Spin-unsaturated subshells and the nucleon-nucleus optical-model spin-orbit potential

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(Received 15 January 1979)

The role of spin-unsaturated subshells in calculations of optical-model spin-orbit potentials is investigated using a realistic nucleon-nucleon interaction. For projectiles of moderate bombarding energies the one-body spin-orbit potential is predicted to be quenched relative to that for nearby nuclei having only spin-saturated shells. The source of this quenching is the same as that discussed by Scheerbaum in an investigation of the quenching of spin-orbit splittings in bound states. A simple physical picture is given which permits a qualitative understanding of this quenching in terms of the components of the nucleon-nucleon interaction. An estimate of the effect on polarization-like observables is made, and a general problem with currently used tensor forces is reemphasized. The relationship of this problem to a number of other structure and scattering problems is stressed.

[NUCLEAR REACTIONS, NUCLEAR STRUCTURE Calculation of contributions to the optical-model spin-orbit potential from spin-unsaturated subshells.]

I. INTRODUCTION

The general problem of establishing a microscopic basis for the nucleon-nucleus spin-orbit potential is an old one.[1] Most of the work on this problem has been done for bound nucleons primarily within the Hartree-Fock framework.[2-6] More recently Scheerbaum[7-9] has investigated the microscopic origin of the spin-orbit splitting and finds[8] reasonably good agreement between theory and experiment for splittings in nuclei in which both $j=l+\frac{1}{2}$ and $j=l-\frac{1}{2}$ subshells are filled [spin-saturated (SS) shells]. Although the analogous problem for nucleon scattering states has received considerably less attention,[10,11] the results are quite promising. For scattering states the observable most closely parallel to spin-orbit splitting is the induced asymmetry or polarization. For inelastic scattering, the optical-model spin-orbit potential is known[12] to play a dominant role in describing both asymmetries and spin-flip probabilities.

Although the spin-orbit splitting is reasonably well understood for nuclei having SS shells, the situation is quite different[2,3,5] for those nuclei having one or more spin-unsaturated subshells (SUS), that is, for those nuclei in which the $j=l+\frac{1}{2}$ subshell is filled but the $j=l-\frac{1}{2}$ subshell is empty. A typical example is ^{48}Ca in which the $f_{7/2}$ neutron shell is full and the $f_{5/2}$ neutron shell is empty. Theoretical calculations using realistic nucleon-nucleon (N-N) forces invariably predict[2,3,6] too small a splitting of the $j_<$ and $j_>$ levels. This predicted reduction in the spin-orbit splitting has been traced directly to the central and tensor forces which in lowest order contribute[2,3,9] to the spin-orbit splitting only through the SUS due to their dependence on the *product* of the two participating spins. For SS shells spin averaging excludes the participation of the central and tensor forces in first order leaving the one-body spin-orbit potential dependent primarily on the nuclear geometry and the N-N spin-orbit interaction.

A number of workers have investigated the SUS problem (which is still unsolved) in an effort to either solve it or to understand its origin and implications. Wong[2] first clearly pointed out the SUS problem within a Brueckner-Hartree-Fock (BHF) context and in addition discussed its origin. Davies and co-workers[3] have made a series of progressively more sophisticated BHF calculations in SUS nuclei but the spin-orbit splitting remains underestimated. Using the Skyrme force supplemented by a "realistic" tensor force, Stancu *et al.*,[13] on the basis of calculations of spin-orbit splittings in SUS nuclei, conclude that there must be something wrong with the tensor force. Goodman and Borysowicz[14] have very recently invoked the SUS phenomenon to explain the mass dependence of the $l=5$ proton spin-orbit splitting. The recent work of Scheerbaum has been especially helpful in illuminating the relative roles of the central and tensor parts of the N-N force for spin-orbit splittings in SUS nuclei.

The purpose of this work is to explore the role of SUS in the calculation of optical-model spin-orbit potentials for scattering states and to further[3,9] elucidate the roles of SUS and the various components of the internucleon interaction in bound-state problems. In particular, using a realistic N-N interaction we examine the alteration of the optical-model spin-orbit potential expected to arise from specifically SUS effects. The changes in the polarization observables which should attend such an alteration of the spin-orbit potential are studied and compared with experimental data sensitive to such effects. While we anticipate the anomaly persisting to positive energy states, we feel it important to see if its effects there are detectable. If they are, the scattering case may afford a wider variety of experiments in which the anomaly can be studied. Since higher-order corrections to the spin-orbit potential may be calculated rather differently for positive and negative energy states, additional insight into the overall problem may be suggested.

We first outline a derivation of the optical-model spin-orbit potential for nucleons scattering from spherical nuclei having SUS. Although the major results can be obtained by using straightforward angular momentum recoupling techniques, the projection-operator technique suggested by Scheerbaum[9] is used here since it is especially convenient for combining and unifying the central and tensor force contributions. The resulting nonlocal potential is then approximated by a local potential (in closed form) which is shown to exhibit quite clearly the qualitative features discussed earlier[2] for the bound-state problem. A discussion similar to that of Scheerbaum (for spin-orbit *splittings*) is given for the roles of the central and tensor parts of the N-N interaction in the optical-model spin-orbit *potential*. Applications are made for nucleons scattering from ^{28}Si, ^{48}Ca, and ^{12}C. Finally, the relationship between the SUS problem and a number of other important problems in nuclear structure and nuclear reactions is discussed.

II. FORMALISM FOR THE SUS CONTRIBUTION TO THE OPTICAL-MODEL SPIN-ORBIT POTENTIAL

With the exception of the N-N spin-orbit potential, we restrict ourselves to a static N-N effective interaction given by

$$V(1,2)=V_0(r)+V_1(r)\vec{\sigma}_1\cdot\vec{\sigma}_2+V_{LS}(r)\vec{L}\cdot\vec{S}+V_T(r)S_{12}\,. \quad (1)$$

The role of nonstatic forces will become clearer as the formalism unfolds. The above V's should be regarded as matrices in isospin space. To first order in the N-N interaction the scattering wave function satisfies

$$\left[\frac{-\hbar^2}{2\mu}\nabla_1{}^2+U_d(r_1)+U_c(r_1)-E\right]\chi(\vec{r}_1)+U_x\chi(\vec{r}_1)=0\,, \quad (2)$$

where

$$U_d(r_0)=\left\langle\Psi\left|\sum_{i=1}^{A}V(0,i)\right|\Psi\right\rangle \quad (3)$$

and

$$U_x\chi(r_0)=\left\langle\Psi\left|-\sum_{i=1}^{A}V(0,i)P_{i0}\right|\Psi(1\cdots A)\right\rangle\chi(0)\,. \quad (4)$$

Here Ψ is the spherical ($\vec{I}=0$) ground-state wave function of the target which is fully antisymmetrized and U_d and U_x are defined as the direct and exchange "potentials," respectively. P_{i0} denotes the exchange operator for *all* of the coordinates $(\vec{r},\sigma,\tau)$ having the labels $(i,0)$. For the direct term only V_0 and V_{LS} will contribute independent of the state of spin saturation when $\vec{I}=0$. This arises directly from parity conservation[9] and our assumption of static forces. Only the V_{LS} component of the N-N interaction contributes to the direct part of the optical-model spin-orbit potential (U_d^{LS}). (For nuclei with SUS, V_{LS} will contribute a small correction to the spin-independent part of the optical potential.) Since we are interested here in those contributions to the spin-orbit part of the optical potential peculiar to SUS, we focus our attention on the (nonlocal) exchange potential.

If the ground state is assumed to consist of a single Slater determinant (composed of the single-particle states ψ_i), the action of the exchange potential may be expressed as

$$U_x\chi(\vec{r}_1)=\sum_{i=\text{occ}}\int d2\langle\psi_i(2)|-V(1,2)P_{12}|\psi_i(2)\rangle\chi(1)\,, \quad (5)$$

where an "integration" over the coordinates labeled by 2 is implied. We next note using $P_{12}=P^xP^\sigma P^\tau$ that

$$-P_{12}=P^x[\lambda_{SE}+\lambda_{TE}-\lambda_{SO}-\lambda_{TO}]\,, \quad (6)$$

where P^x is the space exchange operator and λ_{SE}, for example, is a projection operator onto the singlet-even subspace of nucleons 1 and 2. Consequently, if V is decomposed into its odd and even parts we can rewrite Eq. (5) as

$$U_x\chi(\vec{r}_1)=\sum_{i=\text{occ}}\int d^3r_2(\psi_i(2)\,|\,\hat{V}(1,2)\,|\,\psi_i(\vec{r}_1,\sigma_2,\tau_2))\times\chi(\vec{r}_2,\sigma_1,\tau_1)\,. \quad (7)$$

We use the bold parentheses to indicate integration over only the spin and isospin coordinates, and $\hat{V}$ indicates the N-N interaction which is operative for the exchange term. From Eqs. (6) and (7), we get $\hat{V}$ from V by simply changing the sign of the odd-state parts of the force. From Eq. (7) we define

$$U_x(\vec{r}_1,\vec{r}_2)=\sum_{i=\text{occ}}(\psi_i(\vec{r}_2,\sigma_2)\,|\,\hat{V}(1,2)\,|\,\psi_i(\vec{r}_1,\sigma_2))\,, \quad (8)$$

where we suppress isospin labels for brevity. Equation (1) indicates two types of terms. $\hat{V}_0$ and that part of $\hat{V}_{LS}$ containing $\vec{\sigma}_2$ can only contribute to the spin-independent part of the optical potential.

$\hat{V}_1$, $\hat{V}_T$, and that part of V_{LS} containing $\vec{\sigma}_1$ can only contribute to the spin-orbit part of the optical potential. Since that part of $\hat{V}_{LS}$ which contains $\vec{\sigma}_1$ does not act on the spin or the target, it contributes whether SUS are present or not. Consequently, the contribution to the spin-orbit optical potential which is peculiar to a SUS of angular momentum j is given by

$$\Delta U_j^{LS}(\vec{r}_1,\vec{r}_2)=\sum_{\mu}(\psi_{nlj,\mu}(\vec{r}_2,\sigma_2)\,|\,\hat{V}_1\vec{\sigma}_1\cdot\vec{\sigma}_2+\hat{V}_T S_{12}\,|\,\psi_{nlj,\mu}(\vec{r}_1,\sigma_2)). \tag{9}$$

If there is more than one SUS, we sum over the relevant (n,l,j).

Consider first the contribution of the contral force $\hat{V}_1$. Following the suggestion of Scheerbaum[9] we introduce the projection operator

$$P^+=\frac{(l+1)+\vec{\sigma}_2\cdot\vec{l}_1}{2l+1}, \tag{10}$$

which projects onto the $j=l+\frac{1}{2}$ subspace and permits the sum over $j=l\pm\frac{1}{2}$ as well as μ. Only the $\vec{\sigma}_2\cdot\vec{l}_1$ term survives and when combined with the $\vec{\sigma}_1\cdot\vec{\sigma}_2$ factor and recoupled leads to

$$\begin{aligned}\Delta U_j^{LS}(\vec{r}_1,\vec{r}_2)&=(-)^l\left[\frac{4l(l+1)}{3(2l+1)}\right]^{1/2}\\&\quad\times\hat{V}_1(r_{12})[\phi_l(\vec{r}_1)\times\phi_l(\vec{r}_2)]^1\cdot\vec{\sigma}_1\\&\equiv\hat{V}_1(r_{12})\vec{\rho}_1(\vec{r}_1,\vec{r}_2)\cdot\vec{\sigma}_1.\end{aligned} \tag{11}$$

Here $\phi_{lm}(\vec{r})=u_{nl}(r)Y_{lm}(\hat{r}), u_{nl}(r)$ is the complete radial part of the single-particle orbital (nlj), and Eq. (11) defines the mixed spin density used in Ref. 15. Equation (11) is also equivalent to Eq. (8) of Scheerbaum.[9] The structure of Eq. (11) should be noted. ΔU is a scalar overall (and of even parity) but is of rank 1 in coordinate and spin space. If, as in the direct term $\vec{r}_1=\vec{r}_2$, we see immediately that parity considerations eliminate this term. If the $j=l-\frac{1}{2}$ level were also full (and the radial parts of the $l\pm\frac{1}{2}$ wave functions were the same) it would contribute a $\Delta U_{j<}=-\Delta U_{j>}$ as can readily be seen by noting that the only contributing part of P^+ is the $\vec{\sigma}_2\cdot\vec{l}_1$ term which enters P^- (the projection operator for $j_<$) with the opposite sign. Clearly s-state orbitals do not contribute to ΔU_j. It is interesting to note that ΔU_j^{LS} of Eq. (11) is proportional to (and derivable from) the $G_{110}(\vec{r}_1,\vec{r}_2)$ defined in Ref. 16 for use in "inelastic" scattering where the triad (110) denotes spin and orbital angular momentum transfers of 1 with a net angular momentum transfer of $J=0$. Indeed, any mechanism giving rise to a G_{110} will have the geometry appropriate to an effective spin-orbit potential. This point will be pursued later.

We can now include the effects of the tensor force very simply. Using the definition of S_{12} we see that to include the tensor force amounts to making the replacement

$$\hat{V}_1\vec{\sigma}_1\to\hat{V}_T\left[\frac{3(\vec{\sigma}_1\cdot\vec{r}_{12})\vec{r}_{12}}{r_{12}^2}-\vec{\sigma}_1\right] \tag{12}$$

in Eq. (11). Clearly the second term $(-\hat{V}_T)$ contributes just as $\hat{V}_1$. Moreover, since $\vec{r}_{12}=\vec{r}_2-\vec{r}_1$ we must get a pseudoscalar constructed from either $\vec{r}_1$ or $\vec{r}_2$ and $[\phi_l(1)\times\phi_l(2)]^1$. From parity considerations no pseudoscalar can be constructed from these tensors. Therefore to include the effects of the tensor force we simply replace $\hat{V}_1$ by $\hat{V}_1-\hat{V}_T$ in Eq. (11).

Although Eq. (11) is in a form suitable for solving the nonlocal Schrödinger equation (and in fact this will essentially be used), we feel that the physics is made much more transparent by finding an approximate local potential. To derive a local potential we assume that the interaction $\hat{V}_1-\hat{V}_T$ is sufficiently short ranged so that only the leading terms in an expansion of either the interaction or the density need be retained. This is certainly of dubious quantitative validity but will serve to define the simplest form of a "local" potential that can emerge. The approximation may be carried out within the current framework by making the replacement

$$\phi_l(\vec{r}_2)\sim[1+\vec{s}\cdot\nabla_1]\phi_l(\vec{r}_1),\quad\vec{s}=\vec{r}_2-\vec{r}_1. \tag{13}$$

Using the gradient formula, a recoupling transformation, Eq. (22a) of Ref. 18, and a short-range expansion of the interaction we find a local but approximate SUS correction to the spin-orbit potential given by

$$\Delta U_{CT}^{LS}\simeq\frac{\tilde{J}(K^2)}{24\pi}\sum_{nl}2l(l+1)\frac{u_{nl}^2(r_1)}{r_1^2}\vec{L}_1\cdot\vec{\sigma}_1. \tag{14}$$

The subscript CT denotes the contributing part of the N-N force,

$$\tilde{J}(K^2)\equiv4\pi\int_0^\infty s^4ds\,[\hat{V}_1(s)-\hat{V}_T(s)]\left(\frac{3j_1(Ks)}{Ks}\right), \tag{15}$$

$j_1(x)$ is the spherical Bessel function, and K is the momentum variable (such as the local momentum of the incident particle) about which the Fourier transform of the N-N interaction is expanded and in principle is chosen to minimize the importance of higher-order terms in the expansion indicated by Eq. (13). In practice the magnitude of $\tilde{J}(K^2)$ is calibrated by comparison with exact distorted-wave Born approximation (DWBA) calculations using Eq. (11) (see Sec. III). Equation (14) may also be obtained from Eq. (17) of Ref. 19 as a special case of scattering in which the total and orbital angular momentum transfers differ by one unit. Using the somewhat more general result from Ref. 19, ΔU^{LS} can be written as

$$\Delta U_{CT}^{LS} = \frac{\tilde{J}(K^2)}{24\pi} \left\langle \Psi \left| \sum_{i=1}^{A} \frac{\delta(r_i - r_p)}{r_i^2 r_p^2} \vec{L}_i \cdot \vec{\sigma}_i \right| \Psi \right\rangle L_p \cdot \vec{\sigma}_p , \quad (16)$$

where the projectile coordinates are denoted by p. We also note from Ref. 19 that any source of velocity dependence in the N-N force which can be cast in the form of a $[p_{12}{}^2 g(r) + g(r) p_{12}{}^2]\vec{\sigma}_1 \cdot \vec{\sigma}_2$ interaction leads to a similar correction to the spin-orbit potential.

A number of features of ΔU_{CT}^{LS} may be noted. The participating part of the N-N force may be written

$$\hat{V}_1 - \hat{V}_T = \begin{cases} \frac{1}{4}(V_{\rm SE} + V_{\rm TO}) + V_{\rm TNO}, & \text{like nucleons} \\ \frac{1}{8}(V_{\rm TE} - V_{\rm SE} + V_{\rm SO} - V_{\rm TO}) + \frac{1}{2}(V_{\rm TNO} - V_{\rm TNE}), & \text{unlike nucleons} \end{cases} \quad (17)$$

This result is in accord with that of Scheerbaum[9] with the exception of a factor of $-\frac{1}{2}$ which in Ref. 9 is incorporated into the other factors. For realistic forces both the central and tensor force contributions to $\hat{V}_1 - \hat{V}_T$ are positive (repulsive) for both like and unlike interacting nucleons. This observation together with Eqs. (14) and (15) serves to establish the sign of ΔU_{CT}^{LS}. In particular, we see from Eqs. (14) and (15) that the effect of all SUS is opposite that of the usual empirical U^{LS} in that ΔU_{CT}^{LS} makes the optical-model (or shell-model) potential more repulsive (attractive) for nucleon projectiles having $j_p = l_p + \frac{1}{2}$ ($j_p = l_p - \frac{1}{2}$).

A simple semiclassical picture may be used to understand the qualitative aspects of the nature of ΔU_{CT}^{LS}. In particular, consider a nucleon incident on a nucleus having a single (for simplicity) SUS as in Fig. 1. A typical orbit for a bound nucleon having $j = l + \frac{1}{2}$ is shown where the cross and dot denote motion into and out of the figure, respectively. The small arrow denotes spin up for this orbital so that the spin and angular momentum are primarily aligned. For definiteness we take the direction of motion of the incident nucleon to be into the figure and consider first the case in which the projectile spin is up. Roughly speaking two types of collisions will occur which we will label as catch-up and head-on. Catch-up (head-on) collisions will denote those in which the linear momenta of the incident and bound nucleons are primarily parallel (antiparallel). For the bound orbital shown in Fig. 1, catch-up (head-on) collisions occur when the projectile nucleon is incident on the right (left) hemisphere. The exchange matrix elements will be dominated by catch-up rather than head-on collisions since for such collisions a considerably smaller momentum transfer is required of the N-N force. From this observation and the fact that $\hat{V}_1$ and $-\hat{V}_T$ are each repulsive (greater than 0) when the projectile spin is parallel to that of the bound nucleon we see that the nuclear attraction in the right hemisphere (where $\vec{L}_p \cdot \vec{\sigma}_p > 0$) will be reduced by a relatively larger amount than in the left hemisphere (where $\vec{L}_p \cdot \vec{\sigma}_p < 0$). Similarly, for incident nucleons with spin down, $\vec{\sigma} \cdot \vec{\sigma}_p < 0$ on the average yielding a relatively greater attraction for collisions in the right hemisphere where $\vec{L}_p \cdot \vec{\sigma}_p < 0$. Thus for either incident spin direction the net attraction for collisions with $\vec{L}_p \cdot \vec{\sigma}_p > 0$ is reduced relative to those collisions described by $\vec{L}_p \cdot \vec{\sigma}_p < 0$ so that the effect of including the new types of terms arising from SUS is to quench the usual optical-model spin-orbit potential.

Compared with the central ($\vec{\sigma}_i \cdot \vec{\sigma}_2$) force the tensor force is known to be much more effective[20] for transferring relatively large amounts of momentum. As a result it should not be surprising that the tensor force dominates[2,3,9] the quenching. For example, for unlike nucleons, the ratio of the tensor to central contributions to $\tilde{J}(K^2 = 0)$ in Eq. (15) is roughly 5.0 for realistic forces.[21] Moreover, the exchange amplitudes are known to

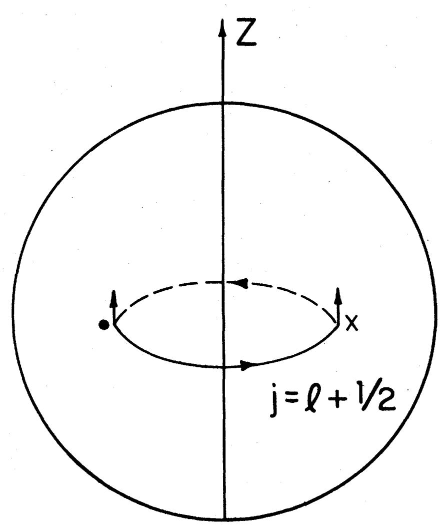

FIG. 1. Schematic diagram of bound single-particle orbital with $j = l + \frac{1}{2}$.

decrease with increasing bombarding energy so that the quenching effect should be most important for relatively small bombarding energies.

At this point it is convenient to reexpress ΔU_{CT}^{LS} in Eq. (16) in a slightly different form. In particular, if we neglect any j dependence of the radial wave functions,

$$\Delta U_{CT}^{LS}=\frac{\tilde{J}(K^2)}{24\pi}\sum_{n,l}\left[lN_{nlj_>}-(l+1)N_{nlj_<}\right]\times\frac{u_{nl}^2(r_p)}{r_p^2}\vec{L}_p\cdot\vec{\sigma}_p\,, \tag{18}$$

where N_{nlj} is the ground-state expectation value of the number operator for the state (nlj). In this approximation we see that ΔU_{CT}^{LS} will peak inside the peak in the density of the SUS nucleons any may be somewhat masked by the absorptive part of the optical potential.

It is interesting to compare the explicit expression [Eq. (18)] for the central and tensor force contribution to ΔU_{CT}^{LS} with that arising from the two-body spin-orbit (TBSO) potential (V_{LS}) for the same orbits. Using the short-range limit for the TBSO force as discussed in Ref. 11 we find

$$\Delta U_{LS}^{LS}\simeq\frac{-J_2^{ls,\text{odd}}}{32\pi}\sum_{n,l}(N_{nlj_>}+N_{nlj_<})\frac{1}{r_p}\frac{\partial}{\partial r_p}\times u_{nl}^2(r_p)\vec{L}_p\cdot\vec{\sigma}_p\,, \tag{19}$$

where

$$J_2^{ls,\text{odd}}\equiv 4\pi\int_0^\infty r^4dr\, V_{LS}^{\text{odd}}(r)\,, \tag{20}$$

and V_{LS}^{odd} is the odd-state part of V_{LS} in Eq. (1). For a single SUS having $n=n_{\min}=1$ we see that this contribution to the one-body spin-orbit potential will change sign at that radius corresponding to the peak value of $u_{nl}^2(r)$. Since $-J_2^{ls,\text{odd}}>0$, inside this radius ΔU_{LS}^{LS} will have the same sign as ΔU_{CT}^{LS}; outside this radius the two terms tend to cancel.

It is also instructive to consider the spin-orbit splitting of some doublet $j'=l'\pm\frac{1}{2}$ induced by a single SUS. This can be calculated relatively easily in closed form if we consider only $n=n'=1$ and use harmonic oscillator orbitals. Using first-order perturbation theory we soon find

$$\Delta E_{LS}^{LS}(l')=-\nu^{5/2}J_2^{\text{odd},ls}f(l,l')\times\left[l-l'-\tfrac{1}{2}\right] \tag{21a}$$

and

$$\Delta E_{CT}^{LS}(l')=\nu^{5/2}\tilde{J}_2(K^2)\tfrac{4}{3}f(l,l')\times l\,, \tag{21b}$$

where ν is the usual oscillator constant in fm^{-2} and

$$f(l,l')=\frac{(l+1)[2(l+l')-1]!!}{(2\pi)^{3/2}2^{l+l'}(2l+1)!!(2l'-1)!!}\,. \tag{22}$$

Wong[2] and Scheerbaum[9] noted that the *predicted* quenching of the one-body spin-orbit potential (or the spin-orbit splitting) due to the *total* contribution of an SUS peaks for that orbit whose l value is one unit smaller than that of the orbitals in the SUS. This result follows readily from our approximate expressions, Eqs. (21a) and (21b). The function $f(l,l')$ peaks (weakly) at $l'=l$ but at this value of l', ΔE_{LS}^{LS} changes from positive to negative (for increasing l') and "interferes" destructively with ΔE_{CT}^{LS} providing the observed diminished quenching of U^{LS} when l' becomes as large as l.

Another measure of the strength of different spin-orbit potentials (one body and two body) used for scattering is often made by comparing their J_2 values where $J_2(f)$ is obtained by replacing $V_{LS}^{\text{odd}}(r)$ by any function $f(r)$ in Eq. (20). For those contributions arising from SUS we find using Eqs. (18) and (19) and assuming that the $j=l+\frac{1}{2}$ subshell is full

$$J_2(\Delta U_{LS}^{LS})=\frac{3(l+1)}{4}J_2^{\text{odd},ls} \tag{23a}$$

and

$$J_2(\Delta U_{CT}^{LS})=\frac{l(l+1)}{3}\tilde{J}(K^2)\,. \tag{23b}$$

Such simple estimates must be used with caution especially when rather different geometries accompany the different ΔU^{LS}. As suggested earlier, this is especially true for the scattering case where the absorptive potential will discriminate among contributions from different spatial regions. Nevertheless, Eqs. (23a) and (23b) [and (21a) and (21b)] suggest that the quenching of the single-particle spin-orbit potential due to SUS should be most important when the unsaturated subshell has a large l value. Moreover, for orbitals of large l small "contaminants" of occupancy of the $j_<$ subshell are relatively less important in quenching ΔU^{LS} than for subshells of smaller l [see Eq. (18)]. This result should consequently affect most strongly the calculation of spin-orbit splittings in relatively heavy nuclei. More specifically, this result poses an acute uncertainty on *any* extrapolation of single-particle spin-orbit potentials to the superheavy region where stability[22] is extremely sensitive to the single-particle spin-orbit splittings. Apart from these considerations we anticipate the largest effect of ΔU_{CT}^{LS} for those nuclei having the largest percentage of SUS nucleons. Although these effects are included in those Hartree-Fock calculations which contain realistic tensor forces, such calculations[3] are known to have difficulty reproducing the observed spin-orbit splitting in *known* nuclei.

III. APPLICATIONS TO ELASTIC SCATTERING

In this section we present some calculations of elastic scattering of protons from nuclei believed to have a spin-unsaturated subshell (SUS). No attempt was made to search on the optical-model parameters to fit the data; that is not the point here. We simply want ot explore the importance of including the contribution of ΔU_{CT}^{LS} in the total optical-model spin-orbit potential. Any phenomenological optical potential which fits the data will clearly include (implicitly) the effects of ΔU_{CT}^{LS}. Ideally one would simply include ΔU_{CT}^{LS} as part of a program to calculate the complete optical potential, as is done by Brieva and Rook[11] for nuclei with spin-saturated subshells (SS). Here we start from phenomenological optical potentials and estimate the size of ΔU_{CT}^{LS} by perturbation theory in the distorted-wave approximation. In particular, we calculate the change in the elastic scattering t matrix via

$$\Delta t^{LS} \simeq \langle \chi^{(-)} | \Delta U^{LS} | \chi^{(+)} \rangle , \tag{24}$$

where $\chi^{(+)}$ are distorted waves generated by some phenomenological potential. Cross sections and asymmetries predicted by t_{phenom} and $t_{\text{phenom}} + \Delta t^{LS}$ can then be compared. This was done using the full nonlocal form of ΔU^{LS} given by Eq. (11) in which case it was unnecessary to introduce a local equivalent potential as in Eq. (18). However, since we believe it is desirable to find an equivalent local potential for purposes of interpretation we also calculated the change in the t matrix due to the approximate local potential ΔU_{CT}^{LS} of Eq. (18) via

$$\Delta t_{CT}^{LS} = \langle \chi^{(-)} | \Delta U_{CT}^{LS} | \chi^{(+)} \rangle . \tag{25}$$

Since the *strength* of ΔU_{CT}^{LS} is not expected to be given correctly by Eq. (18) we calibrated it by normalizing the integrated cross sections predicted by Δt_{CT}^{LS} to those obtained using the nonlocal potential in Eq. (11). It was verified that the amplitudes Δt_{CT}^{LS} and Δt^{LS} are almost completely in phase and that the associated cross sections are very similar in shape. Cross sections and asymmetries were then compared by solving a local Schrödinger equation first with U_{phenom} and then with $U_{\text{phenom}} + \Delta U_{CT}^{LS}$.

The internucleon central and tensor interactions were taken from Ref. 21 with the even- (odd)-state force being that derived from the oscillator matrix elements of the Reid (Elliot) interaction. The regularized one-pion exchange potential (ROPEP) *form* of the tensor force was used. Harmonic oscillator radial functions were used in each case with $\nu(^{12}C) = 0.284\ \text{fm}^{-2}$, $\nu(^{28}Si) = 0.289\ \text{fm}^{-2}$, and $\nu(^{48}Ca) = 0.250\ \text{fm}^{-2}$. Although the use of harmonic oscillator functions may underestimate the effect in the surface region, the calibration procedure discussed above should largely compensate for this.

A. ^{48}Ca + p, $E_p = E_p$ = 15 and 30 MeV

For ^{48}Ca we chose the phenomenological potential to be that given by Becchetti and Greenlees.[23] Structurally, ^{48}Ca is believed to have 8 SUS $f_{7/2}$ neutrons and this property has been used[24] to explain the *apparent* decrease in the proton rms radius as one goes from ^{40}Ca to ^{48}Ca. Indeed, the nuclear matrix element for both processes is proportional to $l(2j+1)$. Figure 2 shows the asymmetries calculated with U_{phenom} and $U_{\text{phenom}} + \Delta U_{CT}^{LS}$ at 30 MeV bombarding energy. Similar results were obtained at $E_p = 15$ MeV. The changes are seen to be non-negligible beyond $\theta_{\text{c.m.}} \sim 50°$. Comparable changes in the cross section only occur for $\theta_{\text{c.m.}} \gtrsim 125°$. Also shown in Fig. 2 is the change in the single-particle spin-orbit potential at $E_p = 30$ MeV produced by the SUS. This translates into cali-

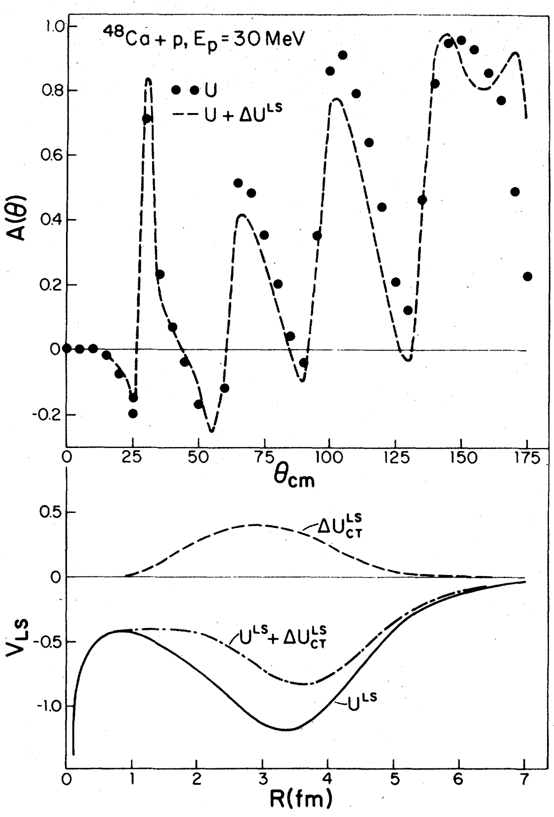

FIG. 2. Effect of SUS on the optical-model spin-orbit potential and on the elastic asymmetry for ^{48}Ca + p with E_p = 30 MeV. The dots denote the results using the phenomenological potential described in the text. The curve labeled $U + \Delta U^{LS}$ denotes the result when the term ΔU_{CT}^{LS} is included.

brated values of $\tilde{J}(K^2)$ of 509 MeV fm^5 and 459 MeV fm^5 at $E_p = 15$ and 30 MeV, respectively. The tensor force contribution to ΔU_{CT}^{LS} is about 14 times that of the central force at each energy. The $\Delta S = 1$ proton-neutron interaction is known[16] to be quite weak.

Since the isospin dependence of the *central* part of the optical potential is often determined[24] by scattering from SUS nuclei, a reliable estimate of ΔU_{CT}^{LS} is clearly desirable. Otherwise, one may extract a spurious isospin dependence which should more correctly be ascribed to SUS effects. Moreover, the tensor force used here would yield an explicit isospin dependent contribution to ΔU_{CT}^{LS} ~40% as large as the isoscalar contribution, with nucleons unlike those in the SUS experiencing a larger quenching of the spin-orbit potential.

B. ^{28}Si+p, $E_p = 30$ and 135 MeV

For ^{28}Si we used the optical potential of Fricke *et al.*[25] at 30 MeV and that of Nadasen[26] at 135 MeV. Following Ref. 27 we assume ground-state occupation probabilities of 10.14 and 0.59 for the $1d_{5/2}$ and $1d_{3/2}$ levels, respectively. This corresponds to an expectation value of $\sum_i l_i \cdot \sigma_i$ of 18.5 which compares favorably with the value of 16.0 found by Kurath[28] using the Nilsson model. The results at $E_p = 30$ MeV are shown in Fig. 3. Although the effect of ΔU^{LS} on the cross section is relatively small forward of ~125°, the effect of ΔU^{LS} on the asymmetry is quite large over the full angular range. For essentially all of the cases studied the effect of ΔU^{LS} on the cross section and asymmetry is qualitatively similar to what one might expect. In particular, since the total U^{LS} is reduced the asymmetries tend to be reduced and the cross section minima tend to be deeper. It should be reemphasized that the $U + \Delta U$ curve does not fit the experimental data, but the difference in the U and $U + \Delta U$ curves does indicate the size of the effect to be expected. Indeed, the phenomenological potential alone provides a reasonable (though not good) fit to the asymmetry and cross section data forward of $\theta_{c.m.} \approx 100°$. An anomalous behavior[29] of the spin-orbit potential for this system has been reported; it would be interesting to reinterpret this behavior when ΔU^{LS} is included explicitly.

Analogous calculations were performed at $E_p = 135$ MeV where there is much new data[26] being taken. The effects are much smaller than at $E_p = 30$ MeV as was anticipated in Sec. II. Using the same internucleon interaction at both energies resulted in a decrease in ΔU_{CT}^{LS} by a factor of 3.6. The difference in the predicted asymmetries at $E_p = 135$ MeV are typically 10–20 % at the extrema.

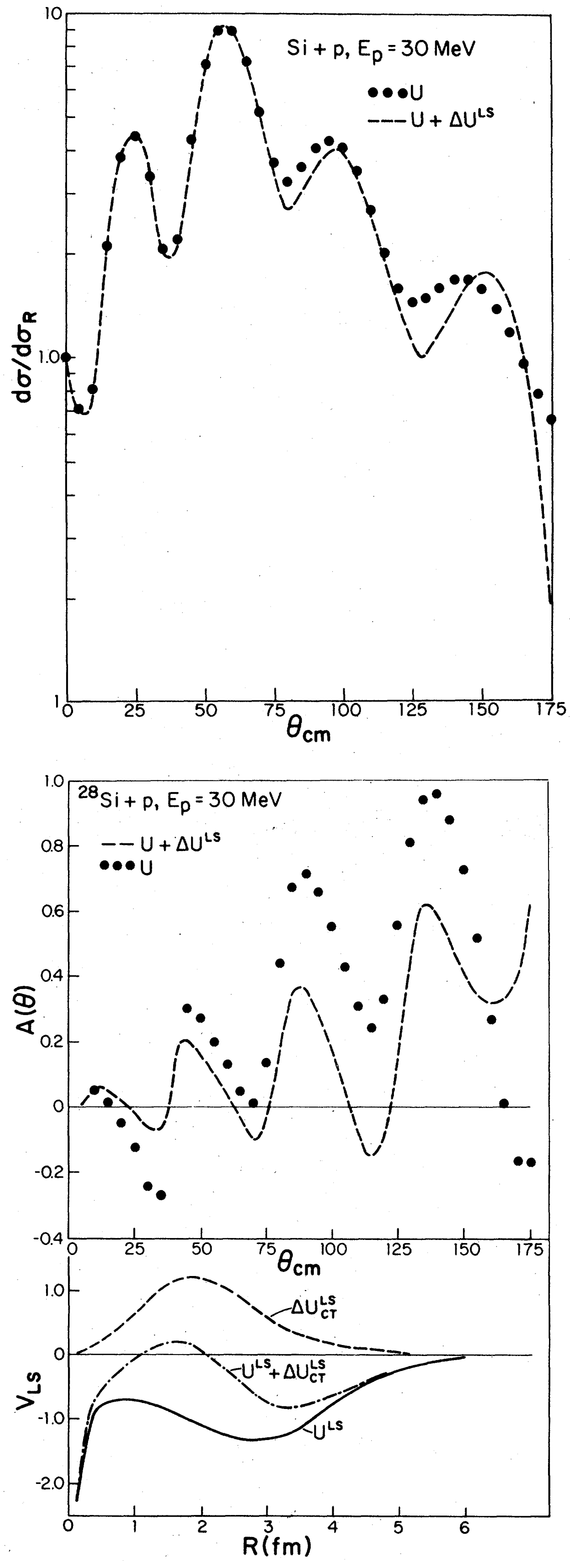

FIG. 3. Effect of SUS on the optical-model spin-orbit potential and on the elastic asymmetry and cross section for ^{28}Si+p at $E_p = 30$ MeV. Legend is as in Fig. 2.

Larger differences do occur in the regions where the asymmetry is most rapidly varying. The *calibrated* values of $\tilde{J}(K^2)$ are 468 and 130 MeV fm^5 at $E_p = 30$ and 135 MeV, respectively. At $E_p = 30$ (135) MeV, the tensor force contribution to ΔU^{LS} is 3.0 (1.5) times that of the central force. The difference between this result and that for ^{48}Ca may be attributed to the isospin dependence of the nucleon-nucleon interaction.

C. $^{12}C + p$, $E_p = 45.5$ MeV

For ^{12}C we used the optical potential *S*2 of Ref. 30 which gives a reasonable fit to both polarization and cross section data. In *j-j* coupling ^{12}C would have eight $1p_{3/2}$ nucleons and would constitute one of the best candidates in which to look for the effects of ΔU^{LS}. To be more realistic we take the *p*-shell occupation probabilities from the work of Cohen and Kurath[31] as provided by McGrory giving $N_{3/2} = 6.48$, $N_{1/2} = 1.52$.

The results for the asymmetry are shown in Fig. 4 together with the experimental data. No attempt was made to optimize the optical-model parameters after ΔU^{LS} was included. The fit to the cross section is made slightly worse. The point here is that ΔU^{LS} can make a *measurable*

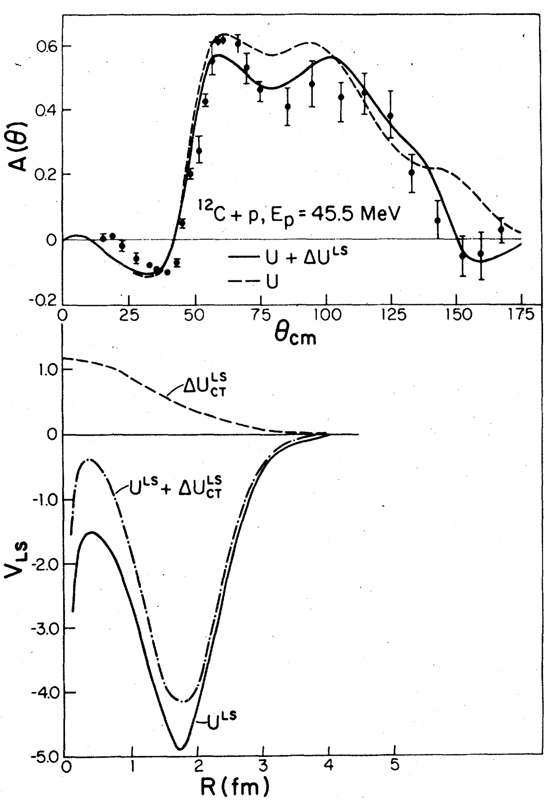

FIG. 4. Effect of SUS on the optical-model spin-orbit potential and on the elastic asymmetry for $^{12}C + p$ at $E_p = 45$ MeV. Legend is as in Fig. 2.

difference in the calculated asymmetries. It would be quite interesting to refit this data including ΔU^{LS} explicitly. The calibrated value of $\tilde{J}(K^2)$ is 396 MeV fm^5 for this transition with the tensor force contribution to ΔU^{LS} being 2.8 times as large as that of the central force.

For each of the systems studied above the values of $\tilde{J}(K^2)$ associated with the important tensor force contribution are typically a factor of ~2 larger than the calibrated values if K^2 is associated with the asymptotic energy. At 135 MeV, however, this asymptotic-energy approximation is correct to within 6%. A local-energy approximation in which K^2 is associated with the local kinetic energy evaluated at the peak of $u_{nl}{}^2$ is accurate to within 25% with the exception of $^{12}C + p$ where the central well depth is unusually large.

IV. DISCUSSION

A. Other sources of ΔU^{LS}

As has been mentioned in Sec. II, any part of an *effective* nucleon-nucleon interaction which is *both* spin and velocity dependent (nonlocal) will contribute to ΔU^{LS} for nuclei with SUS. This occurs because the distinctive contribution of SUS depends on the correlation between a target nucleon's spin and its orbital motion. We have considered only those first-order contributions which arise from an internucleon interaction which is assumed to be intrinsically static. This is believed appropriate since exchange terms of this same type are normally incorporated within the folding model. It is felt that a "completely" successful folding model calculation should include such terms, although the very context in which effective interactions are derived[11] (usually in nuclear matter with much averaging) may preclude an accurate evaluation of such shell effects.

Apart from these first-order effects, many second- (and higher) order processes will contribute to ΔU^{LS}. Generally such contributions will yield a *complex* ΔU^{LS}. A particularly interesting candidate is the two-step $(p,d;d,p)$ mechanism which, if different for the singlet and triplet deuteron, will lead to an *LSJ* transfer of (110) and hence a contribution to ΔU^{LS} for SUS. Unfortunately, it is unclear at present just how important this process is (i.e., how to calculate it reliably). The anomalously large triton spin-orbit potential[32] suggests the importance of such a mechanism. Excitation of giant resonances through the "core-exchange" process[33] may also contribute to ΔU^{LS} in selected energy ranges.

B. Relation of ΔU^{LS} to inelastic scattering

Since the spin-flip probability $[S(\theta)]$ for the excitation of low-lying levels is known[34] to be determined largely by the spin-orbit part of the optical potential, the inclusion of ΔU^{LS} should have a measurable effect on $S(\theta)$ for those nuclei with SUS.

Another inelastic process closely related to the effects of SUS on the spin-orbit part of the optical potential is the excitation of $T=0$ levels having abnormal parity. The processes are related since both receive a large contribution[34,35] from the exchange matrix elements of the tensor force. Excitation of states with $T=1$ are usually dominated by the direct term since the tensor force is largely isovector. Excitation of states with normal parity are preferentially excited by a different part of the force. Recent measurements[34,35] and calculations of cross sections and $S(\theta)$ for the 1^+, $T=0$ level in ^{12}C (12.7 MeV) and for the 2^-, $T=0$ level in ^{16}O (8.88 MeV) suggest that the exchange part of the tensor force mediating $\Delta T=0$ transitions is too strong (but see Ref. 34). This suggests that our estimates of ΔU^{LS} may be too large even though the tensor force used here is a typical "realistic" force. These "interior processes" of low multipolarity should be quite sensitive to any density dependence of the tensor force and this has not been included. An understanding of this type of inelastic process would help clarify the role of the tensor force in estimates of ΔU^{LS}.

C. ΔU^{LS} and M1 resonances

Kurath[28] has shown that for light nuclei with $N=Z$ the energy-weighted $M1$ strength is (semiquantitatively) proportional to the ground-state matrix element of $\sum_i l_i \cdot \sigma_i$. For such nuclei where only a single shell (nl) is unsaturated, we see from Eq. (16) that ΔU_{CT}^{LS} is proportional to this same matrix element. As a result we should look for the effects of ΔU^{LS} in those nuclei where large $M1$ strengths have been identified experimentally. It should be noted that there is a consistency requirement to be met. Within the framework of Kurath's model it is inconsistent to find a large *energy-weighted* $M1$ strength unless the tensor force is either small or of opposite sign to what is believed to be realistic or there is some compensating higher-order correction. Otherwise the $l+\frac{1}{2}$ and $l-\frac{1}{2}$ levels would move closer together and reduce the energy-weighted $M1$ strength. The observation of appreciable strength may place an interesting restriction on the value of the tensor force.

V. SUMMARY

It has been shown explicitly how SUS together with an internucleon force leads in *first order* to a correction to the one-body spin-orbit potential (U^{LS}) relative to the case in which both $l+\frac{1}{2}$ and $l-\frac{1}{2}$ orbits are fully occupied. This correction to U^{LS} for bound states (and its implications for spin-orbit splittings) has been discussed by Scheerbaum.[9] Here this correction has been extended to scattering states with a greater emphasis on the one-body spin-oribt potential itself. The resulting nonlocal potential has been transformed into an "equivalent" energy dependent local potential for interpretative purposes.

Internucleon forces based on nucleon-nucleon scattering data predict a substantial quenching of U^{LS} for nuclei with SUS, particularly for bound states and scattering states of bombarding energy below ~50 MeV. The effects of such quenching has been estimated for the elastic scattering of protons by ^{12}C, ^{28}Si, and ^{48}Ca. Rather large changes in the asymmetries occur when the contribution ΔU^{LS} is included, especially for the two lighter systems. By far, the dominant contribution can be attributed to the tensor force.

It was also pointed out how a number of second-order processes may also yield a correction to U^{LS} which is peculiar to systems with SUS. These corrections would generally be complex.

The close connection between the expected correction to U^{LS} arising from the tensor force and the inelastic excitation of abnormal parity states with $T=0$ was stressed as was the close relationship between ΔU^{LS} and the energy-weighted $M1$ sum rule.

It was also stressed how our present failure to obtain the correct spin-orbit splitting in known SUS nuclei places a severe uncertainty on extrapolations of single-particle potentials to superheavy nuclei.

It should be noted that a cursory glance through tables[36] of optical potentials for many nuclei does not suggest a shell effect (ΔU^{LS}) as calculated here. This is reminiscent of the bound-state problem where such an effect is also largely absent.[6] For the scattering case the spin-orbit potentials were often fixed[36] and variation of other parameters perhaps compensated. It would be interesting to reexamine the spin-orbit part of the optical potential for a number of SUS nuclei with the ΔU^{LS} term included explicitly in the searches. Should the evidence for such a term be definitively absent, the optical- and shell-model anomalies would at least be consistent and constitute a yet more overwhelming "disagreement with theory."[6] This together with the other evidence given here would suggest even more strongly our lack of understanding of the two-body tensor force which arises largely from the one-pion-exchange process.

I am indebted to George Bertsch, G. R. Satchler, and Tom Davies for interesting discussions.

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