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Phenomenological Coulomb interaction for microscopic nuclear structure calculations

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(Received 11 June 1979)

The proton charge distribution, deduced from electron-proton scattering data, was used to calculate the Coulomb interaction between protons in nuclei in the f-p-g shell. The Coulomb energies and shifts between members of the $A = 42$ isotopic spin triplet were calculated.

[NUCLEAR STRUCTURE Derived the Coulomb potential between protons in f-p-g shell. Calculated Coulomb energy in ^{42}Ti and shifts (^{42}Ti-^{42}Ca) and (^{42}Sc-^{42}Ca). Compared with experiment.]

The Coulomb energies of the two valence protons in ^{42}Ti were calculated by Bertsch[1] using the potential of point charges, but corrected for the effects of short range correlations due to the hard core repulsion of the strong interaction. The details involving the hard core repulsion were further investigated by Anderson, Wilson, and Goldhammer,[2] McCarthy and Walker,[3] and by Goldhammer.[4] The calculation of the Coulomb displacement energies of the ground states of ^{18}Ne-^{18}O and ^{42}Ti-^{42}Ca were made by Shlomo and Bertsch,[5] using the Coulomb interaction between two protons with finite charge distributions given by Okamoto and Lucas.[6] This potential was also used by Shlomo[7] to investigate the ground state energy differences of mirror nuclei. There was some controversy concerning the approach to the problem of including the short range correlations. The inclusion of these effects in the work of Refs. 2 and 4 made a large difference in the results, while a much smaller effect was observed in the results given in Ref. 3. It was later shown by Bertsch and Shlomo[8] that the inclusion of the effects of short range correlations does not significantly change the Coulomb matrix elements, in agreement with the earlier results of McCarthy and Walker.[3] In addition, Bertsch and Shlomo[8] recalculated the Coulomb matrix elements using the Coulomb interaction of a finite proton charge distribution and obtained Coulomb shifts which were even smaller than those of their earlier work. The conclusion at this point was that not only were the expected increases in the Coulomb shifts, between ^{42}Ti and ^{42}Ca, for example, due to short range correlations far too small to explain the experimental shifts, but further reduction in the theoretical shifts was found when finite proton charge distributions were used.

In the present investigation, we have taken the point of view that the effects of short range correlations are small, and hence can be neglected, while more recent interpretations of electron-proton scattering data made by Borkowski *et al.*[9] which include a significant amount of new data both at low and high momentum transfer, have been used. In addition, we have included corrections for the 2p, 4p2h, and 3p1h renormalizations, where in each, one interaction is electromagnetic, while one is strong. The nuclear interaction was taken to be the Sussex interaction.[10]

The common interpretation of the relation between the electric form factor $G_E(q^2)$, where q is the momentum transfer, and the charge density $\rho(r)$ is given by the Fourier transform

$$\rho(r) = \frac{1}{(2\pi)^3} \int G_E(q^2) e^{-i\vec{q}\cdot\vec{r}} d^3q \,, \tag{1}$$

where the form factor $G_E(q^2)$ is obtained from electron-proton scattering data. A crucial point in the interpretation of charge distributions arises concerning the high q^2 limit of the form factor. If $G_E(q^2)$ approaches a nonzero asymptotic value at high q^2, this implies that the charge distribution has a smoothly varying part, similar to a Gaussian, for example, and a delta function part at the center of the proton. An example of such a distri-

bution was given by Berkelman *et al.*[11] The form factor given by Borkowski and his co-workers has a zero asymptotic value, implying a smooth charge distribution with no point charge core.

The charge distribution of the proton was derived from Borkowski's form factor, and the Coulomb potential energy between two protons was calculated numerically from the distributions. A large number of integrations of the energy of the overlapping spherical charge distributions was made for many relative distances r between charge centers. Throughout the calculations, the approximation was made that the charge distributions remained unperturbed by the presence of the other particle. This should tend to overestimate the Coulomb interaction, but far less than the use of the point charge assumption would. A plot of the resulting potential energy as a function of r is shown in Fig. 1 (curve *A*) along with that used earlier by Shlomo (curve *B*) and that of two point charges (curve *C*). In addition, the charge distribution derived from Borkowski's form factor is shown in arbitrary units (curve *D*).

The Coulomb potential was fitted to second order polynomials over several regions of relative distance r and can be written as follows:

$$
\begin{aligned}
&(r<0.8)\,, \quad V_c = 0.997 \text{ MeV}\,,\\
&(0.8 \le r \le 1.4)\,, \quad V_c = 0.94 + 0.24r - 0.21r^2\,,\\
&(1.4 \le r \le 2.0)\,, \quad V_c = 1.36 - 0.38r + 0.03r^2\,,\\
&(2.0 \le r \le 2.4)\,, \quad V_c = 1.42 - 0.47r + 0.05r^2\,,\\
&(r>2.4)\,, \quad V_c = 1.44r^{-1}\,,
\end{aligned}
\tag{2}
$$

where r is in fermis and V_c is given in MeV.

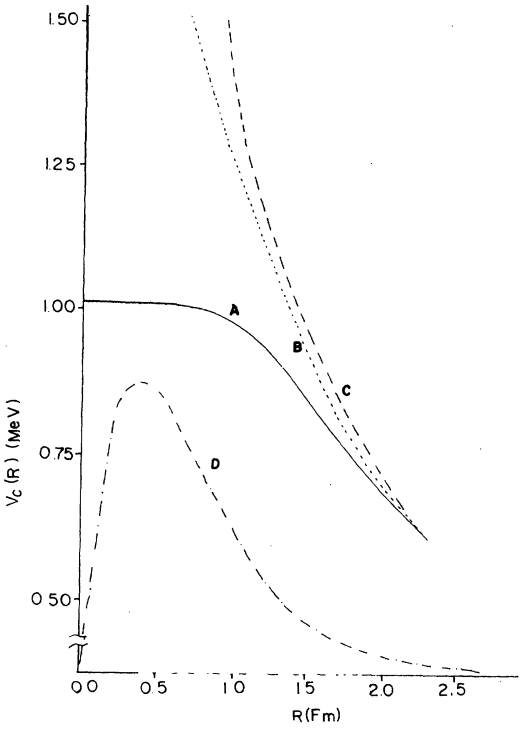

FIG. 1. Coulomb potential energy between protons. Curve *A* is the potential derived in this investigation. Curve *B* is that deduced earlier by Shlomo (Ref. 7) and curve *C* is that between two point charges. Curve *D* is the charge distribution of the proton derived in this work and is given in arbitrary units.

The Coulomb radial matrix elements were calculated in the harmonic oscillator basis with $\hbar\omega = 10.5$ MeV. They were coupled to form the two-body shell model matrix elements $\langle j_3 j_4 | V_c | j_1 j_2 \rangle_{JT}$ and then corrected for the 2p, 4p2h, and 3p1h graphs in which one interaction represents the strong interaction and one represents the electromagnetic interaction. The similarly coupled and renormalized Sussex interaction matrix elements were then corrected with the Coulomb matrix elements, and the energies of the various levels of the nuclei ^{42}Ca, ^{42}Sc, and ^{42}Ti were calculated with and without the correction for the Coulomb interaction. These calculations were made by simply diagonalizing the effective Hamiltonian of two nucleons in the *f-p-g* shell. The resulting shifts were calculated using both the ^{41}Ca single particle levels and ^{41}Sc single particle levels. The results are presented in Tables I and II for both the proton charge distribution derived in the present investigation and the point charge proton. Our calculations also revealed that the 3p1h correction, with one interaction line representing the nuclear interaction and one representing the electromagnetic interaction, is not negligible and should be included.

Several features are striking from reviewing the

TABLE I. Coulomb shifts using ^{41}Ca single particle levels for protons.

	E(Coul. ^{42}Ti)[a]		(^{42}Sc − ^{42}Ca)		(^{42}Ti − ^{42}Ca)	
J	Theor.	Exp.	Theor.	Exp.	Theor.	Exp.
	Finite proton charge distribution					
0	488	550	0	90	488	368
2	425	457	0	28	425	399
4	341	342	0	24	341	292
6	313	307	0	43	313	219
	Point proton charge					
0	591	550	0	90	591	368
2	448	457	0	28	448	399
4	354	342	0	24	354	292
6	318	307	0	43	318	219

[a] E(Coul. ^{42}Ti) ≡ BE ($T_z = 1$) + BE ($T_z = -1$) − 2BE ($T_z = 0$), where BE is the binding energy.

TABLE II. Coulomb shifts using ^{41}Sc single particle levels for protons.

J	E(Coul. ^{42}Ti)[a] Theor.	Exp.	(^{42}Sc − ^{42}Ca) Theor.	Exp.	(^{42}Ti − ^{42}Ca) Theor.	Exp.
			Finite proton charge distribution			
0	638	550	137	90	364	368
2	522	457	84	28	354	399
4	391	342	43	24	305	292
6	344	307	18	43	298	219
			Point proton charge			
0	746	550	137	90	472	368
2	566	457	84	28	378	399
4	405	342	43	24	319	292
6	340	307	18	43	304	219

[a] E(Coul. ^{42}Ti) ≡ BE ($T_z = 1$) + BE ($T_z = -1$) − 2BE ($T_z = 0$), where BE is the binding energy.

results. First, the selection of single particle energies appears as important as any input data. It severely affects the total Coulomb energies as well as the (^{42}Sc − ^{42}Ca) shift. The (^{42}Ti − ^{42}Ca) shift of the ground states, using ^{41}Sc single particle energies, is very promising. The $J = 2$ shift is somewhat disappointing in this case; however, we should point out that the effective single particle energy levels for protons in the f-p-g shell are not nearly as well established as the ^{41}Ca levels for neutrons. In particular, the $f_{5/2}$ level, which plays a strong role in the $J = 2$ state, is not well established. In any case, our results for the proton-proton Coulomb potential is useful at present; however, it appears that the most important future development to test this and other potentials would be the clear establishment of the proton single particle levels in the f-p-g shell. The present form of the proton-proton Coulomb potential is an independent quantity which could be used with correlated wave functions, even though in this shell this effect is small, and would in any case be better tested only after clear establishment of a reliable set of proton single particle energies in the f-p-g shell, particularly the $f_{5/2}$ level.

The important issue of the possible charge dependence of the nucleon-nucleon interaction, which has been raised again recently,[12] is only further clouded by the uncertainty in the proton single particle energies in this shell. By reference to Tables I and II, one notes that the present calculation of the Coulomb energy of ^{42}Ti is underestimated when one uses the ^{41}Ca energy levels while the relative (^{42}Ti − ^{42}Ca) shift is overestimated and the total ^{42}Ti Coulomb energy is overestimated. When one uses the ^{41}Sc single particle energies for protons, however, the ^{42}Ti Coulomb energy is overestimated while the relative (^{42}Ti − ^{42}Ca) shift is very close to the experimental value for the ground state. This further emphasizes the importance of the role of the single particle levels used in the calculations.

The similarity of our results compared to those based on the charge distribution of Ref. 6 is at first surprising when one sees the large difference in the Coulomb interaction potentials shown in Fig. 1. The reason for this similarity is that the coupled matrix elements involved consist of a large number of radial matrix elements with $l > 0, 1$; hence, the Coulomb shifts are not very sensitive to the interior charge structure of the proton which is also the most probable reason that short range correlations are not important in this shell. The correction of the nucleon-nucleon scattering phase shifts, at the higher energy end of the range, would be far more sensitive to the inner charge structure; hence, we suggest the use of our prescription for calculating the Coulomb contribution to the matrix elements for such applications. In addition, the use of correlated wave functions might also be important in such cases and can easily be incorporated in a numerical calculation of the matrix elements using the present Coulomb potential.

The authors would like to express their appreciation to Professor George Bertsch for his advice and help concerning the difficulties involved in the selection of proton single particle levels.

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