PHYSICAL REVIEW C VOLUME 20, NUMBER 5 NOVEMBER 1979

Comparison of optical-model and Lane-model analyses of sub-Coulomb protons on ^{92,94}Zr

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(Received 22 June 1979)

Accurate proton elastic-scattering cross sections were measured with enriched targets of ^{92,94}Zr from E_p = 2.0 to 6.5 MeV. The elastic-scattering cross sections, together with absorption cross sections, were analyzed with a Lane model which employed the optical potential of Johnson *et al.* The resulting parameters were compared with those obtained with a single-channel optical model and negligible differences were found. Significant differences between the ^{92}Zr and ^{94}Zr real diffusenesses resulted from the inclusion of the (p,p) data in the analyses.

[NUCLEAR REACTIONS ^{92,94}Zr$(p,p)E_p$ = 2.0–6.5 MeV; measured $d\sigma/d\Omega$ at 135° and 165°. Lane-model and optical-model analyses of elastic scattering and absorption cross sections.]

I. INTRODUCTION

Useful information about the nucleon-optical-model potential can be obtained from proton-induced reactions when the incident energy of the protons is below the Coulomb barrier. This is true primarily because the barrier emphasizes the single-particle resonances and their effects dominate the features of the proton absorption. This fact was recently exploited by Johnson *et al.*[1] in an analysis of (p,n) cross sections for 14 nuclides from A = 89 to 130 for an incident proton energy range of 2.5 to 5.8 MeV. It was found that the data could be described by introducing an anomalous A dependence into the depth of the absorptive part of the spherical-optical-model potential. In that analysis no (p,p) data were included, and isobaric analog resonances were ignored.

In the present paper, results of measurements of total (p,n) and differential elastic (p,p) cross sections for ^{92}Zr and ^{94}Zr in the energy range of 2.5 to 6.5 MeV are presented and analyzed. The objectives of the analysis are threefold:

(1) To ascertain the difference between the parameters obtained with a Lane-model[2] analysis, where the isobaric analog states are explicitly considered, and the parameters obtained in an ordinary optical-model analysis.

(2) To ascertain the effects of the inclusion of the (p,p) data on the parameters obtained in such analyses.

(3) To ascertain whether the depths for the absorptive part of the optical-model potential for these two nuclei are consistent with the results of Johnson *et al.*[1]

II. MEASUREMENTS

The measurements were performed with the University of Kentucky model CN Van de Graaff accelerator and the associated facilities. The (p,n) cross sections were previously presented[3] in conjunction with other (p,n) measurements; the resulting estimated uncertainties were ±7%. The elastic scattering yields were carefully measured in order to minimize systematic energy-dependent errors. At the available bombarding energies only 10–20% deviations from the Rutherford cross sections were expected and thus, in order to extract information about the model parameters, it was necessary to measure the (p,p) excitation functions as precisely as possible.

The yields were corrected for experimental resolution due to energy loss in the target (< 3% correction) and dead time in the electronics (< 2% correction). A more difficult problem was maintaining a constant target thickness. This was accomplished by precisely moving the target, in the plane perpendicular to the beam, and measuring the proton yields to determine relative thicknesses across the face of the target. The target was then left in a position where the thickness was at an extremum; thus when the beam position fluctuated on the target, the thickness changed an insignificant amount. The measurements were later repeated at several selected incident energies in order to verify the precision of the original measurements. In these later measurements an alternative technique was used: The yields at back angles were divided by the sum of yields at ±30°. The yields at ±30° obeyed the Rutherford E^{-2} de-

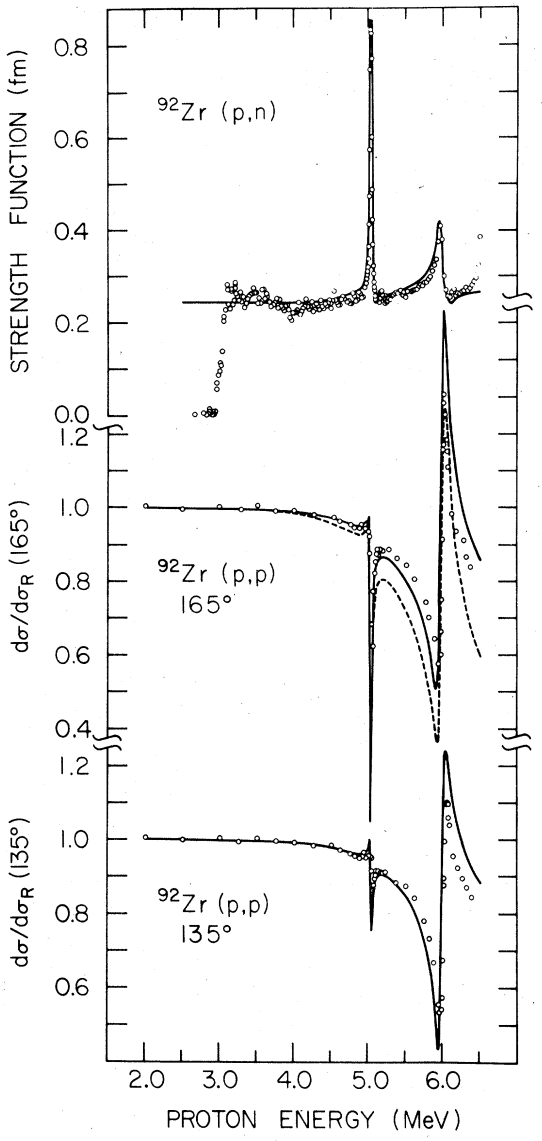

FIG. 1. $^{92}Zr(p,n)$ strength function and $^{92}Zr(p,p)$ differential elastic cross sections normalized to Rutherford. The solid curve is the result of the coupled-channels calculation described in the text. The dashed curve is the result of a coupled-channels calculation performed with real diffuseness of 0.73 fm.

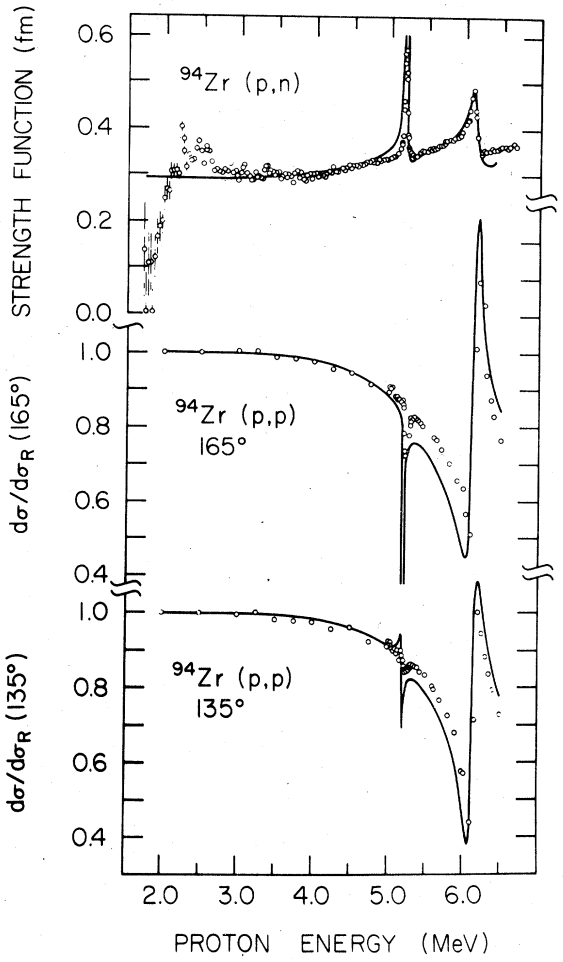

FIG. 2. $^{94}Zr(p,n)$ strength function and $^{94}Zr(p,p)$ differential elastic cross sections normalized to Rutherford. The solid curve is the result of the coupled-channels calculation described in the text.

pendence and the obtained ratio was thus directly proportional to $d\sigma/d\sigma_{\text{Rutherford}}$ and independent of the target thickness. In order to test the assumption that the ±30° yields obeyed the E^{-2} rule, upon completion of the analysis the ±30° cross sections were calculated with the coupled-channels model; it was ascertained that the error introduced into the cross-section ratio was < 1%. Considering the agreement between the independent sets of (p,p) measurements and the reproducibility of the original measurements, the overall measurement precision is estimated to be ±1.5%.

The original (p,p) yields, corrected for experimental resolution and dead time, were multiplied by E^2 and normalized to 1.0 at lower energies. This yielded ratios $d\sigma/d\sigma_{\text{Rutherford}}$ which are presented in Figs. 1 and 2. The corresponding (p,n) cross sections are presented in the form of l-averaged strength functions[1] in the same figures. The resonances which appear in the data for both isotopes are the $d_{5/2}$ and $s_{1/2}$ isobaric analog resonances.

III. ANALYSIS

The optical-model potential assumed is essentially the same potential that Johnson *et al.*[1] used to fit the (p,n) cross sections of nuclei in the A = 89 to 130 region:

$$V(r) = -V_R(E) f(r, R_R, a_R) + V_c(r, R_c)$$
$$-\frac{U_0(N-Z)}{A} f(r, R_R, a_R)$$
$$+ i 4 a_D W_D \frac{d}{dr} f(r, R_D, a_D)$$
$$+ V_{SO} \frac{\vec{\sigma}\cdot\vec{l}}{r} \left(\frac{\hbar}{m_\pi c}\right)^2 \frac{d}{dr} f(r, R_{SO}, a_{SO}),$$

where $V_c(r, R_c)$ is the Coulomb potential for a uniformly charged sphere,

$$f(r, R, a) = \{1 + \exp[(r-R)/a]\}^{-1},$$
$$V_R(E) = V_0 + 0.45Z/A^{1/3} - 0.32E,$$

and

$$R_x = r_x A^{1/3}.$$

The values of the parameters which were not varied are $r_R = 1.2$ fm, $a_R = 0.73$ fm, $r_D = 1.3$ fm, $a_D = 0.4$ fm, $V_{SO} = 6.4$ MeV, $r_{SO} = 1.03$ fm, $a_{SO} = 0.63$ fm, and $R_c = 1.23$ fm. This potential was used for the ordinary optical-model analysis to be described later.

This same potential was also used in the coupled-channels scheme, first introduced by Lane[2], by defining the coupled-channels potentials in terms of the parameters and form factors of $V(r)$. The coupled equations are not repeated here but are expressed in Eqs. (1), (2), (6), and (7) of Ref. 4. Equation (6) of said reference describes the physical channel of proton plus target (with coupling to the neutron plus analog of the target represented by the right-hand side). The following definitions were adopted to make the potential in the left-hand side of Eq. (6) identical to the potential defined above:

$$V^{\pm} = -V_R(E) f(r, R_R, a_R) + iW^{\pm}$$
$$+ V_{SO} \frac{\vec{\sigma}\cdot\vec{l}}{r} \left(\frac{\hbar}{m_\pi c}\right)^2 \frac{d}{dr} f(r, R_{SO}, a_{SO}),$$
$$U = \frac{(4)U_0}{A} f(r, R_R, a_R),$$
$$W^- = \frac{2T_0 + 1}{2T_0} 4 a_D W_D \frac{d}{dr} f(r, R_D, a_D),$$

and

$$W^+ = 0.$$

In addition, the parameter V_0 was allowed to be l-dependent.

Optical-model and coupled-channels calculations were then performed according to the procedures outlined below. In performing the optical-model calculations, a Hauser-Feshbach[5] analysis was always included and showed that the difference between the absorption cross sections and the absolute, total (p,n) cross sections was less than 2% starting at energies just a few hundred kilovolts above the (p,n) threshold. Hence, in doing the coupled-channels calculations, this difference was ignored and, furthermore, the compound elastic contributions to the differential elastic (p,p) cross sections were neglected.

The calulations were performed according to the following procedures.

(1) Ordinary optical-model calculations were performed until optimum fits (lowest χ^2) were obtained to selected data points for the (p,n) off resonance (background) strength functions without reference to the (p,p) data. In order to obtain a direct comparison with the work of Johnson *et al.*[1], the only parameters which were varied were V_0 and W_D. The other parameters were all taken to have the values used in Ref. 1 with one exception: Johnson *et al.*[1] assumed the value of U_0 to be equal to zero for nuclei within a given isotopic series and equal to 24 MeV from one series to another. Since U_0 could not be assumed to be zero in subsequent coupled-channels calculations, its value was chosen to be 24 MeV throughout these analyses. The optimum values obtained for V_0 and W_D are listed under (1) in Table I.

(2) The parameters obtained according to procedure (1) were then used in the coupled-channels calculations, with the following differences. The values of V_0 for $l=2$, $l=0$, and $l=4$ were determined by the positions of the $d_{5/2}$, $s_{1/2}$, and $g_{7/2}$ isobaric analog resonances,[6] respectively. A consequence of this is that the s-wave giant-resonance position was shifted slightly compared to its position ascertained by procedure (1). To maintain the same quality fit, it was then necessary to change the values of W_D and V_0 for $l=1$. The value of V_0 for $l=3$ was arbitrarily assumed to be the same as that for $l=1$. The parameters obtained in this way are listed under (2) in Table I.

(3) The next step was to recalculate, including the (p,p) data. Since significant deviation from Rutherford scattering does not occur until energies close to the onset of the isobaric analog resonances, the single-channel model was not employed. Using the coupled-channels model it was possible to determine whether a given set of parameters yielded a fit whose overall background quality was good over the isobaric analog regions. This procedure yielded good-quality fits for ^{94}Zr by varying V_0 and W_D only. However, this was not the case for ^{92}Zr. All such attempts yielded results which were too low compared to the (p,p) experimental data. An example of such an effort is shown as the dashed line in Fig. 1. As a consequence, other parameters were varied and it

TABLE I. Values of the parameters obtained with the different procedures discussed in Sec. III.

		^{92}Zr			^{94}Zr		
		1	2	3	1	2	3
V_0 (MeV)	$l=0$	54.3	53.8	54.3	53.8	53.5	53.5
	$l=1$	54.3	53.8	54.8	53.8	53.8	53.3
	$l=2$	54.3	52.9	53.0	53.8	52.6	52.6
	$l=3$	54.3	53.8	54.8	53.8	53.8	53.3
	$l=4$	54.3	55.1	54.0	53.8	54.6	54.6
a_R (fm)		0.73	0.73	0.63	0.73	0.73	0.73
W_D (MeV)		5.5	5.2	10.0	10.6	9.4	10.8

was found that good results could be obtained by lowering the value of a_R. The values obtained are shown under (3) in Table I. The results of the calculations using those values are shown in Figs. 1 and 2. Because of the greater inherent difficulty of this procedure, the values shown in column 3 have greater uncertainty than those in columns 1 and 2. Also note that the calculated strength functions shown in Figs. 1 and 2 are total proton strength functions, whereas the measurements shown are only (p,n) strength functions: They are not expected to be the same in the vicinity of and below the threshold energies.

IV. CONCLUSIONS

Spectroscopic factors for the $d_{5/2}$ and $s_{1/2}$ parent states of the isobaric analog resonances have been obtained by Cohen and Chubinsky.[7] Their results are 0.54 and 0.30 for the $d_{5/2}$ states and 0.91 and 0.89 for the $s_{1/2}$ states in ^{93}Zr and ^{95}Zr, respectively. From these results it is evident that, especially in the case of the $d_{5/2}$ states, the coupled-channels formalism is not expected to yield good results for the height and width of the resonances. This is indeed the case. Nevertheless, by choosing appropriate values of V_0 the positions of the resonances can be made to correspond to their experimental values. When this was done, only small differences were obtained between these V_0's and the one obtained from the optical-model analysis of the (p,n) background only. In fact, had the latter V_0 been used in the coupled-channels analysis, the positions for the resonances would have been close to their experimental values. For example, in ^{92}Zr, the $d_{5/2}$ resonance would have appeared at ~4.0 MeV, compared to its experimental position at ~5 MeV. In this respect, it is worthwhile pointing out that a shift of 1 MeV in V_0 caused a shift of ~1 MeV in the position of the resonance in question.

The difference referred to above was especially small in the case of the $s_{1/2}$ resonances, as can be seen from comparing the values for V_0 ($l=0$) in columns 1 and 2 of Table I. Therefore, explicit inclusion of the isobaric analog resonances in the analysis did not result in a major shift of the position of the s-wave single-particle resonances. This contributed to the result that inclusion of the isobaric analog resonances did not have a major effect on the values obtained for W_D.

For ^{94}Zr, inclusion of the (p,p) data did not significantly alter the values of the parameters obtained by considering the (p,n) data alone. However, inclusion of the (p,p) data produced a significant effect in the case of ^{92}Zr. It was impossible to get a reasonable fit to the (p,p) background by varying V_0 and W_D only. It was also impossible to get a reasonable fit by varying the geometry of the absorptive potential. Satisfactory fits could only be obtained by varying the real potential geometry. Since several parameter combinations are possible, an attempt was made to ascertain the single most sensitive parameter, in addition to V_0 and W_D. This turned out to be the diffuseness parameter of the real potential, a_R. To produce a satisfactory fit to both the (p,n), and (p,p) data for ^{92}Zr this parameter had to be decreased about 15%. As a result, the value of W_D obtained changed dramatically from about 5 to 10 MeV. Insofar as the difference in a_R for ^{92}Zr and ^{94}Zr is concerned, it is known[8] that these isotopes do have some collective $E2$ excitation strength with a $\beta_2 \approx 0.1$. This difference, therefore, may be understood in terms of weak collective properties or, perhaps, shell-closure effects.

In summary, the results of this analysis do indicate that inclusion of the (p,p) data can significantly affect the values obtained for the potential parameters. However, they did not produce any evidence that explicitly coupling the upper and lower isospin states would significantly affect the results of Johnson *et al.*,[1] who found an anomalous behavior for the absorptive potential W_D. Al-

though none of the values obtained for W_D in this analysis are inconsistent with an anomalous behavior of the absorptive potential, further measurements and analyses which include the elastic scattering are needed to settle the question of whether or not it is necessary to vary W_D anomalously over the mass 103 region.

ACKNOWLEDGMENTS

The authors wish to thank Dr. C. H. Johnson and Dr. J. P. Schiffer for helpful duscussions. This work was supported in part by a grant from the National Science Foundation.

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