Strong-interaction effects in kaonic hydrogen

A. Deloff* and J. Law
Physics Department, University of Guelph, Guelph, Ontario, Canada N1G 2W1
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It is argued that Coulomb corrections to the K^-p scattering length may affect the latter significantly and reduce the strong-interaction effects in kaonic hydrogen to below the observable level.

[NUCLEAR REACTIONS kaon-baryon interactions.]

Davies *et al.*[1] have recently reported on the observation of x-ray deexcitation of kaonic hydrogen. The experiment was carried out to see the K^-p strong-interaction effects in the electromagnetic transitions. They observe the $2p$-$1s$ transition at an energy of (6.52 ± 0.06) keV, which is consistent with a strong-interaction shift of the line of less than about 60 eV and width $\Gamma < 230$ eV. Since the experiment is to be repeated the purpose of this note is to provide a theoretical understanding of this and future results.

The strong-interaction effect manifests itself in a shift (ϵ) and a broadening (Γ) of the $1s$ energy level of kaonic hydrogen, and can be directly related to the complex K^-p scattering length A_C by[2]

$$\epsilon + i\tfrac{1}{2}\Gamma = 2\alpha^3\mu^2 A_C , \tag{1}$$

where μ is the K^-p reduced mass, $\alpha^{-1} = 137.036$, and we use $\hbar = c = 1$. It should be stressed that A_C is the Coulomb corrected scattering length and is not simply equal to the isospin averaged scattering length

$$A_p = \tfrac{1}{2}(A_0 + A_1) , \tag{2}$$

where A_0, and A_1 are the isospin singlet and triplet scattering lengths,[3] respectively. However, if we neglect corrections and simply set $A_C \simeq A_p$ in (1), we obtain

$$(\epsilon, \Gamma) = (-270, 581)\ \text{eV} \tag{3}$$

as a first estimate. The first correction to be taken into account is the threshold difference between the K^-p and the charge exchange $\bar{K}^0 n$ channel, the inclusion of which gives us the scattering length[4]

$$A_t = \frac{A_p + |k_0| A_0 A_1}{1 + |k_0| A_p} , \tag{4}$$

where $|k_0|$ is the $\bar{K}^0 n$ center of mass momentum evaluated at the K^-p threshold. Using $A_C \simeq A_t$ in (1) yields

$$(\epsilon, \Gamma) = (-410, 682)\ \text{eV} . \tag{5}$$

In addition to the threshold correction, the Coulomb correction following Dalitz and Tuan[4,5] leads to the following expression for the scattering length:

$$A_{DT} = \frac{A_t}{1 - 2\pi A_t \lambda / B} , \tag{6}$$

where B is the corresponding Bohr radius ($\simeq 84$ fm) for kaonic hydrogen, and $\lambda = -[2\gamma + \ln(2R/B)]/\pi$ with γ being the Euler constant. The customary choice of the matching radius $R = 0.4$ fm is inserted in λ. The choice $A_C \simeq A_{DT}$ yields

$$(\epsilon, \Gamma) = (-397, 579)\ \text{eV} . \tag{7}$$

As can be seen from the above estimates, the predicted values of (ϵ, Γ) are large enough to have been revealed by the experiment of Davies *et al.* Since the experiment seems to suggest an upper limit on (ϵ, Γ) to be about an order of magnitude smaller than the theoretical predictions, we address ourselves to the question of how this apparent discrepancy may be resolved on theoretical grounds. We feel that the possible discrepancy may be attributed to the incomplete treatment of the Coulomb correction. This difficulty is well known in pp scattering as demonstrated explicitly by Sauer.[6] Since the Coulomb correction depends in an essential way upon the behavior of the wave function at small distances, there is *a priori* no model-independent way to extract the pp scattering length from the nn scattering length under the assumption of charge symmetry because the size of the Coulomb correction depends upon the nature of the strong interaction. In order to see how the ambiguity arises in the corresponding K^-p case, we will rederive Sauer's result using the boundary condition model approach of Dalitz and Tuan.

Assuming that the strong K^-p interaction can be represented by a short range complex potential V_s which comprises all strong-interaction effects

and generates (in the absence of the Coulomb potential V_C) the scattering length A_t, we want to calculate the Coulomb corrected scattering length A_C in terms of A_t. If $u(r)$ and $v(r)$ are the regular solutions of the Schrödinger equations with potentials (V_s+V_c) and V_s, respectively, then A_c and A_t will be given by

$$A_C=\lim_{k\to 0}\left[\frac{\Phi(R)}{\theta(R)}\cdot\frac{u'(R)/u(R)-\Phi'(R)/\Phi(R)}{u'(R)/u(R)-\theta'(R)/\theta(R)}\right] \quad (8)$$

$$A_t=\lim_{k\to 0}\left[R-v(R)/v'(R)\right], \quad (9)$$

where R is the matching radius, and the Coulomb functions[7] Φ and Θ in the limit $k\to 0$ (i.e., at threshold) are given in terms of Bessel functions as

$$\lim_{k\to 0}\Phi(R)=\frac{B}{2}\left(\frac{2R}{B}\right)^{1/2}J_1\left[2\left(\frac{2R}{B}\right)^{1/2}\right]$$

$$\simeq R\left[1-\frac{R}{B}\right]+O(R^2/B^2), \quad (10)$$

$$\lim_{k\to 0}\Theta(R)=-\pi\left(\frac{2R}{B}\right)^{1/2}Y_1\left[2\left(\frac{2R}{B}\right)^{1/2}\right]$$

$$\simeq 1+\frac{2R}{B}(1+\pi\lambda)+O(R^2/B^2). \quad (11)$$

We have also expanded the above expressions in powers of B^{-1}, since the Bohr radius B is much larger than the range of the strong interaction and the matching radius R. The derivative of the Wronskian $W[u,v]$ may be obtained from the wave equations and if integrated in the range $(0,R)$ yields in the limit $k\to 0$

$$R\frac{u'(R)}{u(R)}-R\frac{v'(R)}{v(R)}=\Delta(r), \quad (12)$$

where

$$\Delta(R)=-\frac{2R}{B}\int_0^R\frac{u(r)}{u(R)}\frac{v(r)}{v(R)}\frac{dr}{r}. \quad (13)$$

Eliminating the log derivatives between (12), (8), and (9), we obtain the following relationship between A_C and A_t:

$$A_C=-(\Phi/\Theta)\frac{1+(\Delta-R\Phi'/\Phi)(1+A_t/R)}{1+(\Delta-R\Theta'/\Theta)(1+A_t/R)}. \quad (14)$$

Using the approximations of (10) and (11), accurate to power $1/B$, we reduce (14) to

$$A_C=\frac{A_t-R\Delta(1-R/B)(1+A_t/R)-R(R+2A_t)/B}{1-2\pi\lambda A_t/B+2R/B+\Delta(1+A_t/R)[1+2R(1+\pi\lambda)/B]}. \quad (15)$$

The important point to be made is that in (15), R has to be at the least set to the range of the $\bar{K}N$ force. In principle, the matching radius R may be chosen arbitrarily outside the range of the strong interaction d (for sake of argument we can assume that the $\bar{K}N$ force cuts off sharply at $r=d$). However, (14) has to be independent of R while it does, of course, depend on d, in particular if $d\to 0$, the strong interaction vanishes. Comparing (15) and (6), we note that it is the Δ dependence that is missing in the Daltiz and Tuan formula (6) (other than minor terms of order R/B). The essence of Sauer's result is that the actual size of the Δ correction depends on the behavior of the integrand within the range of the forces, i.e., on the detailed structure of V_s at small distances.[8] Unfortunately, it is not possible to obtain Δ in a model-independent way, without a detailed knowledge of the dynamics of the system. If the latter is such that Δ is negligible, then (6) correctly accounts for the Coulomb field and ϵ and Γ are of the order of ~ 0.5 keV. On the other hand, if Δ turns out to be not negligible, then A_C can in fact take on any value and in particular A_C may be reduced to such an extent that the resulting (ϵ,Γ) may be too small to produce observable effects. For instance, if $\Delta=1.47-i\,0.02$ with $R=0.4$ fm, we obtain $A_C=(-0.099+i\,0.080)$ fm, giving

$$(\epsilon,\Gamma)=(-41,66)\ \text{eV}, \quad (16)$$

and the strong-interaction effects would not have been observable in the Davies *et al.* experiment. This would in turn have a significant effect on related analyses of kaonic atoms.

It should be emphasized that since the K^-p cross section depends on Δ, the final fitted values of A_0 and A_1 would be significantly affected if Δ were to be appreciable. In other words, if the experiment (which is to be repeated at CERN) confirms the lack of observable strong-interaction effects in the 1s level of kaonic hydrogen, it would mean that Δ is large and the $\bar{K}N$ scattering analysis to extract A_0 and A_1 would have to be repeated including the additional parameter Δ. Thus, it is the task of the experiment to tell us how large the parameter Δ is, and the most direct way to obtain an estimate of Δ is a measure of the level shift and width of kaonic hydrogen. Ideally, the $\bar{K}N$ scattering analysis should be performed jointly, including the data from kaonic hydrogen.

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*On leave of absence from Institute for Nuclear Research, Warsaw, Poland.

[1]J. Davies *et al.*, *2nd International Conference on Meson-Nuclear Physics, Houston, 1979*; Phys. Lett. 83B, 55 (1979).

[2]A. Deloff, Phys. Rev. C 13, 730 (1976). For our discussion, we have disregarded a small relativistic correction to the energy shift formula.

[3]We take $A_0 = (-1.66 + i\,0.75)$ fm, $A_1 = (0.35 + i\,0.66)$ fm from the $\bar{K}N$ scattering analysis of A. Martin, Phys. Lett. 65B, 346 (1976).

[4]R. H. Dalitz and S. F. Tuan, Ann. Phys. (N.Y.) 10, 307 (1960).

[5]This expression arises by taking the matching radius $R \to 0$ except in the log term in λ, where R is set equal to 0.4 fm.

[6]P. U. Sauer, Phys. Rev. Lett. 32, 626 (1974).

[7]E. Lambert, Helv. Phys. Acta 42, 667 (1969). These functions are related to the standard Coulomb functions $F(r)$ and $G(r)$ by $F(r) = Ck\Phi(r)$,

$$\Theta(r) = C\,G(r) + (2/kBC)\left[\left[\mathrm{Re}\psi\left(\frac{i}{kB}\right) + \ln(kB)\right]\right]F(r)$$

with $C^2 = (2\pi/kB)\,[1 - \exp(-2\pi/kB)]^{-1}$, $\psi(z) = \Gamma'(z)/\Gamma(z)$.

[8]The knowledge of A_t is insufficient to determine V_s in a unique way, and this ambiguity is manifested by the presence of Δ. One is always free to introduce a phase shift equivalent transformation which leaves A_t intact but dramatically changes the behavior of the wave function at small separations so that Δ can be made very large.