# Single-folding model for the spin-orbit potential between $^{19}$F and $^{28}$Si

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A general expression for the spin-orbit potential between two heavy ions is derived using the single-folding method. This expression is applied to the calculation of the spin-orbit potential between $^{19}$F and $^{28}$Si, using single-particle as well as cluster model wave functions for $^{19}$F. In the cluster model, the magnitude of the $^{19}$F + $^{28}$Si spin-orbit potential at the strong absorption radius is enhanced by a factor of 50 over the single-particle model estimate.

[NUCLEAR REACTIONS $^{19}$F+$^{28}$Si spin-orbit potential: single-folding method, single-particle, and cluster model wave functions for $^{19}$F.]

There is currently a lot of interest[1-6] in the magnitude of the spin-orbit potential for heavy ions. The transfer reaction data[1] and the polarization data[2] can only be reproduced by adding a significant spin-orbit term to the optical potential. However, in some cases such as $^{19}$F + $^{28}$Si, the empirical result[1] is two orders of magnitude larger than the value obtained theoretically.[4,5] So far, all the calculations[4,5] were performed assuming a single-particle harmonic oscillator wave function for $^{19}$F.

In the present note, we derive a general expression for the spin-orbit potential between two heavy ions using the single-folding model. We apply this expression to $^{19}$F, assuming three different types of wave functions: a single-particle harmonic oscillator eigenfunction, a single-particle Woods-Saxon eigenfunction, and a $t + ^{16}$O cluster model wave function. The first two give approximately the same result for the $^{19}$F + $^{28}$Si spin-orbit potential at the strong absorption radius.[7] Clustering enhances the value of the spin-orbit potential by a factor of 50.

Let us briefly review the model. Consider a target of mass $A_t$ and spin zero, and a projectile of mass $A_p$ and spin $\vec{S}$. Suppose that the projectile can be described in terms of a cluster of mass $A_c$ and spin $\vec{s}$, moving in a relative $Ns$ state $\psi_c(\vec{r})$ around a spin zero core. $N$ is the number of nodes allowed by the shell model for the relative motion. When the spin-orbit potential $v_{ls}(x)$ between the cluster and the target is known, the spin-orbit potential $V_{LS}(R)$ between the projectile and the target can be evaluated in the single-folding model. Extending the method of Ref. 3 to take into account any cluster of mass $A_c$, we have

$$\vec{L}\cdot\vec{S}V_{LS}(R)=\vec{l}\cdot\vec{s}\int d^3r\,|\psi_c(\vec{r})|^2 v_{ls}(x)\,, \tag{1}$$

where the coordinates $\vec{R}$, $\vec{r}$, and $\vec{x}$ are shown in Fig. 1, $\vec{L}$ is the orbital angular momentum of the projectile with respect to the target, and $\vec{l}$ is the orbital angular momentum of the cluster with respect to the target.

Neglecting the velocity of the cluster around the projectile core, we can approximate the orbital angular momentum of the cluster by

$$\vec{l}=\frac{A_c}{A_p}\vec{P}\times\vec{R}+\frac{A_c(A_p-A_c)}{A_p^2}\vec{P}\times\vec{r}\,, \tag{2}$$

where $\vec{P}$ is the projectile momentum.

The first term in Eq. (2) is proportional to the projectile angular momentum $\vec{L}$. We shall denote by $V_{LS,1}(R)$ the contribution of this term to the heavy-ion spin-orbit potential

$$V_{LS,1}(R)=\frac{A_c}{A_p}\int d^3r\,|\psi_c(\vec{r})|^2 v_{ls}(x)\,. \tag{3}$$

The contribution of the second term can be easily evaluated using the heavy-ion approximation

$$\vec{r}\cong\frac{(\vec{R}\cdot\vec{r})}{R^2}\vec{R}\,,$$

which leads to

$$V_{LS,2}(R)\cong\frac{A_c(A_p-A_c)}{A_p^2R}\int d^3r\,|\psi_c(\vec{r})|^2 r\cos\theta v_{ls}(x)\,. \tag{4}$$

The spin-orbit potential $V_{LS}(R)$ between the projectile and the target is given by the sum

$$V_{LS}(R)=V_{LS,1}(R)+V_{LS,2}(R)\,. \tag{5}$$

For the cluster-target spin-orbit potential $v_{ls}(x)$, we use the standard parametrization

$$v_{ls}(x)=2\left(\frac{\hbar}{m_\pi c}\right)^2\frac{V_0}{x}\frac{df}{dx}\,, \tag{6a}$$

where

 

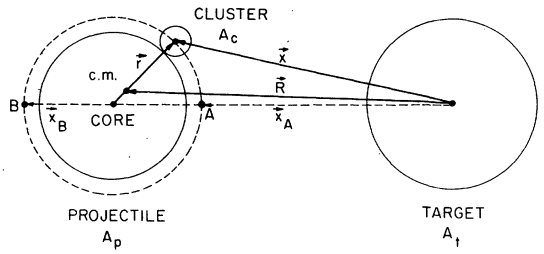


FIG. 1. Notation used for the coordinates $\vec{R}$, $\vec{r}$, $\vec{x}$, $\vec{x}_A$, and $\vec{x}_B$ throughout this work.

$$f(x)=\frac{1}{1+\exp[(x-R_{ls})/a_{ls}]}. \tag{6b}$$

Inserting this expression for $v_{ls}(x)$ in Eqs. (3) and (4), and performing the integrations, we obtain

$$V_{LS,1}(R)=\left(\frac{\hbar}{m_\pi c}\right)^2\frac{A_c}{(A_p-A_c)}[f(x_B)-f(x_A)] \times\frac{V_0}{R}\int_0^\infty r|\mathcal{R}_c(r)|^2dr \tag{7}$$

and

$$V_{LS,2}(R)\cong-\frac{1}{4}\left(\frac{\hbar}{m_\pi c}\right)^2\frac{A_c(A_p-A_c)}{A_p^2}\frac{1}{x_A}\frac{df}{dx_A} \times\frac{V_0}{R}\int_0^\infty r^3|\mathcal{R}_c(r)|^2dr\,, \tag{8}$$

where $\mathcal{R}_c(r)$ is the radial part of $\psi_c(\vec{r})$,

$$x_A=R-\frac{A_p-A_c}{A_p}r \tag{9a}$$

and

$$x_B=R+\frac{A_p-A_c}{A_p}r. \tag{9b}$$

Let us now apply this model to the $^{19}$F projectile and $^{28}$Si target.

We use initially a $p+^{18}$O single-particle wave function for $^{19}$F, setting $A_c=1$ in Eqs. (7) and (8). We adopt a $1s_{1/2}$ harmonic-oscillator eigenfunction for the proton, and determine the oscillator constant $\alpha=0.645$ fm$^{-1}$ by fitting the experimental rms radius[8] of $^{19}$F, $r=2.9$ fm. For the $v_{ls}(x)$ potential between the proton and the $^{28}$Si target, we use the standard parametrization,[9] $V_0=6.0$ MeV, $R_{ls}=3.067$ fm, and $a_{ls}=0.75$ fm. At the strong absorption radius,[7] $R_s=1.5(A_p^{1/3}+A_t^{1/3})$ fm, the $^{19}$F + $^{28}$Si spin-orbit potential of Eq. (5) is

$$V_{LS}(R_s)=(-1.2+0.3)\text{ keV}=-0.9\text{ keV}. \tag{10}$$

The behavior of $V_{LS}(R)$ in the vicinity of the strong absorption radius is shown in Fig. 2.

In order to investigate whether this result is strongly dependent on the type of wave function

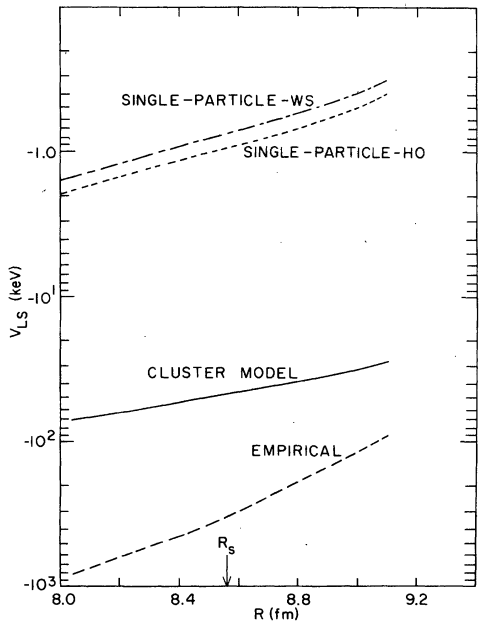


FIG. 2. The spin-orbit potential for $^{19}$F+$^{28}$Si in the vicinity of the strong absorption radius. The dash-dotted line and the dotted line are obtained using a single-particle wave function for $^{19}$F. The dash-dotted line corresponds to a Woods-Saxon eigenfunction, and the dotted line to a harmonic-oscillator eigenfunction. The solid line is obtained assuming a cluster-model wave function for $^{19}$F, and the dashed line is the empirical[1] spin-orbit potential for $^{19}$F+$^{28}$Si.

used for the proton, we perform the same calculation using a $1s_{1/2}$ Woods-Saxon eigenfunction. We set the radius of the Woods-Saxon well at 2.9 fm and the diffusivity at 0.64 fm. We adjust the depth of the Woods-Saxon well to reproduce the 8 MeV separation energy of the proton in $^{19}$F, taking into account the Coulomb potential of a spherical charge distribution of radius 2.9 fm. With this new single-particle wave function for $^{19}$F, the $^{19}$F + $^{28}$Si spin-orbit potential at the strong absorption radius is

$$V_{LS}(R_s)=(-1.0+0.3)\text{ keV}=-0.7\text{ keV}. \tag{11}$$

Therefore, the type of wave function used for the proton does not seem to have a large effect on the heavy-ion spin-orbit potential (see Fig. 2).

From earlier work,[10-12] it is well known that clustering effects are significant in $^{19}$F. For instance, the cluster model[12] accounts successfully for the excitation energies, charge radii, quadrupole moments, and $E2$ and $M1$ transition rates in $^{19}$F that cannot be explained in terms of a single-particle wave function. Therefore, it is also important to investigate the effect of clustering on the $^{19}$F + $^{28}$Si spin-orbit potential.

In the single-folding model developed above, we can easily introduce the $t+{}^{16}O$ wave function.[12] The triton spin-orbit potential $v_{ls}(x)$ is known from the recent asymmetry measurements on a variety of targets.[13,14] The parametrization which leads to the smallest $\chi^2$ per point in the analyzing power and the cross section data is of the form[13,14] $V_0 = 6$ MeV, $R_{ls} = 5.15$ fm, and $a_{ls} = 0.8$ fm. (If the depth $V_0$ were lowered[15] to 2 MeV, or raised to 12 MeV, and the geometry kept fixed, the value of $\chi^2$ per point would increase by a factor of 4,[14] nearly independent of the target.) In order to calculate the wave function $\psi_c(\vec{r})$, we make the standard assumption that the triton cluster is moving around the $^{16}O$ core in a $3s_{1/2}$ orbit with binding energy 11.73 MeV, and we adopt the cluster-core potential of Ref. 12. The value of $A_c$ in Eqs. (7) and (8) is now $A_c = 3$, and the cluster-core rms radius[12] is $r = 4.0$ fm. The resulting $^{19}F + {}^{28}Si$ spin-orbit potential at the strong absorption radius is

$$V_{LS}(R_s) = (-57 + 10)\ \text{keV} = -47\ \text{keV}\,, \qquad (12)$$

which is 50 times stronger than the value obtained in Eqs. (10) and (11) using the single-particle model (see Fig. 2).

This large enhancement in $V_{LS}(R_s)$ is due, in part, to a larger cluster mass. It is also due to the geometry of the triton-target spin-orbit potential combined with the cluster-core distance in the $^{19}F$ wave function.

In conclusion, our phenomenological approach indicates that the $t+{}^{16}O$ model wave function for $^{19}F$ enhances the $^{19}F + {}^{28}Si$ spin-orbit potential by a factor of 50 over the single-particle estimate near the strong absorption radius. However, the cluster model result is still about a factor of 7 smaller than the empirical[1] spin-orbit strength.

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