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Phenomenological study of lambda-nucleon interaction

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A joint fit has been made to the binding energy data of the $A = 3$-4 hypernuclei (ground and excited states) together with the Λ-p scattering data in order to deduce depth and range parameters of a phenomenological Λ-nucleon potential. Having determined the potential parameters we were able to infer the scattering lengths and effective ranges for the Λ-p and Λ-n systems.

[NUCLEAR REACTIONS Λ-nucleon potential inferred from the data; effective range parameters calculated.]

The CERN-Lyon-Warsaw collaboration[1] has recently reported the observation of two γ rays at 1.04 ± 0.04 and 1.15 ± 0.04 MeV, following K^- capture at rest in lithium targets. This measurement is consistent with the previous CERN experiment[2,3] in which a line at 1.09 MeV had been seen,[4] and a rather convincing interpretation is that these γ rays result from the formation and a subsequent γ decay of the excited hypernuclear systems ${}^4_\Lambda\mathrm{He}^*$ and ${}^4_\Lambda\mathrm{H}^*$. To be able to resolve the problem of the proper assignment of these two γ transitions, the appropriate ground states were identified[1] by detecting the ground state decay pions in coincidence with the γ's. Accordingly, the new CERN experiment has revealed that the ultimate assignment is the following: The line at 1.04 MeV corresponds to the deexcitation of ${}^4_\Lambda\mathrm{H}^*$, and that at 1.15 MeV to ${}^4_\Lambda\mathrm{He}^*$, respectively.

The measurement of the excitation energies of the $A = 4$ hypernuclei has added a very important piece of information from which further insight can be gained concerning the nature of the Λ-nucleon interaction.

Prompted by the new experimental evidence, we have analyzed the binding energy data of the $A = 3$–4 hypernuclei together with the Λ-proton scattering data in the energy region 0–20 MeV, using a simple model which employs a phenomenological Λ-nucleon potential. Following previous analyses,[5,6] this potential is chosen to be central with a hard core of radius 0.45 or 0.60 fm. In order to account for the binding energy difference for the hypernuclei ${}^4_\Lambda\mathrm{H}$ and ${}^4_\Lambda\mathrm{He}$, the potential has a charge symmetry breaking (CSB) component built in. The latter is also central but spin dependence is admitted. The shape of all components is chosen to be the same of the exponential form

$$\begin{aligned} &V(r) = \infty, \quad r < r_c \\ &V(r) = -[(U_s + \tau_3 W_s)P_s + (U_t + \tau_3 W_t)P_t] \\ &\qquad \times \exp[-\lambda(r - r_c)], \quad r > r_c \end{aligned} \tag{1}$$

where τ_3 is an isospin operator with the eigenvalues $(+1)$ and (-1) for the proton and neutron, respectively; U_s, U_t are charge symmetric (CS), and correspondingly W_s, W_t are CSB depth parameters, P_s and P_t are singlet and triplet spin projection operators. The parameter λ specifies the range of the interaction[7] (1) and for a fixed core radius r_c there are, in total, five parameters to be determined, U_s, U_t, W_s, W_t, and the intrinsic range b.

In the early analyzes,[5,6] when only the ground state hypernuclei data were available, the depth parameters in (1) were evaluated from the binding energies of the $A = 3$ and $A = 4$ hypernuclei. However, knowing three ground state binding energies only, the problem is underdetermined, and for a given shape of the potential (1), three rather than four depth parmeters might have been inferred. To make the problem soluble, arbitrary assumptions concerning the spin dependence of the CSB component were made so as to reduce the number of CSB depths to one. Accordingly, the two CS depths and one CSB depth were adjusted to the ground state binding energies of ${}^3_\Lambda\mathrm{H}$, ${}^4_\Lambda\mathrm{H}$, and ${}^4_\Lambda\mathrm{He}$. In order to provide an additional check and discriminate between the various shapes for every potential adjusted to the binding energies, the low energy Λ-p cross section was calculated and compared with experiment. As a quantitative measure of the goodness of fit the usual χ^2 criterion was adopted and potentials which resulted in large values of χ^2 were rejected from further consideration.

The observation of the 1.09 MeV line,[2] ascribed alternatively to ${}^4_\Lambda\mathrm{H}^*$ or to ${}^4_\Lambda\mathrm{He}^*$ deexcitation, imposed an extra constraint and from the four binding energies available all four depth parameters could have been obtained for the first time. For a number of potential shapes the parameters were adjusted this way[2] and for each of them the Λ-p cross sections were calculated together with the corresponding χ^2. Since for all these shapes the binding energies could be made to fit the binding energy data, the low value of χ^2 was again regarded as the ultimate criterion of acceptability.

With the recent CERN measurement of the two excitation energies, the Λ-nucleon interaction problem is overdetermined, since there are four depth parameters to be inferred from five binding energies of the following hypernuclear species: ${}^3_\Lambda\mathrm{H}$, ${}^4_\Lambda\mathrm{H}$, ${}^4_\Lambda\mathrm{H}^*$, ${}^4_\Lambda\mathrm{He}$, and ${}^4_\Lambda\mathrm{He}^*$. Obviously, the Λ-nucleon potential (1) may still be determined by making a joint fit to all the experimental data available, which comprise both the above binding energies and the Λ-p total cross sections. As a matter of fact, such a procedure appears to be even more satisfactory in that all experimental data are treated equally. Thus, the total χ^2 to be minimized can be written as

$$\chi^2(U_s, U_t, W_s, W_t, b) = \sum_{j=1}^{5} [(B_j^{\mathrm{exp}} - B_j^{\mathrm{cal}})/\Delta B_j]^2 + \sum_{j=1}^{12} [(\sigma_j^{\mathrm{exp}} - \sigma_j^{\mathrm{cal}})/\Delta\sigma_j]^2 , \quad (2)$$

where B_j are the binding energies, σ_j are the Λ-p cross sections, the superscipts exp and cal denote experimental and calculated value, respectively, ΔB_j and $\Delta\sigma_j$ are the associated experimental errors, and finally, the summation index j runs over the experimental data points considered. [In the first sum in (2) we took 5 binding energies[8] and the second sum accounts for 12 data points taken from Refs. 9 and 10.] The parameters U_s, U_t, W_s, W_t, and b in (2) are regarded as variables and the B_j^{cal} and σ_j^{cal} quantities are to be understood as functions of those variables. It should also be stressed that although we allow b to vary, the hard core radius must be fixed, i.e., we may adjust only the shape of the potential outside the hard core.

In principle, it is a very difficult task to obtain B_j^{cal} as a function of the parameters specifying the ΛN potential since in each case either a three-body or a four-body bound state problem has to be solved. It should also be noted that in order to find a minimum of χ^2 which is a function of five variables, the function has to be evaluated many times so that at first sight the problem does not appear to be numerically feasible at all. Fortunately, one can get away from solving the inherent many-body problem by relying instead on interpolating formulas derived by extensive variational calculations.[5,6] These formulas relate the lambda binding energy with the spin averaged effective well depth for the given hypernucleus. The latter depth, denoted hereafter as V_j, is a linear combination of the parameters U_s, U_t, W_s, W_t and can be easily derived for all hypernuclear species under consideration. The different effective depths V_j are in turn related to B_j^{cal} as follows:

$$V_j = a_j(b) + b_j(b)(B_j^{\mathrm{cal}})^{1/2} + c_j(b)B_j^{\mathrm{cal}} , \quad (3)$$

where the coefficients a_j, b_j, and c_j have been tabulated in Refs. 5 and 6 for several values of b so that by interpolation one can obtain the functional dependence on b. The last step is to solve Eq. (3) with respect B_j^{cal} and insert this quantity in (2). In practice this completes the evaluation of χ^2, because the calculation of σ_j^{cal} is straightforward, of course.

For the hypernuclei ${}^4_\Lambda\mathrm{He}$ and ${}^4_\Lambda\mathrm{He}^*$ the theoretical binding energies that enter formula (3) have to be corrected by adding the appropriate Coulomb energies, i.e., the amount by which the Coulomb energy of the ${}^3\mathrm{He}$ core nucleus is going to be changed as a result of attaching a Λ particle to it. However, there seems to be a controversy in the literature concerning this point. The value 200–250 keV, obtained in Refs. 5 and 6 on the basis of a variational calculation, is an order of magnitude larger than the estimate given in Ref. 11. The argument which is put forward in Ref. 11 is that by attaching a neutron to ${}^3\mathrm{He}$ the appropriate Coulomb energy will be changed by about 70 keV (by comparing the Coulomb energies of ${}^4\mathrm{He}$ and ${}^3\mathrm{He}$). Since the neutron binding energy in ${}^4\mathrm{He}$ is roughly 10 times larger than the binding energy of Λ in ${}^4_\Lambda\mathrm{He}$, the authors[11] conclude that the Coulomb correction in ${}^4_\Lambda\mathrm{He}$ should be about 10–20 keV. In view of this apparent discrepancy, we have used two different values for the Coulomb energy, setting $E_{\mathrm{Coul}} = -200$ keV, or $E_{\mathrm{Coul}} = -20$ keV. It turns out, however, that the results obtained do not depend dramatically upon the particular choice of E_{Coul}.

We began our data fitting procedure with a four parameter search. The shape of the potential (1) was then kept fixed by setting b to a constant, and we varied only the depth parameters U_s, U_t, W_s, W_t. The potential shapes considered are specified in Table I. The potentials C, D, F, G are from Ref. 5, while the potential denoted DHT is from

TABLE I. Parameters of the Λ-nucleon potential (1).

Potential type	b (fm)	r_C (fm)	λ^{-1} (fm)
C	1.5	0.45	0.169 434
DHT	2.0	0.45	0.310 655
X	1.502	0.45	0.170 058
D	1.5	0.60	0.084 717
F	2.0	0.60	0.225 887
G	2.5	0.60	0.367 107
Y	1.5	0.60	0.084 674

TABLE II. The χ^2 values for the various potentials for two different values of the Coulomb energy.

Potential type	χ^2 $E_{\text{Coul}}=-200$ keV	$E_{\text{Coul}}=-20$ keV
C	5.22	11.4
DHT	14.16	17.7
X	4.54	4.83
D	5.37	8.06
F	37.5	27.9
G	117.1	97.9
Y	5.30	5.86

TABLE III. Optimal set of parameters for the different potentials ($E_{\text{Coul}}=-200$ keV).

Potential type	U_s (MeV)	U_t (MeV)	W_s (MeV)	W_t (MeV)
C	1635	1492	38.5	32.9
DHT	456.4	390.8	15.6	15.4
X	1629	1487	35.4	31.9
D	7123	6836	70.3	63.7
F	937.7	839.2	16.0	21.6
G	344.0	296.3	5.5	11.1
Y	7126	6838	72.2	64.4

Ref. 6, and their shapes have been fixed once and for all. By contrast, the potentials designated as X and Y, have variable intrinsic range b but fixed hard core radii equal 0.45 and 0.60 fm, respectively, and their geometric parameters (depths and ranges) have been inferred from a five parameter fit. In practice, after completing a four parameter search, we tried to improve the fit by allowing b to vary (for fixed r_c) using in the corresponding five parameter fit the optimal set of values from the four parameter fit as starting values in the appropriate minimizing routine. In Table II we have presented the corresponding χ^2 values. As seen from Tables I and II, the variation of b brings only a minor improvement to the fit, which for the potentials C, D, X, and Y is rather good ($\chi^2=4$ to 5 for 17 data points). Thus, the potentials with $b\approx 1.5$ fm are definitely favored.[12] In Table III we have envisaged the optimal sets of the depth parameters. The CSB part always has the same sign as the CS part and constitutes only a small correction about 1 to 3% of the CS component, in accordance with the electromagnetic origin of the CSB potential.[13] In Table IV we compare the predicted binding energies with experiment. As seen from Table IV, the potentials with $b\gtrsim 2$ fm invariably give rise to an overbound $^3_\Lambda$H system, with the trend to aggravate further the discrepancy for increasing b values. It appears that the potentials with b values around 1.5 fm afford the best chance of yielding a satisfactory representation of the binding energy data. This conclusion receives further support when one compares the Λ-p total cross sections $\sigma_{\Lambda p}$ with experiment. Table V contains the calculated values of $\sigma_{\Lambda p}$ together with the experimental data from Ref 9. Potentials DHT, F, G with $b\gtrsim 2$ fm give too rapid falloff of the cross section regarded as a function of laboratory lambda momentum p_Λ, and the larger the b value is, the steeper is the cross section. It is interesting to note that the $\sigma_{\Lambda p}$ values obtained by using the potentials C, D, X, and Y are all very close to each other and the calculated values agree with the data quite well.

In Table VI we have presented the effective range parameters evaluated from the potentials under consideration. Again, the values of these parameters for the potentials C, D, X, and Y are remarkably close to each other. The potentials with $b\gtrsim 2$ fm yield considerably larger values for the scattering lengths, and in consequence, the cross section becomes too steep, as noticed before. The dependence of the effective range parameters on the particular choice of E_{Coul} does not appear to be essential. For illustration we have appended in Table VI the corresponding values of the effective range parameters evaluated under the assumption that $E_{\text{Coul}}=-20$ keV. The corresponding entries in

TABLE IV. Predicted binding energies for the $A=3$–4 hypernuclei ($E_{\text{Coul}}=-200$ keV). All entries are in MeV.

Hypernucleus	C	DHT	X	D	F	G	Y	Experiment (Refs. 1 and 8)
$^3_\Lambda$H ($J=\frac{1}{2}$)	0.10	0.25	0.10	0.08	0.35	0.50	0.08	0.13 ± 0.05
$^4_\Lambda$H ($J=0$)	2.04	2.02	2.04	2.05	1.98	1.92	2.05	2.04 ± 0.04
$^4_\Lambda$H ($J=1$)	0.97	1.00	0.99	0.99	1.09	1.16	0.98	1.00 ± 0.05
$^4_\Lambda$He ($J=0$)	2.39	2.38	2.39	2.39	2.36	2.32	2.39	2.39 ± 0.03
$^4_\Lambda$He ($J=1$)	1.24	1.25	1.23	1.22	1.22	1.23	1.23	1.24 ± 0.04

TABLE V. Predicted Λ-p cross section ($E_{\text{Coul}} = -200$ keV). All entries are in mb.

p_Λ (MeV/c)	C	DHT	X	D	F	G	Y	Experiment (Ref. 9)
145	176	212	183	184	237	278	182	180 ± 22
185	132	143	137	138	158	168	137	130 ± 17
210	110	112	114	115	123	122	114	118 ± 16
230	95	92	98	99	100	94	98	101 ± 12
250	82	75	84	85	82	72	84	83 ± 9
290	61	50	62	63	56	42	62	57 ± 9

Table VI have been put in parentheses.

The results presented in Table III and VI exclude, of course, the possibility that the potential (1) might be capable of supporting a Λ-nucleon bound state. Furthermore, the dominant attractive CS part is considerably weaker in the triplet state in comparison with the singlet state, so that the deduced triplet potential is in all cases too weak to yield a particle stable excited state of ${}^3_\Lambda$H with $J=\frac{3}{2}$. Finally, it should, perhaps, be mentioned that the overbinding problem[14] of ${}^5_\Lambda$He has not been alleviated; the calculated binding energies for this hypernucleus come up roughly by about 2 MeV larger than the experimental value.

The Λ-p scattering data of Refs. 9 and 10 also give some information on the angular distribution. Up to about 10 MeV, these distributions are essentially isotropic and as the c.m. energy increases from 10 to 20 MeV a small forward-backward asymmetry is seen. Obviously the forward-to-backward ratio F/B depends on the interaction in states with relative Λ-p momenta greater than zero, but assuming that the Λ-p potential (1) holds not only for $l = 0$ but is adequate for all partial waves, F/B can be readily calculated. The potentials with $b \approx 2$ fm do not yield agreement with experiment because large intrinsic ranges give rise to significant p-wave contribution in the differential cross section and the calculated F/B ratio exhibits a too rapid increase with energy than would be necessary to explain the data. This is a rather well known difficulty which in the earlier analyses[5,6] was remedied by introducing an extra parameter x reducing the strength of the interaction in the odd parity states relative to that in the even parity states. To bring the F/B ratio into agreement with the data it was necessary to reduce by 60% the p-wave interaction. Since the best fit potentials inferred in this work have all $b \approx 1.5$ fm, the above difficulty does not occur and the potentials C, D, X, and Y all yield a rather weak p-wave scattering so that the resulting F/B ratio is a slowly increasing function of the c.m. energy. For the potentials D and Y (with $r_c = 0.6$ fm) F/B is almost constant in the interval 0–20 MeV, indicating a too weak attraction in the p states. However, for the potentials C and X (with $r_c = 0.45$ fm) the predicted F/B ratio is slowly rising from 1.05 at $E = 10$ MeV to 1.13 at $E = 20$ MeV. The latter behavior is roughly consistent with the crude experimental data and no additional adjustment of the p-state interaction seems to be required.

TABLE VI. Predicted Λ-p effective range parameters. The values in parentheses have been calculated assuming $E_{\text{Coul}} = -20$ keV; the remaining values are obtained for $E_{\text{Coul}} = -200$ keV.

Potential type	$-a_s$ (fm)	r_{0s} (fm)	$-a_t$ (fm)	r_{0t} (fm)
C	2.51	2.02	1.26	2.62
	(2.39)	(2.04)	(1.18)	(2.70)
DHT	3.39	2.70	1.58	3.64
	(3.23)	(2.74)	(1.47)	(3.78)
X	2.57	2.01	1.30	2.59
	(2.58)	(2.01)	(1.30)	(2.60)
D	2.54	1.99	1.31	2.53
	(2.42)	(2.02)	(1.23)	(2.61)
F	3.77	2.60	1.71	3.45
	(3.68)	(2.62)	(1.58)	(3.59)
G	4.98	3.23	2.33	4.19
	(4.78)	(3.26)	(2.13)	(4.36)
Y	2.52	2.00	1.30	2.54
	(2.53)	(2.00)	(1.30)	(2.55)

Summarizing, we have made a joint fit to the binding energy data of the $A = 3$–4 hypernuclei (ground and excited states) and the Λ-p scattering data using a number of effective, central Λ-nucleon potentials of various geometrical shapes (hard core radii,[15] $r_c = 0.45$ fm and $r_c = 0.60$ fm, and intrinsic ranges $b = 1.5$–2.5 fm) whose ranges and depths were regarded as adjustable parameters. We have determined four potentials (C, D, X, and Y), all of which yield a very good fit to the data, and the calculated binding energies and cross sections have been found in all cases to lie within the limits of the experimental errors. These potentials have very similar features except for different hard core radii. Thus, in all cases the intrinsic range is close to 1.5 fm,[16] there is a pronounced spin dependence in the CS component and a small degree of spin dependence in the CSB component. Since the CSB correction in (1) is of the same sign (cf. Table III) as the dominant CS part, the Λ-n potential will always be slightly weaker than the Λ-p potential, for both singlet and triplet interaction. The resulting effective range parameters calculated from potentials C, D, X, and Y are remarkably close to each other with the following average values:

Λ-proton: $a_s = -2.54$ fm, $r_{0s} = 2.0$ fm,
$a_t = -1.29$ fm, $r_{0t} = 2.57$ fm;
Λ-neutron: $a_s = -1.79$ fm, $r_{0s} = 2.34$ fm,
$a_t = -1.0$ fm, $r_{0t} = 2.94$ fm.

Concluding, we would like to emphasize that these parameters have been deduced under the simplifying assumption that formula (1) provides adequate representation of the Λ-nucleon interaction. Admittedly, there would be a number of effects which have not been accounted for in (1) and which might influence our results. To mention but a few, the range of the potential (1) does not have to be the same in all four components; there may be a noncentral contribution in (1), a coupling to the Σ channel, three body ΛNN forces, etc. All these extensions, however, require formidable computational work to be done.

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[15] We have also considered potentials with smaller hard core radii $r_c = 0-0.3$ fm but the results were discouraging. The cross sections were systematically too flat and the CSB contribution was rather large, 5–10% of the CS part.

[16] It is interesting to note that the Λ-p potential (1) with $b \approx 1.5$ fm centered at origin has a range that roughly corresponds to a potential generated via a two-pion-exchange mechanism which is believed to be a major contributor to the Λ-nucleon interaction.