PHYSICAL REVIEW C VOLUME 20, NUMBER 4 OCTOBER 1979

^{68}Cu in the core-coupling model

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(Received 6 June 1979)

Energy levels in ^{68}Cu are calculated using the core-coupling model. Levels in the odd nuclei ^{65}Cu, ^{67}Cu, and ^{69}Zn are first calculated to find the appropriate core-coupling parameters. These parameters are then used along with a zero-range proton-neutron interaction to calculate levels in ^{68}Cu. In the model the second excited state is predicted to be a 3^+ level rather than the (3^-) level currently assigned in the nuclear data sheets. The ordering of the quartet formed by coupling the $p_{3/2}$ proton to the $g_{9/2}$ neutron is predicted to be 6^-, 3^-, 4^-, 5^-.

[NUCLEAR STRUCTURE Core-coupling model calculation of energy levels in ^{68}Cu, ^{65}Cu, ^{67}Cu, ^{69}Zn, ^{67}Ni.]

The core-coupling model of Thankappan and True[1] has been used to fit a wide range of nuclei. Most of these nuclei have been odd, with the exception of the odd-odd nucleus ^{90}Y.[2] In this paper. we report the results of using the core-coupling model to fit ^{68}Cu.

When using the core-coupling model to fit an odd-odd nucleus, there are a plethora of parameters. In order to make an orderly choice of parameters, the corresponding odd nuclei must first be fitted. Thus, we first found the parameters for the 29th proton by doing calculations for ^{65}Cu and ^{67}Cu. These were found to fit the experimental data for the low-lying states very well. The parameters for the 39th neutron were not so easily ascertained. The odd nucleus ^{67}Ni has no known levels, so ^{69}Zn was fitted to find the appropriate neutron parameters. The proton-neutron interaction was chosen to be a zero-range interaction to cut down on the number of parameters and to correspond to previous work on the odd-odd nucleus ^{90}Y.

The low-lying energy levels of ^{68}Cu are believed to be due to shell model states of the 29th proton ($p_{3/2}$, $f_{5/2}$, and $p_{1/2}$) and the 39th neutron ($p_{1/2}$ and $g_{9/2}$) coupled to the 0^+ and 2^+ states of the doubly magic ^{66}Ni core. The ground state doublet consisting of 1^+ and 2^+ states at 0 and 84 keV is assumed to arise from coupling the $p_{3/2}$ proton and $p_{1/2}$ neutron.

A multiplet consisting of 3^-, 6^-, 4^-, and 5^- formed from the $p_{3/2}$ proton and $g_{9/2}$ neutron has previously been thought to form some of the next highest levels in ^{68}Cu.[3] The 6^- member of this multiplet is thought to form the metastable state (lifetime of 3.8 min) at 716 keV. This is the third excited state in ^{68}Cu. The second excited state at 606 keV is currently assigned a spin and parity of (3^-) in the nuclear data sheets[4] due to the work of Ref. 3. Recent study of the ^{68}Zn$(t, {}^3$He$)^{68}$Cu reaction[5] indicates that the second excited state has a much lower cross section at 30° in the $(t, {}^3$He$)$ reaction than do states at 716, 772, and 950 keV. This leads one to suspect that the state at 606 keV is not a member of the 3^-–6^- multiplet. One of the aims of doing our core-coupling calculation was to ascertain the ordering of the 3^-–6^- multiplet and to predict the spin and parity of the state at 606 keV.

Our calculation proceeded along the lines outlined in Ref. 2. We will outline only the basic equations here and the reader is referred to Ref. 2 for details. The Hamiltonian for the odd-odd nucleus is

$$H = H_c + H_p + H_n + H_{\mathrm{pc}} + H_{\mathrm{nc}} + H_{\mathrm{pn}} ,$$

where H_c, H_p, and H_n are the Hamiltonians which yield the core energy, single-particle proton energy and single-particle neutron energy. The H_{pc} and H_{nc} are the proton-core and neutron-core Hamiltonians of Thankappan and True[1] which are

$$H_{\mathrm{pc}} = -\xi_p \vec{J}_c \cdot \vec{j}_p - \eta_p \overleftrightarrow{Q}_c \cdot \overleftrightarrow{Q}_p ,$$

$$H_{\mathrm{nc}} = -\xi_n \vec{J}_c \cdot \vec{j}_n - \eta_n \overleftrightarrow{Q}_c \cdot \overleftrightarrow{Q}_n ,$$

where $\vec{J}_c$, $\vec{j}_p$, and $\vec{j}_n$ are the angular momenta of the core, the proton, and the neutron respectively and $\overleftrightarrow{Q}_c$, $\overleftrightarrow{Q}_p$, and $\overleftrightarrow{Q}_n$ are the quadrupole operators of the core, proton, and neutron respectively. The parameters ξ and η are the strength parameters for the dipole-dipole and quadrupole-quadrupole terms, respectively. The reduced matrix elements of $\overleftrightarrow{Q}_c$ and the strength of the quadrupole interaction are further parametrized in the form

$$\chi_{1p} = \eta_p \langle 0^+ \| Q_c \| 2^+ \rangle \, ,$$
$$\chi_{1n} = \eta_n \langle 0^+ \| Q_c \| 2^+ \rangle \, ,$$
$$\chi_{2p} = \eta_p \langle 2^+ \| Q_c \| 2^+ \rangle \, ,$$
$$\chi_{2n} = \eta_n \langle 2^+ \| Q_c \| 2^+ \rangle \, .$$

Thus, there are three core-coupling parameters ξ, χ_1, and χ_2 for both the proton and the neutron. The proton-neutron Hamiltonian is given by the zero-range interaction

$$H_{\rm pn} = V_0[(1-\alpha) + \alpha(\vec{\sigma}_p \cdot \vec{\sigma}_n)]\delta(\vec{r}_p - \vec{r}_n) \, .$$

When calculating energy levels in ^{68}Cu there are twelve parameters. These consist of the energy of the ^{66}Ni 2^+ core, $E(2+)$, the energies of the excited single-particle proton and neutron states $E(f_{5/2})$ and $E(p_{1/2})$ for the proton and $E(g_{9/2})$ for the neutron, six strength parameters ξ_p, ξ_n, χ_{1p}, χ_{1n}, χ_{2p}, and χ_{2n}, and two proton-neutron interaction parameters, V_0 and α. We have not included the energies of the ground state single-particle levels $E(p_{3/2})$ for the proton and $E(p_{1/2})$ for the neutron. These were set equal to zero.

In order to find reasonable values for all these parameters, we first did calculations on appropriate odd nuclei. The ideal odd nuclei would be ^{67}Cu and ^{67}Ni which consist of the 29th proton and 39th neutron outside a ^{66}Ni core. Unfortunately, no levels are known in ^{67}Ni and only two levels in ^{67}Cu have spins and parities identified. The odd nuclei ^{65}Cu and ^{69}Zn were chosen for study because they consist of the 29th proton outside an ^{64}Ni core and the 39th neutron outside a ^{68}Zn core. If the ^{64}Ni core and the ^{68}Zn core are similar to the ^{66}Ni core, our core-coupling parameters found from fitting ^{65}Cu and ^{69}Zn should also apply to ^{67}Cu and ^{67}Ni. The ^{66}Ni and ^{64}Ni cores seem to be fairly similar. They have 2^+ first excited states at 1.42 and 1.35 MeV, respectively. Thus, we expect similar core-coupling parameters to fit both ^{67}Cu and ^{65}Cu. In the case of the 39th neutron the situation is less ideal. The ^{68}Zn core has a 2^+ first excited state at 1.08 MeV. This indicates that the addition of two extra protons into an ^{66}Ni core has a substantial effect, and we do not expect that core-coupling parameters found for ^{69}Zn would necessarily work for states in ^{67}Ni. However, not enough is known about high-spin positive-parity states in ^{69}Zn to do a high quality analysis of states in ^{69}Zn anyway, and our values for the odd neutron parameters must remain partially ambiguous.

Once the core-coupling parameters were ascertained, the proton-neutron interaction parameters V_0 and α were determined by setting V_0 at -40 MeV fm^3, which was the value found for ^{90}Y, and varying α to give the correct splitting for the 1^+-2^+ ground state doublet. After this was done, only one or two parameters needed to be varied to reproduce levels in ^{68}Cu.

The core-coupling model fits states in the odd copper isotopes very well.[6] We updated the fit to ^{65}Cu by using more recent data[7] which gave improved spectroscopic factors for the 29th proton. Our fit to ^{65}Cu is shown in Fig. 1. In this figure we have plotted energy levels and designated spectroscopic factors above or below each low-lying level in ^{65}Cu. The parameters used in our best fit to ^{65}Cu along with all parameters used in all our fits are given in Table I.

Figure 1 also shows the known experimental levels in ^{67}Cu.[8] The six lowest levels line up nicely with levels in ^{65}Cu. Although the spins and parities of four of these levels are unknown, we fit ^{67}Cu assuming that the ordering of levels was the same as in ^{65}Cu. This led to only slight variations of the core-coupling parameters, but the $p_{1/2}$ single-particle energy did change noticeably from 1.18 to 1.67 MeV. Our fit to the ^{67}Cu levels is shown in Fig. 1, and the parameters used are given in Table I.

Figure 2 shows the experimental levels known in ^{69}Zn (Ref. 9) and ^{67}Ni (Ref. 8) along with our core-coupling fits. The levels in ^{69}Zn indicate that the ground state of the 39th neutron is the $p_{1/2}$ level and the first excited state is the $g_{9/2}$ shell model state. The second and third excited states can be described as the $p_{1/2}$ state coupled to the 2^+ core state. The core-coupling model parameter which splits these states is ξ_n. The parameters χ_{1n} and χ_{2n} do not affect these states since the single-particle quadrupole tensor $\overset{\leftrightarrow}{Q}_n$ cannot connect $\frac{1}{2}^-$ states from the Wigner-Eckart theorem. If we make

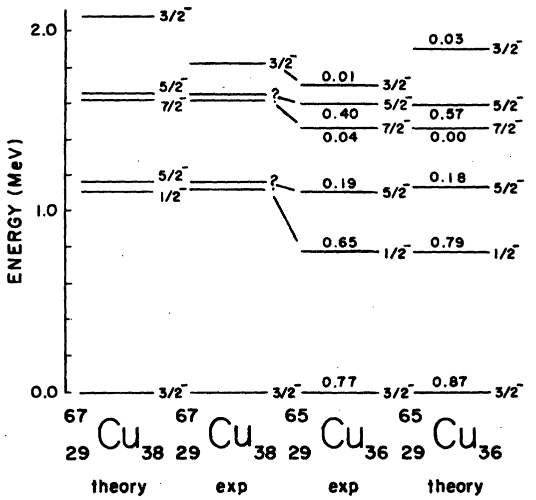

FIG. 1. Energy levels in ^{65}Cu and ^{67}Cu. Columns 2 and 3 are the experimental values from Refs. 8 and 7, respectively. Columns 1 and 4 are the core-coupling calculations. The numbers next to the ^{65}Cu levels are the single-particle spectroscopic factors.

TABLE I. Core-coupling parameters for ^{65}Cu, ^{67}Cu, ^{69}Zn, ^{67}Ni, and ^{68}Cu. Note: Same means parameter is identical to that of column next left.

Proton	^{65}Cu	^{67}Cu	^{68}Cu
$E(p_{3/2})$	0.00	same	same
$E(p_{1/2})$ (MeV)	1.18	1.67	same
$E(f_{5/2})$	1.65	1.58	same
$E(2+)$	1.35	1.42	1.12
χ_{1p} MeV fm^{-2}	0.40	same	same
χ_{2p}	0.42	same	same
ξ_p (MeV)	0.19	0.16	same
Neutron	^{69}Zn	^{67}Ni	^{68}Cu
$E(p_{1/2})$	0.00	same	same
$E(g_{9/2})$ (MeV)	0.53	same	0.43
$E(2+)$	1.08	1.42	1.12
χ_{1n} MeV fm^{-2}	0.10	same	same
χ_{2n}	0.10	same	same
ξ_n (MeV)	0.12	same	same
V_{pn}		^{90}Y	^{68}Cu
V_0 (MeV fm^3)		−40.0	same
α		0.25	0.08

reasonable guesses for the parameters χ_{1n} and χ_{2n} as indicated in Table I, we get the theoretical levels shown in Fig. 2. The high-spin states which are predicted in the model have not yet been seen experimentally. If we apply the same parameters to ^{67}Ni except for a 2^+ core energy of 1.42 MeV, we get the predicted levels shown in Fig. 2. It will be interesting to see if a low-lying $\frac{13}{2}^+$ state is eventually found in either ^{69}Zn or ^{67}Ni.

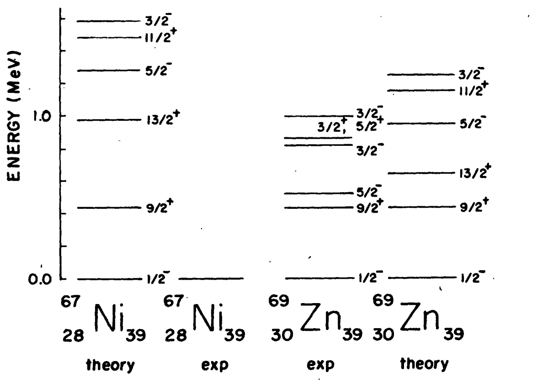

FIG. 2. Energy levels in ^{67}Ni and ^{69}Zn. Columns 2 and 3 are the experimental values from Refs. 8 and 9, respectively. Columns 1 and 4 are the core-coupling calculations.

In order to fit states in ^{68}Cu, only a few adjustments were needed in the parameters. This had also been the case with ^{90}Y.[2] The ^{68}Cu parameters used are given in Table I. Figure 3 shows the ^{68}Cu experimental levels,[4] the ^{68}Zn$(t, ^3$He$)^{68}$Cu cross section at 30° (Ref. 5) and the core-coupling calculation.

The splitting of the ground state doublet in ^{68}Cu was reproduced by keeping V_0 at −40 MeV fm^3 as in ^{90}Y and then adjusting the spin-spin interaction parameter α to 0.08. The ordering of the 3^-–6^- quartet then became 6^-, 3^-, 4^-, 5^- with the 4^- and 5^- states very close together. The single-particle energy for the $g_{9/2}$ neutron was adjusted from 0.53 to 0.43 MeV to put the 6^- calculated state at the correct energy. This corresponded to a downward shift in the proton $g_{9/2}$ single-particle energy which was necessary when fitting ^{90}Y.[2]

An examination of Fig. 3 leads one to speculate that possibly the strongly excited $(t, ^3$He) level just above the 6^- state is the 3^- state predicted from the calculation. The next very strongly excited $(t, ^3$He) state at 0.950 MeV may be a $4^-, 5^-$ doublet which has not been resolved. The state at 0.606 MeV which has previously been assigned as (3^-) is found to be a 3^+ state in the core-coupling calculation. This state is formed by coupling the 2^+ core state to the ground state doublet. Two 3^+ states are formed in this fashion. When these two states mix, one of them is pushed down and becomes the lowest positive parity state other than the ground state doublet. It should be pointed out

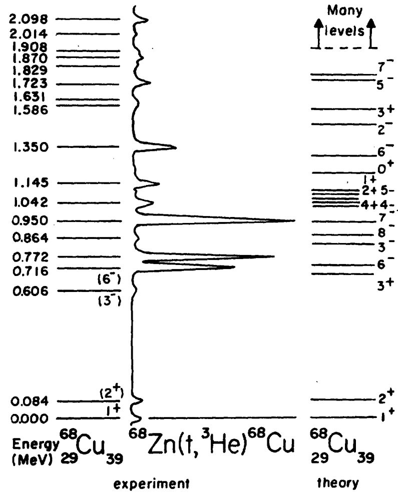

FIG. 3. Energy levels in ^{68}Cu. Column 1 shows the experimental levels from Refs. 3, 4, and 5. Column 2 shows the $(t, ^3$He) cross section at 30° from Ref. 5. Column 3 is the core-coupling calculation using the parameters in Table I.

that in order to get the 3^+ core-coupled state low enough in energy, the energy of the 2^+ core state had to be lowered to 1.12 MeV. Again, this was similar to the ^{90}Y calculation where the 2^+ core energy was lowered.[2] The energy of 1.12 MeV is not much different from the energy of 1.08 MeV found for ^{68}Zn.

The 4^+ level formed by coupling the 2^+ core state to the 2^+ first excited state also is seen at a fairly low energy. Other low energy positive parity states are a 2^+ state formed in a manner similar to the lowest 3^+ state, and the $1^+, 0^+$ doublet formed by coupling the $p_{1/2}$ excited proton state to the $p_{1/2}$ neutron ground state.

According to the core-coupling model, the high spin core-coupled states are pushed down in energy. This leads to low-lying 8^- and 7^- states calculated at about 0.90 MeV and another 6^- state calculated at about 1.3 MeV. This is not at all unreasonable since an (8^-) state has been observed in ^{68}Zn (Ref. 10) only 332 keV above the lowest 6^- state. These low-lying 8^- and 7^- states are the highest spin states which occur when coupling the 2^+ core to the 3^-–6^- quartet.

Figure 3 also shows that the core-coupling model predicts about the right number of states below 1.8 MeV. Some of the calculated states are probably only very weakly excited in the $(t, {}^3\text{He})$ experiment, or they could be weakly excited states which are buried under strongly excited states lying close by.

In summary, we think that the $(t, {}^3\text{He})$ data combined with the core-coupling calculations strongly indicate that the level at 0.606 MeV in ^{68}Cu is a 3^+ state and not (3^-) as shown on the nuclear data sheets.[4] We further believe that the ordering of the 3^-–6^- quartet is $6^-, 3^-, 4^-, 5^-$ with the $4^-, 5^-$ states very close together. We also predict states of spin $8^-, 7^-$ at about 1 MeV in ^{68}Cu.

The author would like to express appreciation to Dr. Peter Hoffmann-Pinther for his generous assistance throughout this work.

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