

Proton scattering from ^{24}Mg at 0.8 GeV

G. Blanpied

New Mexico State University, Las Cruces, New Mexico 88003

N. M. Hintz, G. S. Kyle, and J. W. Palm

University of Minnesota, Minneapolis, Minnesota 55455

R. Liljestrand,* M. Barlett, and C. Harvey

University of Texas, Austin, Texas 78712

G. W. Hoffmann

University of Texas, Austin, Texas 78712

and Los Alamos Scientific Laboratory, Los Alamos, New Mexico 87545

L. Ray and D. G. Madland

Los Alamos Scientific Laboratory, Los Alamos, New Mexico 87545

(Received 22 February 1979)

Angular distributions for the elastic and inelastic scattering of 800 MeV protons from ^{24}Mg are presented. Results of distorted-wave Born-approximation (DWBA) and coupled-channels (CC) analyses of the data for the 0_1^+ , 2_1^+ , 4_1^+ , and 6_1^+ members of the ground state rotational band, the 2_2^+ , 3_2^+ , 4_2^+ , and 5_2^+ members of the γ band and the 0^+ member of the β band are discussed, and the effects of ground state deformation and multistep contributions are assessed. The DWBA calculations for the 4_1^+ and 6_1^+ angular distributions fail to even qualitatively reproduce the data, while a CC calculation, using a symmetric rotator model with quadrupole and hexadecapole ground state deformations, provides good fits for the 0_1^+ , 2_1^+ , and 4_1^+ angular distributions. A CC calculation in which the ground band is coupled to the γ band, with an assumed asymmetric vibrational amplitude, gives a fair fit to the data for the 2_2^+ state, but fails to fit the 3_2^+ , 4_2^+ , and 5_2^+ data. This failure in the calculation may be due to the lack of a direct step to the 4_2^+ state, as well as neglect of the spin-flip process. The result of coupling the ground band with the 6.43 MeV 0_β^+ state indicates that this axially symmetric deformed vibrational model may be essentially correct, but also seems to indicate that a direct $0_1^+ \rightarrow 0_\beta^+$ step needs to be considered, based on the behavior of the forward angle 0_β^+ data. Additional coupling of a 2_β^+ state does not dramatically improve the fit to the 0_β^+ data. The results obtained here from analyses of the 800 MeV data are compared with those obtained through analyses of lower energy data.

NUCLEAR REACTIONS $^{24}\text{Mg}(p, p')$ $E = 0.8$ GeV, measured $\sigma(\theta)$; enriched target; resolution ≥ 80 keV, $\theta_{\text{cm}} = 8^\circ$ to 32° , $\Delta\theta = 0.1^\circ$. Optical model potential, DWBA analysis, coupled-channels analysis, symmetric and asymmetric rotator model, coupling parameters.

I. INTRODUCTION

Inelastic scattering experiments, using beams of protons, neutrons, deuterons, α 's, electrons, and pions, have provided considerable information concerning the excited state structure of ^{24}Mg .¹⁻¹⁴ In general, coupled-channels (CC) analyses which were made for some of the low energy data, and which used the symmetric rotator or the deformed-vibrator model, were able to reproduce the magnitudes of the various 4.12 MeV 4^+ angular distributions, but failed to even qualitatively fit the shapes. On the other hand, a CC analysis of recent 800 MeV $p + ^{12}\text{C}$ inelastic data has shown¹⁵ that at medium energies the angular distributions for the 2^+ and 4^+ members of the ground state ro-

tational band are very sensitive to the ground state quadrupole and hexadecapole deformations; in addition, excellent fits to both the shapes and magnitudes were obtained. One would like to know if this success is singular, or if it is a general feature of the CC description of medium energy inelastic proton scattering from deformed nuclei.

Reported here are new data for 800 MeV $p + ^{24}\text{Mg}$ elastic and inelastic scattering and the results of theoretical analyses aimed at answering the question posed in the above paragraph. The experimental data consist of angular distributions for the (1.37, 2_1^+), (4.12, 4_1^+), (8.11, 6_1^+) members of the ground band, the (4.24, 2_2^+), (5.24, 3_2^+), (6.01, 4_2^+), (7.81, 5_2^+) members of the γ band, and the (6.43, 0_β^+) member of the β band. As shown below,

the CC analysis of the data provides further evidence of the great sensitivity of the medium energy inelastic data to the ground state deformation of the target nucleus. As for the case of 800 MeV proton scattering from ^{12}C , an excellent description of the angular distribution for the first 4^+ state in ^{24}Mg is obtained.

II. EXPERIMENTAL METHOD

The data were obtained using the high resolution spectrometer (HRS) facility of the Los Alamos Clinton P. Anderson Meson Physics Facility (LAMPF). The HRS has been described elsewhere.¹⁶ The target consisted of a 19.58 mg/cm² ^{24}Mg foil, enriched to 99.94%. The overall experimental energy resolution was typically 80–110 keV for full HRS acceptance. The spectrum obtained at $\theta_{\text{lab}} = 29^\circ$ is shown in Fig. 1, and the states of interest in this paper are indicated. Although at this angle excitation of the 4.24, 2^+ state is seen to be weak compared to the 4.12, 4^+ state, at more forward angles where the strengths of these two states are comparable, the ~ 100 keV resolution allowed extraction of both angular distributions.

Data were also obtained for $p + ^{12}\text{C}$ elastic and inelastic scattering, and the ^{24}Mg angular distributions were normalized relative to the ^{12}C results which in turn were normalized to the ^{12}C data reported in Refs. 15 and 16. The absolute normalization of the ^{24}Mg data is accurate to $\pm 15\%$. The angle calibration was determined to $\pm 0.1^\circ$ by comparing the new ^{12}C data with that reported in Refs. 15–16. The 800 MeV $p + ^{24}\text{Mg}$ experimental angular distributions, along with theoretical curves to be discussed below, are presented in Figs. 2–6.

III. DWBA ANALYSIS

The elastic angular distribution was fit using the optical model with a spherical Woods-Saxon potential given in the low-energy notation¹⁷ V , W , W_D , r , a , r_w , a_w , r_D , a_D , and r_C by -7.0 , 84.7 , and 32.3 MeV and 0.968 , 0.539 , 0.915 , 0.677 , 0.441 , 0.478 , and 1.05 fm, respectively. The fit is shown as the solid curve in Fig. 2. The spin-orbit potential was neglected since in DWBA analyses of 800 MeV proton inelastic angular distributions for low-lying natural parity states, it is known that equivalent predictions of inelastic angular distributions result when spin-orbit terms are omitted or fully included (both in the diagonal and nondiagonal parts of the optical potential, i.e., a deformed spin-orbit potential is included), provided that equivalent fits to the elastic angular distributions are obtained.¹⁸ Since the CC code used for the present calculations (JUPITER¹⁹) does not include provisions for spin-

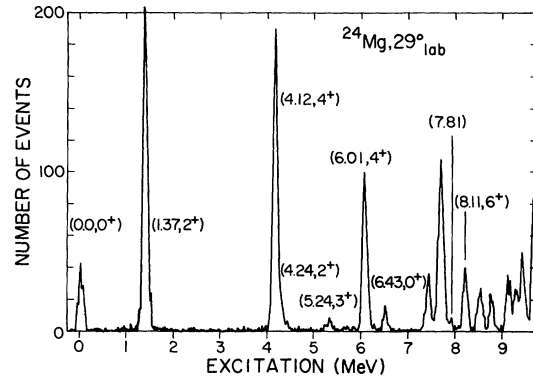


FIG. 1. The $\theta_L = 29^\circ$ spectrum for $^{24}\text{Mg}(p, p')$ at 800 MeV.

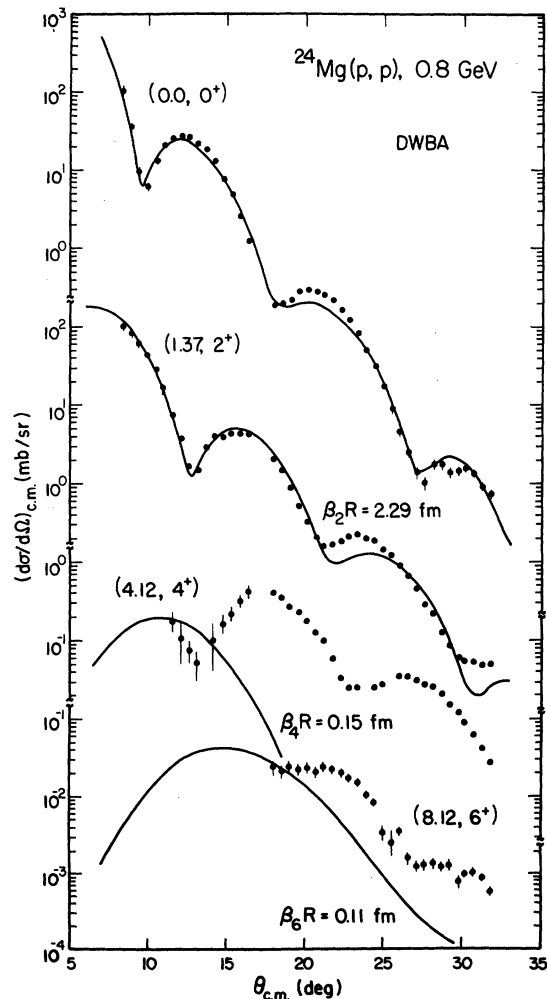


FIG. 2. Optical model and DWBA results for the ^{24}Mg ground state rotational band (solid curves) are compared with the data.

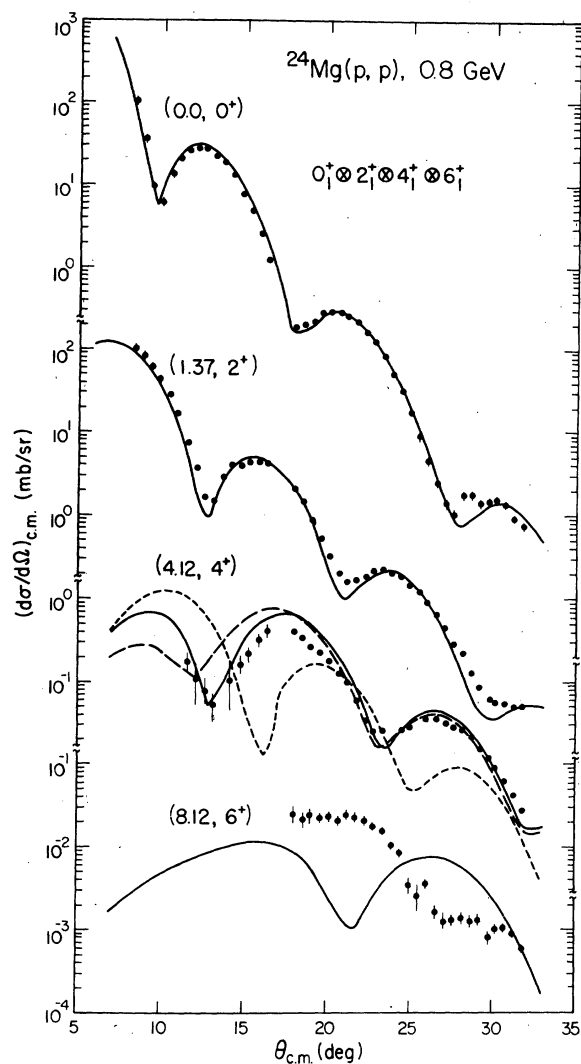


FIG. 3. Shown as solid curves are CC predictions using the deformed symmetric rotator collective model, and coupling the 0_1^+ , 2_1^+ , 4_1^+ , and 6_1^+ states, with $\beta_2 R = +1.61$ fm and $\beta_4 R = -0.05$ fm. The long-dash curve for the 4^+ state results when $\beta_4 R = -0.15$ fm, while the short-dash curve for the 4^+ state is the result when the coupling to the 2^+ state is omitted.

orbit deformation, and since one of the purposes of the present study is to compare DWBA and CC predictions, the spin-orbit potential was therefore omitted from the DWBA analysis. Thus for the 3_2^+ and 5_2^+ angular distributions (unnatural parity states) no direct spin-flip mechanism is allowed in the calculations discussed here. Such a refinement in the analysis will be deferred to a later time.

DWBA calculations were done using a version of the program VENUS,²⁰⁻²¹ modified to include rela-

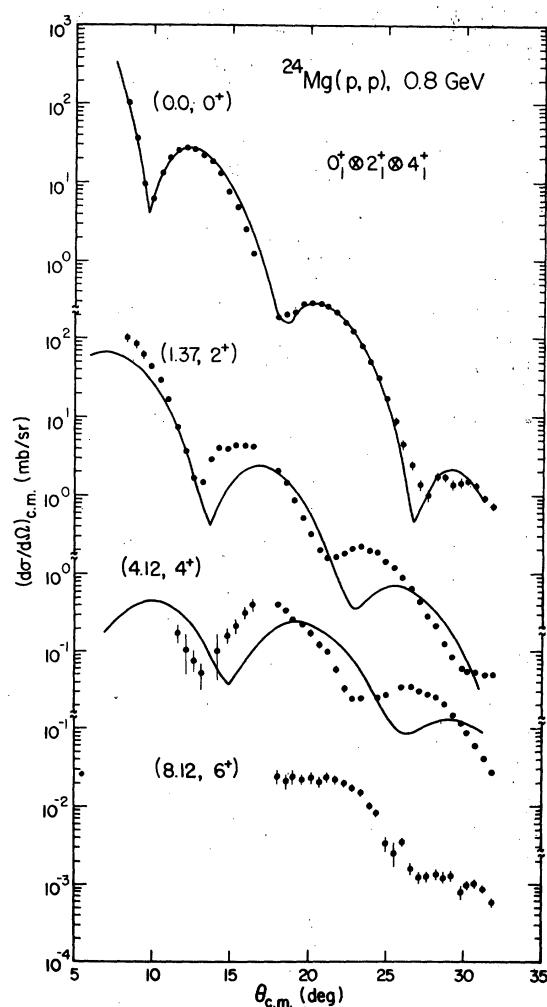


FIG. 4. The results for coupling the 0_1^+ , 2_1^+ , and 4_1^+ states, with $\beta_2 R = -1.61$ fm and $\beta_4 R = -0.05$ fm.

tivistic kinematics,²⁰ and the optical potential obtained from the analysis of the elastic data. The results for the $(1,37, 2_1^+)$, $(4,12, 4_1^+)$, and $(8,11, 6_1^+)$ angular distributions are shown in Fig. 2. A good fit to the 2_1^+ data is obtained with $|\beta_2 R| = 2.29$ fm, where the length R is given by $r_0 A^{1/3}$. However, poor fits are obtained for the 4_1^+ and 6_1^+ data. The failure of the DWBA calculation to reproduce the shape of the 4_1^+ angular distribution is similar to the result obtained for the 4^+ state in ^{12}C .¹⁵ Since the angular distribution for the 4_1^+ transition in ^{24}Mg is qualitatively similar to that for the 4^+ transition in ^{12}C , multistep and deformation effects, found to be important for ^{12}C , are also expected to be important for ^{24}Mg . This expectation led to the CC calculations discussed below.

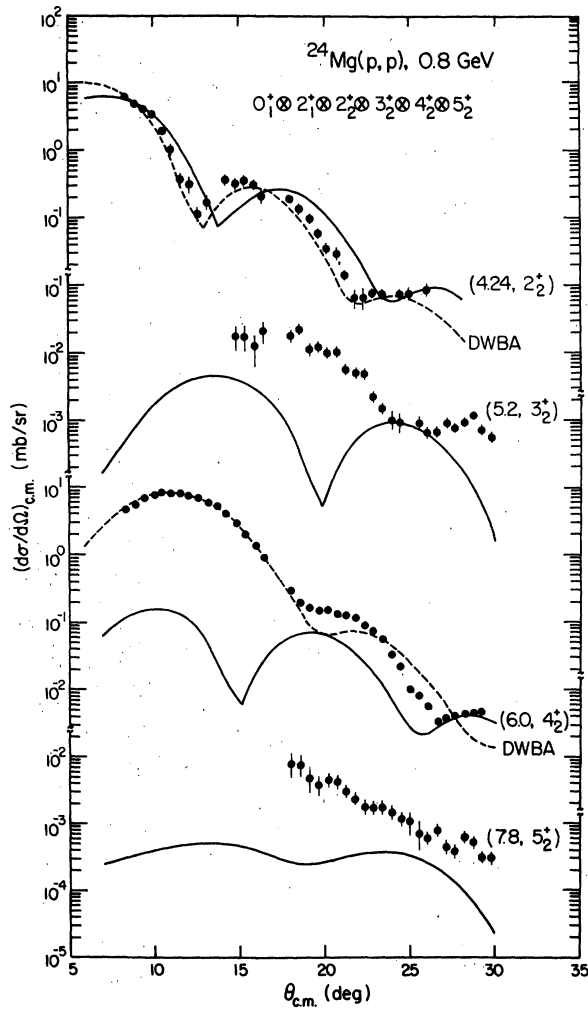


FIG. 5. The solid curves result from a CC calculation which couples the 0_1^+ , 2_1^+ , 2_2^+ , 3_2^+ , 4_2^+ , and 5_2^+ states and uses an asymmetry parameter of $\gamma = 20^\circ$. The dashed curves for the 2_2^+ and 4_2^+ states are the results of DWBA calculations with $\beta_2 R = 0.54$ fm and $\beta_4 R = 0.96$ fm.

IV. COUPLED-CHANNELS RESULTS

The calculations reported in this section were done using a corrected version of the program JUPITER,¹⁹ modified to include relativistic kinematics. As in Ref. 15, the deformation of the potential shape was treated using the Legendre-polynomial-expansion procedure described by Tamura.¹⁹ The axially symmetric collective rotational model with quadrupole and hexadecapole deformations was used to describe the ground state band. The γ band and β band states were assumed to correspond to γ vibrations in which the nucleus retains the same equilibrium, spheroidal deformation, but in addition oscillates such that ellipsoidal

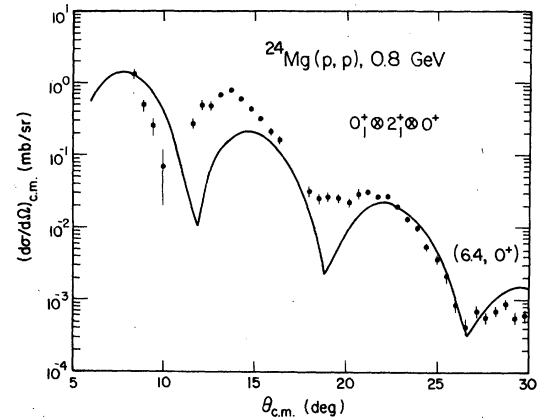


FIG. 6. The angular distribution for excitation of the $(6, 4, 0_2^+)$ state is compared to a CC prediction which couples the 0_1^+ , 2_1^+ , and 0_2^+ states. The coupling parameter for the β band is $|\beta - \beta_2| \times R = 0.4$ fm.

shapes are produced^{9, 22} ($K = 2$ band) and β vibrations in which the nucleus oscillates about a given equilibrium deformation, always retaining its axial symmetry ($K = 0$ band).

A. Ground-state rotational band

The 0^+ , 2^+ , 4^+ , and 6^+ states were treated as members of the ground state, $K = 0$ rotational band, and coupling between all four channels (except for direct $0^+ - 6^+$ coupling, due to code limitations) was assumed. The optical potential and the ground state quadrupole and hexadecapole deformations were adjusted to optimize simultaneously the fits to the 0^+ , 2^+ , and 4^+ angular distributions. Although an angular distribution for the 6^+ transition was predicted, the 6^+ data were not considered in the optimization procedure since no direct $0^+ - 6^+$ transition could be considered. The best fit is given by the solid curves in Fig. 3. The potential parameters, in the same notation as before, are $-5.3, 100.0, 16.0$ MeV, $0.929, 0.453, 0.929, 0.545, 0.446, 0.397$, and 1.05 fm, while the deformation lengths are $\beta_2 R = +1.61$ fm (prolate) and $\beta_4 R = -0.05$ fm. These deformation lengths compare favorably with results of CC analyses of ^{24}Mg inelastic data obtained using various probes at lower energies, as seen in Table I. Deformation lengths, $\beta_i R$, rather than deformation parameters, β_i , are compared in Table I since several previous analyses¹⁵⁻¹⁸ of inelastic proton scattering data obtained at $E_p \sim 1$ GeV have demonstrated that the deformation length is more compatible with low energy results than the deformation parameter. Note also that the deformation length of the predominant part of the complex optical potential is tabulated in Table I, although in general a weighted

TABLE I. Deformation parameters used in coupled channels analyses with the symmetric rotator model, and with the asymmetric rotator model in which the ground band is coupled to the γ band.

$\beta_2 R$ (fm)	$\beta_4 R$ (fm)	$\beta_6 R$ (fm)	γ (deg)	$\beta_2 \gamma$ (rad)	Reaction reference (MeV)
1.68					e, e' 187 13
1.65	-0.22				e, e' 183-250 13
1.35					d, d' 26 8
1.73			36	0.21	α, α' 29 9
1.26			24	0.13	α, α' 42 10
1.36	-0.05				α, α' 104 11
1.56	-0.19	+0.18	22	0.18	p, p' 20 6
1.74 \pm 0.10			21 \pm 1	0.19 \pm 0.02	p, p' 23-29 4
1.65	-0.18 \pm 0.29				p, p' 25 2
1.72			23	0.20	p, p' 30 3
1.68	-0.11				p, p' 40 5
1.58 (DWBA)			32	0.21	p, p' 50 1
1.61	-0.05		20	0.21	p, p' 800 this work

average of the real and imaginary components of the transition potential should be given. Such a refinement would be insignificant for the optical potentials employed in the analyses referred to in Table I.

At 800 MeV, the fit to the elastic data is primarily influenced by the imaginary volume potential although the "surface" derivative imaginary potential was found to be necessary in both the CC and DWBA calculations to help fill in the second minimum near 18° . The fits shown in Fig. 3, to the 2^+ and 4^+ angular distributions are good, while the fit to the 6^+ data is poor, suggesting the possible importance of the direct $0^+ \rightarrow 6^+$ transition. The coupling of the 6^+ state in this calculation only influences the calculated angular distribution of the 4^+ state past about 25° .

The sensitivity of the calculation to both the signs and magnitudes of the deformation parameters is striking. For example, repeating the above calculation with $\beta_4 R = -0.15$ fm leads to the predicted 4^+ angular distribution given by the long-dash curve in Fig. 3, while the predicted 0^+ and 2^+ angular distributions remain essentially unchanged. The sensitivity of the CC prediction to the sign of the quadrupole deformation is demonstrated in Fig. 4 where the results of a calculation with $\beta_2 R = -1.61$ fm (oblate deformation) are indicated by the solid curves (here, $W = 90$ MeV to restore the fit to the elastic data). Notice in Fig. 4 that the predicted cross sections for the 2^+ and 4^+ states miss the data even at the first minima and become increasingly out of phase with increasing angle. Thus, the choice of a prolate deformed shape for ^{24}Mg is required by the present analysis.

In order to study the importance of multistep ex-

citation of the 4^+ state, the CC calculation was repeated using the deformation lengths corresponding to the solid curve in Fig. 3, but omitting the coupling of the ground and 4^+ channels to the 2^+ channel. The 4^+ channel is thus reached from the entrance channel directly via the $\beta_4 Y_4$ term in the coupling potential and "indirectly" via the $(\beta_2 Y_2)^2$ term that appears in the Legendre polynomial expansion of the deformed potential.¹⁹ The omission of the 2^+ channel requires a reduction in W of about 10% in order to recover the fit to the elastic cross section. The result of this calculation for the 4^+ angular distribution is given by the short-dash curve in Fig. 3. By comparing the DWBA prediction in Fig. 2 and the solid and short-dash curves in Fig. 3 for the 4^+ angular distributions, it is seen that while deformation alone is sufficient to produce maxima and minima structure which is qualitatively similar to the data, coupling to the 2^+ channel is required in order to produce the correct magnitudes and angular positions of the maxima and minima observed experimentally.

Future modifications to JUPITER are planned which will allow a $0^+ \rightarrow 6^+$ coupling term as well as nonzero β_6 and β_8 deformations. This may improve the agreement of the calculation with the data for the 6^+ state and could slightly alter the choice of the best value of β_4 from that found here.

B. γ band

For these calculations the $(4.24, 2_2^+)$, $(5.24, 3_2^+)$, $(6.01, 4_2^+)$, and $(7.81, 5_2^+)$ states were considered as members of the γ band. Previous CC calculations have included the 2^+ and 3^+ states,^{3,9-10} or the 2^+ , 3^+ , and 4^+ states,^{4,6} but not the 5^+ state. In a

recent tabulation, Endt and Van der Leun²³ assign $J^\pi = 3^+, 4^-, 5^+$ to the 7.81 MeV state, and for the calculation reported here, $J^\pi = 5^+$ was assumed.

The calculation is similar to one reported by Tamura⁹ for α inelastic scattering, with the addition here of the β_4 deformation of the ground state potential, and the coupling of the 4_2^+ and 5_2^+ states to the 0_1^+ , 2_1^+ , 2_2^+ , and 3_2^+ coupling scheme. A calculation coupling the 0_1^+ , 2_1^+ , 4_1^+ , 2_2^+ , 3_2^+ , and 4_2^+ states demonstrated that the inclusion of the 4_1^+ state had no effect on the predicted angular distributions (out to 30°) for states in the γ band. The parameters for the calculation were those corresponding to the solid curves in Fig. 3, while the product $\beta_2\gamma = 0.21$ ($\gamma = 20^\circ$) was chosen to reproduce the magnitude of the first maximum of the (4_2^+ , 2_2^+) cross section. The calculated angular distributions for the 0_1^+ and 2_1^+ states are unchanged from the solid curves of Fig. 3, while those for the states in the γ band are given by the solid curves in Fig. 5. As seen in Table I, the value of $\beta_2\gamma$ agrees well with that used in previous calculations. Hartree-Fock calculations predict that $\gamma = 18^\circ$ for ^{24}Mg .²⁴

The fit to the 2_2^+ angular distribution is only fair; the calculation misses the position of the first minimum by about a degree and is generally slightly out of phase with the data. A DWBA calculation using $|\beta_2 R| = 0.54$ fm produced the dashed curve shown in Fig. 5, and is seen to be in better agreement with the data. This value compares favorably with values obtained through previous DWBA analyses (0.53–0.59 fm).^{5-6,12}

The CC predictions for the 3_2^+ , 4_2^+ , and 5_2^+ angular distributions in general do not reproduce the magnitudes or shapes of the data. The 3_2^+ prediction is a factor of 5 lower than the data at forward angles, while that for the 4_2^+ is a factor of 50 too low. Since the 3_2^+ and 5_2^+ states are only excited by two-step processes in the present calculation, the coupling to the 4_2^+ is very important. If the 4_2^+ forward angle cross section could be increased, the cross sections for the 3_2^+ and 5_2^+ states would also increase. Thus, a direct step from the ground state to the 4_2^+ state may be required to fit the data. (At present, JUPITER has no provisions for this step.) Also, calculations which include a direct $0^+ \rightarrow 3^+$, 5^+ coupling via a spin-flip ($S=1$) process should be included before drawing final conclusions regarding the success or failure of the deformed-vibrational model in describing these data.

Concerning the direct $0^+ \rightarrow 4_2^+$ transition, Rush and Ganguly¹ showed that a DWBA calculation with $\beta_2 R = 0.68 \pm 0.10$ fm and $\beta\gamma = 0.21$ gave a reasonable fit to 50 MeV (p, p') data. The result of a DWBA calculation for the 800 MeV data is indicated by the short-dash curve in Fig. 5. This calculation re-

produces the data at forward angles, but becomes out of phase by about 20° where the magnitude of the CC calculation begins to be comparable to the data. The value of $|\beta_4 R| = 0.96$ fm used in this DWBA calculation compares favorably with the values (0.90, 0.91 fm)^{5,6} used to fit low energy proton scattering data, but disagrees with the values of 0.58–0.69 fm¹² found from fitting α scattering data. This disagreement with the α scattering result for the 4_2^+ state is interesting considering the agreement for the 2_2^+ state and is perhaps an indication of a difference in the reaction mechanism.

The differences between the CC results and the experimental data of factors of 5 for the 3_2^+ and of 50 for the 4_2^+ state are contrasted with factors of 30 (40) for the 3_2^+ and 10 (20) for the 4_2^+ for the low energy proton results given in Ref. 6 (4). The calculations are similar except Ref. 4 includes a non-zero spin-orbit term, and Ref. 6 includes a non-zero spin-orbit term with a deformation different from that of the central and imaginary potentials. When the calculation in Ref. 6 is repeated with the spin-orbit term set to zero, the predicted 3_2^+ cross section is reduced by about a factor of 2 while the other cross sections are essentially unchanged. Such a large difference in results from similar calculations, performed at low and intermediate energies, perhaps indicates a difference in the reaction mechanism, or in the importance of spin-flip processes for 20–30 MeV protons and 800 MeV protons in the inelastic excitation of these non-natural parity states.

C. β band

The possibility of describing the 0^+ state at 6.4 MeV as the head of a β -vibrational band in ^{24}Mg was investigated in a calculation coupling the ground band 0_1^+ , 2_1^+ states to the 0_2^+ state. Since a β vibration corresponds to an oscillation of the axially symmetric ground state deformation, the appropriate parameter is $|\beta - \beta_2|$.²² A value of $|\beta - \beta_2| R = 0.40$ fm ($\beta_2 R = 1.6$ fm) was found to reproduce the magnitude of the first maximum in the angular distribution for the (6.43, 0^+) state. The calculated angular distributions for the 0_1^+ and 2_1^+ states are unchanged from Fig. 3, while that for the 6.43 MeV state is shown in Fig. 6. The overall agreement with the data is fair. Additional coupling to the 2_2^+ state of the γ band in the above calculation had no effect on the predicted cross sections. As was the case for the calculations for the γ band, the theoretical predictions are in best agreement with the data beyond about 20° , after missing the position of the first minimum. The lack of agreement at forward angles may be due to the lack of a direct step $0_1^+ \rightarrow 0_2^+$ in the calcula-

tion or may result from the omission of higher excited states in the β band.

In order to investigate the possible effect on the 0^+_β angular distribution of coupling to other members in the β band, a CC calculation was made which included coupling to the 2^+_β state at 7.4 MeV (no data are presently available for this state). The resulting 0^+ angular distribution differs slightly from that shown in Fig. 6. The first minimum is shifted in by 1° , the second by 0.5° , and the third by 0.4° . The value of the cross sections at the first three maxima change by -18% , 0% , and $+23\%$, while the depths of the minima remain essentially unchanged. Both calculations fail to reproduce the second maximum and the shallow second minimum in the data.

DWBA calculations for 40 MeV proton inelastic excitation of this state, which employ a "vibrating-diffuseness" macroscopic form factor, a "breathing-mode" form factor, and a microscopic form factor all fail to reproduce the general diffraction pattern or slope of the data.⁵ DWBA calculations made in an attempt to fit 20 MeV proton inelastic data, using Soyeur's and Wildenthal's shell-model wave functions, also fail.⁶ A DWBA calculation for 42 MeV α scattering¹² with $\beta_0 R = 0.42$ fm roughly reproduces the data except for the prediction of deep minima. Rush and Ganguly¹ were able to achieve some success with the second derivative of the optical potential as a form factor and with a vibrational formalism that considered the 0^+ level to be a member of a two-phonon triplet. A microscopic calculation, for the excitation of this state by 23.5 MeV α 's, was able to fit the data by empirically adjusting the $1p_{1/2}$ hole and $1d_{3/2}$ particle components in the wave functions.²⁵

V. CONCLUSIONS

New data for 800 MeV proton inelastic excitation of the ground state, the γ , and the β bands in ^{24}Mg have been compared with DWBA and CC predictions. DWBA fits to the angular distributions for the 4^+_1 and 6^+_1 members of the ground band fail to reproduce the data. A coupled channels calculation

using an axially symmetric collective rotational model with quadrupole and hexadecapole ground state deformation provides good fits to the cross sections for the 0^+_1 , 2^+_1 , and 4^+_1 states. A complete examination of the data for the 6^+_1 state in which direct 0^+-6^+ coupling and a β_6 deformation in the ground state are included, will be the subject of future study.

A CC calculation for inelastic scattering to states in the γ band, in which the 0^+_1 , 2^+_1 , 2^+_2 , 3^+_2 , 4^+_2 , and 5^+_2 states are coupled assuming an asymmetric vibrational amplitude $\beta_2\gamma$, gave a fair representation of the 2^+_2 data, but failed for the 3^+_2 , 4^+_2 , and 5^+_2 angular distributions. This failure can perhaps be remedied by the inclusion of a direct step to the 4^+_2 through an additional 0^+-4^+ coupling term and by the inclusion of spin-flip processes. The fact that the DWBA calculations for the 2^+_2 and 4^+_2 states fit the data much better than does the CC calculation suggests that such a direct step is needed. Blair has pointed out that a large direct step to the 4^+_2 is expected on the basis of the overlap of Nilsson wave functions.^{12,26}

The result for the angular distribution of the 0^+ member of the β band, obtained by coupling the 0^+_1 , 2^+_1 , and $(6.43, 0^+_2)$ states, indicates that the axially symmetric deformed vibrator model may be essentially correct, but also seems to indicate that a direct $0^+_1-0^+_2$ step should be considered. The coupling of the 2^+_2 state was considered but an improved fit to the 6.43, 0^+ data was not obtained.

Finally, the excellent fits obtained to the data for the 2^+_1 and 4^+_1 states for $^{12}\text{C}^{15}$ and ^{24}Mg with 800 MeV protons seem to indicate that similar data and analyses of other p - and s - d shell deformed nuclei will provide a more accurate description of the intrinsic ground state deformation of these nuclei.

A complete tabulation of the numerical data is on deposit in PAPS.²⁷

We wish to thank J. S. Blair for helpful correspondence during the preparation of this manuscript.

This work was supported in part by the U. S. Department of Energy and The Robert A. Welch Foundation.

*Present address: TRIUMF, University of British Columbia, Vancouver B. C. Canada V6T 1W5.

¹A. A. Rush and N. K. Ganguly, Nucl. Phys. **A117**, 101 (1968).

²R. de Swiniarski *et al.*, Phys. Rev. Lett. **23**, 317 (1969).

³J. Eenmaa, R. K. Cole, C. N. Waddell, H. S. Sandhu, and R. R. Dittman, Nucl. Phys. **A218**, 125 (1974).

⁴I. Lovas, M. Rogge, U. Schwimm, P. Turek, and D. Ing-

ham, Nucl. Phys. **A286**, 12 (1977).

⁵B. Zwiaglinski, G. M. Crawley, H. Nann, and J. A. Nolen, Jr., Phys. Rev. C **17**, 872 (1978); B. Zwiaglinski, G. M. Crawley, W. Chung, H. Nann, and J. A. Nolen, Jr., *ibid.* **18**, 1228 (1978).

⁶R. M. Lombard, J. L. Escudie, and M. Soyeur, Phys. Rev. C **18**, 42 (1978).

⁷P. H. Stelson, R. L. Robinson, H. J. Kim, J. Rapaport,

- and G. R. Satchler, Nucl. Phys. 68, 97 (1956).
- ⁸H. R. E. Tjin *et al.*, Nucl. Phys. A106, 85 (1968).
- ⁹T. Tamura, Nucl. Phys. 73, 241 (1965); J. Kokame, K. Fukunaga, N. Inoue, and H. Nakamura, Phys. Lett. 8, 342 (1964).
- ¹⁰J. S. Vincent, E. T. Boschitz, and J. R. Priest, Phys. Lett. 25B, 81 (1967).
- ¹¹H. Rebel, *et al.*, Nucl. Phys. A182, 145 (1972).
- ¹²I. M. Naqib and J. S. Blair, Phys. Rev. 165, 1250 (1968).
- ¹³Y. Horikawa *et al.*, Phys. Lett. 36B, 9 (1971); R. Helm, Phys. Rev. 104, 1466 (1956).
- ¹⁴C. A. Wiedner *et al.*, Phys. Lett. 78B, 26 (1978).
- ¹⁵L. Ray, G. S. Blanpied, W. R. Coker, R. P. Liljestrand, and G. W. Hoffmann, Phys. Rev. Lett. 40, 1547 (1978).
- ¹⁶G. S. Blanpied *et al.*, Phys. Rev. Lett. 39, 1447 (1977); G. S. Blanpied *et al.*, Phys. Rev. C 18, 1436 (1978).
- ¹⁷C. M. Perey and F. G. Perey, At. Data Nucl. Data Tables 13, 293 (1974).
- ¹⁸L. Ray and W. R. Coker, Phys. Lett. 79B, 182 (1978).
- ¹⁹T. Tamura, Rev. Mod. Phys. 37, 679 (1965); Annu. Rev. Nucl. Sci. 19, 99 (1969).
- ²⁰W. R. Coker, L. Ray, and G. W. Hoffmann, Phys. Lett. 64B, 403 (1976).
- ²¹T. Tamura, W. R. Coker, and F. Rybicki, Comput. Phys. Commun. 2, 94 (1971).
- ²²For a discussion of γ bands and β bands in nuclei see, for example, J. D. Rogers, Annu. Rev. Nucl. Sci. 15, 241 (1965); R. K. Sheline, Rev. Mod. Phys. 32, 1 (1960); and J. P. Davidson, *ibid.* 37, 105 (1965).
- ²³P. M. Endt and C. Van der Leun, Nucl. Phys. A310, 1 (1978).
- ²⁴M. K. Pal and A. P. Stamp, Phys. Rev. 158, 924 (1967).
- ²⁵H. P. Morsch, D. Dehnhard, and T. K. Li, Phys. Rev. Lett. 34, 1527 (1975); and H. P. Morsch and P. Decowski, Phys. Lett. 82B, 1 (1979).
- ²⁶J. S. Blair, private communication.
- ²⁷See AIP document No. PAPS PRVCA 20-1490-6 for six pages of numerical data. Order by PAPS number and journal reference from American Institute of Physics, Physics Auxiliary Publications Service, 335 East 45th Street, New York, New York 10017. The price is \$1.50 for microfiche or \$5 for photocopies. Airmail additional. Make check payable to the American Institute of Physics. This Material also appears in *Current Physics Microfilm*, the monthly microfilm edition of the complete set of journals published by AIP, on frames immediately following this journal article.