PHYSICAL REVIEW C VOLUME 20, NUMBER 4 OCTOBER 1979

"Optimal" approximation to elastic projectile-nucleus scattering

S. A. Gurvitz

*Physics Department, Weizmann Institute of Science, Rehovot, Israel
and Swiss Institute for Nuclear Research, CH-5234 Villigen, Switzerland*

(Received 22 November 1978)

An approximation which minimizes binding and recoil corrections is derived for projectile nucleus elastic scattering. We generalize previous work on single scattering and consider now multiple scattering amplitudes in this approximation. These are expressed in terms of amplitudes for elastic scattering off on-shell target nucleons and form factors. The first nonvanishing correction terms are estimated.

[NUCLEAR REACTIONS Multiple scattering amplitudes, minimization of corrections, factorization, reflections, higher order corrections.]

I. INTRODUCTION

The projectile-nucleus elastic scattering amplitude is usually expressed in terms of a multiple scattering expansion.[1] The ingredients of every multiple scattering term are the matrix τ_i describing the scattering on a bound nucleon i and the Green's function G describing the propagation of the projectile between scattering. τ_i and G are many-body operators; therefore, we need approximations for practical applications of multiple scattering expansion. In a previous paper[2] we developed the "optimal" approximation for the first term of this expansion—the single scattering amplitude. For elastic projectile scattering we found that in the approximation, designed to minimize corrections, the single scattering amplitude factors into a form factor of the nucleus and an *on-shell* projectile-nucleon amplitude with the energy argument increasing with momentum transfer.

In this paper we develop a similar optimal prescription for evaluation of multiple scattering terms; that is, we find approximations for τ_i and G in every multiple scattering amplitude such that the first order correction is zero. As a bonus we find that for our choice of approximations the integrands in multiple scattering integrals factor into a multiparticle target density in momentum representation and a part dependent on the projectile on-shell nucleon input. We find also the first nonvanishing correction and estimate its magnitude.

The plan of this paper is as follows: In Sec. II we derive our result for multiple scattering of a projectile off different struck nucleons. In Sec. III we consider the lowest order reflection term when the projectile rescatters on the same nucleon. In this Section we also formulate the general rules for calculation of any multiple scattering amplitude in optimal approximation. In Sec. IV we discuss the correction terms. Pauli corrections are not considered here.

II. MULTIPLE SCATTERING TERMS OF DIFFERENT STRUCK NUCLEONS

The projectile-nucleus scattering t matrix T_A satisfies the Lippmann-Schwinger equation

$$T_A = \sum_{i=1}^{A} V_i + \sum_{i=1}^{A} V_i G T_A, \tag{1}$$

where V_i is the potential between the projectile and nucleon i, and G is the full Green's function

$$G^{-1} = E - H - K_p, \tag{2}$$

where E is the total energy, H the full target Hamiltonian, and K_p the projectile kinetic energy. The first step in the treatment of Eq. (1) is to construct the t matrix for the scattering of the projectile from *one* nucleon bound in the field of the others. This t matrix τ satisfies the equation

$$\tau_i = V_i + V_i G \tau_i. \tag{3}$$

Then, introducing the auxiliary t matrices T_A^i

$$T_A^i = V_i + V_i G T_A, \tag{4}$$

in terms of which

$$T_A = \sum_i T_A^i, \tag{5}$$

we relate the scattering operators T_A^i and τ through a system of coupled equations[1]:

$$T_A^i = \tau_i + \tau_i G \sum_{j \neq i} T_A^j. \tag{6}$$

Equations (5) and (6) are exact expressions of the projectile-nucleus scattering operator T_A. The formal iterative solution of Eqs. (5) and (6) is

$$T_A = \sum_i \tau_i + \sum_{i\neq j} \tau_i G\tau_j + \sum_{\substack{i\neq j\\ j\neq k}} \tau_i G\tau_j G\tau_k + \cdots . \quad (7)$$

Equation (7) expresses T_A as a series of single ($T_A^{(1)}$), double ($T_A^{(2)}$), ..., n-order scatterings ($T_A^{(n)}$) of the projectile on the nucleons bound in the nucleus. As such, this formal series is a good starting point for many approximate solutions of the projectile-nucleus scattering problem. However, every term of the multiple scattering series includes the operators τ, which are solutions of the many-body scattering equation (3), and Green's functions G containing the full target Hamiltonian. Hence, for the practical treatment of multiple scattering series we look for approximations to τ and G.

In a previous paper[2] we sought an approximation for the first term of the multiple scattering series (7)— τ_i. In the following we will concentrate on the optimal approximation for the multiple scattering terms $T_A^{(n)}$ ($n \geq 2$) in the multiple scattering series (7). We show how best to choose the approximations for the Green's functions G and operators τ, which define the approximation $T_a^{(n)}$ to each multiple scattering term $T_A^{(n)}$, so that the first correction to $T_A^{(n)} - T_a^{(n)}$ vanishes for the elastic scattering from the target ground state. We also derive expressions for the higher order nonvanishing correction terms and discuss their magnitude relative to the main terms $T_a^{(n)}$.

Instead of carrying out the argument for the general case—with all the resultant notational complications—we will present it for a special example and state the generalization later.

Consider the scattering of a projectile of mass μ and energy E_p from a target of two nucleons, each of mass m, bound to an infinitely massive core, allowing for their mutual interactions. We take the potential between the projectile and nucleon i to be V_i, that between target nucleon i and the core to be $\tilde{V}_i$, and the one between target nucleons $\tilde{V}_{12}$. All the interactions are taken to be local and we assume that there is no projectile-core interaction.

Equation (7) reads in this case

$$T_A \equiv T = \tau_1 + \tau_2 + \tau_1 G\tau_2 + \tau_2 G\tau_1 + \tau_1 G\tau_2 G\tau_1 + \cdots . \quad (8)$$

We consider separately the terms describing the scattering on different struck nucleons and the terms which include rescattering on the same nucleon (reflections). In our example the reflections are the triple and higher order term in the multiple scattering series (8).

The double scattering amplitude of Eq. (8) for elastic scattering from the target bound state reads

$$\begin{aligned}\langle\psi_0,\vec{p}\,|T^{(2)}(E_p)|\,\psi_0,\vec{p}\,'\rangle = \int &\psi_0(\vec{P}_1-\vec{p},\vec{P}_2-\vec{p})\langle\vec{p},\vec{P}_1,\vec{P}_2|\tau_1|\vec{p}_1,\vec{P}_1'',\vec{p}_2''\rangle\\ &\times\langle\vec{p}_1,\vec{P}_1'',\vec{P}_2''|G|\vec{p}_2,\vec{P}_1''',\vec{P}_2'''\rangle\langle\vec{p}_2,\vec{P}_1''',\vec{P}_2'''|\tau_2|\vec{p}\,',\vec{P}_1',\vec{P}_2'\rangle\psi_0^*(\vec{P}_1'-\vec{p}\,',\vec{P}_2'-\vec{p}\,')\\ &\times d^3P_1d^3P_2d^3P_1'd^3P_2'd^3P_1''d^3P_2''d^3P_1'''d^3P_2'''d^3p_1d^3p_2\,, \end{aligned} \quad (9)$$

where $\vec{p}$ and $\vec{p}\,'$ are initial and final projectile momenta, $E_p = p^2/2\mu = p'^2/2\mu$ is the projectile energy, and $\vec{P}_i$ is the total projectile plus nucleon i momentum ($\vec{P}_i = \vec{p} + \vec{Q}_i$). ψ_0 is the bound state wave function with binding energy B. It satisfies $H\psi_0 = -B\psi_0$, which reads in momentum space

$$\begin{aligned}\left(\frac{Q_1^2}{2m}+\frac{Q_2^2}{2m}\right)\psi_0(\vec{Q}_1,\vec{Q}_2) + \int \{&\tilde{V}_1(\vec{Q}_1-\vec{Q}_1')\delta^3(\vec{Q}_2-\vec{Q}_2') + \tilde{V}_2(\vec{Q}_2-\vec{Q}_2')\delta^3(\vec{Q}_1-\vec{Q}_1')\\ &+\tilde{V}_{12}(\vec{Q}_1-\vec{Q}_1')\delta^3(\vec{Q}_1+\vec{Q}_2-\vec{Q}_1'-\vec{Q}_2')\}\psi_0(\vec{Q}_1',\vec{Q}_2')d^3Q_1'd^3Q_2' = -B\psi_0(\vec{Q}_1,\vec{Q}_2)\,,\end{aligned} \quad (10)$$

where we have used explicitly the fact that the nucleon potentials $\tilde{V}$ are local. G is the full Green's function defined by Eq. (2) with $E = E_p - B$. (It is diagonal in the projectile momentum $\vec{p}_{1(2)}$.) We will approximate the full Green's function G by a Green's function G_a, which does not involve the total Hamiltonian. In the same way as in the previous paper[2] we look for G_a in the form

$$\langle\vec{p}_1,\vec{P}_1'',\vec{P}_2''|G_a|\vec{p}_2,\vec{P}_1''',\vec{P}_2'''\rangle = \frac{\delta^3(\vec{P}_1''-\vec{P}_1''')\,\delta^3(\vec{P}_2''-\vec{P}_2''')\,\delta^3(\vec{p}_1-\vec{p}_2)}{\epsilon - p_1^2/2\mu}, \quad (11)$$

where ϵ is to be determined so that the first order correction to $T^{(2)} - T_a^{(2)}$ is zero. The quantity ϵ may depend on the projectile momentum $\vec{p}_1$ and on external parameters, but it does not depend on the total projectile-nucleon momentum $\vec{P}_i$. The full Green's function G can be written as an expansion in the operator hG_a,

$$G = G_a + G_a hG_a + G_a hG_a hG_a + \cdots , \quad (12)$$

where

$$h = G_a^{-1} - G^{-1} \equiv \epsilon - E_p + B + H. \tag{13}$$

The operators τ_i in Eq. (9) we approximate by a scattering matrix t_i of the form

$$t_i = V_i + V_i G_a^{(i)} t_i , \tag{14}$$

where the Green's function $G_a^{(i)}$ is taken in the form

$$\langle \vec{p}, \vec{P}_1, \vec{P}_2 | G_a^{(i)} | \vec{p}', \vec{P}_1, \vec{P}_2 \rangle = \frac{\delta^3(\vec{P}_1 - \vec{P}_1')\, \delta^3(\vec{P}_2 - \vec{P}_2')\, \delta^3(\vec{p} - \vec{p}')}{\epsilon_i - p^2/2\mu} . \tag{14a}$$

The quantity ϵ_i is also to be determined so that the first correction to $T^{(2)} - T_a^{(2)}$ is zero and ϵ_i may depend on the projectile momentum $\vec{p}$ and on external parameters, but it does not depend on the total projectile-nucleon momentum $\vec{P}_i$. The dependence of ϵ_i on the projectile momentum and external parameters may in principle be quite different from that of quantity ϵ. Given (3) and (14) we can write[2]

$$\tau_i = t_i + t_i G_a^{(i)} h_i G_a^{(i)} t_i + t_i G_a^{(i)} h_i G_a^{(i)} h_i G_a^{(i)} t_i + t_i G_a^{(i)} h_i G_a^{(i)} t_i G_a^{(i)} h_i G_a^{(i)} t_i + \cdots , \tag{15}$$

where

$$h_i = G_a^{(i)-1} - G^{-1} \equiv \epsilon_i - E_p + B + H. \tag{16}$$

The scattering matrix t_i [Eq. (14] will conserve total projectile-nucleon i momentum $\vec{P}_i$ and nucleon momentum $\vec{Q}_j = \vec{P}_j - \vec{p}$ $(j \neq i)$ and be independent of $\vec{P}_i$ and $\vec{Q}_j$ so we can write[2]

$$\langle \vec{p}, \vec{P}_i, \vec{Q}_j | t_i | \vec{p}_1, \vec{P}_i', \vec{Q}_j' \rangle = \langle \vec{p} | \hat{t}_i | \vec{p}_1 \rangle \delta^3(\vec{P}_i - \vec{P}_i') \delta^3(\vec{Q}_j - \vec{Q}_j'). \tag{17}$$

Substituting (12) and (15) into (8) we see that the double scattering term can be expressed as a power series in operators hG_a and $G_a^{(i)}\, t_i$ of which the zero order term and the terms including the first power of the operator h are

$$\begin{aligned} T^{(2)} = \tau_1 G \tau_2 &= t_1 G_a t_2 + t_1 G_a^{(1)} h_1 G_a^{(1)} t_1 G_a t_2 + t_1 G_a h G_a t_2 + t_1 G_a t_2 G_a^{(2)} h_2 G_a^{(2)} t_2 + \cdots \\ &\equiv t_1 G_a t_2 + \Delta_1' T^{(2)} + \Delta_1'' T^{(2)} + \Delta_1''' T^{(2)} + \cdots . \end{aligned} \tag{18}$$

Consider the matrix element of $\Delta_1' T^{(2)}$ for elastic scattering from the target bound state. (The condition of vanishing of this matrix element will determine the quantity ϵ_1 in the Green's function $G_a^{(1)}$ defining t_1.) Using (14a), (16), and (17) it reads

$$\begin{aligned} \langle \Psi_0, \vec{p} | \Delta_1' T^{(2)} | \Psi_0, \vec{p}' \rangle = \int & \psi_0(\vec{P}_1 - \vec{p}, \vec{P}_2 - \vec{p}) \langle \vec{p} | \hat{t}_1 | \vec{p}_1' \rangle \delta^3(\vec{P}_2 - \vec{p} - \vec{P}_2'' + \vec{p}_1') \\ & \times \frac{\langle \vec{p}_1', \vec{P}_1, \vec{P}_2'' | h_1 | \vec{p}_1', \vec{P}_1'', \vec{P}_2''' \rangle}{(\epsilon_1 - p_1'^2/2\mu)^2} \langle \vec{p}_1' | \hat{t}_1 | \vec{p}_1 \rangle \delta^3(\vec{P}_2''' - \vec{p}_1' - \vec{P}_2' + \vec{p}_1) \\ & \times \frac{\langle \vec{p}_1 | \hat{t}_2 | \vec{p}' \rangle}{\epsilon - p_1^2/2\mu} \delta^3(\vec{P}_1'' - \vec{p}_1 - \vec{P}_1' + \vec{p}') \psi_0^* (\vec{P}_1' - \vec{p}', \vec{P}_2' - \vec{p}') d^3p_1 d^3p_1' d^3P_1 \cdots d^3P_1'' d^3P_2 \cdots d^3P_2''' , \end{aligned} \tag{19}$$

where we used the fact that h_1 [Eq. (16)] is diagonal in the projectile momentum $\vec{p}_1'$. For the matrix element of h_1 in Eq. (19) we have

$$\begin{aligned} \langle \vec{p}_1', \vec{P}_1, \vec{P}_2'' | \epsilon_1 - E_p + B + H | \vec{p}_1', \vec{P}_1'', \vec{P}_2''' \rangle = & \left[\epsilon_1 - E_p + B + \frac{(\vec{P}_1 - \vec{p}_1')^2}{2m} + \frac{(\vec{P}_2'' - \vec{p}_1')^2}{2m} \right] \\ & \times \delta^3(\vec{P}_1 - \vec{P}_1'') \delta^3(\vec{P}_2'' - \vec{P}_2''') + \tilde{V}(\vec{P}_1 - \vec{P}_1'', \vec{P}_2'' - \vec{P}_2''') , \end{aligned} \tag{20}$$

where

$$\tilde{V}(\vec{P}_1 - \vec{P}_1'', \vec{P}_2'' - \vec{P}_2''') \equiv \tilde{V}_1(\vec{P}_1 - \vec{P}_1'') \delta^3(\vec{P}_2'' - \vec{P}_2''') + \tilde{V}_2(\vec{P}_2'' - \vec{P}_2''') \delta^3(\vec{P}_1 - \vec{P}_1'') + \tilde{V}_{12}(\vec{P}_1 - \vec{P}_1'') \delta^3(\vec{P}_1 + \vec{P}_2'' - \vec{P}_1'' - \vec{P}_2''') . \tag{20a}$$

After substituting (20) into (19) we use the Schrödinger equation (10) to eliminate $\tilde{V}$:

$$\int \psi_0(\vec{P}_1 - \vec{p}, \vec{P}_2'' - \vec{p}_1') \tilde{V}(\vec{P}_1 - \vec{P}_1'', \vec{P}_2'' - \vec{P}_2''') d^3P_1 d^3P_2'' = \left[-B - \frac{(\vec{P}_1'' - \vec{p})^2}{2m} - \frac{(\vec{P}_2''' - \vec{p}_1')^2}{2m} \right] \psi_0(\vec{P}_1'' - \vec{p}, \vec{P}_2''' - \vec{p}_1') . \tag{20b}$$

Then we can rewrite Eq. (19) as

$$\langle\psi_0,\vec{p}\,|\Delta_1' T^{(2)}|\,\psi_0,\vec{p}'\rangle = \int \psi_0(\vec{P}_1''-\vec{p},\vec{P}_2'-\vec{p}_1)\left[\epsilon_1 - E_p + \frac{(\vec{P}_1''-\vec{p}_1')^2}{2m} - \frac{(\vec{P}_1''-\vec{p})^2}{2m}\right] \times \psi_0^*(\vec{P}_1''-\vec{p}_1,\vec{P}_2'-\vec{p}')\,R(\vec{p}_1,\vec{p}_1')\,d^3p_1 d^3p_1' d^3P_1'' d^3P_2' . \quad (21)$$

In (21) $R(\vec{p}_1,\vec{p}_2)$ is the remaining part of integral (19), which does not depend on $\vec{P}_i$ variables. If we further use the fact that ψ_0 has a definite parity

$$\psi_0(\vec{Q}_1,\vec{Q}_2)\psi_0^*(\vec{Q}_1',\vec{Q}_2') = \psi_0(-\vec{Q}_1,-\vec{Q}_2)\psi_0^*(-\vec{Q}_1',-\vec{Q}_2') , \quad (22)$$

we find that

$$\int \psi_0(\vec{P}_1''-\vec{p},\vec{P}_2'-\vec{p}_1)\,\frac{\vec{P}_1''\cdot(\vec{p}-\vec{p}_1')}{m}\,\psi_0^*(\vec{P}_1''-\vec{p}_1,\vec{P}_2'-\vec{p}')\,d^3P_1''d^3P_2' = \frac{(\vec{p}+\vec{p}_1)\cdot(\vec{p}-\vec{p}_1')}{2m}\int \psi_0(\vec{P}_1''-\vec{p},\vec{P}_2'-\vec{p}_1) \times \psi_0^*(\vec{P}_1''-\vec{p}_1,\vec{P}_2'-\vec{p}')\,d^3P_1''d^3P_2' . \quad (22a)$$

We now choose ϵ_1 to be a function of $\vec{p}_1'$ and two *external parameters* $\vec{K}_1$, q_1 which has the form

$$\epsilon_1 = E_p - (\vec{K}_1-\vec{p}_1')^2\frac{1}{2m} + \left(\frac{q_1}{2}\right)^2\frac{1}{2m} . \quad (23)$$

Using (22a), and (23), one sees that Eq. (21) vanishes if

$$\vec{K}_1 = \frac{\vec{p}+\vec{p}_1}{2},\quad q_1 = |\vec{p}-\vec{p}_1| . \quad (23a)$$

Equation (23) gives the special form ϵ_1 should have in the propagator $G_a^{(1)}$ of Eq. (11) in order that the scattering operator t_1 of Eq. (14) be defined so that the elastic scattering matrix element of $\Delta_1' T^{(2)}$ vanishes. When Eq. (14) with such a propagator $G_a^{(1)}$ is considered, one sees that the matrix element $\langle\vec{p}\,|\hat{t}_1|\,\vec{p}_1\rangle$ is just the elementary projectile-nucleon scattering amplitude expressed in the Breit frame [Fig. 1(a)] at the total energy

$$E_1 = E_p + \frac{(\vec{p}-\vec{p}_1)^2}{8m} = \frac{p^2}{2\mu} + \frac{(\vec{p}-\vec{p}_1)^2}{8m} , \quad (24)$$

defined as if the struck nucleon is on the mass shell (cf Ref. 2). The parameters $\vec{K}_1$ and q_1 [Eq. 23(a)] define the total projectile-nucleon momentum and the struck nucleon kinetic energy $(\frac{1}{2}q_1)^2(1/2m)$. Since in general $|\vec{p}_1| \neq |\vec{p}|$, this amplitude is a half off-shell one.

The condition of vanishing of the matrix element $\langle\psi_0,\vec{p}\,|\Delta_1''' T^{(2)}|\,\vec{p}',\psi_0\rangle$ will determine the quantity ϵ_2 in the Green's function $G_a^{(2)}$ defining t_2. The matrix element $\langle\vec{p}_1|\hat{t}_2|\vec{p}'\rangle$ appears to be the elementary projectile-nucleon scattering amplitude [Fig. 1(b)] at the total energy

$$E_2 = E_p + \frac{(\vec{p}_1-\vec{p}')^2}{8m} = \frac{p^2}{2\mu} + \frac{(\vec{p}_1-\vec{p}')^2}{8m} . \quad (24a)$$

In terms of the more familiar t matrix, depending on relative momenta and c.m. energy we can write

$$\langle\vec{p}\,|\hat{t}_1|\,\vec{p}_1\rangle = t_1(E_{\rm eff}^{(1)},\vec{p}-\eta\vec{K}_1,\vec{p}_1-\eta\vec{K}_1) ,$$
$$\langle\vec{p}_1|\hat{t}_2|\vec{p}'\rangle = t_2(E_{\rm eff}^{(2)},\vec{p}_1-\eta\vec{K}_2,\vec{p}'-\eta\vec{K}_2) , \quad (25)$$

where $\vec{K}_1 = (\vec{p}+\vec{p}_1)/2$, $E_{\rm eff}^{(1)} = p^2/2\mu - K_1^2/2(m+\mu) + (\vec{p}-\vec{p}_1)^2/8m$, $\eta = \mu/(m+\mu)$, and $\vec{K}_2$, $E_{\rm eff}^{(2)}$ are defined by the same expressions with $\vec{p}\to\vec{p}'$.

The vanishing of the elastic projectile-target bound state scattering matrix element of the operator $\Delta_1'' T^{(2)}$ [Eq. (18)] will determine the quantity ϵ in the Green's function G_a. Using (11), (12), and (17) we can write

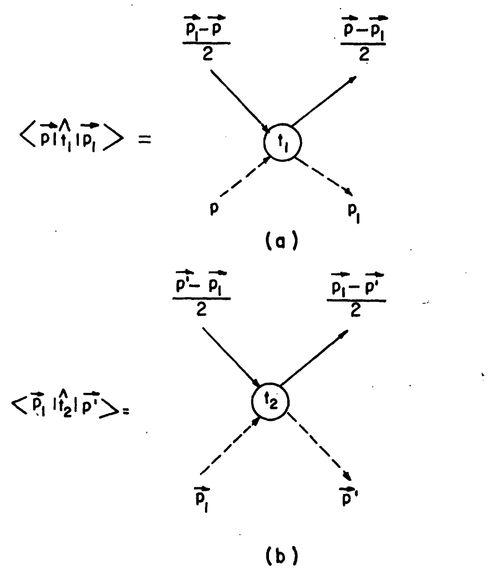

FIG. 1. (a,b) Schematic representation of the matrix elements $\langle\vec{p}\,|\hat{t}_1|\,\vec{p}_1\rangle$ and $\langle\vec{p}_1|\hat{t}_2|\vec{p}'\rangle$ as an amplitude for the projectile-(dashed line) nucleon (solid line) scattering.

$$\langle\psi_0,\vec{p}|\Delta_1''T^{(2)}|\vec{p}',\psi_0\rangle=\int\psi_0(\vec{P}_1-\vec{p},\vec{P}_2-\vec{p}_1)\frac{\langle\vec{p}|\hat{t}_1|\vec{p}_1\rangle}{\epsilon-p_1^2/2\mu}\langle\vec{p}_1,\vec{P}_1,\vec{P}_2|\epsilon-E_p+B+H|\vec{p}_1,\vec{P}_1',\vec{P}_2'\rangle$$
$$\times\frac{\langle\vec{p}_1|\hat{t}_2|\vec{p}'\rangle}{\epsilon-p_1^2/2\mu}\psi_0^*(\vec{P}_1'-\vec{p}_1,\vec{P}_2'-\vec{p}')\,d^3p_1d^3P_1d^3P_1'd^3P_2d^3P_2',\tag{26}$$

Using the Schrödinger Eq. (10) to eliminate $\tilde{V}$ in the Hamiltonian H we find

$$\langle\psi_0,\vec{p}|\Delta_1''T^{(2)}|\vec{p}',\psi_0\rangle=\int\psi_0(\vec{P}_1-\vec{p},\vec{P}_2-\vec{p}_1)\left[\epsilon-E_p+\frac{(\vec{P}_1-\vec{p}_1)^2}{2m}-\frac{(\vec{P}_1-\vec{p})^2}{2m}\right]\psi_0^*(\vec{P}_1-\vec{p}_1,\vec{P}_2-\vec{p}')$$
$$\times R(\vec{p}_1)\,d^3p_1d^3P_1d^3P_2,\tag{27}$$

where $R(\vec{p}_1)$ is the remaining part of the integrand (26), which does not depend on $\vec{P}_i$ variables. Using again the definite parity of ψ_0 [Eq. (22)] we find that Eq. (27) vanishes if we choose

$$\epsilon=E_p=\frac{p^2}{2\mu}.\tag{28}$$

This quantity defines the Green's function G_a [Eq. (11)] which reads now

$$\langle\vec{p}_1,\vec{P}_1'',\vec{P}_2''|G_a|\vec{p}_2,\vec{P}_1''',\vec{P}_2'''\rangle=\frac{\delta^3(\vec{P}_1''-\vec{P}_1''')\delta^3(\vec{P}_2''-\vec{P}_2''')\delta^3(\vec{p}_1-\vec{p}_2)}{p^2/2\mu-p_1^2/2\mu}.\tag{29}$$

Let us now see the consequence of using our scattering operators t_1, t_2 and Green's function G_a as an approximation for the t matrices τ_1, τ_2 and Green's function G in Eq. (9) for the double scattering term. We have for $T^{(2)}\sim T_a^{(2)}$

$$\langle\psi_0,\vec{p}|T_a^{(2)}|\vec{p}',\psi_0\rangle=\int\psi_0(\vec{P}_1-\vec{p},\vec{P}_2-\vec{p}_1)\frac{\langle\vec{p}|\hat{t}_1|\vec{p}_1\rangle\langle\vec{p}_1|\hat{t}_2|\vec{p}'\rangle}{p^2/2\mu-p_1^2/2\mu}\psi_0^*(\vec{P}_1-\vec{p}_1,\vec{P}_2-\vec{p}')\,d^3P_1d^3P_2d^3p_1$$
$$=\int\frac{t_1(E_{\text{eff}}^{(1)},\vec{p}-\eta\vec{K}_1,\vec{p}_1-\eta\vec{K}_1)\,t_2(E_{\text{eff}}^{(2)},\vec{p}_1-\eta\vec{K}_2,\vec{p}'-\eta\vec{K}_2)}{p^2/2\mu-p_1^2/2\mu}S_{00}(\vec{p}-\vec{p}_1,\vec{p}_1-\vec{p}')\,d^3p_1,\tag{30}$$

where S_{00} is

$$S_{00}(\vec{p}-\vec{p}_1,\vec{p}_1-\vec{p}')=\int\psi_0(\vec{P}_1-\vec{p},\vec{P}_2-\vec{p}_1)\psi_0^*(\vec{P}_1-\vec{p}_1,\vec{P}_2-\vec{p}')\,d^3P_1d^3P_2.\tag{31}$$

One can easily see that this quantity is two-nucleon density in momentum representation. In the independent nucleon model S_{00} equals the product of single nucleon form factors:

$$S_{00}(\vec{p}-\vec{p}_1,\vec{p}_1-\vec{p}')=S_{00}(\vec{p}-\vec{p}_1)S_{00}(\vec{p}_1-\vec{p}').\tag{32}$$

The integrand (30) for our optimal approximation of the double scattering amplitude factors into two-nucleon density containing nuclear information and a part dependent only on the projectile-nucleon input and this comes about from the fact that $\hat{t}_1$, $\hat{t}_2$, and G_a are independent of the total projectile-nucleon momentum $\vec{P}_i$.

The double scattering amplitude in the optimal approximation may be represented by the usual diagram, Fig. 2(a). However, contrary to standard treatment, it is calculated at total projectile-nucleon momenta $\vec{P}_1=(\vec{p}+\vec{p}_1)/2$, $\vec{P}_2=(\vec{p}_1+\vec{p}')/2$. The averaging over ground state wave functions is standard [Fig. 2(b)]. The amplitudes t_i in Fig. 2(b) are taken at energies E_i [Eqs. (24), and (24a)] set equal to the sum of the *on-shell* nucleon energy and the energy of the projectile.

The extension of our method to the case of finite nuclear mass (M) is straightforward. The derivation is simplest in the projectile-nucleus Breit frame. It requires the replacement of the nucleon

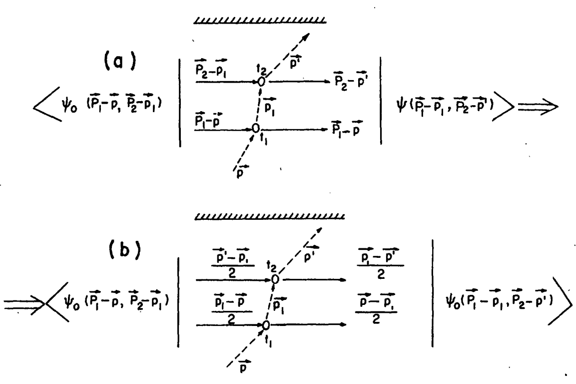

FIG. 2. (a, b) Schematic representation for the double scattering amplitude in the optimal approximation. The nucleons are bound to an infinitely massive core.

momentum by the relative nucleon-nucleus momentum, so the nuclear wave functions for initial and final states are

$$\psi_0\left(\vec{P}_1-\vec{p}+\frac{\vec{q}}{2M}m,\vec{P}_2-\vec{p}+\frac{\vec{q}}{2M}m\right)$$

and

$$\psi_0\left(\vec{P}_1'-\vec{p}'-\frac{\vec{q}}{2M}m,\vec{P}_2'-\vec{p}'-\frac{\vec{q}}{2M}m\right),$$

where $\vec{q}=\vec{p}-\vec{p}'$, and the nuclear Hamiltonian now contains the kinetic energy of mass $M-2m$. This changes our result (30) only by replacing $S_{00}(p-p_1,p_1-p')$ with

$$S_{00}\left(\vec{p}-\vec{p}_1-\frac{\vec{q}}{M}m,\vec{p}_1-\vec{p}'-\frac{\vec{q}}{M}m\right).$$

The extension of our procedure to a many-body target for the n-order multiple scattering amplitude with *different* struck nucleons is simple. Performing the same procedure as for the double scattering term we find that the optimal approximation $T_a^{(n)}$ for the n-order amplitude $T_A^{(n)}$ can be written in the projectile-nucleus Breit frame as

$$\langle\psi_0,\vec{p}\,|T_a^{(n)}|\,\vec{p}',\psi_0\rangle=\int\frac{t_1(E_{\rm eff}^{(1)},\vec{p}-\eta\vec{K}_1,\vec{p}_1-\eta\vec{K}_1)\,t_2(E_{\rm eff}^{(2)},\vec{p}_1-\eta\vec{K}_2,\vec{p}_2-\eta\vec{K}_2)\cdots t_n(E_{\rm eff}^{(n)},\vec{p}_{n-1}-\eta\vec{K}_n,\vec{p}'-\eta\vec{K}_n)}{(p^2/2\mu-p_1^2/2\mu)(p^2/2\mu-p_2^2/2\mu)\cdots(p^2/2\mu-p_{n-1}^2/2\mu)}$$
$$\times S_{00}\left(\vec{p}-\vec{p}_1-\frac{\vec{q}}{M}m,\vec{p}_1-\vec{p}_2-\frac{\vec{q}}{M}m,\ldots,\vec{p}_{n-1}-\vec{p}'-\frac{\vec{q}}{M}m\right)d^3p\cdots d^3p_{n-1}\,, \qquad (33)$$

where

$$\vec{K}_1=\frac{\vec{p}+\vec{p}_1}{2},\quad \vec{K}_{j\neq1,n}=\frac{\vec{p}_{j-1}+\vec{p}_j}{2},\quad \vec{K}_n=\frac{\vec{p}_{n-1}+\vec{p}'}{2},\quad E_{\rm eff}^{(j)}=\frac{p^2}{2m}-\frac{K_j^2}{2(m+\mu)}+\frac{(\vec{K}_j-\vec{p}_j)^2}{2m}$$

($\vec{p}_n\equiv\vec{p}'$, $\vec{p}$, and $\vec{p}'$ are the projectile momenta in the projectile-nucleus Breit frame), and

$$S_{00}\left(\vec{p}-\vec{p}_1-\frac{\vec{q}}{M}m,\ldots,\vec{p}_{n-1}-\vec{p}_n-\frac{\vec{q}}{M}m\right)=\int\psi_0\left(\vec{P}_1-\vec{p}+\frac{\vec{q}}{2M}m,\ldots,\vec{P}_n-\vec{p}_{n-1}+\frac{\vec{q}}{2M}m\right)$$
$$\times\psi_0^*\left(\vec{P}_1-\vec{p}_1-\frac{\vec{q}}{2M}m,\ldots,\vec{P}_n-\vec{p}'-\frac{\vec{q}}{2M}m\right)d^3P_1\cdots d^3P_n\,. \qquad (34)$$

We see that the integrand (33) also factors into multiparticle nuclear density in momentum representation S_{00} containing only nuclear information and the projectile part dependent on the projectile-nucleon input. The amplitude $T_a^{(n)}$ [Eq. (33)] may be represented by the diagram [Fig. 3(a)] which is calculated at nucleon momenta corresponding to

$$\vec{P}_1\to\frac{\vec{p}+\vec{p}_1}{2},\ldots,\vec{P}_n\to\frac{\vec{p}_{n-1}+\vec{p}'}{2},$$

but averaged over ground state wave functions in the usual way [Fig. 3(b)]. The amplitude t_i [in Fig. 3(b)] is taken at an energy equal to the sum of the projectile energy after $(i-1)$ collisions, which is $p^2/2\mu$ as can be easily seen from Fig. 3(b), and the on-shell energy $[(\vec{p}_i-\vec{p}_{i-1})/2]^2\,(1/2m)$ of nucleon i. One sees from Fig. 3(b) that the optimal approximation for the n-order scattering amplitude corresponds to scattering off n on-shell nucleons which have all the momentum of the nucleus while the core remains at rest.

Now we comment on the relation between our optimal approximation $T_a^{(n)}$ to the multiple scattering amplitudes $T_A^{(n)}$ and the first order optical potential. Applying the usual arguments for the derivation of the standard first order optical potential from the multiple scattering series, i.e., approximating the projectile-nucleus elastic amplitude $T_A^{00}(E,\vec{p},\vec{p}')$ by the sum of multiple scattering terms off different struck nucleons and assuming the nucleons to be uncorrelated,[3] we easily find that the equation for T_A^{00} in our optimal approximation reads

$$\frac{A-1}{A}T_A^{00}(E,\vec{p},\vec{p}')=U_a^{\rm opt}(E,\vec{p},\vec{p}')+\int U_a^{\rm opt}(E,\vec{p},\vec{p}'')\frac{d^3p''}{p^2/2\mu-p''^2/2\mu}\times\left(\frac{A-1}{A}\right)T_A^{00}(E,\vec{p}'',\vec{p}'), \qquad (35)$$

where $E=p^2/2\mu$ and

$$U_a^{\rm opt}(E,\vec{p},\vec{p}')=(A-1)t(E^{\rm eff},\vec{p}-\eta\vec{K},\vec{p}'-\eta\vec{K})S_{00}(\vec{p}-\vec{p}')\,, \qquad (36)$$

with $\vec{K}=(\vec{p}+\vec{p}')/2$,

$$E^{\rm eff}=E-\left(\frac{\vec{p}+\vec{p}'}{2}\right)^2\frac{1}{2(m+\mu)}+\left(\frac{\vec{p}-\vec{p}'}{2}\right)^2\frac{1}{2m},$$

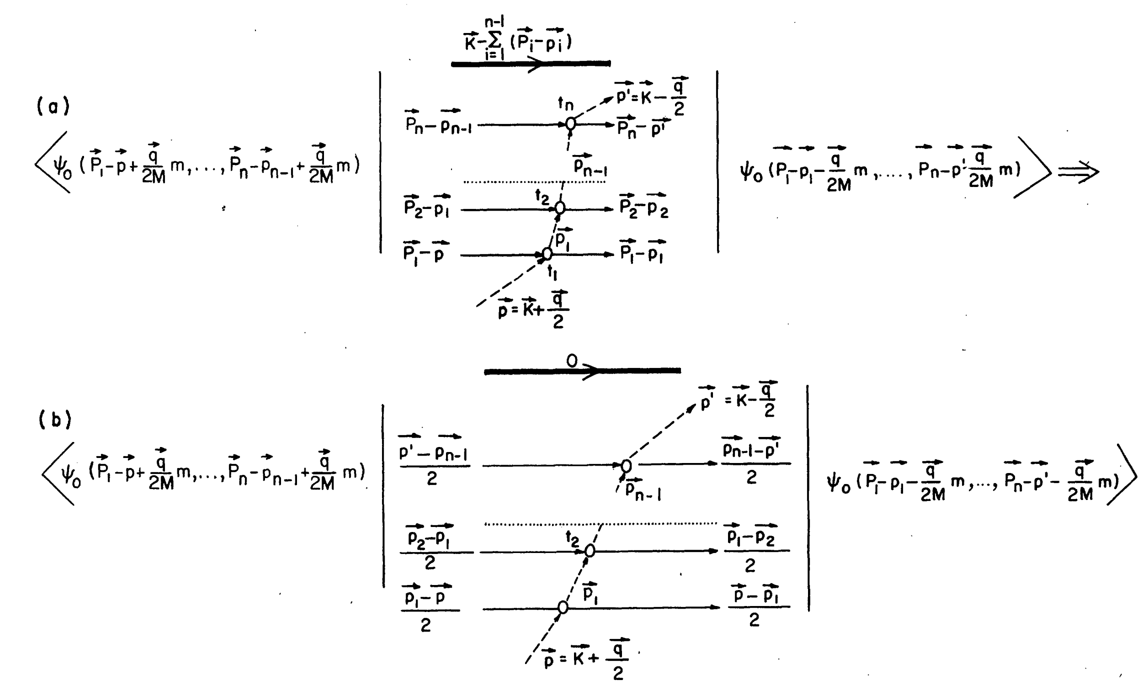

FIG. 3. (a,b) Schematic representation for the n-order multiple scattering amplitude in the optimal approximation for the case of finite nuclear mass.

and S_{00} is the single nucleon form factor.

III. REFLECTION TERMS

We now consider the multiple scattering terms which include rescattering on the same nucleon. We take for simplicity the previous model of a projectile scattering on two nucleons bound to an infinitely heavy core. The first reflection term the multiple scattering series (8) is the triple scattering term $T^{(3)} = \tau_1 G \tau_2 G \tau_1$. We again approximate the operators τ_i and G by t_i and G_a [Eqs. (11) and (14)] which will be defined so that the first correction to $T_a^{(3)} - T^{(3)}$ vanishes. It can be written as

$$\langle \psi_0, \vec{p}\,|\Delta_1 T^{(3)}|\,\vec{p}\,', \psi_0\rangle = \langle \psi_0, \vec{p}\,|\,\delta\tau_1 G_a t_2 G_a t_1 + t_1 \delta G t_2 G_a t_1 + t_1 G_a \delta\tau_2 G_a t_1 + t_1 G_a t_2 \delta G t_1 + t_1 G_a t_2 G_a \delta\tau_1\,|\,\psi_0, \vec{p}\,'\rangle = 0\,, \quad (37)$$

where $\delta\tau_i = t_i h_i G_a^2 t_i$, $\delta G = G_a h G_a$, and h and h_i are given by Eqs. (13) and (16). Setting the matrix element of each term in Eq. (37) to be equal to zero, we determine the operators t_i and G_a. By a similar procedure to that for the double scattering term above and using the definite parity of ψ_0 [Eq. (22)] we find finally after some algebra for the $T_a^{(3)} \sim T^{(3)}$ amplitude

$$\langle \psi_0, \vec{p}\,|\,T_a^{(3)}\,|\,\vec{p}\,', \psi_0\rangle = \int \frac{t_1(E_{eff}^{(1)}, \vec{p} - \eta\vec{K}_1, \vec{p}_1 - \eta\vec{K}_1)\, t_2(E_{eff}^{(2)}, \vec{p}_1 - \eta\vec{K}_2, \vec{p}_2 - \eta\vec{K}_2)}{[p^2/2\mu - p_1^2/2\mu + (\vec{p} - \vec{p}_1)\cdot(\vec{p}_2 - \vec{p}\,')/2m]\,[p^2/2\mu - p_2^2/2\mu + (\vec{p} - \vec{p}_1)(\vec{p}_2 - \vec{p}\,')/2m]}$$

$$\times\, t_1(E_{eff}^{(3)}, \vec{p}_2 - \eta\vec{K}_3, \vec{p}\,' - \eta\vec{K}_3)\, S_{00}(\vec{p} - \vec{p}_1 + \vec{p}_2 - \vec{p}\,', \vec{p}_1 - \vec{p}_2)\, d^3p_1\, d^3p_2\,, \quad (38)$$

where S_{00} is two-nucleon density in momentum representation defined by (31) and the amplitudes t_i are written in terms of the c.m. energy and relative momenta, so that

$$\vec{K}_1 = \frac{\vec{p} + \vec{p}_1 - \vec{p}_2 + \vec{p}\,'}{2}\,, \quad E_{eff}^{(1)} = \frac{p^2}{2\mu} - \frac{K_1^2}{2(m+\mu)} + \left(\frac{\vec{p} - \vec{p}_1 + \vec{p}_2 - \vec{p}\,'}{2}\right)^2 \frac{1}{2m}\,,$$

$$\vec{K}_2 = \frac{\vec{p}_1 + \vec{p}_2}{2}\,, \quad E_{eff}^{(2)} = \frac{p^2}{2\mu} - \frac{K_2^2}{2(m+\mu)} + \frac{(\vec{p}_1 - \vec{p}_2)^2}{8m} + \frac{(\vec{p} - \vec{p}_1)\cdot(\vec{p}_2 - \vec{p}\,')}{2m}\,, \quad (39)$$

$$\vec{K}_3 = \frac{\vec{p} - \vec{p}_1 + \vec{p}_2 + \vec{p}\,'}{2}\,, \quad E_{eff}^{(3)} = \frac{p^2}{2\mu} - \frac{K_3^2}{2(m+\mu)} + \left(\frac{\vec{p} - \vec{p}_1 + \vec{p}_2 - \vec{p}\,'}{2}\right)^2 \frac{1}{2m}\,.$$

The integrand (38) again has a factorized form since $\hat{t}_i$ and G_a are independent of the total projectile-nucleon momenta $\vec{P}_i$. Using the identities for the propagators in Eq. (38)

$$\frac{p^2}{2\mu}-\frac{p_i^2}{2\mu}+\frac{(\vec{p}-\vec{p}_1)\cdot(\vec{p}_2-\vec{p}')}{2m}\equiv\frac{p^2}{2\mu}+\left(\frac{-\vec{p}+\vec{p}_1-\vec{p}_2+\vec{p}'}{2}\right)^2\frac{1}{2m}-\frac{p_i^2}{2\mu}-\left(\frac{\vec{p}-\vec{p}_1-\vec{p}_2+\vec{p}'}{2}\right)^2\frac{1}{2m}, \quad (40)$$

we can easily show that the amplitude $T_a^{(3)}$ may be represented by the usual diagram [Fig. 4(a)] which is calculated, however, at values of the total projectile-nucleon i momenta $\vec{P}_i$ chosen in such a way that the initial momentum of nucleon i equals its final momentum but with opposite sign [Fig. 4(b)], and is averaged over the ground state wave function in a standard way. The amplitude t_i, where index i is the number of the collision, is taken at an energy E_i $[E_i = E_{\text{eff}}^{(i)} + K_i^2/2(m+\mu)]$ equal to the sum of the on-shell energy $E_{\text{kin}}^{(j)}$ of nucleon i and the energy of the projectile after $i-1$ collision, i.e.,

$$E_i = E_{\text{kin}}^{(i)} + \frac{p^2}{2\mu} + \sum_{j=1}^{i-1}(E_{\text{kin}}^{(j)} - E_{\text{kin}}^{(j)\prime}), \quad (41)$$

where $E_{\text{kin}}^{(j)}$ and $E_{\text{kin}}^{(j)\prime}$ are the kinetic energies of the nucleon before and after collision j.

We can show that these rules, which we have found for the calculation of the optimal approximation to the first reflection term and also to the multiple scattering terms off different struck nucleons, are valid for any term of the multiple scattering series. In the case of a finite nuclear mass these rules remain the same if we calculate the multiple scattering amplitude in the projectile-nucleus Breit frame, except that the nucleon momenta in the nuclear wave function should be replaced by the relative nucleon-nucleus momenta. [One sees easily that these rules require that the struck nucleons have all the momentum of the nucleus during the collision, cf Fig. 3(b).]

We note that such factorized forms for optimal approximation of multiple scattering terms of *different* struck nucleons [Eq. (33)] have been used before[4] without derivation and demonstration that they are the first terms of a systematic expansion designed to minimize corrections. However, the optimal approximation formulas for the reflection terms [such as Eq. (38)] have not been used before to our knowledge. The comparison of our expressions for the reflection terms with those calculated in the fixed scatterer approximation (as in the Brueckner model[5] shows that the optimal approximation results in the modification of the energies in the elementary amplitudes and also in the modification of the propagators of the projectile.

IV. CORRECTION TERMS

In this section we investigate the first nonvanishing corrections to $T_A^{(n)} - T_a^{(n)}$ with our optimal choice of t_i and G_a in $T_a^{(n)}$. We study the double scattering term $T^{(2)}$ in the previous model of a projectile scattering on two nucleons bound to an infinitely heavy core. Consider the expansions (12), (15), and (18) defining $T^{(2)}$ and keep the terms up to second order in the operators h. Since the operators G_a and h commute,[2] the matrix element for the first nonvanishing correction can be written as

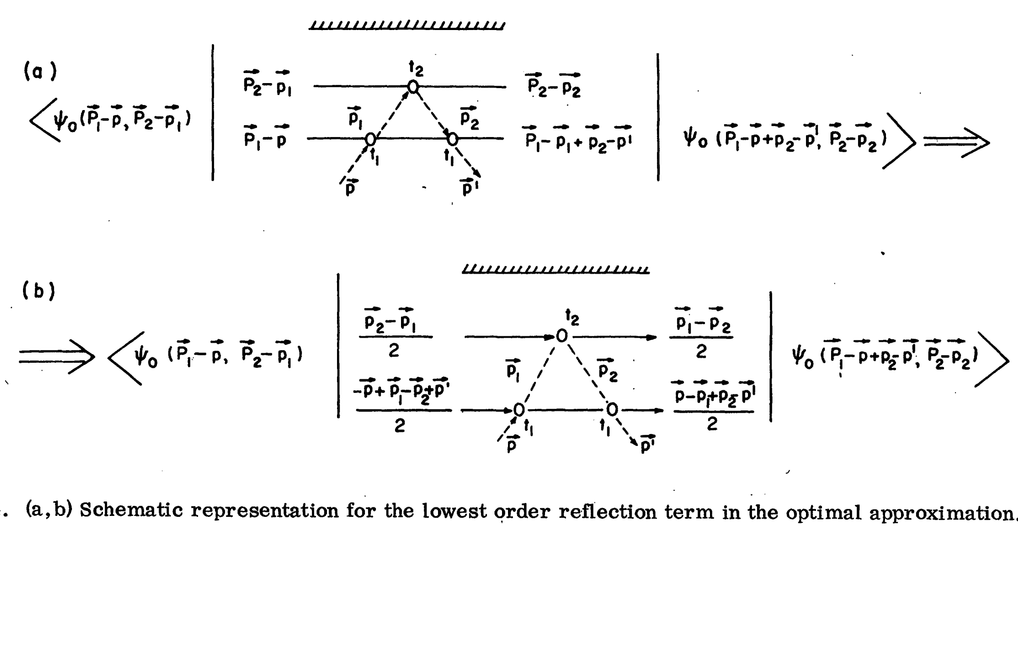

FIG. 4. (a,b) Schematic representation for the lowest order reflection term in the optimal approximation.

$$\begin{aligned}\langle\psi_0,\vec{p}\,|\Delta_2 T^{(2)}|\,\psi_0,\vec{p}\,'\rangle = &\langle\psi_0,\vec{p}\,|\,\hat{t}_1 h_1 G_a^{(1)} + (G_a^{(1)} + G_a^{(1)}\hat{t}_1 G_a^{(1)})\,G_a^{(1)} h_1 \hat{t}_1 G_a \hat{t}_2\,|\,\psi_0,\vec{p}\,'\rangle \\ &+ \langle\psi_0,\vec{p}\,|\,\hat{t}_1 G_a \hat{t}_2 h_2 G_a^{(2)}(G_a^{(2)} + G_a^{(2)}\hat{t}_2 G_a^{(2)})G_a^{(2)} h_2 \hat{t}_2\,|\,\psi_0,\vec{p}\,'\rangle \\ &+ \langle\psi_0,\vec{p}\,|\,\hat{t}_1 h_1 (G_a^{(1)})^2 \hat{t}_1 h G_a{}^2 \hat{t}_2 + \hat{t}_1 h G_a{}^2 \hat{t}_2 h_2 (G_a^{(2)})^2 \hat{t}_2\,|\,\psi_0,\vec{p}\,'\rangle \\ &+ \langle\psi_0,\vec{p}\,|\,\hat{t}_1 h_1 (G_a^{(1)})^2 \hat{t}_1 G_a \hat{t}_2 h_2 (G_a^{(2)})^2 \hat{t}_2\,|\,\psi_0,\vec{p}\,'\rangle + \langle\psi_0,\vec{p}\,|\,\hat{t}_1 G_a{}^3 h^2 \hat{t}_2\,|\,\psi_0,\vec{p}\,'\rangle\,. \end{aligned} \tag{42}$$

Assuming definite parity of ψ_0 *separately in each nucleon momentum* which holds in general for the independent nuclear model

$$\psi_0(\vec{Q}_1,\vec{Q}_2)\psi_0^*(\vec{Q}_1',\vec{Q}_2') = \psi_0(-\vec{Q}_1,\vec{Q}_2)\psi_0^*(-\vec{Q}_1',\vec{Q}_2')$$

and the same for momentum $\vec{Q}_2$, we find that only the first two matrix elements in Eq. (42) contribute. All the other terms vanish. Indeed, consider for example the last term $\Delta_2^{\rm V} T^{(2)}$ of Eq. (42) which reads

$$\begin{aligned}\langle\psi_0,\vec{p}\,|\Delta_2^{\rm V} T^{(2)}|\,\psi_0,\vec{p}\,'\rangle = &\int \psi_0(\vec{P}_1-\vec{p},\vec{P}_2-\vec{p}_1)\,\langle\vec{p}\,|\,\hat{t}_1\,|\,\vec{p}_1\rangle\langle\vec{p}_1,\vec{P}_1,\vec{P}_2\,|\,h\,|\,\vec{p}_1,\vec{P}_1'',\vec{P}_2''\rangle\langle\vec{p}_1,\vec{P}_1'',\vec{P}_2''\,|\,h\,|\,\vec{p}_1,\vec{P}_1',\vec{P}_2'\rangle \\ &\cdot \frac{d^3p_1}{(p^2/2\mu - p_1{}^2/2\mu)^3}\langle\vec{p}_1\,|\,\hat{t}_2\,|\,\vec{p}\,'\rangle\psi_0^*(\vec{P}_1'-\vec{p}_1,\vec{P}_2'-\vec{p}\,')\,d^3P_1\cdots d^3P_2''\,. \end{aligned} \tag{43}$$

Using Eqs. (10) and (20) and performing the $\vec{P}_i$ integration we find

$$\begin{aligned}\langle\psi_0,\vec{p}\,|\Delta_2^{\rm V} T^{(2)}|\,\psi_0,\vec{p}\,'\rangle = &\int \psi_0(\vec{P}_1-\vec{p},\vec{P}_2-\vec{p}_1)\left(\frac{(\vec{P}_1-\vec{p}_1)^2}{2m} - \frac{(\vec{P}_1-\vec{p})^2}{2m}\right)\left(\frac{(\vec{P}_2-\vec{p}_1)^2}{2m} - \frac{(\vec{P}_2-\vec{p}\,')^2}{2m}\right) \\ &\times \psi_0^*(\vec{P}_1-\vec{p}_1,\vec{P}_2-\vec{p}\,')\,R(\vec{p}_1)\,d^3P_1\,d^3P_2 = 0\,. \end{aligned} \tag{44}$$

$R(\vec{p}_1)$ is the $\vec{P}_i$-independent part of the integrand (44). The treatment of the first two nonvanishing terms $\Delta_2^{\rm I(II)} T^{(2)}$ in Eq. (42) is the same as the treatment of the correction term in our previous paper.[2] Evaluation of the terms requires some off-shell information for the elementary amplitude t_i. However, an estimation of these terms can be done without detailed input. One finds after some algebra (as in Ref. 2)

$$\frac{\Delta_2^{\rm I(II)} T^{(2)}}{T_a^{(2)}} \cong V_N{}^2 \bar{\mu} T_0\,; \tag{45}$$

where V_N is the rms velocity of the bound nucleon and T_0 is the projective-nucleon scattering time delay $[T_0 = (1/t)(dt/dE)]$. If t is relatively slow varying we can approximate T_0 as $1/E$ and $\Delta_2^{\rm I(II)} T^{(2)}$ is of order $(V_N/V_x)^2$ relative to the main term, where V_x is the projectile velocity.

The assumption of definite parity of ψ_0 for each nucleon variable is necessary for the vanishing of the last three terms in Eq. (42). (We notice that the vanishing of the first order corrections to the multiple scattering terms $T_a^{(n)}$ only requires definite parity of ψ_0 when all nucleon momenta change sign, which always holds for ψ_0.) Therefore, these terms do not equal zero, in principle, if one takes into account the nucleon correlations in the wave function. If, for example, the wave function ψ_0 depends on the sum and on the difference of nucleon momenta

$$\psi_0(\vec{Q}_1,\vec{Q}_2) \to \phi_0\left(\frac{\vec{Q}_1-\vec{Q}_2}{2},\ \vec{Q}_1+\vec{Q}_2\right) \equiv \phi_0(\vec{x},\vec{y})$$

and has a definite parity in each of these variables

$$\begin{aligned}\phi_0(\vec{x},\vec{y})\phi_0^*(\vec{x}\,',\vec{y}\,') &= \phi_0(-\vec{x},\vec{y})\phi_0^*(-\vec{x}\,',\vec{y}\,') \\ &= \phi_0(\vec{x}_1,-\vec{y})\phi_0^*(\vec{x}_1',-\vec{y}\,')\end{aligned} \tag{46}$$

all matrix elements in Eq. (42) may be easily evaluated. The first two terms remain of order $V_N{}^2\bar{\mu}T_0$ relative to the main term. The exact evaluation of the third and fourth terms of Eq. (42) requires off-shell input for the elementary t_i amplitudes (like that of the first two terms); however, the estimation shows that these terms are of the same order of magnitude as the first two terms.

The last term $\Delta_2^{\rm V} T^{(2)}$ of Eq. (42) corresponds to the correction from projectile propagation in the intermediate state. We now turn to a more detailed analysis of this correction term. We consider a more realistic example of a finite mass nucleus M. As was done above for the first order correction $\Delta_1 T^{(2)}$, we evaluate $\Delta_2^{\rm V} T^{(2)}$ in the projectile-nucleus Breit frame. In this case Eq. (43) only requires the replacement

$$\psi_0(\vec{P}_1-\vec{p},\vec{P}_2-\vec{p}_1)\to\psi_0\left(\vec{P}_1-\vec{p}+\frac{\vec{q}}{2M}m,\vec{P}_2-\vec{p}_1+\frac{\vec{q}}{2M}m\right)\equiv\phi_0\left(\frac{\vec{P}_1-\vec{P}_2+\vec{p}_1-\vec{p}}{2},\vec{P}_1+\vec{P}_2-\vec{p}-\vec{p}_1+\frac{\vec{q}}{M}m\right) \tag{47}$$

and

$$\psi_0(\vec{P}_1'-\vec{p}_1,\vec{P}_2'-\vec{p}')\to\phi_0\left(\frac{\vec{P}_1'-\vec{P}_2'+\vec{p}'-\vec{p}_1}{2},\vec{P}_1'+\vec{P}_2'-\vec{p}_1-\vec{p}_2-\frac{\vec{q}}{M}m\right),\quad \vec{q}=\vec{p}-\vec{p}', \tag{48}$$

and the operator h now contains the kinetic energy of mass $M-2m$. Using Eqs. (10) and (20), performing the $\vec{P}_i$ integration, and replacing

$$\vec{P}_1=\frac{\vec{p}+\vec{p}_1}{2}+\frac{\vec{\tilde{Q}}}{2}+\vec{Q};\quad \vec{P}_2=\frac{\vec{p}_1+\vec{p}'}{2}+\frac{\vec{\tilde{Q}}}{2}-\vec{Q},$$

we find after some algebra

$$\begin{aligned}\langle\psi_0,\vec{p}|\,\Delta_2^V T^{(2)}|\psi_0,\vec{p}'\rangle=&\int\frac{t_1(E_{\rm eff}^{(1)},\vec{p}-\eta\vec{K}_1,\vec{p}_1-\eta\vec{K}_1)\,t_2(E_{\rm eff}^{(2)},\vec{p}_1-\eta\vec{K}_2,\vec{p}'-\eta\vec{K}_2)}{(p^2/2\mu-p_1^2/2\mu)^3}d^3p_1\\ &\times\int\phi_0\left(\vec{\tilde{Q}}+\frac{\vec{p}_1-\vec{K}}{2},\vec{Q}-\frac{M-2m}{M}\vec{q}\right)\phi_0^*\left(\vec{\tilde{Q}}-\frac{\vec{p}_1-\vec{K}}{2},\vec{Q}+\frac{M-2m}{M}\vec{q}\right)\\ &\times\frac{(\vec{\tilde{Q}}/2+\vec{Q})\cdot(\vec{p}-\vec{p}_1)}{m}\,\frac{(\vec{\tilde{Q}}/2-\vec{Q})\cdot(\vec{p}'-\vec{p}_1)}{m}d^3\tilde{Q}\,d^3Q,\end{aligned} \tag{49}$$

where $\vec{p}$ and $\vec{p}'$ are the projectile momenta in the Breit projectile-nucleus system, $E_{\rm eff}$, $\vec{K}_1$, and $\vec{K}_2$ are defined by Eq. (25) and $\vec{K}=(\vec{p}+\vec{p}')/2$. To simplify the following discussion we consider scattering on the deuteron. In this case the integrand (49) must be multiplied by $\delta^{(3)}(\tilde{Q})$ which correctly accounts for the total momentum of the nucleons being $\mp\frac{1}{2}\vec{q}$ in the initial and final state. One sees also that

$$S_{00}(\vec{p}-\vec{p}_1,\vec{p}_1-\vec{p}')\to S_0\left(\vec{p}_1-\frac{\vec{p}+\vec{p}'}{2}\right)\equiv S_0(\vec{p}_1-\vec{K})$$

in the main term [Eq. (30)] where $S_0(\vec{Q})$ is the body form factor of the deuteron. Now we study the part of the integral (49) that depends upon the bound state wave function. We have integrals of the form

$$A_{ij}(\vec{p}_1-\vec{K})=-\int\phi_0\left(\vec{Q}+\frac{\vec{p}_1-\vec{K}}{2}\right)\phi_0^*\left(\vec{Q}-\frac{\vec{p}_1-\vec{K}}{2}\right)\frac{Q_iQ_j\,d^3Q}{m^2}=\frac{S_2(\vec{p}_1-\vec{K})\delta_{ij}}{3m^2}, \tag{50}$$

where we have used the symmetry of the wave functions to obtain δ_{ij} and introduced the quantity[2]

$$S_2(\vec{p}_1-\vec{K})=\int\phi_0\left(\vec{Q}+\frac{\vec{p}_1-\vec{K}}{2}\right)\phi_0^*\left(\vec{Q}-\frac{\vec{p}_1-\vec{K}}{2}\right)Q^2\,d^3Q. \tag{51}$$

$S_2(\vec{Q})$ is a second moment of the deuteron form factor $S_0(\vec{Q})$ and we can estimate it for small $|\vec{Q}|$ as

$$S_2(\vec{Q})\cong\langle Q^2\rangle\,S_0(\vec{Q}), \tag{52}$$

which is exact for Gaussian wave functions. Substituting (50) into (49) we obtain for the sum of the term $T_a^{(2)}$ and the correction term from the projectile propagation $\Delta_2^V T^{(2)}$

$$\langle\psi_0,\vec{p}|\,T_a^{(2)}+\Delta_2^V T^{(2)}|\vec{p}',\psi_0\rangle=\int t_1(E_{\rm eff}^{(1)},\vec{p}-\eta\vec{K}_1,\vec{p}_1-\eta\vec{K}_1)\,t_2(E_{\rm eff}^{(2)},\vec{p}_1-\eta\vec{K}_2,\vec{p}'-\eta\vec{K}_2)\,\tilde{G}(\vec{p}_1)d^3p_1, \tag{53}$$

where

$$\begin{aligned}\tilde{G}(\vec{p}_1)&=\frac{1}{p^2/2\mu-p_1^2/2\mu}\left[S_0(\vec{p}_1-\vec{K})-\frac{S_2(\vec{p}_1-\vec{K})}{(p^2/2\mu-p_1^2/2\mu)^2}\,\frac{(\vec{p}-\vec{p}_1)\cdot(\vec{p}'-\vec{p}_1)}{3m^2}\right]\\ &\cong\frac{S_0(\vec{p}_1-\vec{K})}{p^2/2\mu-p_1^2/2\mu+\dfrac{S_2(\vec{p}_1-\vec{K})/S_0(\vec{p}_1-\vec{K})}{p^2/2\mu-p_1^2/2\mu}\cdot\dfrac{(\vec{p}_1-\vec{K})^2-\frac{1}{4}q^2}{3m^2}}.\end{aligned} \tag{54}$$

Since the form factor $S_0(Q)$ decreases rapidly with Q, only $\vec{p}_1\sim\vec{K}$ contributes essentially in the integral (53) and we neglect $(\vec{p}_1-\vec{K})^2$ with respect to $\frac{1}{4}q^2$ in (54) and use Eq. (52) to write $\tilde{G}(\vec{p}_1)$ as

$$\tilde{G}(\vec{p}_1) \cong \frac{S_0(\vec{p}_1 - \vec{K})}{p^2/2\mu - p_1{}^2/2\mu - \dfrac{\langle Q^2\rangle}{p^2/2\mu - p_1{}^2/2\mu}\dfrac{q^2}{12m^2}}$$

$$\cong \tfrac{1}{2} S_0(\vec{p}_1 - \vec{K})\left[\frac{1}{p^2/2\mu - p_1{}^2/2\mu + (q/2m)(\langle Q^2\rangle/3)^{1/2}} + \frac{1}{p^2/2\mu - p_1{}^2/2\mu - (q/2m)(\langle Q^2\rangle/3)^{1/2}}\right]. \quad (55)$$

Therefore, we have finally found for the double scattering term

$$\langle \psi_0, \vec{p}\,|T_a^{(2)} + \Delta_2^V T^{(2)}|\,\vec{p}\,', \psi_0\rangle = \int t_1(E_{\text{eff}}^{(1)}, \vec{p} - \eta \vec{K}_1, \vec{p}_1 - \eta\vec{K}_1) t_2(E_{\text{eff}}^{(2)}, \vec{p}_1 - \eta\vec{K}_2, \vec{p}\,' - \eta\vec{K}_2) \times \frac{S_0(\vec{p}_1 - \vec{K})}{2}\left[\frac{1}{p^2/2\mu - p_1{}^2/2\mu + qV_N/2\sqrt{3}} + \frac{1}{p^2/2\mu - p_1{}^2/2\mu - qV_N/2\sqrt{3}}\right] d^3p_1 . \quad (56)$$

Comparing Eq. (56) with the main term [Eq. (30)] we see that the nonvanishing correction from projectile propagation results in double poles in the Green's function with their positions shifted from $p_1 = \pm p$ to the values $p_1 = \pm p \pm \delta$, where

$$\delta = \frac{q\mu V_N}{2\sqrt{3}p} = \frac{q}{2\sqrt{3}}\frac{V_N}{V_x} . \quad (57)$$

As was pointed out above, the deuteron has been taken for simplicity. The same arguments can be carried out for the correction to the double scattering term given by Eq. (49) and result in the same answer which we have found for the deuteron.

The corrections to the reflection and higher order terms can be found in the same way as has been followed here. We can show that their magnitude relative to the main term is of the same order of magnitude that we have found for the double scattering term.

The numerical consequences of our approach for hadron-nucleus elastic scattering will be presented in a separate publication.

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