptive potential $W(R)$ is assumed to have a surface-derivative Woods-Saxon form, i.e.,

$$W(R) = W_I \left[4a_I \frac{d}{dR} f_I(R) \right], \tag{3}$$

with

$$f_I = \{1 + \exp[(R - R_I)/a_I]\}^{-1}. \tag{3'}$$

The values of the geometry parameters R_I and a_I are chosen to be the same as those used in the neighboring system $\alpha + ^{16}$O; these values are[1]

$$R_I = 4.2 \text{ fm}, \quad a_I = 0.6 \text{ fm}. \tag{4}$$

With these parameters fixed, the only adjustable quantity is the depth parameter W_I; this parameter is then adjusted at each energy to yield a best fit with experimental differential-cross-section result.

The exchange-mixture parameter u in the nucleon-nucleon potential [see Eq. (117) of Ref. 1] is determined by fitting the α-particle separation energy of 3.90 MeV (Ref. 18) in the first excited $\frac{1}{2}^-$ state of ^{19}F which has a rather pure $\alpha + ^{15}$N cluster configuration.[16] The resultant value turns out to be equal to 0.873. In comparing with the value of 0.907 obtained in a single-channel $t + ^{16}$O resonating-group calculation[1] to fit the experimental triton separation energy in the ground state of ^{19}F, one notes that the present value is somewhat smaller. This is a reasonable finding, because specific distortion effects[19] are expected to be less important in the $\alpha + ^{15}$N system than in the $t + ^{16}$O system, and in the single-channel approximation such effects are approximately taken into account by a phenomenological adjustment in u. As for the spin-orbit parameter J_λ, the value found in the $t + ^{16}$O case[1] to fit the experimentally determined energy splitting between the lowest $\frac{5}{2}^+$ and $\frac{3}{2}^+$ states in ^{19}F is 12.4 MeV fm^5. In the present calculation, we shall simply adopt this value to compute energy splittings of all ^{19}F bound and resonance levels.

III. RESULTS

In this section, we discuss level-structure, phase-shift, and cross-section results obtained from both the present investigation and a previous resonating-group $t + ^{16}$O calculation.[1] Although some of the results from the latter calculation have already been reported in Ref. 1, we feel that a simultaneous presentation of both sets of results is desirable and will give a more complete description of the general characteristics of the compound system ^{19}F.

Phase shifts from 0 to 30 MeV, obtained with J_λ and W_I set as zero, are shown in Fig. 1. From this figure, it is seen that there exist many resonance levels which have predominantly $\alpha + ^{15}$N or $t + ^{16}$O cluster configurations. Also, by comparing the behavior of $\alpha + ^{15}$N phases with that of $\alpha + ^{16}$O phases depicted in Fig. 3 of Ref. 1, one readily notes that there is a large degree of similarity. This indicates that the effective interaction in the $\alpha + ^{15}$N system is quite similar to that in the $\alpha + ^{16}$O system, suggesting that the 1p hole in the ^{15}N target plays a relatively minor role.

In Fig. 2, we show a comparison between calculated and experimental[15] $\alpha + ^{15}$N and $t + ^{16}$O cluster states in ^{19}F. To obtain the calculated results, we have performed both bound-state and scattering calculations with $W_I = 0$ and the J_λ value given in the previous section. As is seen, the agreement is generally quite satisfactory, except that the calculated excitation energies of the negative-parity $t + ^{16}$O levels seem too large. At present, we are not sure about the origin of this discrepancy, although we suspect that it comes from our use of a simplified single-Gaussian triton spatial wave function which results in a relatively inadequate

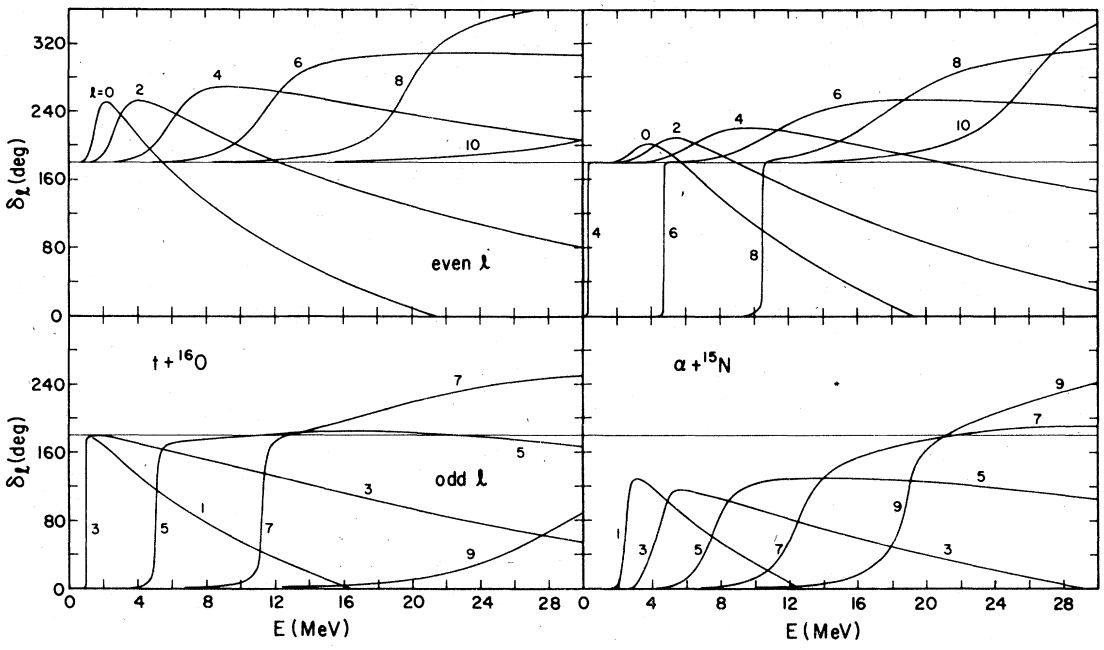

FIG. 1. Calculated phase shifts for $t + {}^{16}O$ and $\alpha + {}^{15}N$ scattering.

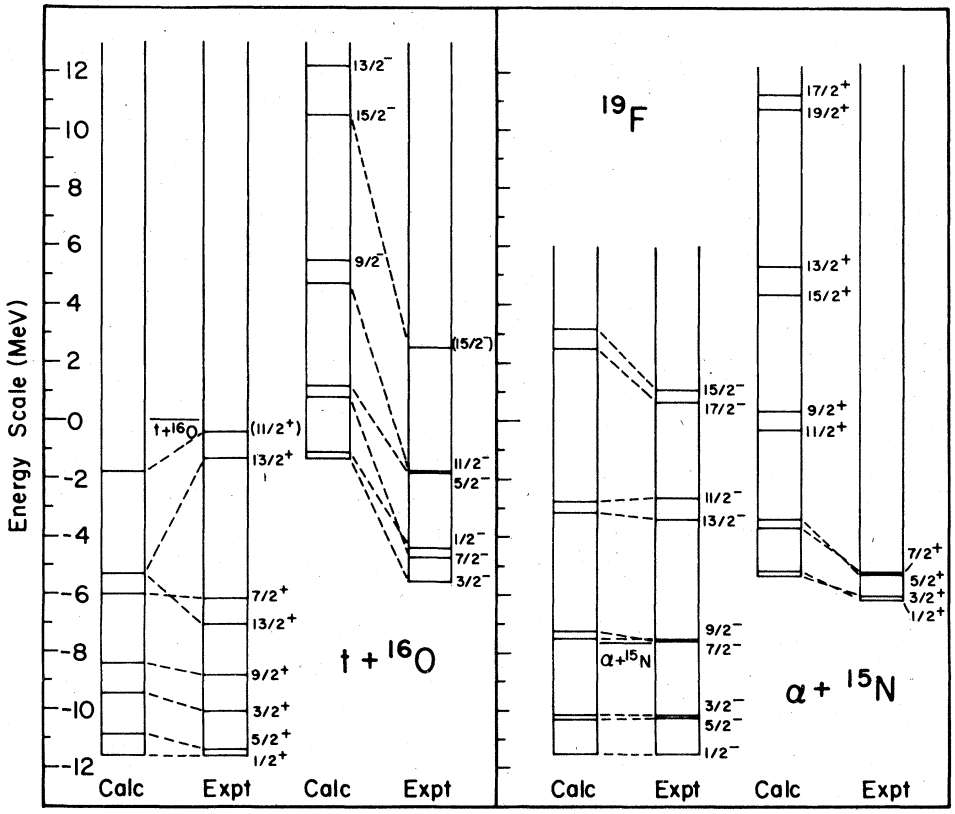

FIG. 2. Comparison of calculated and experimental energy spectra of ^{19}F. The levels shown are those which have predominantly $t + {}^{16}O$ or $\alpha + {}^{15}N$ cluster configurations.

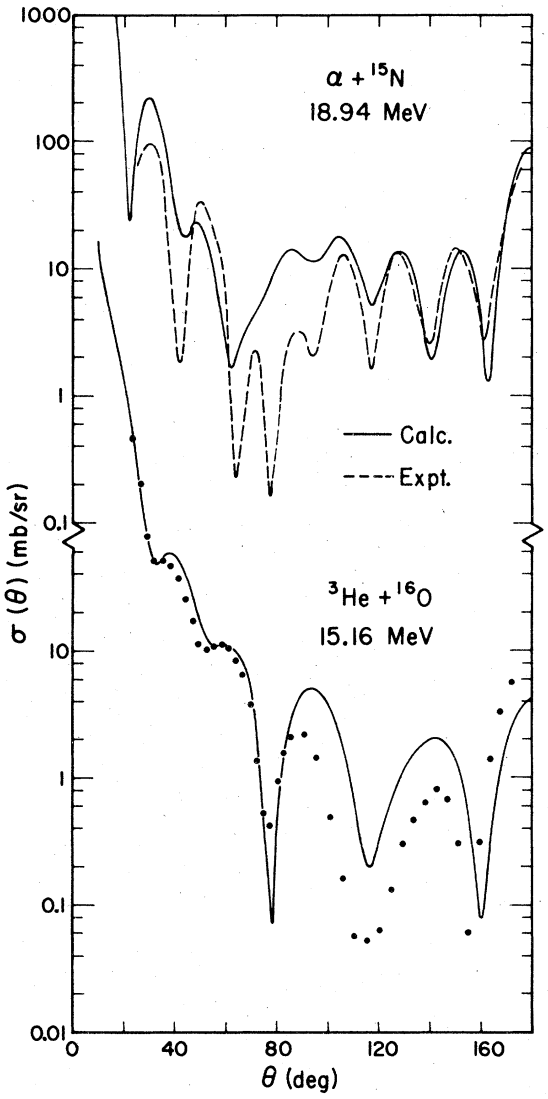

FIG. 3. Comparison of calculated and experimental $\alpha+{}^{15}N$ and ${}^{3}He+{}^{16}O$ differential cross sections at an energy of 6 MeV/nucleon.

description of the long-range part of the $t+{}^{16}O$ effective interaction.

The $\alpha+{}^{15}N$ differential cross section at 18.94 MeV (i.e., 6 MeV/nucleon), calculated with $J_\lambda = 12.4$ MeV fm^5 and $W_I = 2.4$ MeV, is shown in Fig. 3 together with the experimental data of Oeschler *et al.*[20] For completeness, we show also in this figure the calculated and experimental results for ${}^{3}He+{}^{16}O$ scattering at the same energy per nucleon.[1] Here one notes that, even though the oscillatory features of the experimental cross sections are quite different, resonating-group calculations are capable of yielding rather satisfactory descriptions of both sets of these experimental data.

IV. CONCLUSION

The present investigation, together with a previous resonation-group study of the $t+{}^{16}O$ system, shows that with the use of a reasonable nucleon-nucleon potential, even single-channel resonating-group calculations can yield a rather satisfactory description of many interesting features of the compound system ${}^{19}F$. From the results of these calculations, we have been able to identify many bound and resonance states which have predominantly $\alpha+{}^{15}N$ or $t+{}^{16}O$ cluster configurations. In addition, by the introduction of a phenomenological imaginary potential, the calculations also yield differential cross sections which agree fairly well with experimental data.

From the many resonating-group calculations so far performed, one can systematically learn the important characteristics of the $\alpha+$nucleus effective interaction. For example, it is found here that the $\alpha+{}^{15}N$ and $\alpha+{}^{16}O$ effective interactions are similar in nature, thus indicating that the extra hole in ${}^{15}N$ does not make a major contribution.

The encouraging results obtained here suggest that one should now proceed to perform a coupled-channel resonating-group calculation involving both $\alpha+{}^{15}N$ and $t+{}^{16}O$ cluster configurations in order to gain a better understanding about the level structure of the nucleus ${}^{19}F$. Quite obviously, this will not be an easy task; however, with the use of the complex-generator-coordinate technique described in Ref. 1, it is our belief that the calculation will be computationally feasible, although laborious.

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