

# Time-dependent Hartree-Fock calculations of $^{86}\text{Kr} + ^{139}\text{La}$ at $E_{\text{lab}} = 505, 610, \text{ and } 710 \text{ MeV}$

K. T. R. Davies

*Physics Division, Oak Ridge National Laboratory, Oak Ridge, Tennessee 37830*

K. R. Sandhya Devi

*Department of Theoretical Physics, University of Manchester, Manchester M13 9PL, United Kingdom*

M. R. Strayer

*Daresbury Laboratory, Science Research Council, Daresbury, Warrington WA4 4AD, United Kingdom  
and Department of Physics, Texas A & M University, College Station, Texas 77843*

(Received 9 May 1979)

Time-dependent mean-field calculations, using the axially symmetric, rotating frame approximation, have been performed for the reaction  $^{86}\text{Kr} + ^{139}\text{La}$  at  $E_{\text{lab}} = 505, 610, \text{ and } 710 \text{ MeV}$ . This is a new intermediate mass region for scattering heretofore relatively unexplored in time-dependent Hartree-Fock studies, and we obtain some results not observed in either lighter or heavier nuclear systems. At all three energies the final kinetic energy loss and angular dependence are in fair agreement with experiment, the most serious discrepancies occurring for the large-angle (small angular momentum) fully damped events whose calculated kinetic energies are about 50 MeV above the experimental ridge on the Wilczyński plots. Fusion is *not* seen at the two lower energies studied. However, at  $E_{\text{lab}} = 710 \text{ MeV}$  fusion is observed for angular momenta from 0 to  $65 \hbar$ , and just beyond the upper fusion limit we observe positive-angle scattering, in contrast to the negative-angle scattering always seen beyond the fusion limit for reactions of lighter ions. It is suggested that the fusion behavior for this reaction may be a definitive test of whether heavy-ion scattering is better described by a microscopic mean-field theory or macroscopic fluid dynamics. At  $E_{\text{lab}} = 710 \text{ MeV}$  emission of alpha-like particles from the neck at scission is observed for a small range of angular momenta near  $100 \hbar$ .

NUCLEAR REACTIONS  $^{86}\text{Kr} + ^{139}\text{La}$  at  $E_{\text{lab}} = 505, 610, \text{ and } 710 \text{ MeV}$  in the time-dependent Hartree-Fock approximation. Fusion and strongly damped collisions. Pre-equilibrium alpha-like emission from the neck.

## I. INTRODUCTION

Over the last few years, a great number of time-dependent Hartree-Fock (TDHF) calculations have been performed for a variety of nuclear systems using several types of one-, two-, and three-dimensional models. (See Refs. 1 and 2 for comprehensive lists of references.) In these studies, a wide range of nuclear phenomena have been observed, including fission, fusion, deep-inelastic scattering, and resonances.

Most of the TDHF calculations have involved collisions of light ions, e.g.,  $^{16}\text{O} + ^{16}\text{O}$ ,<sup>1-7</sup>  $^{40}\text{Ca} + ^{40}\text{Ca}$ ,<sup>2-4,6,7</sup>  $^{14}\text{N} + ^{12}\text{C}$ ,<sup>5,8</sup> and  $^{16}\text{O} + ^{40}\text{Ca}$ .<sup>9</sup> In these light systems we have observed orbiting behavior for large impact parameters and highly inelastic scattering associated with vibrational instability for nearly head-on collisions, but we have been especially concerned with an intermediate region of impact parameters where one usually finds fusion.<sup>1,4,6,7,9,10</sup> It has been found that the TDHF fusion cross sections are in good agreement with the experimental results,<sup>4,9,10</sup> but additional com-

parisons between theory and experiment are needed.

Several TDHF studies have been performed for heavier nuclear systems, including the induced fission of  $^{236}\text{U}$ .<sup>11</sup> Recently calculations have been made for the  $^{208}\text{Pb} + ^{208}\text{Pb}$  reaction,<sup>12</sup> for the scattering of  $^{84}\text{Kr}$  on  $^{208}\text{Pb}$  and  $^{209}\text{Bi}$ ,<sup>13</sup> for  $^{118}\text{Pd} + ^{118}\text{Pd}$ ,<sup>14</sup> and for the reaction  $^{238}\text{U} + ^{238}\text{U}$ .<sup>15</sup> In these collisions, it is found that the calculated final fragment kinetic energies, scattering angles, and mean masses are in good agreement with experiment, but the mass and charge distribution widths are an order of magnitude too small.<sup>12,13</sup> For the very heavy-ion reactions,<sup>12,13,15</sup> we observe no fusion or any indication of orbiting, even though there is considerable strong damping signifying deep-inelastic behavior. For all angular momenta studied, the projectile-like ion after the scattering was observed to be "reflected" from (rather than "passing through") the target.<sup>13</sup> The Coulomb force seems to strongly influence the behavior of these reactions,<sup>12,13</sup> giving results very different from those obtained in studies of lighter ions.

It is of interest to study collisions of heavy ions for various mass regions intermediate between those corresponding to very light-ion scattering and to reactions such as  $^{84}\text{Kr} + ^{209}\text{Bi}$ . It is important to understand changes in nuclear systematics as we make the transition from light-ion scattering, which is dominated by orbiting and fusion, to collisions of very heavy ions where deep-inelastic processes with no fusion are observed. An excellent example of such an intermediate system is  $^{86}\text{Kr} + ^{139}\text{La}$  which has recently been studied experimentally at  $E_{\text{lab}} = 505, 610, \text{ and } 710 \text{ MeV}$ .<sup>16</sup> The experimental results for this system indicate a considerable amount of both fully and partially damped events, with perhaps a small amount of fusion. (For each energy, the upper limit on the fusion cross sections is less than 25% of the total reaction cross section.)

We have performed TDHF calculations for  $^{86}\text{Kr} + ^{139}\text{La}$  at the three experimental energies. The model used for our calculations is the two-dimensional, axially symmetric, rigid clutching approximation.<sup>2,3</sup> There are reservations regarding the physical applicability of this model since it neglects nonaxial degrees of freedom, and thus it might underestimate the interaction time and the amount of energy dissipation. However, by comparison with fully three-dimensional calculations,<sup>2</sup> we have been able to roughly establish the energy range of validity of the model for light-ion scattering. Assuming that this validity criterion is applicable to heavier systems, we conclude that our results should very well reproduce the three-dimensional calculations at  $E_{\text{lab}} = 505 \text{ MeV}$  and probably at  $E_{\text{lab}} = 610 \text{ MeV}$ , while at  $E_{\text{lab}} = 710 \text{ MeV}$ , it is expected that our calculations *may* not be reliable. Nevertheless, as we shall see in Sec. II, the agreement between theory and experiment at  $E_{\text{lab}} = 710 \text{ MeV}$  is no worse than at the lower two energies. Better two-dimensional calculations<sup>17</sup> are now being performed with the more reliable separable approximation,<sup>7,18</sup> and it will be interesting to see how closely the two types of two-dimensional calculations agree with one another in this mass regime.

The details of our calculation have been described in other papers.<sup>2,3,8,11,13</sup> We use the finite-range, nonlocal Skyrme-type potential described in Ref. 11. Spin-orbit splitting is not included, neutron and proton states are treated separately, and Coulomb exchange is included in an approximate way.<sup>19</sup> For the clutching approximation, we use prescription R2 of Ref. 2. The initial target and projectile wave functions were calculated using the "filling approximation," in which one assumes uniform fractional occupancy of nonclosed spherical orbitals, and no pairing

was included in either the static or dynamical calculations.

In Sec. II we present the main results of our calculations. Then in Sec. III we conclude the paper with a discussion of how various limitations of our model and of the mean-field approximation may affect the reliability of some of our results. We also speculate on whether the fusion behavior of  $^{86}\text{Kr} + ^{139}\text{La}$  might be a definitive test to distinguish between TDHF theory and macroscopic fluid dynamics.<sup>20</sup>

## II. RESULTS

In Figs. 1, 2, and 3 we present the results for our final center-of-mass energies and angles on the experimental Wilczyński plots<sup>16</sup> for  $E_{\text{lab}} = 505, 610, \text{ and } 710 \text{ MeV}$ , respectively. The experimental contours for the double differential cross section  $d^2\sigma/dE_{\text{c.m.}}d\theta$  are also displayed. It should be emphasized that the TDHF results do *not* refer to differential cross sections. For each initial orbital angular momentum,<sup>21</sup> we place a point corresponding to the final center-of-mass energy and angle. To display cross sections, one would have to go beyond the mean-field approximation.

These results are qualitatively similar to those obtained in the TDHF calculations of  $^{84}\text{Kr} + ^{208}\text{Pb}$  and  $^{84}\text{Kr} + ^{209}\text{Bi}$ .<sup>13</sup> We obtain fair correspondence with the experimental contours. Each curve reproduces the energy and angle in the grazing region, bends down over the quasielastic ridge to the left towards the deep-inelastic peak, and finally turns to the right, approaching large scattering angles for small impact parameters. For all of the energies the experimental energy losses and scattering angles are fairly well reproduced for  $l$  values in the quasielastic region. However, near the deep-inelastic peak, there are discrepancies between theory and experiment, and the calculated large-angle events (for small impact parameters) are about 50 MeV above the experimental ridge. This is a serious disagreement with experiment. This discrepancy may not be quite as large as it appears since the corrected experimental energies may be somewhat low. As the incident bombarding energy decreases, the uncertainty in absolute value of the experimental kinetic energy gets larger, reaching values as high as 12% for  $E_{\text{lab}} = 505 \text{ MeV}$ .<sup>22</sup> By comparing Figs. 1, 2, and 3, we notice that the final center-of-mass energy for intermediate to small  $l$  values is roughly independent of the incident bombarding energy, an effect previously observed.<sup>13</sup>

In Fig. 1 it is seen that for  $E_{\text{lab}} = 505 \text{ MeV}$  the TDHF curve misses the deep-inelastic peak, since the turning point at  $l = 150$  is about 30 MeV

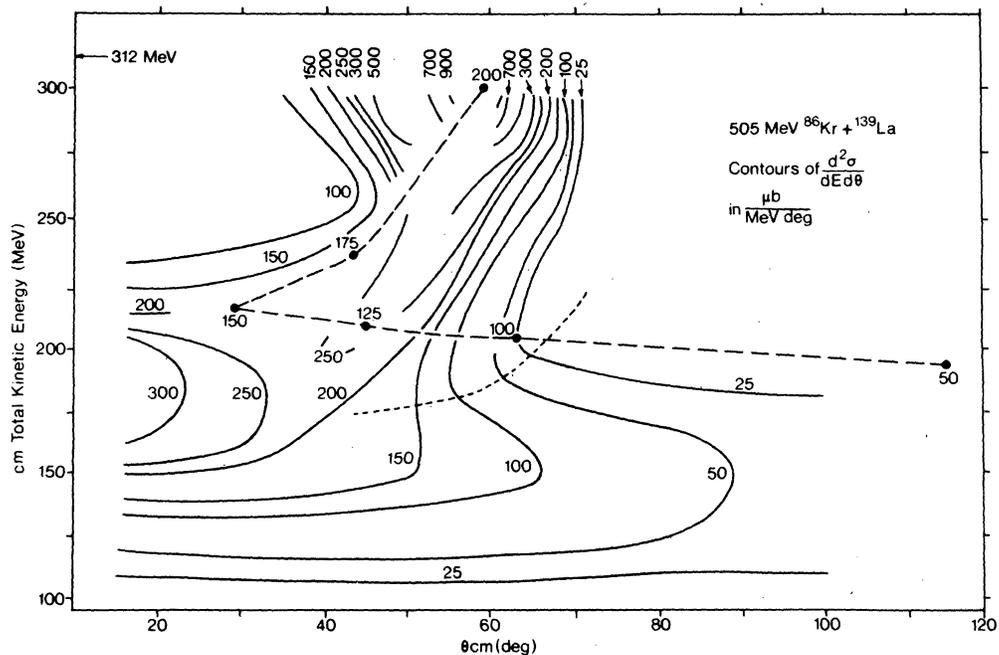


FIG. 1. Comparison of calculated points, labeled by the initial orbital angular momentum in units of  $\hbar$ , with the experimental Wilczyński plot from Ref. 16 for  $^{86}\text{Kr} + ^{139}\text{La}$  at  $E_{\text{lab}} = 505$  MeV. The short-dashed line indicates an approximate division (Ref. 16) into partially and fully damped events. The ordinate, extracted from Ref. 16, is slightly non-linear, owing to corrections for neutron emission. The top left arrow indicates elastic scattering.

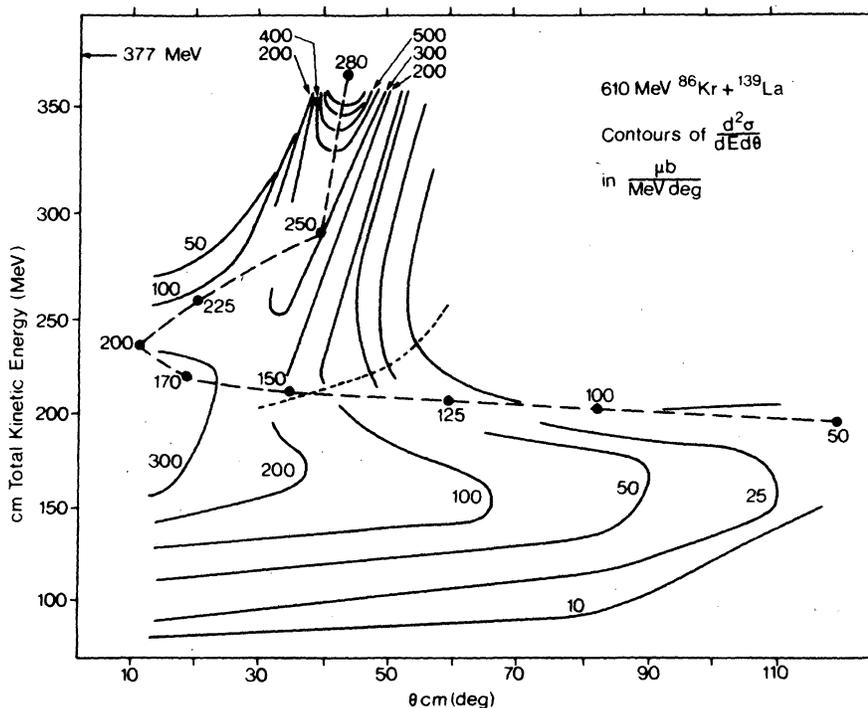


FIG. 2. Similar to Fig. 1 for  $^{86}\text{Kr} + ^{139}\text{La}$  at  $E_{\text{lab}} = 610$  MeV.

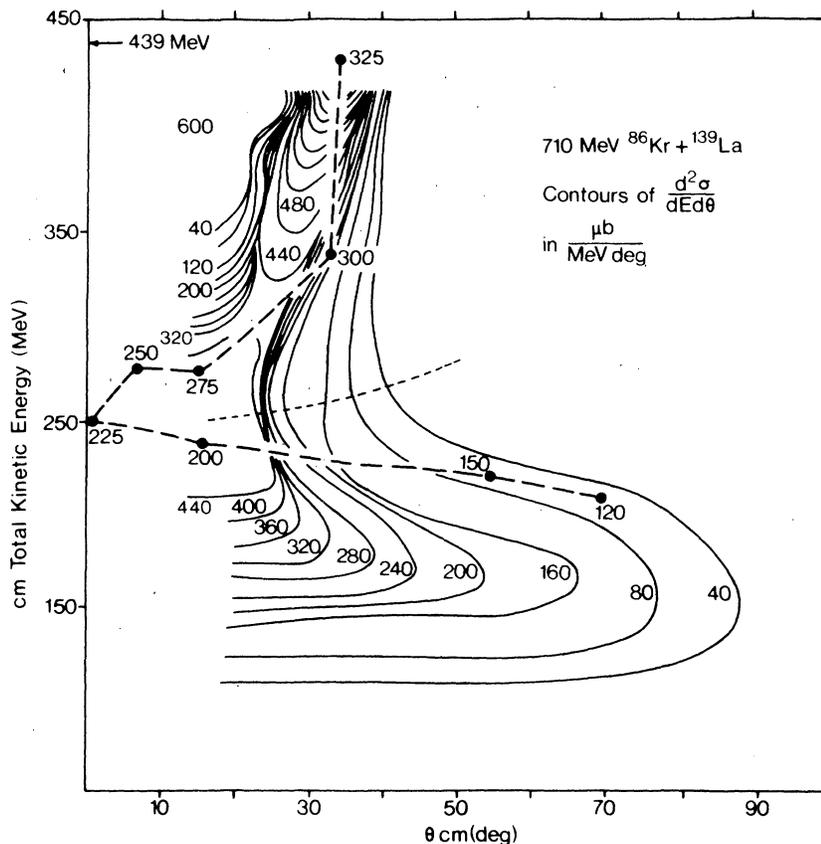


FIG. 3. Similar to Fig. 1 for  $^{86}\text{Kr} + ^{139}\text{La}$  at  $E_{\text{lab}} = 710$  MeV. For angular momenta between 110 and 95, three-body behavior dominates, and for  $l$  values below 65 fusion is observed.

too high in energy and about 15 degrees too high in angle. Also, as mentioned, for small  $l$  the TDHF curve is 50 MeV above the experimental ridge. However, it is instructive to calculate contributions to various parts of the total cross section using the formula

$$\sigma = \pi \sum_j [(b_j^{(\text{max})})^2 - (b_j^{(\text{min})})^2], \quad (1)$$

where  $b_j^{(\text{max})}$  and  $b_j^{(\text{min})}$  are the maximum and minimum impact parameters along a particular branch which contributes to the given partial cross section. In Ref. 16 an approximate separation was made of the partially and fully damped events. The region *above* the dotted line in Fig. 1 was assumed to be *partially damped* and that *below* the line, *fully damped*. Since it was not possible to unambiguously distinguish between the two types of events, it was assumed that for angles less than  $\approx 42^\circ$  and for angles greater than  $\approx 72^\circ$ , the scattering is all fully damped.<sup>16</sup> Therefore, using this division and comparing with our TDHF points, we see that there are two partially damped TDHF branches: angular momenta from  $l \approx 175$  to  $\approx 200$

and from  $l \approx 100$  to  $\approx 125$ . Also, there are two fully damped branches: angular momenta from  $l \approx 125$  to  $\approx 175$  and  $l$  values below  $\approx 100$ . Then from Eq. (1) we find that

$$\sigma(\text{partially damped}) \approx 745 \text{ mb}$$

and

$$\sigma(\text{fully damped}) \approx 975 \text{ mb},$$

which compare very well with the experimental values<sup>16</sup> of  $770 \pm 165$  and  $1020 \pm 200$ , respectively.

In Fig. 2 we see that for  $E_{\text{lab}} = 610$  MeV the turning point for the TDHF curve occurs at  $l = 200$  and lies right at the edge of the deep-inelastic peak. Figure 3 shows that at  $E_{\text{lab}} = 710$  MeV a range of angular momenta from about 190 to 280 all lie on the broad deep-inelastic peak. Also, for  $E_{\text{lab}} = 710$  MeV there seems to be somewhat more fluctuation in the TDHF curve than at the two lower energies. Thus, as the energy increases, one sees more structure as the TDHF curve more strongly penetrates the deep-inelastic region, an effect which has been observed in collisions of heavier systems.<sup>13</sup>

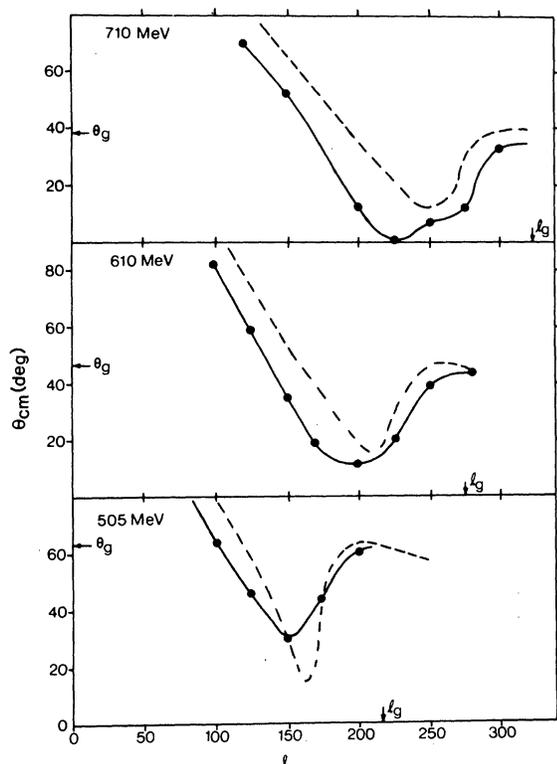


FIG. 4. Deflection functions for  $E_{\text{lab}} = 505, 610,$  and  $710$  MeV. The full curves connect the calculated TDHF points. The dashed curves are obtained using a proximity potential (Ref. 16). The arrows indicate  $\theta_g$  and  $l_g$ , the grazing angle and angular momentum determined experimentally (Ref. 16).

In Ref. 16 classical deflection functions were calculated for both an adiabatic approximation and assuming a fast reaction using the proximity potential.<sup>23</sup> For the former, the strength of the interaction for zero separation is determined by the energy of the compound system relative to that of two infinitely separated spherical ions. For the latter, the reaction is assumed to be sufficiently fast so that the ions retain their original sizes and shapes, and the proximity potential is obtained from an interaction energy between curved surfaces, expressed as a product of a geometrical factor and a universal separation function.<sup>23</sup> This universal function is related to the surface energy coefficient and is calculated using the Thomas-Fermi approximation. The adiabatic approximation yields a deflection function containing a region in which the scattering angles become negative, while the proximity potential exhibits only positive-angle scattering. As we see from Fig. 4, our TDHF scattering angles are always positive for all three energies, and our deflection functions closely resemble the curves<sup>16</sup> obtained from the proximity potential.

Other calculated quantities of interest are displayed in Figs. 5 and 6. As in previous TDHF studies,<sup>3,12,13</sup> the full widths at half maximum of the mass and charge distributions are small. The charge widths vary between 2.5 and 4.0 mass units and the mass widths between 5.5 and 7.5 units. The final mass or charge transfer is very small, in no case amounting to more than 2 mass

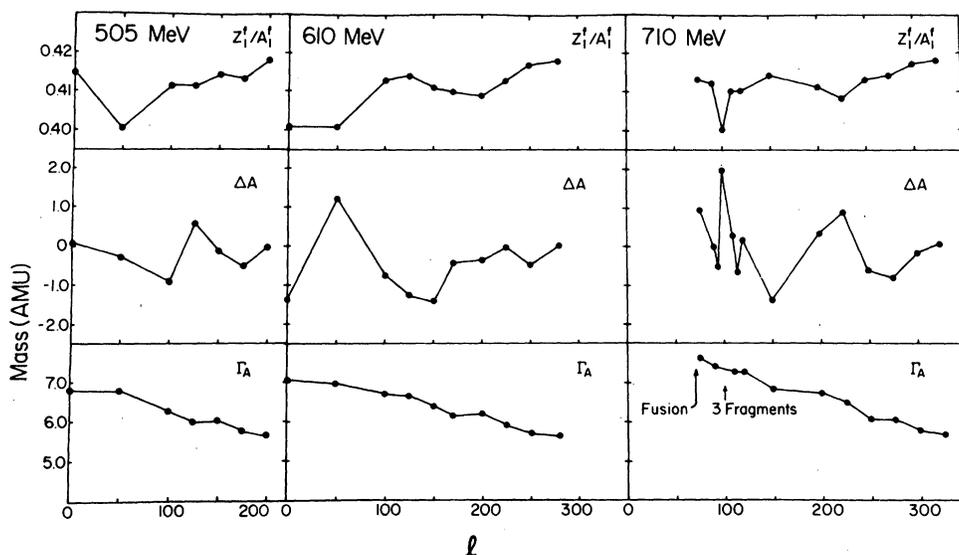


FIG. 5. Summary of mass distribution and charge equilibration quantities for  $^{86}\text{Kr} + ^{138}\text{La}$  for  $E_{\text{lab}} = 505, 610,$  and  $710$  MeV. Plotted as a function of the initial orbital angular momentum are the ratio ( $Z_f^*/A_f^*$ ),  $\Delta A$ , the net mass change, and  $\Gamma_A$ , the full width at half maximum of the mass distribution, all for the Kr-like fragment after the collision.

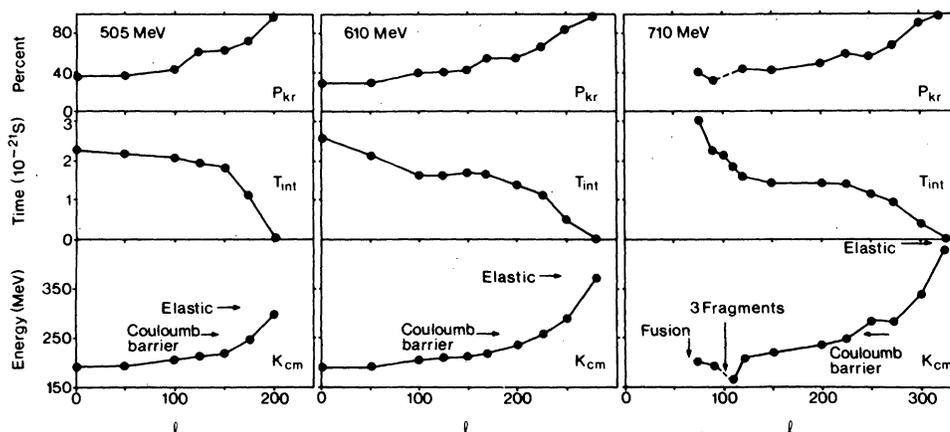


FIG. 6. Various dynamical quantities for  $^{86}\text{Kr} + ^{139}\text{La}$  at  $E_{\text{lab}} = 505, 610,$  and  $710$  MeV. Plotted as a function of the initial orbital angular momentum are  $P_{\text{Kr}}$ , the percentage of Kr orbitals remaining in the light fragment after the collision,  $T_{\text{int}}$ , the contact time in units of  $10^{-21}$  sec, and  $K_{\text{cm}}$ , the final center-of-mass kinetic energy of the fragments in MeV. The Coulomb barrier  $V_C = Z_1 Z_2 e^2 / [r_0 (A_1^{1/3} + A_2^{1/3})]$ , with  $r_0 = 1.2$ .

units. Since  $\Delta A$  is very small for all  $l$  values, there seems to be little evidence in these calculations for the fractionation of angular momentum along the mass-asymmetry degree of freedom.<sup>24</sup> In Fig. 5 we show the ratio of  $Z_1/A_1$  after the collision for the Kr-like fragment. This ratio is a measure of charge equilibration. For grazing impact parameters, this quantity approaches the initial  $Z_1/A_1$  value of 0.419. For smaller angular momenta, this quantity fluctuates somewhat but has values close to 0.413, the compound system ratio  $(Z_1 + Z_2)/(A_1 + A_2)$ .

In Fig. 6 we display the quantity  $P_{\text{Kr}}$ ,<sup>13</sup> which is defined as the percentage of Kr orbitals remaining in the light fragment after the collision. That is, initially we have a number of occupied orbitals which are localized in the projectile; we follow them throughout the collision and then determine what fraction of them still remains in the scattered Kr-like ion. This function is a measure of single-particle activity. Although  $P_{\text{Kr}}$  approaches 100% for grazing collisions, the mixing of single-particle orbitals is quite dramatic at smaller impact parameters, with  $P_{\text{Kr}}$  decreasing to values of 30 to 40%. Thus, even though there is a strong mixing of single-particle orbits, the compound system tends to "remember" the original configuration, and masses very close to the original Kr and La ions emerge after the scattering.<sup>13</sup>

We next discuss the TDHF results obtained at  $E_{\text{lab}} = 710$  MeV for values of  $l$  between 95 and 110 where one observes a pronounced three-body behavior. In this  $l$  region a small body of matter less than or equal to 4 mass units forms in the neck just prior to scission and begins to break away from the two larger fragments. In most

cases the small body coalesces with one of the two larger ions when the neck finally ruptures, as we show in Fig. 7 for  $l = 97.5$ . However, for a small range of angular momenta in the interval  $97.5 < l < 102.5$  the small fragment is able to separate completely, and we observe alpha-particle-like emission from the neck. In Fig. 8 we show density contour plots at six times (in units of  $10^{-21}$  sec) during the collision for  $l = 100$ . At  $t = 0.15$  the two ions are approaching one another, at  $t = 0.35$  they have completely coalesced, and at  $t = 2.10$  they have begun to separate. At  $t = 2.45$  a small, distinct fragment of matter forms in the neck region, and at  $t = 2.50$  the neck ruptures with the small body still slightly attached to the La-like fragment on the right. Then at  $t = 2.60$  the small alpha-like fragment completely separates from the right-hand ion and remains stationary in the center-of-mass system as the two larger ions move away.

Three-body breakup was previously seen in a two-dimensional TDHF calculation of rods on rods scattering.<sup>25</sup> Alpha-like-particle emission from the neck was also observed in a TDHF calculation of fission.<sup>11</sup> This study included BCS effects and ternary fission occurred for a small value of the pairing gap. The emission of alpha particles from the neck in fission is a phenomenon which experimentally has a very low probability of occurrence.<sup>11</sup> However, in deep-inelastic scattering of moderately heavy systems the cross section for alpha emission from the neck is found to be quite large.<sup>26</sup> It should also be mentioned that since we observe a very narrow  $l$  window for alpha-like emission at  $E_{\text{lab}} = 710$  MeV and since we sample widely spaced values of  $l$ , we cannot rule out the

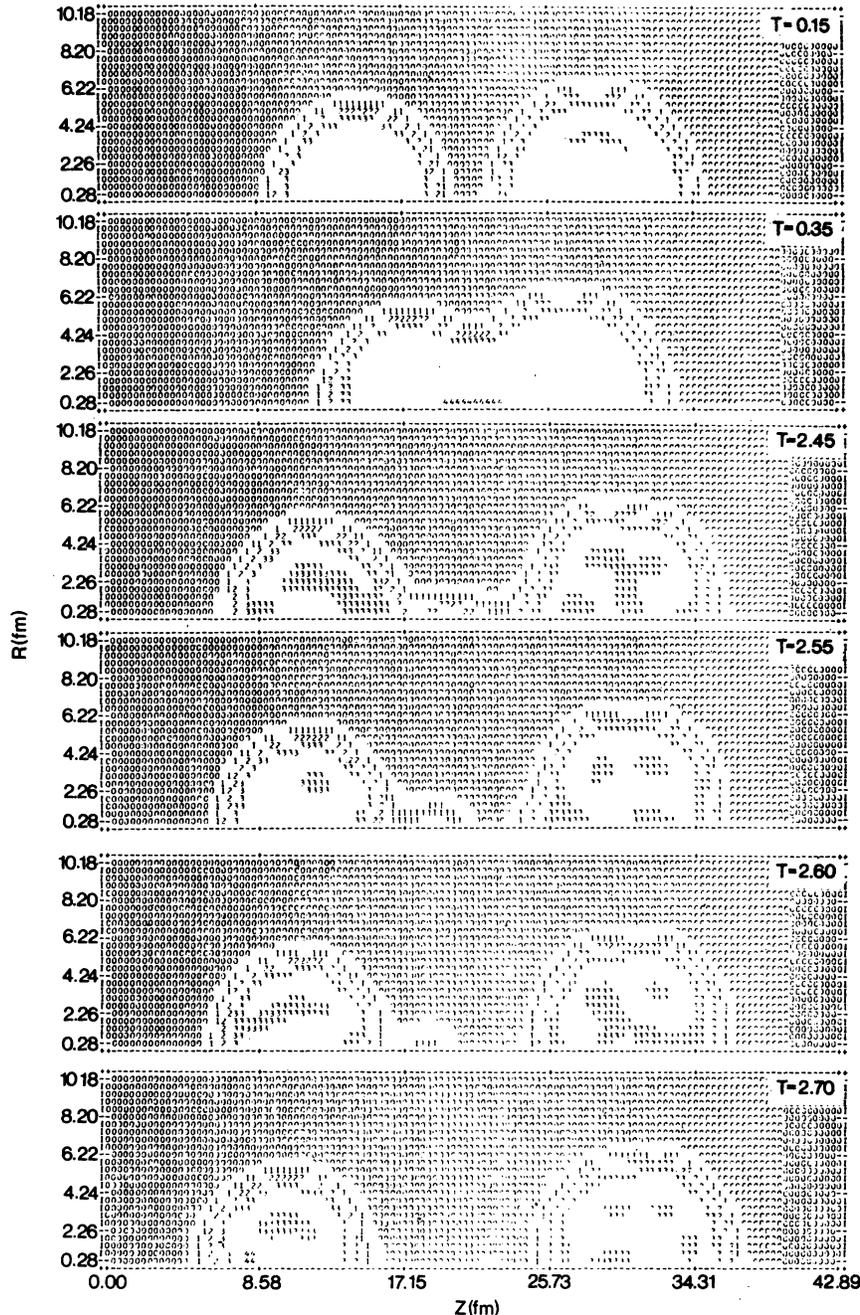


FIG. 7. Equidensity contours are displayed at various times during the  $^{86}\text{Kr} + ^{139}\text{La}$  reaction for  $E_{\text{lab}} = 710$  MeV and  $l = 97.5$ . In each case the abscissa ( $z$  axis) lies along the line joining the mass centers of the projectile and target. The axially symmetric density is plotted as a function of the cylindrical coordinates  $z$  and  $\rho$  (ordinate). The times are in units of  $10^{-21}$  sec.

possibility that pre-equilibrium alpha-like emission could occur at the lower two energies. In particular, for  $E_{\text{lab}} = 610$  MeV and  $l = 0$  the density profiles are suggestive of three-body breakup, since the behavior is very similar to the reaction

shown in Fig. 7. For this case, near scission at  $t = 2.55$  a small, distinct fragment is attached to the Kr-like ion, but the two bodies coalesce as time proceeds.

Finally, we discuss the fusion behavior of the

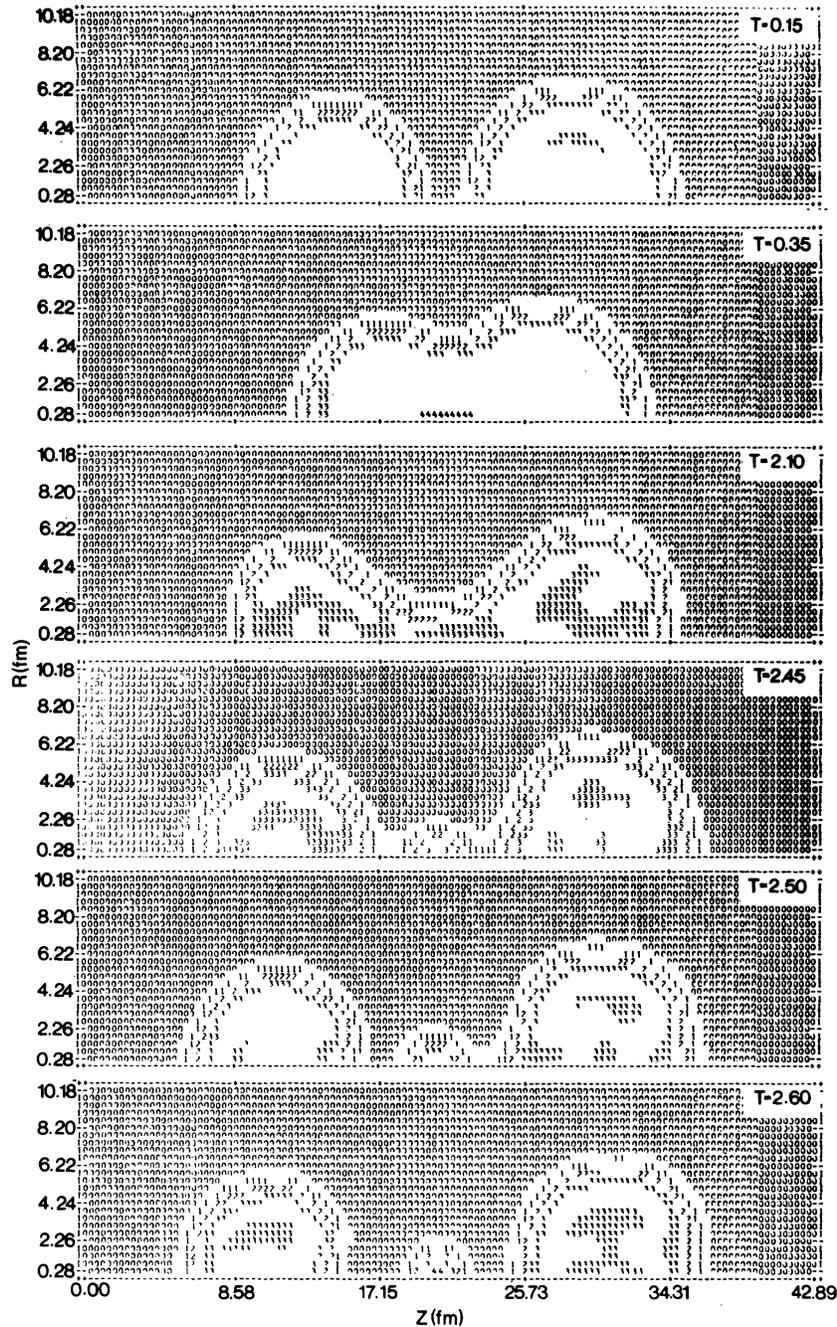


FIG. 8. Similar to Fig. 7 for  $^{86}\text{Kr} + ^{139}\text{La}$  at  $E_{\text{lab}} = 710$  MeV and  $l = 100$ .

$^{84}\text{Kr} + ^{139}\text{La}$  system. No fusion was observed for any impact parameter at both  $E_{\text{lab}} = 505$  and 610 MeV. At  $E_{\text{lab}} = 710$  MeV we obtain fusion for  $l$  values from 0 to  $\approx 65$ , giving a fusion cross section of about 118 mb which is well below the experimental upper limit of 500 mb.<sup>16</sup> In our studies the dynamical path for  $l$  values just above the fusion limit has a somewhat different character than that

of lighter systems. Just beyond the upper limit of 65, we observe positive-angle scattering, in contrast to the negative-angle scattering always seen beyond the fusion limit in reactions of lighter ions.<sup>1,2,7,9</sup>

We emphasize that at *zero* impact parameter we obtain no fusion for  $E_{\text{lab}} = 505$  and 610 MeV but do observe fusion at  $E_{\text{lab}} = 710$  MeV. Since the rigid

clutching model is exact for head-on collisions, it is expected that this same general type of fusion behavior should persist in improved two-dimensional or three-dimensional TDHF calculations. This energy dependence for fusion is interesting and can be related to an effect first studied by Nix and Sierk<sup>20</sup> using macroscopic fluid dynamics. They pointed out that, in order for fusion to occur at energies above the interaction barrier, the dynamical path in a multidimensional configuration space must pass inside the fission saddle point<sup>27</sup> for the compound system. This condition becomes more difficult to satisfy as the mass of the compound system increases, as the angular momentum increases, and as the energy above the barrier decreases.<sup>20</sup> For a moderately heavy system at zero impact parameter and at a relatively small energy above the barrier, the dynamical path may bend back beyond the saddle point so that fusion does not occur. This would qualitatively explain why we observe fusion at  $E_{lab} = 710$  MeV, but not at the two lower energies. A reaction for which an energy threshold for fusion is first seen in a macroscopic model is the symmetric system  $^{110}\text{Pd} + ^{110}\text{Pd} \rightarrow ^{220}\text{U}$ ,<sup>20</sup> and this compound system has a mass very close to the value of 225 for the  $^{86}\text{Kr} + ^{139}\text{La}$  reaction. However, we only obtain qualitative agreement with the macroscopic results. The energies above the barrier at which fusion is first seen in Ref. 20 are less than 10 MeV, while in our TDHF calculations of  $^{86}\text{Kr} + ^{139}\text{La}$  this threshold occurs between 120 and 200 MeV. It is not possible yet to make a direct comparison between the TDHF results and the macroscopic studies, which were calculated for mass-symmetric systems assuming no viscosity.<sup>20</sup> Using reasonable values of one- or two-body viscosity coefficients<sup>29,30</sup> should further inhibit the formation of fusion in the macroscopic calculations, since viscosity will cause the dynamical paths to bend away from the fission saddle points. Also, in our TDHF calculations, the absence of fusion at the two lower energies may be partly related to our underestimation of the experimental energy loss for small  $l$ , as seen in Figs. 1–3.

### III. CONCLUDING REMARKS

There are a number of factors which limit the reliability of our results. For example, since there is no spin-orbit splitting in our calculations and we use the “filling approximation” for the nonclosed shells, our nuclei do not have the correct stiffness with respect to quadrupole deformations of their shape. Other variations of the clutching prescription have been studied<sup>2</sup> and may be used in future calculations. In particular, at

the beginning of the calculation if one assumes “gradual” clutching (model R4 in Ref. 2), one expects to obtain a less positive scattering angle with more energy loss than we find in our present calculation which uses a “sudden” clutching prescription. (However, at  $E_{lab} = 505$  MeV preliminary calculations using the “gradual” clutching model give results almost identical to those reported in Sec. II.<sup>31</sup>) Also, we know that different versions of the force yield different dynamical behavior.<sup>4,6,9</sup> All of these effects deserve further study.

However, we want to mainly focus on two limitations of the theory which we feel may be most important in affecting the reliability of our results.

(i) *Use of an axially symmetric, rigid clutching model.* By comparing the two-dimensional rigid clutching model with fully three-dimensional calculations for light-ion scattering<sup>2,6,9</sup> we have found that the axially symmetric model is a very good approximation at moderate energies above the interaction barrier. In lighter systems this model seems to break down at about the energy at which the three-dimensional fusion window first appears.<sup>2,6,9</sup> The validity of the model for reactions of heavier ions has not been determined yet. When the results presented here are compared with improved calculations<sup>17</sup> using the separable approximation,<sup>32</sup> we shall undoubtedly gain some understanding regarding the range of validity of the rigid clutching model in heavier systems.

Based on comparisons made in light-ion scattering,<sup>2,6,9</sup> it is expected that our results at  $E_{lab} = 505$  MeV and probably at  $E_{lab} = 610$  MeV should agree reasonably well with three-dimensional calculations. The 710-MeV laboratory energy is likely too high to expect the axial approximation to be very accurate. Therefore, the results at this energy may differ from three-dimensional calculations and should be considered qualitative and perhaps tentative.

However, as we have seen in Sec. II, the agreement between theory and experiment at  $E_{lab} = 710$  MeV is no worse than that at  $E_{lab} = 505$  and 610 MeV. Moreover, in all previous comparisons with three-dimensional calculations,<sup>2,6,9</sup> the rigid clutching model always *underestimates* the amount of fusion and of energy dissipation. Therefore, one would predict that improved calculations of the  $^{86}\text{Kr} + ^{139}\text{La}$  system should exhibit a variety of deep-inelastic processes, similar to those reported here.

As indicated in the previous section it is *not* expected that the basic fusion behavior will change in improved calculations. This is because our results for head-on collisions should exactly

agree with three-dimensional calculations. At  $E_{1ab}=710$  MeV, it is possible that a larger fusion region will be found. Since we did not see fusion for  $l=0$  at both  $E_{1ab}=505$  and 610 MeV, it seems unlikely that any fusion will be observed in the improved calculations unless angular momentum fusion windows exist at these energies. If such windows exist at lower energies, then the window must vanish at  $E_{1ab}=710$  MeV, which would be an anomalous behavior never seen in TDHF studies of lighter systems.<sup>4,6,9,10</sup>

(ii) *Validity of TDHF theory.* There are, of course, limitations of our results due to the use of the mean-field approximation. One test of the TDHF theory is a comparison with experiment to verify the existence of the TDHF fusion window.<sup>4,9,10</sup> Also, some time ago it was proposed<sup>33</sup> that we search for a definitive test to determine whether TDHF or macroscopic fluid dynamics<sup>20</sup> gives a better representation of heavy-ion scattering at moderate energies above the interaction barrier. The results presented here for the fusion behavior of the  $^{86}\text{Kr} + ^{139}\text{La}$  system suggest that a discrepancy may exist between these two theories. As indicated in the preceding section, for masses of the compound system in the neighborhood of  $A=220$ , both TDHF and macroscopic fluid dynamics<sup>20</sup> predict fusion thresholds at energies above the interaction barrier. However, these thresholds differ by an order of magnitude, indicating that there may be a substantial quantitative discrepancy between the predictions of the two theories. In order to resolve this question, we first need to obtain more reliable TDHF results and to perform calculations of the  $^{86}\text{Kr} + ^{139}\text{La}$  system using a macroscopic theory which includes the effects

of viscosity. Also, it is important to determine experimentally the magnitude of the fusion threshold and to obtain accurate measurements of the fusion cross sections to be compared with the theoretical predictions.

In summary, we have demonstrated that TDHF calculations of  $^{86}\text{Kr} + ^{139}\text{La}$  exhibit a number of interesting new results. Our calculations show a rich variety of dynamical behavior, including deep-inelastic scattering, fusion with an accompanying energy threshold, and three-body breakup. The agreement with experiment is only fair since there is a discrepancy between the calculated and experimental energy loss, which becomes especially serious for large-angle scattering. The origin of this problem needs to be understood. It is hoped that our study will stimulate additional microscopic and macroscopic calculations of the  $^{86}\text{Kr} + ^{139}\text{La}$  system.

#### ACKNOWLEDGMENTS

This research was supported in part by the Division of Physical Research, U. S. Dept. of Energy, under Contract No. W-7405-eng-26 with the Union Carbide Corporation, and by the National Science Foundation under Grant No. Phy 78-11577. One of us (M.R.S.) would like to thank the Oak Ridge Associated Universities for supporting visits to the Oak Ridge National Laboratory. We are very indebted to J. M. Irvine and to the SRC Daresbury Laboratory for providing most of the resources to perform our calculations. We would like to acknowledge useful discussions with P. Bonche, D. M. Brink, E. Halbert, A. K. Kerman, S. E. Koonin, J. W. Negele, J. R. Nix, and R. Vandenbosch.

<sup>1</sup>H. Flocard, S. E. Koonin, and M. S. Weiss, Phys. Rev. C **17**, 1682 (1978).

<sup>2</sup>K. T. R. Davies, H. T. Feldmeier, H. Flocard, and M. S. Weiss, Phys. Rev. C **18**, 2631 (1978).

<sup>3</sup>S. E. Koonin, K. T. R. Davies, V. Maruhn-Rezwani, H. Feldmeier, S. J. Krieger, and J. W. Negele, Phys. Rev. C **15**, 1359 (1977).

<sup>4</sup>P. Bonche, B. Grammaticos, and S. E. Koonin, Phys. Rev. C **17**, 1700 (1978).

<sup>5</sup>R. Y. Cusson, J. A. Maruhn, and H. W. Meldner, Phys. Rev. C **18**, 2589 (1978).

<sup>6</sup>S. J. Krieger and K. T. R. Davies, Phys. Rev. C **18**, 2567 (1978).

<sup>7</sup>S. E. Koonin, B. Flanders, H. Flocard, and M. S. Weiss, Phys. Lett. **B77**, 13 (1978).

<sup>8</sup>V. Maruhn-Rezwani, K. T. R. Davies, and S. E. Koonin, Phys. Lett. **67B**, 134 (1977).

<sup>9</sup>P. Bonche, K. T. R. Davies, B. Flanders, H. Flocard, B. Grammaticos, S. E. Koonin, S. J. Krieger, and

M. S. Weiss, Phys. Rev. C (to be published).

<sup>10</sup>S. J. Krieger and K. T. R. Davies, Phys. Rev. C **20**, 167 (1979).

<sup>11</sup>J. W. Negele, S. E. Koonin, P. Möller, J. R. Nix, and A. J. Sierk, Phys. Rev. C **17**, 1098 (1978).

<sup>12</sup>A. K. Dhar and B. S. Nilsson, Phys. Lett. **77B**, 50 (1978); A. K. Dhar, Phys. Rev. Lett. **42**, 1124 (1979).

<sup>13</sup>K. T. R. Davies, V. Maruhn-Rezwani, S. E. Koonin, and J. W. Negele, Phys. Rev. Lett. **41**, 632 (1978); K. T. R. Davies and S. E. Koonin (unpublished).

<sup>14</sup>R. Y. Cusson and H. W. Meldner, Phys. Rev. Lett. **42**, 694 (1979).

<sup>15</sup>R. Y. Cusson, J. A. Maruhn, and H. Stocker (unpublished).

<sup>16</sup>R. Vandenbosch, M. P. Webb, P. Dyer, R. J. Pugh, R. Weisfield, T. D. Thomas, and M. S. Zisman, Phys. Rev. C **17**, 1672 (1978).

<sup>17</sup>B. Flanders, P. Bonche, S. E. Koonin, and M. S. Weiss (unpublished).

- <sup>18</sup>K. R. Sandhya Devi and M. R. Strayer, *J. Phys. G* **4**, L97 (1978); *Phys. Lett.* **77B**, 135 (1978).
- <sup>19</sup>J. W. Negele and D. Vautherin, *Phys. Rev. C* **5**, 1472 (1972).
- <sup>20</sup>J. R. Nix and A. J. Sierk, *Phys. Rev. C* **15**, 2072 (1977).
- <sup>21</sup>We shall refer to the initial orbital angular momentum as  $l\hbar$ . In the actual calculations the precise value of the angular momentum is given by  $L \equiv (l + \frac{1}{2})\hbar$ .
- <sup>22</sup>R. Vandenbosch, private communication.
- <sup>23</sup>J. Błocki, J. Randrup, W. J. Swiatecki, and C. F. Tsang, *Ann. Phys. (N.Y.)* **105**, 427 (1977).
- <sup>24</sup>R. Regimbart, A. N. Behkami, G. L. Wozniak, R. P. Schmitt, J. S. Sventek, and L. G. Moretto, *Phys. Rev. Lett.* **41**, 1355 (1978).
- <sup>25</sup>P. Bonche, B. Grammaticos, and A. Jaffrin, in *Proceedings of the European Conference on Nuclear Physics with Heavy Ions*, Caen, 1976 (unpublished).
- <sup>26</sup>J. M. Miller, G. L. Catchen, D. Logan, M. Rajagopalan, J. M. Alexander, M. Kaplan, and M. S. Zisman, *Phys. Rev. Lett.* **40**, 100 (1978).
- <sup>27</sup>The fission saddle points of Ref. 20 also include the effects of angular momentum (Ref. 28).
- <sup>28</sup>S. Cohen, F. Plasil, and W. J. Swiatecki, *Ann. Phys. (N.Y.)* **82**, 557 (1974).
- <sup>29</sup>K. T. R. Davies, R. A. Managan, J. R. Nix, and A. J. Sierk, *Phys. Rev. C* **16**, 1890 (1977).
- <sup>30</sup>A. J. Sierk, S. E. Koonin, and J. R. Nix, *Phys. Rev. C* **17**, 646 (1978).
- <sup>31</sup>K. T. R. Davies, K. R. Sandhya Devi, and M. R. Strayer (unpublished).
- <sup>32</sup>The two-dimensional separable approximation (Refs. 7, 9, 18) gives almost perfect agreement with the three-dimensional calculations at all energies studied in lighter systems.
- <sup>33</sup>J. R. Nix, private communication.