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Particle-number-projected mass tensor

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A general method is presented for eliminating, in the theoretical values of the nuclear mass-tensor components, the unphysical effects due to the nuclear number fluctuation allowed by the usual BCS wave functions. General formulas for number-projected orthonormalized pairing wave functions and their energies are established in the quasiparticle representation. Analytical expressions are given for the pertinent pairing correlation matrix elements and overlap integrals, with strict nucleon number conservation. The importance and validity of the physical and mathematical approximations used are discussed. The tensor so obtained, without number dispersion spurious effects, is compared to the pure BCS tensor.

[NUCLEAR STRUCTURE Inertial-mass tensor calculated in cranking model with number-projected BCS functions; SBCS theory; fission.]

I. INTRODUCTION

The deformation energy and the tensor of effective mass B_{ij} are the most important pieces of information required for a dynamic description of the fissioning nucleus.

Different techniques in the calculation of the shell correction energy,[1-4] due essentially but not solely to Strutinsky,[3,4] have permitted the calculation of the collective potential energy with a high degree of precision.

Little conclusive progress has been made, however, in the evaluation of mass parameters since the initial calculation of Inglis.[5] We should mention, however, without any claim to completeness, the following improvements:

(i) The collective mass parameters can be calculated with the random-phase approximation (RPA) corrections.[6,7]

(ii) They can also be evaluated by taking account of the nonadiabatic correction,[8,9] whether this is negligible[10,11] or not.[8,9]

(iii) The components of the effective mass tensor can and must be calculated for nuclear shapes which have reflection asymmetry.[10,12,13]

(iv) The self-cranked generator coordinate method,[14] together with constrained Hartree-Fock-Bogoliubov wave functions, may be used to calculate the mass parameter for quadrupole deformations.[15]

(v) Different variations of the theory of superfluidity have been introduced in the calculation of mass parameters.

Jensen and Miranda investigated the effects of monopole and quadrupole pairing on the fission mass parameter[16] and found that the deformation dependence of the pairing gap (on which the mass parameter essentially depends) depends in a fundamental way on the definition of the matrix elements of the quadrupole pairing interaction. A brief study of the influence of the particle-number fluctuation in the usual BCS theory on nuclear moments of inertia has been made in the framework of the projected BCS (PBCS) and fixed BCS (FBCS) theories. In an investigation of the Coriolis antipairing effect[18] Frauendorf estimated the projection operator given in integral form[19] and showed that the elimination of the unphysical components of the BCS functions was indispensable if one desired to find a reliable estimate of the value of the angular momentum by which the system passes from the superfluid state to the normal state.

To the best of our knowledge a similar study of the spurious effects of the dispersion in the number of nucleons has never been made for the mass-tensor parameters. Such a study appears, however, to us to be indispensable for two reasons: on the one hand, because the inertial-mass tensor plays a primordial role in the determination of the behavior of the dynamical system and is the key factor in the evaluation of the half-lives for spontaneous fission and of the reduced widths, and therefore of the cross sections in the case of induced fission (via the kinetic energy associated with a slow aperiodic increase in the nuclear distortion leading finally to fission); on the other hand, because the values of the inertial parameters depend in an extremely acute way on the pairing parameters, such as the pairing gap[10,16,20] and the pairing-energy strength,[20,21] and consequently on the unphysical effects due to the particle-number fluctuation allowed by the usual BCS wave functions. In what follows, we calculate the inertia tensor parameters in the general formalism of the strict particle-number-projected BCS theory[22-26] (SBCS theory). This formalism is confirmed to be extremely powerful in the extraction from the BCS functions of the component corresponding to a well-fixed number of nucleons, and it is also well adapted to the numerical calculations on computers.[23-26] The many-quasiparticle-number-projected pairing wave functions and the corresponding energies are given in Sec. II. However, because these projected wave functions are not orthonormal, the orthonormalization procedure of Schmidt is applied to them and the result is indicated in Sec. III as well as the energies corresponding to these new orthonormal states.

In Sec. IV the number-projected adiabatic cranking model is developed, the matrix elements and overlap integrals are calculated in the quasiparticle representation using the projected orthonormalized states, and the components of the mass tensor, from which all the unphysical effects (due to the nonstrict conservation of the number of nucleons) have been withdrawn, are compared to the pure BCS components given in the Appendix.

II. MANY-QUASIPARTICLE PROJECTED STATES AND THEIR ENERGIES

In order to evaluate the mass parameters in the cranking model[5] we need the energies of the states with 0, 2, and 4 quasiparticles projected in the occupation-number space. We start, therefore, by establishing the expression of these energies in the framework of the strict particle-number-projected BCS theory.

A. Sharp number-projected BCS theory (SBCS theory)

We content ourselves with recalling here the main results of the SBCS formalism and refer to Refs. 23–26 for more detailed explanations.

Let us recall, in order to make our notation clear, that in the usual BCS theory the intrinsic movement of a system of P pairs of paired particles (neutrons or protons) is described by the intrinsic Hamiltonian

$$H=\sum_{\nu>0} \epsilon_\nu(a_\nu^\dagger a_\nu + a_{\tilde\nu}^\dagger a_{\tilde\nu}) - G\sum_{\nu,\mu>0} a_\nu^\dagger a_{\tilde\nu}^\dagger a_{\tilde\mu} a_\mu \,. \tag{1}$$

We designate by $|\tilde\nu\rangle = a_{\tilde\nu}^\dagger|0\rangle$ the time-reversed partner of $|\nu\rangle = a_\nu^\dagger|0\rangle$.

The ground state is then described by

$$|\psi\rangle = |\mathrm{BCS}\rangle = \prod_{\nu>0}(u_\nu + v_\nu a_\nu^\dagger a_{\tilde\nu}^\dagger)|0\rangle \,. \tag{2}$$

This ground state behaves as a vacuum for quasiparticles created and destroyed by the usual $c_\nu^\dagger$ and c_ν operators:

$$c_\nu|\mathrm{BCS}\rangle = 0\,, \text{ for all } \nu\,.$$

In this representation the excited states possess an even number of quasiparticles. These states are no longer eigenfunctions of the particle-number operator,

$$N=\sum_{\nu>0}(a_\nu^\dagger a_\nu + a_{\tilde\nu}^\dagger a_{\tilde\nu})\,, \tag{3}$$

since only the mean value of the operator N is assumed constant and equal to the real number of particles. The states represent, rather, a superposition of states describing several nuclei differing between themselves by an even number of particles, and corresponding to the same chemical potential λ and to the same half-width of the pairing gap Δ. It is clearly shown in the particle representation that the states of the quasiparticle representation are described by a superposition of wave functions with 0, 2, 4, ..., 2Ω particles, where Ω is the total degeneracy of the system.

In view of the increasing number of applications of the mass tensor, particularly in the heavy nuclei where the pairing correlations are extremely important, it seemed worthwhile to study more extensively the errors expected as a result of the nonconservation of particle number.

It has been shown[22,23] that the sequence of states (normalized by the constant C_n),

$$|\psi\rangle_n = C_n\left[\sum_{k=0}^{n+1} \xi_k z_k^{-P}\prod_{\nu>0}(u_\nu + z_k v_\nu a_\nu^\dagger a_{\tilde\nu}^\dagger) + \mathrm{c.c.}\right]|0\rangle\,, \tag{4}$$

where

$$z_k = \exp\left(i\,\frac{k\pi}{n+1}\right)$$

and

$$\xi_k = \begin{cases} 1, & \text{if } 0<k<n+1 \\ \frac{1}{2}, & \text{if } k=0 \text{ or } n+1 \end{cases}$$

(see Ref. 27), converges towards the states $|\mathrm{PBCS}\rangle$[28] or $|\mathrm{FBCS}\rangle$,[29,30] according to whether the variational parameters u_ν and v_ν are variationally determined before or after projection. The advantage of this method is that the convergence is practically realized for $n=2$ or 3 (Refs. 23–26, 31, 32), and it is theoretically total for $2(n+1) > \mathrm{Max}\,(P, \Omega - P)$.

B. SBCS many-quasiparticle states

The spurious effects of the dispersion in the number of nucleons on many-quasiparticle BCS states may easily be eliminated within the SBCS theory. Let us give the final results: The projections in the occupation-number space of the two-quasiparticle states $|\nu\nu\rangle = c_\nu^\dagger c_{\tilde\nu}^\dagger|\psi\rangle$ and $|\nu\mu\rangle = c_\nu^\dagger c_{\tilde\mu}^\dagger|\psi\rangle$, and of the four-quasiparticle states $|\nu\nu\mu\mu\rangle = c_\nu^\dagger c_{\tilde\nu}^\dagger c_\mu^\dagger c_{\tilde\mu}^\dagger|\psi\rangle$, $|\nu\nu\mu\eta\rangle = c_\nu^\dagger c_{\tilde\nu}^\dagger c_\mu^\dagger c_{\tilde\eta}^\dagger|\psi\rangle$, and $|\nu\mu\eta\rho\rangle = c_\nu^\dagger c_{\tilde\mu}^\dagger c_\eta^\dagger c_{\tilde\rho}^\dagger|\psi\rangle$ are given by the following expressions if $2(n+1) > \mathrm{Max}(P, \Omega - P)$,

$$|(\nu\nu)_n\rangle = C_n^\nu\left[\sum_{k=0}^{n+1}\xi_k z_k^{-P}(-v_\nu + z_k u_\nu a_\nu^\dagger a_{\tilde\nu}^\dagger)\prod_{\substack{\gamma\\(\neq\nu)}}(u_\gamma + z_k v_\gamma a_\gamma^\dagger a_{\tilde\gamma}^\dagger) + \text{c.c.}\right]|0\rangle\,, \tag{5a}$$

$$|(\nu\mu)_n\rangle = C_n^{\nu\mu}\left[\sum_{k=0}^{n+1}\xi_k z_k^{-(P-1)}a_\nu^\dagger a_{\tilde\mu}^\dagger\prod_{\substack{\gamma\\(\neq\nu,\mu)}}(u_\gamma + z_k v_\gamma a_\gamma^\dagger a_{\tilde\gamma}^\dagger) + \text{c.c.}\right]|0\rangle\,, \tag{5b}$$

$$|(\nu\nu\mu\mu)_n\rangle = C_n^{\nu\nu\mu\mu}\left[\sum_{k=0}^{n+1}\xi_k z_k^{-P}(-v_\nu + z_k u_\nu a_\nu^\dagger a_{\tilde\nu}^\dagger)(-v_\mu + z_k u_\mu a_\mu^\dagger a_{\tilde\mu}^\dagger)\prod_{\substack{\gamma\\(\neq\nu,\mu)}}(u_\gamma + z_k v_\gamma a_\gamma^\dagger a_{\tilde\gamma}^\dagger) + \text{c.c.}\right]|0\rangle\,, \tag{5c}$$

$$|(\nu\nu\mu\eta)_n\rangle = C_n^{\nu\nu\mu\eta}\left[\sum_{k=0}^{n+1}\xi_k z_k^{-(P-1)}(-v_\nu + z_k u_\nu a_\nu^\dagger a_{\tilde\nu}^\dagger)a_\mu^\dagger a_{\tilde\eta}^\dagger\prod_{\substack{\gamma\\(\neq\nu,\mu,\eta)}}(u_\gamma + z_k v_\gamma a_\gamma^\dagger a_{\tilde\gamma}^\dagger) + \text{c.c.}\right]|0\rangle\,, \tag{5d}$$

$$|(\nu\mu\eta\rho)_n\rangle = C_n^{\nu\mu\eta\rho}\left[\sum_{k=0}^{n+1}\xi_k z_k^{-(P-2)}a_\nu^\dagger a_{\tilde\mu}^\dagger a_\eta^\dagger a_{\tilde\rho}^\dagger\prod_{\substack{\gamma\\(\neq\nu,\mu,\eta,\rho)}}(u_\gamma + z_k v_\gamma a_\gamma^\dagger a_{\tilde\gamma}^\dagger) + \text{c.c.}\right]|0\rangle\,. \tag{5e}$$

The energies of the ground state and of the projected states with two and four quasiparticles are expressed in terms of the matrix elements of the total Hamiltonian, in the truncated basis given by Eqs. (4)–(5).

These matrix elements are calculated with the least difficulty in the quasiparticle (qp) representation of Bogoliubov-Valatin. We must therefore express the kets of Eqs. (4) and (5) in the qp representation. To this end we employ the fact that every ket $|\phi\rangle$ belonging to the Hilbert space $\mathcal{H}$ of the states of the system described by the BCS theory admits the following expansion:

$$|\phi\rangle = |\psi\rangle\langle\psi|\phi\rangle + \sum_{\nu,\mu>0} c_\nu^\dagger c_{\tilde\mu}^\dagger|\psi\rangle\langle\psi|c_{\tilde\mu}c_\nu|\phi\rangle + \sum_{\nu\mu\eta\rho} c_\nu^\dagger c_{\tilde\mu}^\dagger c_\eta^\dagger c_{\tilde\rho}^\dagger|\psi\rangle\langle\psi|c_{\tilde\rho}c_\eta c_{\tilde\mu}c_\nu|\phi\rangle + \cdots\,.$$

When all is calculated, we obtain the following slightly cumbersome result:

$$|\psi_n\rangle = C_n\left\{\sum_{k=0}^{n+1}\xi_k z_k^{-P}\prod_{\nu>0}(u_\nu^2 + z_k v_\nu^2)\left[1 + (z_k - 1)\sum_\nu \frac{u_\nu v_\nu}{u_\nu^2 + z_k v_\nu^2}A_\nu^\dagger + \frac{(z_k-1)^2}{2}\sum_{\mu\neq\nu}\frac{u_\nu v_\nu u_\mu v_\mu}{(u_\nu^2 + z_k v_\nu^2)(u_\mu^2 + z_k v_\mu^2)}A_\nu^\dagger A_\mu^\dagger + \cdots\right] + \text{c.c.}\right\}|\psi\rangle\,, \tag{6a}$$

$$\begin{aligned}|(\nu\nu)_n\rangle = C_n^{\nu\nu}\Bigg\{\sum_{k=0}^{n+1}\xi_k z_k^{-P}\prod_{\substack{\gamma\\(\neq\nu)}}(u_\gamma^2 + z_\gamma v_\gamma^2)\Bigg[&u_\gamma v_\gamma(z_k - 1) + A_\nu^\dagger \\ &+ u_\nu v_\nu(z_k-1)^2\sum_{\substack{\mu\\(\neq\nu)}}\frac{u_\mu v_\mu}{u_\mu^2 + z_k v_\mu^2}A_\mu^\dagger + (z_k - 1)\sum_{\substack{\mu\\(\neq\nu)}}\frac{u_\mu v_\mu}{u_\mu^2 + z_k v_\mu^2}A_\nu^\dagger A_\mu^\dagger \\ &+ \tfrac{1}{2}u_\nu v_\nu(z_k-1)^3\sum_{\substack{\mu\neq\gamma\\(\neq\nu)}}\frac{u_\mu v_\mu u_\gamma v_\gamma}{(u_\mu^2 + z_k v_\mu^2)(u_\gamma^2 + z_k v_\gamma^2)}A_\mu^\dagger A_\gamma^\dagger + \cdots\Bigg] + \text{c.c.}\Bigg\}|\psi\rangle\,,\end{aligned} \tag{6b}$$

$$|(\nu\mu)_n\rangle = C_n^{\nu\mu}\left\{\sum_{k=0}^{n+1}\xi_k z_k^{-(P-1)}\prod_{\substack{\gamma\\(\neq\nu,\mu)}}(u_\gamma^2 + z_k v_\gamma^2)\left[1 + (z_k - 1)\sum_{\substack{\gamma\\(\neq\nu,\mu)}}\frac{u_\gamma v_\gamma}{u_\gamma^2 + z_k v_\gamma^2}A_\gamma^\dagger + \cdots\right] + \text{c.c.}\right\}c_\nu^\dagger c_{\tilde\mu}^\dagger|\psi\rangle\,, \tag{6c}$$

$$|(\nu\nu\mu\mu)_n\rangle = C_n^{\nu\nu\mu\mu}\Bigg\{\sum_{k=0}^{n+1}\xi_k z_k^{-P}\prod_{\gamma(\neq\nu,\mu)}(u_\gamma^2+z_k v_\gamma^2)\Bigg[(z_k-1)^2u_\nu v_\nu u_\mu u_\mu+(z_k-1)u_\mu v_\mu A_\nu^\dagger+(z_k-1)u_\nu v_\nu A_\mu^\dagger+A_\nu^\dagger A_\mu^\dagger$$
$$+(z_k-1)^2u_\mu v_\mu\sum_{\gamma(\neq\nu,\mu)}\frac{u_\gamma v_\gamma}{u_\gamma^2+z_k v_\gamma^2}A_\nu^\dagger A_\gamma^\dagger$$
$$+(z_k-1)^2u_\nu v_\nu\sum_{\gamma(\neq\nu,\mu)}\frac{u_\gamma v_\gamma}{u_\gamma^2+z_k v_\gamma^2}A_\mu^\dagger A_\gamma^\dagger+\cdots\Bigg]+\text{c.c.}\Bigg\}|\psi\rangle, \quad (6d)$$

$$|(\nu\nu\mu\eta)_n\rangle = C_n^{\nu\nu\mu\eta}\Bigg\{\sum_{k=0}^{n+1}\xi_k z_k^{-(P-1)}\prod_{\gamma(\neq\nu,\mu,\eta)}(u_\gamma^2+z_k v_\gamma^2)\Bigg[(z_k-1)u_\nu v_\nu c_\mu^\dagger c_{\tilde\eta}^\dagger$$
$$+c_\nu^\dagger c_{\tilde\nu}^\dagger c_\mu^\dagger c_{\tilde\mu}^\dagger+u_\nu v_\nu(z_k-1)^2\sum_{\gamma(\neq\nu,\mu,\eta)}\frac{u_\gamma v_\gamma}{(u_\gamma^2+z_k v_\gamma^2)}c_\mu^\dagger c_{\tilde\eta}^\dagger A_\gamma^\dagger$$
$$+(z_k-1)\sum_{\gamma(\neq\nu,\mu,\eta)}\frac{u_\gamma v_\gamma}{u_\gamma^2+z_k v_\gamma^2}A_\gamma^\dagger c_\nu^\dagger c_{\tilde\nu}^\dagger c_\mu^\dagger c_{\tilde\mu}^\dagger+\cdots\Bigg]+\text{c.c.}\Bigg\}|\psi\rangle, \quad (6e)$$

$$|(\nu\mu\eta\rho)_n\rangle = C_n^{\nu\mu\eta\rho}\Bigg\{\sum_{k=0}^{n+1}\xi_k z_k^{-(P-2)}\prod_{\gamma(\neq\nu,\mu,\eta,\rho)}(u_\gamma^2+z_k v_\gamma^2)\Bigg[1+(z_k-1)\sum_{\gamma(\neq\nu,\mu,\eta,\rho)}\frac{u_\gamma v_\gamma}{u_\gamma^2+z_k v_\gamma^2}A_\gamma^\dagger+\cdots\Bigg]+\text{c.c.}\Bigg\}c_\nu^\dagger c_{\tilde\mu}^\dagger c_\eta^\dagger c_{\tilde\rho}^\dagger|\psi\rangle. \quad (6f)$$

The pair creation operator $A_\nu^\dagger = c_\nu^\dagger c_{\tilde\nu}^\dagger$ may easily be shown to be invariant under time reversal.[23]

C. SBCS many-qp energies

The Hamiltonian (1) and its canonical transform in qp representation,[23]

$$H=\sum_{i,j=0}^{4}H_{ij}, \quad |i-j|=0,2$$

conserve the number of particles and only connect the components of Eqs. (5) and (6) corresponding to the same number of particles. This fact considerably simplifies the calculation of the matrix elements of H. We find, in fact,

$$\mathcal{E}_n^0=\langle\psi_n|H|\psi_n\rangle=2(n+1)C_n\langle\psi_n|H|\psi\rangle, \quad (7a)$$
$$\mathcal{E}_n^{\nu\nu}=\langle(\nu\nu)_n|H|(\nu\nu)_n\rangle=2(n+1)C_n^{\nu}\langle\psi_n|Hc_\nu^\dagger c_{\tilde\nu}^\dagger|\psi\rangle, \quad (7b)$$
$$\mathcal{E}_n^{\nu\mu}=\langle(\nu\mu)_n|H|(\nu\mu)_n\rangle=2(n+1)C_n^{\nu\mu}\langle\psi_n|Hc_\nu^\dagger c_{\tilde\mu}^\dagger|\psi\rangle, \quad (7c)$$
$$\mathcal{E}_n^{\nu\nu\mu\mu}=\langle(\nu\nu\mu\mu)_n|H|(\nu\nu\mu\mu)_n\rangle=2(n+1)C_n^{\nu\nu\mu\mu}\langle\psi_n|Hc_\nu^\dagger c_{\tilde\nu}^\dagger c_\mu^\dagger c_{\tilde\mu}^\dagger|\psi\rangle, \quad (7d)$$
$$\mathcal{E}_n^{\nu\nu\mu\eta}=\langle(\nu\nu\mu\eta)_n|H|(\nu\nu\mu\eta)_n=2(n+1)C_n^{\nu\nu\mu\eta}\langle\psi_n|Hc_\nu^\dagger c_{\tilde\nu}^\dagger c_\mu^\dagger c_{\tilde\eta}^\dagger|\psi\rangle, \quad (7e)$$
$$\mathcal{E}_n^{\nu\mu\eta\rho}=\langle(\nu\mu\eta\rho)_n|H|(\nu\mu\eta\rho)_n\rangle=2(n+1)C_n^{\nu\mu\eta\rho}\langle\psi_n|Hc_\nu^\dagger c_{\tilde\mu}^\dagger c_\eta^\dagger c_{\tilde\rho}^\dagger|\psi\rangle. \quad (7f)$$

1. Zero-qp SBCS energy

It is easy to see that the only components of H giving a nonvanishing contribution to $\mathcal{E}_n^0$ are H_{00} and H_{40}. Let us recall that the variational determination of the probability amplitudes u_ν and v_ν cancels the term H_{20}. More explicitly,

$$\mathcal{E}_n^0=H_{00}-G\sum_{\nu,\mu>0}A_n^{\nu\mu}u_\nu^2 v_\mu^2, \quad (8)$$

where the BCS energy is simply

$$H_{00}=2\sum_{\nu>0}(\epsilon_\nu-\lambda-Gv_\nu^2)v_\nu^2-G\sum_{\nu,\mu>0}u_\nu v_\nu u_\mu v_\mu .$$

The $A_n^{\nu\mu}$ functions are defined by

$$A_n^{\nu\mu}=4(n+1)C_n^2\sum_{k=0}^{n+1}\xi_k R_k\sin^2 x_k\{\cos\psi_k[1-2(u_\nu^2v_\mu^2+u_\mu^2v_\nu^2)\sin^2x_k]+(v_\nu^2v_\mu^2-u_\nu^2u_\mu^2)\sin\psi_k\sin(2x_k)\}\frac{\gamma_\nu\gamma_\mu}{\rho_{\nu k}\rho_{\mu k}} .$$

We have, furthermore, used the following definitions:

$$4(n+1)C_n^2\sum_{k=0}^{n+1}\xi_k R_k\cos\psi_k=1,\quad \gamma_\nu=2u_\nu v_\nu,\quad \delta_\nu=u_\nu^2-v_\nu^2,\quad x_k=\frac{k\pi}{2(n+1)},\quad R_k=\prod_\nu \rho_{\nu k},$$

$$\rho_{\nu k}=[1-\gamma_\nu\sin^2x_k]^{1/2},\quad \psi_k=\sum_\nu[\phi_{\nu k}+(\Omega-2P)x_k],\quad \tan\phi_k=-\delta_\nu\tan x_k,\ \text{with}\ |\phi_{\nu k}|\le\pi/2 .$$

2. Two-qp SBCS energies

A calculation similar to the preceding one shows that in the evaluation of $\mathcal{E}_n^{\nu\nu}$ and $\mathcal{E}_n^{\nu\mu}$ [Eqs. (7b) and (7c)] the nonvanishing contributions of H come from the components H_{00}, H_{11}, H_{22}, H_{31}, and H_{40}. We indicate the final results for the two types of two-qp projected states:

$$\mathcal{E}_n^{\nu\nu}=H_{00}+2E_\nu-G+4(n+1)(C_n^{\nu\nu})^2\sum_{k=0}^{n+1}\xi_k\sin^2x_k\sum_{\mu\,(\neq\nu)}GR_k^{\nu\mu}\gamma_\mu(u_\nu^2u_\mu^2+v_\nu^2v_\mu^2)\cos\psi_k^{\nu\mu}, \tag{9a}$$

$$\begin{aligned}\mathcal{E}_n^{\nu\mu}={}&H_{00}+E_\nu+E_\mu-G(u_\nu^4+v_\nu^4)-2Gu_\nu v_\nu u_\mu v_\mu+4(n+1)(C_n^{\nu\mu})^2\sum_{k=0}^{n+1}\xi_k\sum_{\rho\,(\neq\nu,\mu)}R_{\rho k}^{\nu\mu}\sin x_k\gamma_\rho\delta_\rho\cos(\psi_{\rho k}^{\nu\mu}+x_k+\tfrac{1}{2}\pi)\\&+4(n+1)(C_n^{\nu\mu})^2G\sum_{k=0}^{n+1}\xi_k\sin^2x_k\sum_{\substack{\rho\neq\lambda\\(\neq\nu,\mu)}}R_{\rho\lambda k}^{\nu\mu}\gamma_\rho\gamma_\lambda u_\rho^2v_\lambda^2\cos(\psi_{\rho\lambda k}^{\nu\mu}+2x_k+\pi),\end{aligned} \tag{9b}$$

where

$$E_\nu=[(\epsilon_\nu-\lambda-Gv_\nu^2)^2+\Delta^2]^{1/2},\quad \Delta=G\sum_\nu u_\nu v_\nu,\quad 4(n+1)(C_n^{\nu\nu})^2\sum_{k=0}^{n+1}\xi_k^\nu\cos\psi_k^\nu=1,\ \text{and}$$

$$4(n+1)(C_n^{\nu\mu})^2\sum_{k=0}^{n+1}\xi_k R_k^{\nu\mu}\cos\psi_k^{\nu\mu}=1,\quad R_k^\nu=\prod_{\gamma\,(\neq\nu)}\rho_{\gamma k},\quad R_k^{\nu\mu}=\prod_{\gamma\,(\neq\nu,\mu)}\rho_{\gamma k},\quad R_{\gamma k}^{\nu\mu}=\prod_{\lambda\,(\neq\gamma,\nu,\mu)}\rho_{\lambda k},\quad R_{\gamma\lambda k}^{\nu\mu}=\prod_{\rho\,(\neq\gamma,\lambda,\nu,\mu)}\rho_{\rho k},$$

$$\psi_k^\nu=\psi_k-\phi_{\nu k},\quad \psi_k^{\nu\mu}=\psi_k-\phi_{\nu k}-\phi_{\mu k},\quad \psi_{\gamma k}^{\nu\mu}=\psi_k-\phi_{\nu k}-\phi_{\mu k}-\phi_{\gamma k},\quad \psi_{\gamma\lambda k}^{\nu\mu}=\psi_k-\phi_{\nu k}-\phi_{\mu k}-\phi_{\gamma k}-\phi_{\lambda k}.$$

III. ORTHONORMALIZED SBCS STATES AND THEIR ENERGIES

The calculation of the mass parameters in the Inglis cranking model[5] requires a knowledge of all the excited states of the nucleus. Unfortunately the SBCS states determined above are not orthogonal among themselves, nor to the ground state, the projection in the occupation number space having destroyed the orthogonality of the BCS states. Therefore we will replace the preceding SBCS states by linear combinations, which will be normalized and orthogonal among themselves, by employing the well-known Schmidt orthogonalization process and by correctly normalizing the kets so obtained. This orthonormalization procedure is often ignored, although there is no *a priori* reason to neglect it.[33] We employ the following approximations:

(i) The Hilbert space $\mathcal{H}$ associated with the system is restricted to the space $\mathcal{H}_2$ of the two-qp elementary excitations, higher excitations (four and more qp) being very likely negligible, since to excite a single qp we need at least an amount of energy equal to the gap.

(ii) All the terms in (uv) of an order higher to or equal to 3 are neglected.

A. Orthonormalized SBCS states in qp representation

More explicitly, the preceding restrictions signify that the only SBCS states which are considered are the following:

$$|\psi_n\rangle = |\psi\rangle\langle\psi|\psi_n\rangle + \sum_\nu |\nu\nu\rangle\langle\nu\nu|\psi_n\rangle$$
$$= C_n\left\{\sum_{k=0}^{n+1} \xi_k z_k^{-P} \prod_\nu (u_\nu^2 + z_k v_\nu^2)\left[1 + (z_k - 1)\sum_\nu \frac{u_\nu v_\nu}{u_\nu^2 + z_k v_\nu^2} A_\nu^\dagger\right] + \text{c.c.}\right\}|\psi\rangle, \quad (10a)$$

$$|(\nu\nu)_n\rangle = |\psi\rangle\langle\psi|(\nu\nu)_n\rangle + \sum_\mu |\mu\mu\rangle\langle\mu\mu|(\nu\nu)_n\rangle$$
$$= C_n^{\nu\nu}\left\{\sum_{k=0}^{n+1} \xi_k z_k^{-P} \prod_{\substack{\gamma\\(\neq\nu)}} (u_\gamma^2 + z_k v_\gamma^2)\left[(z_k - 1)u_\nu v_\nu + A_\nu^\dagger + (z_k - 1)u_\nu v_\nu \sum_{\substack{\mu\\(\neq\nu)}} \frac{u_\mu v_\mu}{u_\mu^2 + z_k v_\mu^2} A_\mu^\dagger\right] + \text{c.c.}\right\}|\psi\rangle, \quad (10b)$$

$$|(\nu\mu)_n\rangle = |\nu\mu\rangle, \text{ the two-qp BCS state}, \quad (10c)$$

$$|(\nu\nu\mu\mu)_n\rangle = |\psi\rangle\langle\psi|(\nu\nu\mu\mu)_n\rangle + \sum_\eta |\eta\eta\rangle\langle\eta\eta|(\nu\nu\mu\mu)_n\rangle$$
$$= C_n^{\nu\nu\mu\mu}\left\{\sum_{k=0}^{n+1} \xi_k z_k^{-P} \prod_{\substack{\gamma\\(\neq\nu,\mu)}} (u_\gamma + z_k v_\gamma^2)[(z_k - 1)^2 u_\nu v_\nu u_\mu v_\mu + (z_k - 1)u_\mu v_\mu A_\nu^\dagger + (z_k - 1)u_\nu v_\nu A_\mu^\dagger] + \text{c.c.}\right\}|\psi\rangle. \quad (10d)$$

In these equations, account has been taken of the orthogonality of the two-qp state $|\nu\mu\rangle$, with the projected states $|\psi_n\rangle$, $|(\nu\nu)_n\rangle$, and $|(\nu\nu\mu\mu)_n\rangle$, having an even number of qp:

$$\sum_{\nu\neq\mu} |\nu\mu\rangle\langle\nu\mu|\psi_n\rangle = 0, \ldots .$$

These states are obviously not orthogonal.

It is now possible to obtain the overlap integrals of the BCS and SBCS states:

$$\langle\psi|\psi_n\rangle = C_n\left[\sum_{k=0}^{n+1} \xi_k z_k^{-P} \prod_\nu (u_\nu^2 + z_k v_\nu^2) + \text{c.c.}\right], \quad (11a)$$

$$\langle\nu\nu|\psi_n\rangle = u_\nu v_\nu C_n\left[\sum_{k=0}^{n+1} \xi_k z_k^{-P}(z_k - 1) \prod_{\substack{\gamma\\(\neq\nu)}} (u_\gamma^2 + z_k v_\gamma^2) + \text{c.c.}\right], \quad (11b)$$

$$\langle\psi|(\nu\nu)_n\rangle = u_\nu v_\nu C_n^{\nu\nu}\left[\sum_{k=0}^{n+1} \xi_k z_k^{-P}(z_k - 1) \prod_{\substack{\gamma\\(\neq\gamma)}} (u_\gamma^2 + z_k v_\gamma^2) + \text{c.c.}\right], \quad (11c)$$

$$\langle\mu\mu|(\nu\nu)_n\rangle = \delta_{\nu\mu} C_n^{\nu\nu}\left[\sum_{k=0}^{n+1} \xi_k z_k^{-P} \prod_{\substack{\gamma\\(\neq\nu)}} (u_\gamma^2 + z_k v_\gamma^2) + \text{c.c.}\right] \quad (11d)$$

$$+ (1 - \delta_{\nu\mu}) u_\nu v_\nu u_\mu v_\mu C_n^{\nu\nu}\left[\sum_{k=0}^{n+1} \xi_k z_k^{-P}(z_k - 1)^2 \prod_{\substack{\gamma\\(\neq\nu,\mu)}} (u_\gamma^2 + z_k v_\gamma^2) + \text{c.c.}\right], \quad (11e)$$

$$\langle\psi|(\nu\nu\mu\mu)_n\rangle = u_\nu v_\nu u_\mu v_\mu C_n^{\nu\nu\mu\mu}\left[\sum_{k=0}^{n+1} \xi_k z_k^{-P}(z_k - 1)^2 \prod_{\substack{\gamma\\(\neq\nu,\mu)}} (u_\gamma^2 + z_k v_\gamma^2) + \text{c.c.}\right], \quad (11f)$$

$$\langle\eta\eta|(\nu\nu\mu\mu)_n\rangle = [\delta_{\eta\nu} u_\mu v_\mu + \delta_{\eta\mu} u_\nu v_\nu]\, C_n^{\nu\nu\mu\mu}\left[\sum_{k=0}^{n+1} \xi_k z_k^{-P}(z_k - 1) \prod_{\substack{\gamma\\(\neq\nu,\mu)}} (u_\gamma^2 + z_k v_\gamma^2) + \text{c.c.}\right], \quad (11g)$$

$$\langle(\nu\nu\mu\mu)|(\nu\nu\mu\mu)_n\rangle = C_n^{\nu\nu\mu\mu}\left[\sum_{k=0}^{n+1} \xi_k z_k^{-P} \prod_{\substack{\gamma\\(\neq\nu,\mu)}} (u_\gamma^2 + z_k v_\gamma^2) + \text{c.c.}\right]. \quad (11h)$$

The set of orthonormalized states obtained by Schmidt's procedure, beginning with $|\psi_n\rangle$ of Eq. (10a), comprises the following vectors:

$$|(\nu\nu)_n'\rangle = N_n(\nu,\nu)[|(\nu\nu)_n\rangle - |\psi_n\rangle\langle\psi_n|(\nu\nu)_n\rangle], \quad (12a)$$

$$|(\nu\mu)_n'\rangle = |(\nu\mu)_n\rangle = |\nu\mu\rangle, \quad (12b)$$

$$|(\nu\nu\mu\mu)_n'\rangle = M_n(\nu,\mu)\Big\{|(\nu\nu\mu\mu)_n\rangle - \sum_\eta N_n^2(\eta,\eta)|(\eta\eta)_n\rangle\langle(\eta\eta)_n|(\nu\nu\mu\mu)_n\rangle - |\psi_n\rangle\Big[\langle\psi_n|(\nu\nu\mu\mu)_n\rangle + \sum_\eta \langle(\nu\nu\mu\mu)_n|(\eta\eta)_n\rangle\langle(\eta\eta)_n|\psi\rangle\Big]\Big\}, \tag{12c}$$

where the constants of normalization are given by

$$N_n(\nu,\nu) = [1 - |\langle(\nu\nu)_n|\psi_n\rangle|^2]^{-1/2}, \tag{13a}$$

$$M_n(\nu,\mu) = \Big[1 - \sum_\eta N_n^2(\eta,\eta)|\langle(\nu\nu\mu\mu)_n|(\eta\eta)_n\rangle|^2\Big]^{-1/2}. \tag{13b}$$

The different scalar products of the SBCS kets are given either by Eqs. (A3), in which the terms of an order higher than 2 in (uv) are neglected, or by Eqs. (10). The result is given in Eqs. (A4)–(A6).

The same expansions (10) allow us to obtain in $\mathcal{H}_2$ the expressions of the projected and orthonormalized states

$$|(\nu\nu)_n'\rangle = |\psi\rangle L_n(\nu) + \sum_\mu |\mu\mu\rangle P_n(\mu,\nu), \tag{14a}$$

$$|(\nu\nu\mu\mu)_n'\rangle = |\psi\rangle Q_n(\nu,\mu) + \sum_\eta |\eta\eta\rangle S_n(\eta,\nu,\mu), \tag{14b}$$

where the coefficients L_n, P_n, Q_n, and S_n are defined by

$$L_n(\nu) = N_n(\nu,\nu)\Big[\langle\psi|(\nu\nu)_n\rangle - |\langle\psi|\psi_n\rangle|^2\langle\psi|(\nu\nu)_n\rangle + \langle\psi|\psi_n\rangle\sum_\delta\langle\delta\delta|\psi_n\rangle\langle\delta\delta|(\nu\nu)_n\rangle\Big], \tag{15a}$$

$$P_n(\mu,\nu) = N_n(\nu,\nu)\Big[\langle\mu\mu|(\nu\nu)_n\rangle - \langle\mu\mu|\psi_n\rangle\langle\psi|\psi_n\rangle\langle\psi|(\nu\nu)_n\rangle + \langle\mu\mu|\psi_n\rangle\sum_\delta\langle\delta\delta|\psi_n\rangle\langle\delta\delta|(\nu\nu)_n\rangle\Big], \tag{15b}$$

$$Q_n(\nu,\mu) = M_n(\nu,\mu)\Big\{\langle\psi|(\nu\nu\mu\mu)_n\rangle - \langle\psi|\psi_n\rangle\langle\psi_n|(\nu\nu\mu\mu)_n\rangle - \sum_\eta\langle\psi|(\eta\eta)_n\rangle\langle(\eta\eta)_n|(\nu\nu\mu\mu)_n\rangle[N_n^2(\eta,\eta) + \langle\psi|\psi_n\rangle]\Big\}, \tag{15c}$$

$$S_n(\eta,\nu,\mu) = M_n(\nu,\mu)\Big\{\langle\eta\eta|(\nu\nu\mu\mu)_n\rangle - \sum_\delta N_n^2(\delta,\delta)\langle\eta\eta|(\delta\delta)_n\rangle\langle(\delta\delta)_n|(\nu\nu\mu\mu)_n\rangle - \langle\eta\eta|\psi_n\rangle\Big[\langle\psi_n|(\nu\nu\mu\mu)_n\rangle + \sum_\delta\langle(\nu\nu\mu\mu)_n|(\delta\delta)_n\rangle\langle(\delta\delta)_n|\psi\rangle\Big]\Big\}. \tag{15d}$$

These equations define in $\mathcal{H}_2$ a set of projected orthonormalized states.

B. Energies

The energies of the SBCS orthonormalized states previously obtained can be deduced from the matrix elements of H calculated with the zero- and two-qp BCS states.

Let e_n^0, $e_n(\nu)$, $e_n(\nu,\mu)$, $e_n(\nu,\nu,\mu,\mu)$ be the energies of the projected and orthogonalized states [10(a), 12(a)–12(c)], respectively. Then we obtain as the final result

$$e_n(0) = \langle\psi_n|H|\psi_n\rangle = H_{00}\Big(|\langle\psi|\psi_n\rangle|^2 + \sum_\nu|\langle\nu\nu|\psi_n\rangle|^2\Big) + 2\sum_\nu E_\nu|\langle\nu\nu|\psi_n\rangle|^2, \tag{16a}$$

$$e_n(\nu) = \langle(\nu,\nu)_n'|H|(\nu\nu)_n'\rangle = H_{00}|L_n(\nu)|^2 + \sum_\mu(H_{00} + 2E_\mu - 2Gu_\mu^2v_\mu^2)|P_n(\mu,\nu)|^2 - G\sum_{\mu\lambda}(u_\mu^2u_\lambda^2 + v_\mu^2v_\lambda^2)P_n(\mu,\nu)P_n(\lambda,\nu), \tag{16b}$$

$$e_n(\nu,\mu) = \langle\nu\mu|H|\nu\mu\rangle = E_\nu + E_\mu, \tag{16c}$$

$$e_n(\nu,\nu,\mu,\mu)=H_{00}|Q_n(\nu,\mu)|^2+\sum_{\eta}(H_{00}+2E_\eta-2Gu_\eta^2v_\eta^2)|S_n(\eta,\nu,\mu)|^2$$
$$-G\sum_{\eta\delta}(u_\eta^2u_\delta^2+v_\eta^2v_\delta^2)S_n(\eta,\nu,\mu)S_n(\delta,\nu,\mu). \tag{16d}$$

We explicitly give, in spite of their unwieldiness, the main results of our calculation, in view of their usefulness in every numerical calculation of mass parameters, and even in every calculation of SBCS type. We hope that our results will serve to promote and to orient other research in the SBCS theory.

IV. THE FULL INERTIAL-MASS TENSOR

Having now well prepared the foundation of our work, we are prepared to enter upon the computation of the microscopic inertial-mass functions.

A. The SBCS adiabatic cranking model

We suppose that an even-even nucleus formed of A nucleons can be described by a Hamiltonian $\mathcal{H}(\vec{r}_1,\vec{r}_2,\ldots,\vec{r}_A;\beta_1,\ldots,\beta_s)$ depending on the coordinates $\vec{r}_1,\ldots,\vec{r}_A$ of the A nucleons and on a set $\{\beta_i(t)\}$ $i=1,s$ of s parameters depending on time and characterizing the deformation of the nucleus at every moment t.

If we also suppose that the oscillations or the fission process of the nucleus are approximately adiabatic, that is, that the collective speeds $\{\dot\beta_i(t)\}$, $i=1,s$ are small compared with the speeds of the intrinsic movement of the nucleons, the cranking model permits us to write the collective Hamiltonian in the form

$$\mathcal{H}=\frac{1}{2}\sum_{i,j}B_{ij}(\beta_1,\ldots,\beta_s)\dot\beta_i(t)\dot\beta_j(t)+W(\beta_1,\ldots,\beta_s), \tag{17}$$

where $W(\beta_1,\ldots,\beta_s)$ represents the collective potential energy of the nucleus and where the $\frac{1}{2}s(s+1)$ B_{ij} coefficients are the components of the mass tensor. The speeds $\dot\beta_i(t)$ being small, the first-order perturbation theory gives us the B_{ij} in the form

$$B_{ij}=2\hbar^2\sum_{m\neq 0}\frac{\langle\tilde 0|\frac{\partial}{\partial\beta_i}|\tilde m\rangle\langle\tilde m|\frac{\partial}{\partial\beta_j}|\tilde 0\rangle}{\mathcal{E}_m-\mathcal{E}_0}$$
$$=2\hbar^2\sum_{m\neq 0}\frac{\langle\tilde 0|\frac{\partial H}{\partial\beta_i}|\tilde m\rangle\langle\tilde m|\frac{\partial H}{\partial\beta_j}|\tilde 0\rangle}{(\mathcal{E}_m-\mathcal{E}_0)^3}, \tag{18}$$

where H designates the independent particle Hamiltonian with eigenkets $|\tilde m\rangle$ and eigenvalues $\mathcal{E}_m$ for each set $\{\beta_i\}$ of the deformation parameters of the mean field.

If account is explicitly taken of the pairing correlations, the Hamiltonian H is that of Eq. (1) and the ground state $|\tilde 0\rangle$ is given by Eq. (2) or Eq. (4) according to whether the conservation of the number of nucleons is imposed as an average or exactly. In Eq. (17) the summation must extend to all the eigenkets of H, or else in qp representation, to all the states having an even number of qp.

In the BCS theory the only states having non-nil matrix elements are the zero- and two-qp states.[34,35] Here all takes place in $\mathcal{H}_2$ space. In the SBCS theory this is not the case [cf. Eqs. (10)]. In fact, the B_{ij} should be calculated in the Hilbert space spaned by the totality of the states having an even number of qp.

In the usual BCS theory the energy denominator $\mathcal{E}_m-\mathcal{E}_0$ is the energy of the two-qp excited states with respect to the energy of the fundamental $|\psi\rangle$ (BCS vacuum). The matrix elements of Eq. (18) are given in Appendix B for the sake of completeness.

In the SBCS theory the inertia parameter becomes, in the notation of Sec. III,

$$B_{ij}(n)=2\hbar^2\sum_{\nu}\frac{1}{e_n(\nu)-e_n(0)}\left\langle\psi_n\left|\frac{\partial}{\partial\beta_i}\right|(\nu\nu)_n'\right\rangle\left\langle(\nu\nu)_n'\left|\frac{\partial}{\partial\beta_j}\right|\psi_n\right\rangle$$
$$+2\hbar^2\sum_{\nu\neq\mu}\frac{1}{e_n(\nu,\mu)-e_n(0)}\left\langle\psi_n\left|\frac{\partial}{\partial\beta_i}\right|(\nu\mu)_n'\right\rangle\left\langle(\nu\mu)_n'\left|\frac{\partial}{\partial\beta_j}\right|\psi_n\right\rangle$$
$$+2\hbar^2\sum_{\nu\mu}\frac{1}{e_n(\nu,\nu,\mu,\mu)-e_n(0)}\left\langle\psi_n\left|\frac{\partial}{\partial\beta_i}\right|(\nu\nu\mu\mu)_n'\right\rangle\left\langle(\nu\nu\mu\mu)_n'\left|\frac{\partial}{\partial\beta_j}\right|\psi_n\right\rangle+\cdots. \tag{19}$$

Each term of this series is developed in qp representation and only the terms physically important with zero and two qp are retained. By using the algebra briefly set out in Sec. III, that is to say, the SBCS orthonormalized kets (subsection A) and the corresponding energies (subsection B), the B_{ij} components of the mass tensor are thus formally given by

$$B_{ij}(n)=2\hbar^2\sum_{\nu}\frac{1}{e_n(\nu)-e_n(0)}\left\langle\psi_n\left|\frac{\partial}{\partial\beta_i}\right|(\nu\nu)'_n\right\rangle\left\langle(\nu\nu)'_n\left|\frac{\partial}{\partial\beta_j}\right|\psi_n\right\rangle+2\hbar^2\sum_{\nu\neq\mu}\frac{1}{E_\nu+E_\mu}\quad\psi_n\left|\frac{\partial}{\partial\beta_i}\right|\nu\mu\quad\nu\mu\left|\frac{\partial}{\partial\beta_j}\right|\psi_n$$
$$+2\hbar^2\sum_{\nu\neq\mu}\frac{1}{e_n(\nu,\nu,\mu,\mu)-e_n(0)}\left\langle\psi_n\left|\frac{\partial}{\partial\beta_i}\right|(\nu\nu\mu\mu)'_n\right\rangle\left\langle(\nu\nu\mu\mu)'_n\left|\frac{\partial}{\partial\beta_j}\right|\psi_n\right. . \tag{20}$$

By using the anti-Hermiticity of the differential operator,

$$\left\langle\psi_n\left|\frac{\partial}{\partial\beta_i}\right|\phi\right\rangle=-\left\langle\phi\left|\frac{\partial}{\partial\beta_i}\right|\psi_n\right\rangle\forall\langle\phi|, \tag{21}$$

all the derivatives of Eq. (20 can be expressed in terms of the derivatives of $|\psi_n\rangle$ with respect to the deformation parameters. Let us calculate these derivatives.

B. Evaluation of the matrix elements

The matrix elements of the derivation operator $\partial/\partial\beta_i$ are all known if we know $\partial/\partial\beta_i|\psi_n\rangle$. Now in $\mathcal{H}_2$ space the following expansion can formally be written:

$$\frac{\partial}{\partial\beta_i}|\psi_n\rangle=f_n(\beta_i)|\psi\rangle\!\rangle+\sum_{\nu}g_n^{\nu}(\beta_i)|\nu\nu\rangle\!\rangle+\sum_{\nu\neq\mu}h_n^{\nu\mu}(\beta_i)|\nu\mu\rangle\!\rangle, \tag{22}$$

in which the real coefficients f_n, g_n^ν, and $h_n^{\nu\mu}$ are obtained using Eq. (21). The result is

$$f_n(\beta_i)=\left\langle\psi\left|\frac{\partial}{\partial\beta_i}\right|\psi_n\right\rangle=-\left\langle\psi_n\left|\frac{\partial}{\partial\beta_i}\right|\psi\right\rangle, \tag{23a}$$

$$g_n^\nu(\beta_i)=\left\langle\nu\nu\left|\frac{\partial}{\partial\beta_i}\right|\psi_n\right\rangle=-\left\langle\psi_n\left|\frac{\partial}{\partial\beta_i}\right|\nu\nu\right\rangle, \tag{23b}$$

$$h_n^{\nu\mu}(\beta_i)=\left\langle\nu\mu\left|\frac{\partial}{\partial\beta_i}\right|\psi_n\right\rangle=-\left\langle\psi_n\left|\frac{\partial}{\partial\beta_i}\right|\nu\mu\right\rangle. \tag{23c}$$

Thus, it suffices to know the derivatives with respect to the β_i parameters of the zero- and two-qp BCS states. Now it is known[34] that to the first order in β_i the BCS states allow the following expansion in $\mathcal{H}_2$ space:

$$|\psi\rangle\!\rangle=|\psi(0)\rangle\!\rangle+\sum_i\beta_i\left[\sum_\nu\frac{B_\nu(\beta_i)}{u_\nu v_\nu}|(\nu\nu)(0)\rangle\!\rangle-\sum_{\nu\neq\mu}\frac{u_\nu v_\mu+u_\mu v_\nu}{E_\nu+E_\mu}\left\langle\mu\left|\frac{\partial H}{\partial\beta_i}\right|\nu\right\rangle|(\nu\mu)(0)\rangle\!\rangle\right]. \tag{24a}$$

By using the technique of calculation of Bès,[34] which leads to the expansion (24a), we have evaluated the expansion of the states $|\nu\nu\rangle\!\rangle$ and $|\nu\mu\rangle\!\rangle$:

$$|\nu\nu\rangle\!\rangle=|(\nu\nu)(0)\rangle\!\rangle-\sum_i\beta_i\left\{\frac{B_\nu(\beta_i)}{4u_\nu v_\nu}|\psi(0)\rangle\!\rangle+\sum_{\mu(\neq\nu)}\frac{u_\nu u_\mu+v_\nu v_\mu}{\epsilon_\mu-\epsilon_\nu}\left[\left\langle\mu\left|\frac{\partial H}{\partial\beta_i}\right|\nu\right\rangle|(\mu\nu)(0)\rangle\!\rangle+\left\langle\nu\left|\frac{\partial H}{\partial\beta_i}\right|\mu\right\rangle|(\nu\mu)(0)\rangle\!\rangle\right]\right\}, \tag{24b}$$

$$|\nu\mu\rangle\!\rangle=|(\nu\mu)(0)\rangle\!\rangle+\sum_i\beta_i\left[\frac{u_\nu v_\mu+u_\mu v_\nu}{E_\nu+E_\mu}\left\langle\mu\left|\frac{\partial H}{\partial\beta_i}\right|\nu\right\rangle|\psi(0)\rangle\!\rangle-\sum_{\eta\neq\nu}\frac{u_\nu u_\eta+v_\nu v_\eta}{\epsilon_\eta-\epsilon_\nu}\left\langle\eta\left|\frac{\partial H}{\partial\beta_i}\right|\nu\right\rangle|(\eta\mu)(0)\rangle\!\rangle-\sum_{\eta\neq\mu}\frac{u_\mu u_\eta+v_\eta v_\mu}{\epsilon_\eta-\epsilon_\mu}\left\langle\mu\left|\frac{\partial H}{\partial\beta_i}\right|\mu\right\rangle|(\nu\eta)(0)\rangle\!\rangle\right]. \tag{24c}$$

The kets $|\psi(0)\rangle\!\rangle$, $|(\nu\nu)(0)\rangle\!\rangle$, and $|(\nu\mu)(0)\rangle\!\rangle$ describe the zero- and two-qp BCS states for deformation β_i all zero.

The coefficients of ansatz (22) may then easily be computed:

$$f_n(\beta_i)=\sum_\nu\frac{B_\nu(\beta_i)}{4u_\nu v_\nu}\langle\psi_n(0)|(\nu\nu)(0)\rangle\!\rangle, \tag{25a}$$

$$g_n^\nu(\beta_i)=\frac{B_\nu(\beta_i)}{4u_\nu v_\nu}\langle\psi_n(0)|\psi(0)\rangle\!\rangle, \tag{25b}$$

$$h_n^{\nu\mu}(\beta_i)=\frac{u_\nu v_\mu+u_\mu v_\nu}{E_\nu+E_\mu}\left\langle\mu\left|\frac{\partial H}{\partial\beta_i}\right|\nu\right\rangle\langle\psi_n(0)|\psi(0)\rangle\!\rangle. \tag{25c}$$

The final result for the matrix elements of Eq. (20) is

$$\left\langle\psi_n\left|\frac{\partial H}{\partial\beta_i}\right|(\nu\nu)_n'\right\rangle=-\left\langle(\nu\nu)_n'\left|\frac{\partial}{\partial\beta_i}\right|\psi_n\right\rangle=-N_n(\nu,\nu)\Big\{f_n(\beta_i)[\langle(\nu\nu)_n|\psi\rangle-\langle\psi_n|\psi\rangle\langle\psi_n|(\nu\nu)_n\rangle]$$
$$+\sum_\mu g_n^\mu(\beta_i)[\langle(\nu\nu)_n|\mu\mu\rangle-\langle\psi_n|\mu\mu\rangle\langle\psi_n|(\nu\nu)_n\rangle]\Big\},\qquad(26a)$$

$$\left\langle\psi_n\left|\frac{\partial}{\partial\beta_i}\right|\nu\mu\right\rangle=-\left\langle\nu\mu\left|\frac{\partial}{\partial\beta_i}\right|\psi_n\right\rangle=-h_n^{\nu\mu}(\beta_i),\qquad(26b)$$

$$\left\langle\psi_n\left|\frac{\partial}{\partial\beta_i}\right|(\nu\nu\mu\mu)_n'\right\rangle=-\left\langle(\nu\nu\mu\mu)_n'\left|\frac{\partial}{\partial\beta_i}\right|\psi_n\right\rangle=-M_n(\nu,\mu)\Big[\sum_\eta g_n^\eta(\beta_i)\langle(\nu\nu\mu\mu)_n|\eta\eta\rangle$$
$$-\sum_{\eta\delta}N_n(\eta,\eta)g_n^\delta(\beta_i)\langle(\nu\nu\mu\mu)_n|(\eta\eta)_n\rangle\langle(\eta\eta)_n|\delta\delta\rangle\Big].\qquad(26c)$$

C. Mass tensor without spurious number-fluctuation effects

By using the results found in the preceding subsection B, Eq. (20), the number-projected inertial-mass parameters may be written in the form

$$B_{ij}(n)=[B_{ij}(n)]_{\rm BCS}+[B_{ij}(n)]_{\rm PROJ},\qquad(27)$$

where

$$[B_{ij}(n)]_{\rm BCS}=-2\hbar^2\sum_\nu\frac{1}{e_n(\nu)-e_n(0)}\frac{B_\nu(\beta_i)B_\nu(\beta_j)}{(4u_\nu v_\nu)^2}\,|N_n(\nu,\nu)\langle(\nu\nu)_n|\nu\nu\rangle\langle\psi_n|\psi\rangle|^2$$
$$-2\hbar^2\sum_{\nu\neq\mu}\frac{1}{e_n(\nu,\mu)-e_n(0)}\frac{(u_\nu v_\mu+u_\mu v_\nu)^2}{(E_\nu+E_\mu)^2}\left\langle\mu\left|\frac{\partial H}{\partial\beta_i}\right|\nu\right\rangle\left\langle\nu\left|\frac{\partial H}{\partial\beta_j}\right|\mu\right\rangle|\langle\psi_n|\psi\rangle|^2\qquad(28a)$$

and

$$[B_{ij}(n)]_{\rm PROJ}=-2\hbar^2\sum_\nu\frac{1}{e_n(\nu)-e_n(0)}\frac{B_\nu(\beta_i)B_\nu(\beta_j)}{(4u_\nu v_\nu)^2}F_n(\nu)$$
$$-2\hbar^2\sum_{\nu\neq\mu}\frac{1}{e_n(\nu,\nu,\mu,\mu)-e_n(0)}\frac{(u_\nu v_\mu+u_\mu v_\nu)^2}{(E_\nu+E_\mu)^2}\left\langle\mu\left|\frac{\partial H}{\partial\beta_i}\right|\nu\right\rangle\left\langle\nu\left|\frac{\partial H}{\partial\beta_j}\right|\mu\right\rangle G_n(\nu,\mu).\qquad(28b)$$

The F_n and G_n functions are defined by

$$F_n(\nu)=N_n^2(\nu,\nu)\langle(\nu\nu)_n|\nu\nu\rangle\langle\psi_n|\nu\nu\rangle\langle\psi_n|\psi\rangle[\langle(\nu\nu)_n|\psi\rangle-\langle\psi_n|\psi\rangle\langle\psi_n|(\nu\nu)_n\rangle],\qquad(29a)$$

$$G_n(\nu,\mu)=\tfrac{1}{2}M_n^2(\nu,\mu)\sum_{\eta\delta}\langle(\nu\nu\mu\mu)_n|\eta\eta\rangle\langle(\nu\nu\mu\mu)_n|\eta\eta\rangle[1-N_n^2(\delta,\delta)\langle(\delta\delta)_n|\delta\delta\rangle-N_n^2(\eta,\eta)\langle(\eta\eta)_n|\eta\eta\rangle$$
$$+N_n^2(\eta,\eta)N_n^2(\delta,\delta)\langle(\eta\eta)_n|\eta\eta\rangle\langle(\delta\delta)_n|\delta\delta\rangle].\qquad(29b)$$

The first term (28a) of the effective mass-tensor component, Eq. (27), tends to the limit, where there is no projection towards the term $[B_{ij}]_{\rm BCS}$ given by Eq. (B6). The second term, $[B_{ij}(n)]_{\rm PROJ}$, given by Eq. (28b), has no equivalent in the usual BCS theory. It is a corrective term which characterizes the projection and which tends, evidently and visibly, towards 0 when we pass from the SBCS states to the BCS states.

Equations (27)–(29) entirely solve the problem of the extraction of the spurious effects of the dispersion in the number of nucleons on the nuclear mass-tensor components.

V. CONCLUSION

We have presented a generalization of the cranking model thus permitting the obtaining of the mass tensor with orthonormalized wave functions strictly conserving the number of nucleons. Each component of the tensor occurs as a sum of two terms. When the SBCS functions tend towards the BCS functions, that is, when the extraction of the phantom effects, due to the fluctuation of the number of particles, ceases to have effect, the first term becomes the well-known BCS component of the mass tensor and the second term disappears.

All the matrix elements, overlap integrals, and energies necessary to the numerical calculation of the tensor have been given in full detail. To summarize, the present treatment adds another consequential meaning to the physically important problem of evaluation of the mass tensor.

APPENDIX A: SCALAR PRODUCTS OF THE SBCS KETS

The particle-number projection destroys the orthogonality of the kets. The scalar products of the projected kets are given by

$$\langle \psi_n | (\nu\nu)_n \rangle = 2(n+1)C_n C_n^{\nu\nu} \left\{ \sum_{k=0}^{n+1} \xi_k z_k^{-P}(z_k - 1) u_\nu v_\nu \prod_{\substack{\gamma \\ (\neq\nu)}} (u_\gamma^2 + z_k v_\gamma^2) + \text{c.c.} \right\}, \tag{A1}$$

$$\langle \psi_n | (\nu\nu\mu\mu)_n \rangle = 2(n+1)C_n C_n^{\nu\nu\mu\mu} \left\{ \sum_{k=0}^{n+1} \xi_k z_k^{-P}(z_k - 1)^2 u_\nu v_\nu u_\mu v_\mu \prod_{\substack{\gamma \\ (\neq\nu,\mu)}} (u_\gamma^2 + z_k v_\gamma^2) + \text{c.c.} \right\}, \tag{A2}$$

$$\begin{aligned} \langle (\eta\eta)_n | (\nu\nu\mu\mu)_n \rangle = 2(n+1)C_n^{\eta\eta} C_n^{\nu\nu\mu\mu} \Bigg\{ \sum_{k=0}^{n+1} \xi_k z_k^{-P} \prod_{\substack{\gamma \\ (\neq\eta)}} (u_\gamma^2 + z_k v_\gamma^2) \\ \times \Bigg[(z_k - 1)\delta_{\nu\eta} \frac{u_\mu v_\mu}{u_\mu^2 + z_k v_\mu^2} + (z_k - 1)\delta_{\mu\eta} \frac{u_\nu v_\nu}{u_\nu^2 + z_k v_\nu^2} \\ + (z_k - 1)^3 u_\eta v_\eta \frac{u_\nu v_\nu u_\mu v_\mu}{(u_\nu^2 + z_k v_\nu^2)(u_\mu^2 + z_k v_\mu^2)} \Bigg] + \text{c.c.} \Bigg\}. \end{aligned} \tag{A3}$$

Calculated in space $\mathcal{H}_2$ spanned by the components of zero- and two-qp excitations, these scalar products are written

$$\langle (\nu\nu)_n | \psi_n \rangle = \langle \psi | \psi_n \rangle \langle \psi | (\nu\nu)_n \rangle + \sum_\mu \langle \mu\mu | \psi_n \rangle \langle \mu\mu | (\nu\nu)_n \rangle, \tag{A4}$$

$$\langle (\nu\nu\mu\mu)_n | \psi_n \rangle = \langle \psi | \psi_n \rangle \langle \psi | (\nu\nu\mu\mu)_n \rangle + \sum_\eta \langle \eta\eta | (\nu\nu\mu\mu)_n \rangle \langle \eta\eta | \psi_n \rangle, \tag{A5}$$

$$\langle (\eta\eta)_n | (\nu\nu\mu\mu)_n \rangle = \langle \psi | (\eta\eta)_n \rangle \langle \psi | (\nu\nu\mu\mu)_n \rangle + \sum_\delta \langle \delta\delta | (\eta\eta)_n \rangle \langle \delta\delta | (\nu\nu\mu\mu)_n \rangle. \tag{A6}$$

All the terms of order higher than 2 in (uv) have been neglected.

APPENDIX B: BCS MASS TENSOR

With the aim of facilitating the comparison between the BCS and SBCS mass tensor, we recall here the expressions of the BCS matrix elements, entering into the calculation of the mass parameters[2,20,36]:

$$\left\langle \nu\mu \left| \frac{\partial}{\partial\beta_i} \right| \right\rangle = -\frac{u_\nu v_\mu + u_\mu v_\nu}{E_\nu + E_\mu} \left\langle \mu \left| \frac{\partial H}{\partial\beta_i} \right| \nu \right\rangle, \text{ if } \nu \neq \mu \tag{B1}$$

$$\left\langle \nu\nu \left| \frac{\partial}{\partial\beta_i} \right| \mu \right\rangle = \frac{B_\nu(\beta_i)}{4u_\nu v_\nu}, \text{ if } \nu \equiv \mu. \tag{B2}$$

The following definitions have been used[34-36]:

$$B_\nu(\beta_i) = \frac{\Delta(\epsilon_\nu - \lambda)}{E_\nu^3} \frac{\partial\Delta'}{\partial\beta_i} - \frac{\Delta^2}{E_\nu^3} \left\langle \nu \left| \frac{\partial H}{\partial\beta_i} \right| \nu \right\rangle + \frac{\Delta^2}{E_\nu^3} \frac{\partial\lambda'}{\partial\beta_i} \tag{B3}$$

$$\frac{\partial\lambda'}{\partial\beta_i} \frac{ac_i + bd_i}{a^2 + b^2}, \quad \frac{\partial\Delta'}{\partial\beta_i} = \frac{bc_i - ad_i}{a^2 + b^2}, \tag{B4}$$

$$\begin{aligned} a &= \Delta \sum_\nu \frac{1}{E_\nu^3}, \quad b = \sum_\nu \frac{\epsilon_\nu - \lambda}{E_\nu^3}, \\ c_i &= \Delta \sum_\nu \frac{1}{E_\nu^5} \left\langle \nu \left| \frac{\partial H}{\partial\beta_i} \right| \nu \right\rangle, \\ d_i &= \sum_\nu \frac{\epsilon_\nu - \lambda}{E_\nu^3} \left\langle \nu \left| \frac{\partial H}{\partial\beta_i} \right| \nu \right\rangle. \end{aligned} \tag{B5}$$

The derivatives of the pairing gap and the Fermi energy, with respect to deformations, have been obtained by a lowest-order expansion.[34]

As usual, we shall split up the mass parameter into three parts: The leading part carries the index 1; part 2 is independent of the pairing gap and remains, therefore, even if the latter becomes negligible; all the terms dependent on the variation of the pairing gap and of the pairing energy versus the deformations of the nucleus have been grouped together in part 3. Numerically, the two latter parts have the tendency to cancel each other:

$$[B_{ij}]_{\rm BCS}=[B_{ij}]_1+[B_{ij}]_2+[B_{ij}]_3\,, \tag{B6}$$

with

$$[B_{ij}]_1=\tfrac{1}{4}\hbar^2\Delta\sum_\nu \frac{1}{E_\nu^{\,5}}\left\langle\nu\left|\frac{\partial H}{\partial\beta_i}\right|\nu\right\rangle\left\langle\nu\left|\frac{\partial H}{\partial\beta_j}\right|\nu\right\rangle, \tag{B7a}$$

$$[B_{ij}]_2=2\hbar^2\times\sum_{\nu\neq\mu}\frac{(u_\nu v_\mu+u_\mu v_\nu)^2}{(E_\nu+E_\mu)^3}\left\langle\mu\left|\frac{\partial H}{\partial\beta_i}\right|\nu\right\rangle\left\langle\nu\left|\frac{\partial H}{\partial\beta_j}\right|\mu\right\rangle, \tag{B7b}$$

$$[B_{ij}]_3=\tfrac{1}{4}\hbar^2\sum_\nu\frac{1}{E_\nu^{\,5}}\left[-\Delta\left\langle\nu\left|\frac{\partial H}{\partial\beta_i}\right|\nu\right\rangle\left\langle\nu\left|\frac{\partial H}{\partial\beta_j}\right|\nu\right\rangle+\frac{(2E_\nu^{\,3})^2}{\Delta}B_\nu(\beta_i)B_\nu(\beta_j)\right]. \tag{B7c}$$

The last part is identically zero if

$$\frac{\partial\lambda'}{\partial\beta_i}=\frac{\partial\Delta'}{\partial\beta_i}=0\,.$$

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