Weak coupling calculation of ^{212}Po using realistic matrix elements

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The structure of the levels of ^{212}Po has been investigated within the model space of the lowest seven neutron and six proton single particle orbits; the matrix elements of Herling and Kuo were used. Agreement between experiment and theory for the lowest six levels, including the recently discovered 8^+, is excellent. The isomer of 2.93 MeV is suggested to have spin 16^+; the 10^+ is not expected to be an isomer.

[NUCLEAR STRUCTURE ^{212}Po; calculated levels, J^π realistic interaction.]

I. INTRODUCTION

Those nuclei which consist of a doubly closed shell plus two neutrons and two protons have always been of special interest. For such nuclei of mass less than 100, the size of the matrices employed in shell model calculations is not so large as to preclude a reasonable microscopic calculation; such calculations have met with considerable success even though these nuclei exhibit features reminiscent of rotational nuclei, viz., large in-band $E2$ transitions and, for the low angular momentum states, approximately an $L(L+1)$ energy spacing.

The nucleus ^{212}Po is also a member of the select category of closed shell plus four nucleons, although historically it has been studied much less than others; until the advent of the work described in Refs. 1 and 2, only the lowest few states had been assigned spins and only a very few electromagnetic decay rates were known. Since the usual mode of populating the states in ^{212}Po was through the beta decay of the $J^\pi = 1^-$ ground state of ^{212}Bi, only states with low values of angular momentum were known, save for the infamous isomer[3] of spin 16 or 18 at 2.93 MeV. Recently, an investigation of alpha transfer on ^{208}Pb leading to states in ^{212}Po has been initiated,[4] although no new spectroscopic information has yet emerged therefrom.

The theoretical calculations of ^{212}Po have historically been initiated to investigate either the alpha decay rates or the nature of the long-lived isomer.[5,6] The most meticulous calculations heretofore are those of Glendenning and Harada,[7] who employed a purely central phenomenological interaction, thereby omitting the tensor interaction which is known to be essential to explain the level ordering of ^{210}Bi.[8] The angular momentum of the isomer was suggested by Glendenning and Harada to be 18^+, although its predicted energy was too small; a second isomer—so far unobserved—with an angular momentum of 10 was also predicted. No calculations have been performed using realistic matrix elements.

The purpose of this paper is twofold. No shell model calculation has been previously performed in this mass region for nuclei with both neutrons and protons outside ^{208}Pb (save for the case of ^{210}Bi) using realistic matrix elements[9]; it is of considerable interest whether such an interaction can reproduce experimentally observed structure in a several nucleon system and whether collective features will also emerge in ^{212}Po as in the lighter nuclei where the spin-orbit force is less dominant. Calculations of nuclei having several holes in ^{208}Pb using realistic matrix elements have not produced entirely satisfactory agreement with experiment. If the initial calculation described in this paper demonstrates that calculations using realistic interactions can provide an adequate description of spectra for a nucleus having several nucleons outside ^{208}Pb, then more detailed calculations of those nuclear properties which are sensitive to small components of the wave function may be usefully performed. Among such properties are $E2$ transitions and alpha decay rates, both of which are sensitive to small admixtures of orbits with low values of the single particle angular momentum. The calculation will be explained in Sec. II and a comparison with experimental results will be made in Sec. III.

II. CALCULATION

The model space employed in the present calculation consists of the lowest six proton and seven neutron levels outside the assumed closed ^{208}Pb core, Fig. 1. The single particle energies are those of Ref. 9. States constructed within this model space are necessarily nonspurious.

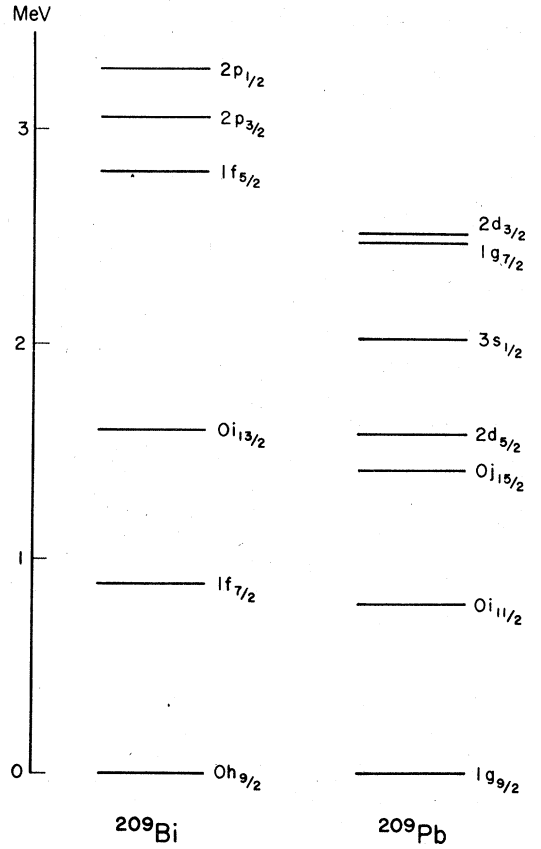

FIG. 1. The single particle levels included in the model space used in the present calculation. The energies are taken from Ref. 9.

This choice is dictated in part by our desire to use the realistic matrix elements of Herling and Kuo[9] which were designed to be used within the model space spanned by these thirteen single particle levels; a calculation within a smaller model space would be fraudulent unless the effects of the omitted levels were compensated for either through the use of perturbation theory (a process which may not converge because of the small energy denominator) or by introducing effective three-body forces.[10] Matrix elements obtained using reaction matrix techniques[11] from an interaction fitted to free N-N scattering data have enjoyed considerable success in correlating experimental information, both in light nuclei and, in particular, in nuclei which have a configuration of several identical particles (or holes) outside ^{208}Pb.[12] The present calculations are the first reported calculations in this mass region employing realistic matrix elements wherein there are both neutrons and protons.

A calculation using a complete basis within our model space would, for many of the allowed values of J, involve matrices of dimensions exceeding several thousand. To circumvent this the calculation employed a weak-coupling basis, similar to the one used in Ref. 7; the basis states are defined as the products of eigenfunctions of two protons and of two neutrons:

$$|^{210}\mathrm{Pb}J_\nu\alpha\times{}^{210}\mathrm{Po}J_\pi\beta;J\rangle \quad \text{and} \quad (1)$$

$$H=H_{\pi\pi}+H_{\nu\nu}+H_{\nu\pi}\,.$$

In the basis state α and β label different two-nucleon states with the same value of J_ν or of J_π. The eigenfunctions of ^{210}Po and ^{210}Pb are expressed in terms of the single nucleon shell model states in Ref. 9.

As the part of the Hamiltonian which applies to the identical nucleons is already diagonal, it only remains to calculate the matrix elements of the neutron-proton interaction $H_{\nu\pi}$; this involves only a recoupling calculation involving 9-j coefficients. Throughout this work the choice of the interaction is interaction B of Herling-Kuo,[9] viz.,

$$G=G_{\mathrm{bare}}+G_{\mathrm{3p\text{-}1h}}\,,$$

with appropriate Coulomb contributions in the case of ^{210}Po. This choice of the Hamiltonian consistently produced the best agreement with experiment, both for the two-nucleon systems[9] and for states of several identical nucleons.[12]

The selection of the appropriate eigenstates of the two-nucleon Hamiltonian for use in the four-nucleon basis, Eq. (1), is of considerable import. In the present work these states were chosen as follows: For two particle states of even angular momentum, the lowest two or three states for each angular momentum were chosen; if other levels with the same J_ν or J_π were nearby, they also were included. For $J_{\nu,\pi}=0$ *all* two-nucleon states were included. It was found that for the states of positive parity, the two neutron and the two proton states of negative parity were unimportant, as might have been anticipated from a consideration of their unperturbed energies; a similar statement applies also to the positive parity two-nucleon states of odd angular momentum, at least for the ^{212}Po states of even angular momentum.

The number of two particle states chosen were therefore two for states of even $J_{\nu,\pi}$ and one for states of odd $J_{\nu,\pi}$, save for $J_{\nu,\pi}=0$ and 2 in which case 5 and 4, and 4 and 3 states were used for the neutron and proton basis, respectively. The adequacy of this truncation was checked by selectively adding other likely two-necleon states, but such additions were found not to affect the energy posi-

tions of the lowest three levels of each J of ^{212}Po. Using this weak-coupling basis, the dimensions of the Hamiltonian matrices were of order 1 to 200.

III. RESULTS

The calculated spectrum along with the known experimental information, both from Ref. 13 and the new results,[1,2] for ^{212}Po are shown in Fig. 2. The position of the ground state 0^+ is shifted slightly so as to line up with the ground state and allow a comparison of the excitation energies.

A cursory glance shows that the agreement between the two spectra is quite astonishing, particularly as there were *no* free parameters available in the calculation. The positions of the first five levels (the "ground state band") all agree with experimental values to a few keV. The energies of the 4^+ and 6^+ are taken from the recent work[1,14] at Stony Brook and that of the 8^+ from Ref. 2. The second 2^+ and the 1^+ states also agree well with known experimental levels with those spins. The level at 1.679 MeV is weakly populated in the beta decay of ^{212}Bi and may be the second 4^+ state, thus satisfactorily accounting for a predicted 4^+ level at this energy. However, there are two excited 0^+ levels predicted to lie below 2 MeV and only one is known; it is possible that the second has so far escaped observation, either because of a very weak beta branch (the 0_2^+ state is expected to have a very small Gamow-Teller matrix element with the ground state of ^{212}Bi, essentially because its structure is based on an excited 0^+ of ^{210}Pb) or because the 0^+ lies above the region accessible by beta decay from ^{212}Bi.

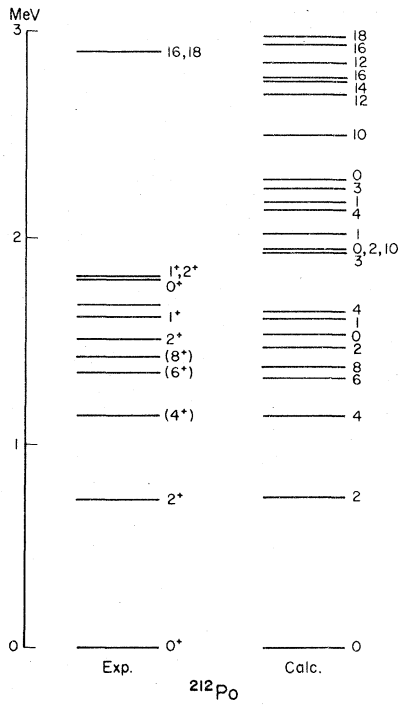

FIG. 2. The experimental and calculated spectra ^{212}Po. Above 2 MeV only selected levels are given. The experimental information is from Refs. 1, 2, and 13.

The present calculation predicts—unlike those of Glendenning[5] or Glendenning and Harada[6]—that the 2.93 MeV isomer has an angular momentum of 16 rather than 18; indeed, two 16^+ states are predicted to have an energy lower than the first 18^+. The calculated energy of the lowest 16^+ is somewhat below 2.93 MeV and is essentially degenerate with the lowest 14^+ level and very near the calculated position of the lowest 12^+ level, implying that an $E6$ (or worse) is the most likely gamma decay mode.

Because of the intrinsic interest in the energies of the 16^+ and 18^+ states, the calculation was repeated with a much larger basis for these two values of the total angular momentum than that allowed by the selection of the two-nucleon basis states given in Sec. III. No consequential change in the energies of either state resulted; we thus conclude that the realistic matrix elements of Herling-Kuo[8] suggest the isomer is most likely a $J=16^+$ state.

The low value for the calculated excitation energy of the 16^+ state is perhaps cause for worry, particularly since the positions of the lower lying states agree so well with experiment. The lowest $J=16^+$ wave function is over 99% $8_1\times 8_1$ [here, as in Eq. (1), the four particle basis state is written as neutron state times proton state]; however, the Herling-Kuo matrix elements predict the 8^+ level of ^{210}Pb some 70 keV too low (the 8^+ of ^{210}Po agrees well with experiment). This directly results in an error of 70 keV in the energy of the 16^+ of ^{212}Po, which is two-thirds of the observed deviation. The 18^+ is $10_1\times 8^+$ and does not suffer from this problem, as the 10^+ of ^{210}Pb is reasonably well predicted by the realistic matrix elements. Shifting the $J=16$ states up by 70 keV results in only one 16^+ lying lower than the 18^+ state.

One of the more striking differences between the results obtained using the realistic matrix elements and those obtained using a phenomenological interaction is the position of the lowest 10^+ state; In Ref. 6 it was predicted to lie beneath the 8^+ level, thus becoming the second isomer in ^{212}Po. The results of the present calculation indicate that not only is it not an isomer, there is a large energy gap between the 8^+ and 10^+ states. The origin of this difference is easily understood. The structure of the lowest 10^+ state is over 99%

TABLE I. The decomposition of the lowest 0^+ and 2^+ is a weak coupling basis. Only components with an intensity greater than 1% are listed. The weak-coupled basis states are listed as in Eq. (1), i.e., neutron times proton.

	0^+ states					
	$0_1 \times 0_1$	$0_2 \times 0_1$	$2_1 \times 2_1$	$4_1 \times 4_1$	$6_1 \times 6_1$	$8_1 \times 8_1$
0_1	0.96	...	0.25	0.11	...	...
0_2	−0.19	−0.70	−0.58	0.27	0.12	...
0_3	−0.18	0.65	−0.58	0.25	...	...
0_4	...	...	0.25	0.26	0.56	0.73
	2^+ states					
	$0_1 \times 2_1$	$2_1 \times 0_1$	$2_2 \times 0_1$	$2_1 \times 2_1$	$2_1 \times 4_1$	$4_1 \times 2_1$
2_1	0.39	−0.86	...	0.18	−0.15	0.20
2_2	0.84	0.44	...	0.14	0.21	...
2_3	...	...	0.97	...	...	...

$10^+ \times 0^+$ in the present calculation and the 10^+ state of ^{210}Pb is—as also in the case of a phenomenological interaction—99% $0i_{11/2}\,1g_{9/2}$. Thus the 10^+ is a two neutron state composed of one particle with $J_1 = l_1 + \frac{1}{2}$ and the second having $J_2 = l_2 - \frac{1}{2}$; it is a general feature of short range interactions that that for such states, the level with maximum angular momentum—in this case 10—has the largest expectation value or, for an attractive interaction, lies lowest. Thus the 10^+ level of ^{210}Pb lies too low in the calculation of Glendenning and Harada and presumably also accounts for the fact that the 18^+ level of ^{212}Po is calculated to be 670 keV lower than they would desire. However, using the realistic matrix elements of Herling and Kuo, the 10^+ of ^{210}Pb lies at the apparent experimental position.

The validity of the weak-coupling model may be seen by examining the eigenvectors of the Hamiltonian. In Table I are given the dominant components of the lowest 0^+ and 2^+ states. In general they are representative of the nature of states of other angular momenta, although the highest angular momenta states tend to be more pure. From an examination of the table one may conclude that although weak coupling is sufficiently valid as to give a zeroth order estimate of the energy and—sometimes the wave function—a complete diagonalization is required to give an accurate calculation.

IV. CONCLUSION

From a comparison of the calculated results presented herein with the known levels of ^{212}Po, it is clear that the realistic matrix elements of Herling and Kuo adequately describe the low-lying levels of ^{212}Po, both in regard to the correct number of states as well as the correct ordering of angular momentum. In particular the results are much improved on previous calculations using a phenomenological interaction. Unlike previous calculations,[5,6] the isomer at 2.93 MeV is here suggested as $J = 16^+$. Further, the 10^+ is not expected to be an isomer.

The success of the present calculations suggests further efforts should be fruitful, both experimentally and theoretically. Experimentally, a knowledge of the spins of several ^{212}Po states and their decay mechanism would be very useful; such information would provide a severe test on the calculated wave functions. Further tests will result from a calculation of the alpha widths, a calculation which may now be more meaningful because of the inclusion within the model space of single particle states of low orbital angular momentum.

Finally, calculations in other nuclei in which both protons and neutrons are active may now be encouraged; recently a plethora of experiments on such nuclei have resulted in a wealth of data and calculations of the Stockholm group[15] suggest the shell model to be remarkably successful in describing the energies of the high spin states.

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