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Medium energy nuclear reactions: "Quasi-two-body scaling" and "hot spots"

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Inclusive spectra for outgoing particles believed *not* to arise from a single-collision process are presented and analyzed as though a single-collision description, "quasi-two-body scaling," were applicable. The characteristic momentum of the momentary distribution k_0 is of order 60 MeV/c and is approximately A independent. An alternative analysis in terms of nuclear temperature is made, suggesting a correlation between the value of k_0 and the nuclear temperature of a "hot spot."

NUCLEAR REACTIONS 90 MeV proton, 140 MeV alpha induced reactions. p, d, t, ^{3}He, and alpha energy spectra interpreted by "quasi-two-body-scaling" and "hot spot"; deduced characteristic parameter.

Frankel has shown the inclusive spectra at backward angles could be related by two-body kinematics.[1] The interpretation of this quasi-two-body scaling (QTBS) is based on a differential cross section for observing a particle of momentum $\vec{q}$ which obeys $d^3\sigma/d\vec{q} = CG(k_{\min})/|\vec{p}-\vec{q}|$. In this equation Frankel originally treated C as a constant (see below). The quantities $\vec{p}$ and $\vec{q}$ are the incident and observed particle momenta and $G(k_{\min})$ is given by $G(k_{\min}) = \int F(k)kdk$. The variable $k_{\min}$ is defined as $k_{\min} = |\vec{p}-\vec{q}| - |\vec{p}\,'|$. $F(k)$ is the internal momentum distribution of the target particle and $\vec{p}\,'$ is the momentum of the unobserved particle. Both Frankel[1] and Amado and Woloshyn[2] have discussed the derivation of this expression. More recently a number of papers[3-6,18] have questioned the assumptions and interpretations of such data in terms of a simple two-body reaction.

In this paper we show that data taken at much lower energies than that of Ref. 1 yield $G(k)$ similar to that at higher energies. In the energy domain dealt with, the mean free path of a nucleon or composite particle is smaller than the nuclear radius and therefore absorption, multiple scattering, and/or pre-equilibrium processes are important. Furthermore, since the quantity $k_{\min}$ is approximately proportional to the outgoing particle energy ϵ at large angles, the exponential behavior of $G(k)$ can be related to an invariant cross section proportional to $e^{-\epsilon/E_0}$, similar to a thermodynamic description of the process, provided E_0 is a quantity which has an understood behavior. We associate E_0 with the temperature of a "hot spot" as described later.

Two significant results came out of the analysis of Ref. 1. First, the function $G(k_{\min})$ could be described by a simple function of the form $G(k) = \exp(-k/k_0)$, and second, $G(k)/A$ was relatively independent of A, the mass number of the target. It should be emphasized that such a momentum distribution deduced is intended to represent high momentum components of the nuclear wave function and need not be relevant at low momenta.[1,4] Frankel applied his analysis to data with incident projectile energies $\geqslant 0.6$ GeV. He indicated that the description he was utilizing extends to momenta $k_{\min}$ as low as 0.1–0.2 GeV/c. Such momenta are involved in lower energy reactions if QTBS is valid.

The forward angle data shown in Refs. 7 and 9 indicate the dominance of quasi-free two-body processes in that the data essentially show two-body scattering ($E_{p'} \simeq E_p \cos^2\theta$) smeared by the internal momentum of the nucleons in the nucleus, consistent with, for example, Refs. 9, 10, and 11. The high energy end of the outgoing proton spectrum has also been analyzed in terms of pre-equilibrium processes and is consistent with the creation of relatively few (~ 5) excitons.[7,8,12]

Figure 1 shows data[7] from 90 MeV protons incident on ^{27}Al, ^{58}Ni, ^{90}Zr, and ^{209}Bi analyzed in the same manner as the data analyzed by Frankel.[1] For these curves $k_{\min}$ was calculated using Eq. (1) of Ref. 1 with a mean excitation energy set equal to zero. References 1, 10, and 11 show that a mean excitation energy of zero is not reasonable. We find that the data plotted in this manner show very nearly an exponential dependence on $k_{\min}$ over nearly the entire range. The only serious exception to this is the data corresponding to outgoing particles from close to, and below, an energy of the order of the Coulomb barrier height.

Figure 2 shows a more critical test of QTBS in that we have plotted the invariant cross section multiplied by $|\vec{p}-\vec{q}|$ and divided by $C(s,t)$ for the case of incident protons for the data at both 140°

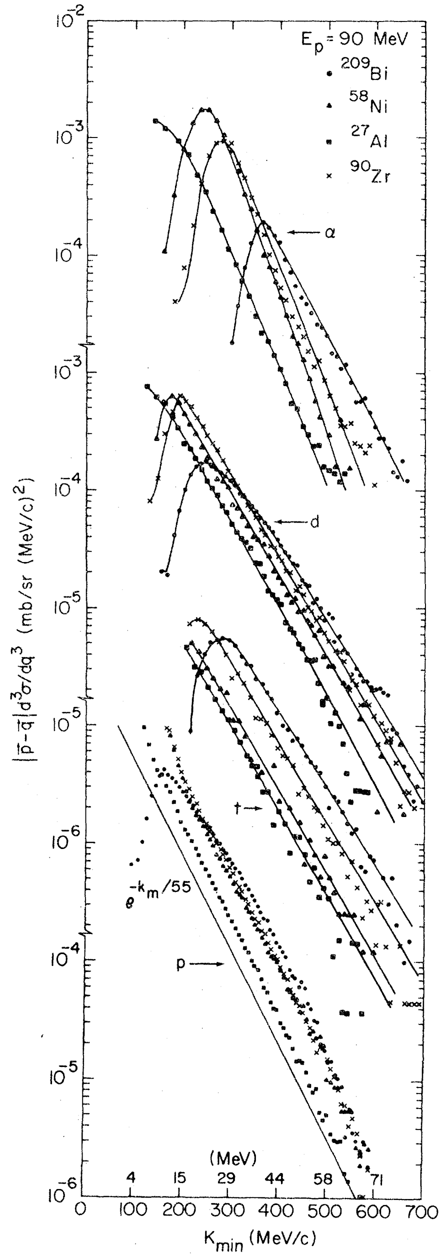

FIG. 1. The quantity $G(k)=|\vec{p}-\vec{q}|\times d^3\sigma/d\vec{q}$ as a function of $k_{\min}$ for incident 90 MeV protons on the targets indicated, and for the indicated observed particles at 140° in the laboratory system. Absolute errors are $\lesssim 30\%$. The energy scale is relevant for protons from a heavy nucleus.

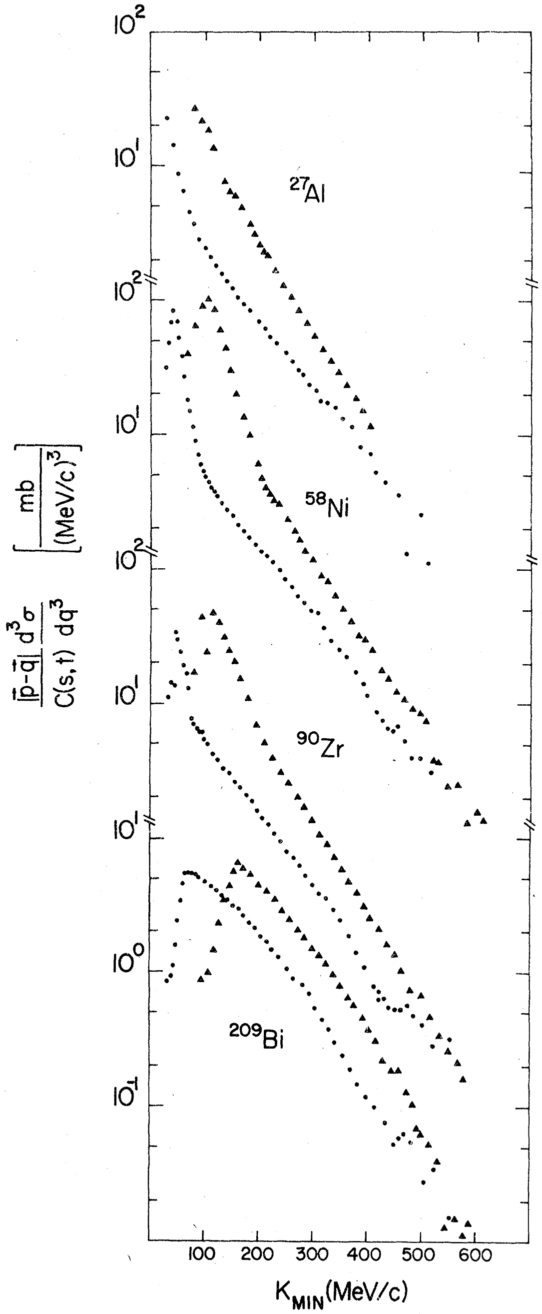

FIG. 2. Same as Fig. 1 except $C(s,t)$ has been divided into $|\vec{p}-\vec{q}|\times d^3\sigma/d\vec{q}$.

and 90°. The equation for $C(s,t)$ is given in Ref. 1. To obtain $d\sigma/dt$ for nucleon-nucleon scattering we transformed $d\sigma/d\Omega$ as given in Ref. 10 or Ref. 13.

Table I gives the values of k_0 obtained from a least square fit to the spectra in the laboratory system at 140° and 90° for 90 MeV protons and 140 MeV α particles incident on the elements indicated. The slopes were deduced from that part of the spectrum corresponding to outgoing particles with energies above the Coulomb barrier. While there is a tendency for the slope k_0 to increase with mass for outgoing hydrogen isotopes, the α particle slope is definitely smaller.

TABLE I. The quantity k_0 deduced from data at 140° and 90° in the laboratory system. $T(\epsilon > 20)$ and $T(\epsilon < 20)$ are deduced from data at 136° and 97° in the c.m. system for the indicated targets. Values are given for the various observed particles. The units of k_0 are MeV/c, and T are MeV. The errors given are statistical and do not reflect any uncertainties from inappropriate treatment of the mean excitation energies, nor do they reflect the deviations from nonlinearity. Overall errors are estimated as $\leqslant \pm 10$ MeV/c.

Incident projectile		90 MeV protons								140 MeV α particles							
Angles		$\theta_{1ab} = 140°$, $\theta_{c.m.} = 136°$				$\theta_{1ab} = 90°$, $\theta_{c.m.} = 97°$				$\theta_{1ab} = 140°$, $\theta_{c.m.}$				$\theta_{1ab} = 90°$, $\theta_{c.m.}$			
Target		^{27}Al	^{58}Ni	^{90}Zr	^{209}Bi	^{27}Al	^{58}Ni	^{90}Zr	^{209}Bi	^{27}Al	^{58}Ni	^{90}Zr	^{209}Bi	^{27}Al	^{58}Ni	^{90}Zr	^{209}Bi
Obs. particle	Param.																
p	K_0	53 ±1	53 ±1	50 ±1	53 ±1	65 ±2	62 ±1	64 ±1	68 ±1	51 ±1	47 ±1	50 ±1	51 ±2	48 ±1	45 ±1	46 ±1	47 ±1
	$T(\epsilon > 20)$	7.9 ±0.1	7.8 ±0.1	7.6 ±0.1	7.7 ±0.1	9.1 ±0.2	8.8 ±0.2	8.4 ±0.1	8.6 ±0.1	7.5 ±0.1	5.0 ±0.1	6.5 ±0.1	6.8 ±0.8	7.6 ±0.1	7.5 ±0.1	7.6 ±0.1	7.6 ±0.1
	$T(\epsilon < 20)$	3.8 ±0.2	2.7 ±0.1	3.1 ±0.2	3.7 ±0.2	4.5 ±0.3	3.0 ±0.2	3.7 ±0.3	4.5 ±0.2	3.6 ±0.2	2.5 ±0.1	2.7 ±0.1	3.1 ±0.1	4.0 ±0.2	2.7 ±0.1	3.0 ±0.2	4.3 ±0.2
d	K_0	61 ±2	70 ±3	64 ±2	66 ±1	78 ±2	85 ±1	85 ±1	88 ±1	62 ±2	68 ±3	71 ±2	86 ±4	65 ±2	71 ±2	76 ±2	90 ±2
	$T(\epsilon > 20)$	6.4 ±0.2	7.0 ±0.3	6.4 ±0.1	6.8 ±0.1	8.9 ±0.1	9.1 ±0.1	8.7 ±0.1	8.6 ±0.1	7.0 ±0.1	5.6 ±0.4	7.6 ±0.2	8.0 ±0.2	8.1 ±0.1	9.2 ±0.1	10.5 ±0.1	10.0 ±0.1
	$T(\epsilon < 20)$	3.8 ±0.2	3.9 ±0.2	3.5 ±0.1	5.1 ±0.2	4.2 ±0.3	4.1 ±0.2	4.0 ±0.3	5.7 ±0.4	4.2 ±0.1	3.6 ±0.2	3.2 ±0.2	4.2 ±0.3	5.2 ±0.3	4.2 ±0.3	3.7 ±0.2	4.3 ±0.2
t	K_0	61 ±6	70 ±9	61 ±3	66 ±2	77 ±5	88 ±4	84 ±2	76 ±1	77 ±6	72 ±7	93 ±4	116 ±9	84 ±2	109 ±3	118 ±3	129 ±3
	$T(\epsilon > 20)$	6.2 ±1.2	4.2 ±0.4	4.9 ±0.3	5.2 ±0.2	7.0 ±0.3	7.8 ±0.7	6.8 ±0.1	6.8 ±0.1	5.7 ±0.2	7.1 ±0.2	8.7 ±0.2	9.2 ±0.9	8.6 ±0.2	12.0 ±0.3	14.0 ±0.2	10.3 ±0.3
	$T(\epsilon < 20)$	2.7 ±0.2	4.2 ±0.3	3.1 ±0.2	2.3 ±0.4	3.2 ±0.4	3.9 ±0.2	3.5 ±0.3	2.5 ±0.5	3.5 ±0.3	2.1 ±0.6	3.0 ±0.5	3.4 ±0.4	3.9 ±0.4	2.2 ±0.7	3.5 ±0.4	3.5 ±0.4
α	K_0	53 ±3	51 ±5	49 ±2	61 ±4	55 ±3	64 ±3	66 ±2	73 ±2	61 ±5	59 ±2	85 ±7	99 ±6	76 ±2	71 ±2	119 ±3	129 ±5
	$T(\epsilon > 20)$	5.3 ±0.2	3.6 ±0.2	4.4 ±0.2	3.5 ±0.2	6.5 ±0.2	5.3 ±0.2	5.4 ±0.1	4.7 ±0.2	5.8 ±0.1	6.4 ±0.1	6.4 ±0.2	4.0 ±0.2	8.8 ±0.1	8.5 ±0.1	10.9 ±0.2	6.8 ±0.3
	$T(\epsilon < 20)$	2.8 ±0.1	2.1 ±0.1	1.6 ±0.1	···	2.8 ±0.1	2.0 ±0.1	1.7 ±0.2	···	3.6 ±0.1	2.2 ±0.2	1.6 ±0.2	···	3.3 ±0.1	2.3 ±0.2	1.9 ±0.2	···

TABLE II. The quantity k_0, in units of MeV/c, for the elements and angles shown for the inclusive reaction $A(p,p')A$. The results analyzed in this table are similar to those shown in Table I, except that the quantity $C(s,t)$ is not treated as a constant, but is deduced from free p-p scattering.

Target	140°	90°
^{27}Al	63 ± 2	80 ± 3
^{58}Ni	66 ± 2	85 ± 3
^{90}Zr	58 ± 2	82 ± 3
^{209}Bi	58 ± 2	81 ± 3

The deduced slopes for outgoing particles are very similar for angles greater than 90°, and the increase in k_0 as far forward as 75° is generally less than about 20 MeV/c.

Table II gives the value of the deduced values of k_0 in the case of observed protons at both 90° and 140° in the laboratory system for 90 MeV incident protons on the indicated nuclei. These values differ from those in Table I, showing the effect of taking into account the factor $C(s,t)$. Over the range of variables s and t for any given nucleus, C is constant to within a factor of ~3. $C(s,t)$ varies little at a given value of $k_{\min}$. These results arise because of the relatively low energy.

The difference of slopes between Tables I and Table II as a function of angle is then arising from the angle dependence in the deduced $G(k)$. As is seen in Fig. 2 there is an angle dependence to the magnitude of the deduced $G(k)$ even when $C(s,t)$ is taken into account. In fact, $C(s,t)$ over the range of $k_{\min}$ is essentially the same at both 90° and 140°. These results suggest the inapplicability of QTBS.[1] That may, however, be too strong a conclusion since distortion has not been taken into account, and furthermore, the differences in k_0 between the two angles is not very large (~20 MeV/c), although it is systematically higher at 90°. We have not analyzed other projectiles because of inadequate representations of the appropriate free particle cross sections.

Figure 1 shows that the quantity $G(k)$ is not A independent. Frankel observed[1] a linear A dependence which again is suggestive of a single-collision process. For ^{27}Al, ^{58}Ni, and ^{90}Zr we find $G(k)/A$ is the same for the reaction (p,p') to within 20%. ^{209}Bi, which is more strongly dominated by the Coulomb barrier, has $G(k)/A$ only about $\frac{1}{3}$ of the average value for the other nuclei. For other outgoing particles $G(k)/A$ is constant to within approximately 20%, except for ^{209}Bi which is systematically low.

An alternative analysis of the data is suggested by the exponential dependence of the differential cross section at energies greater than about 20 MeV. We analyze the large angle data by assuming a mechanism which gives a cross section of the form in the c.m. frame

$$\frac{d^2\sigma}{d\Omega d\epsilon} = C\sigma_{\text{inv}}(\epsilon)\epsilon e^{-\epsilon/T}.$$

In Fig. 3, we plot

$$R = \frac{d^2\sigma}{d\Omega d\epsilon}\Big/\epsilon\sigma_{\text{inv}}(\epsilon)e^{-\epsilon/T}$$

as a function of ϵ, the outgoing particle energy. The c.m. differential cross sections were obtained from the laboratory differential cross section using the relation

$$\frac{1}{p}\,\frac{d^2\sigma}{d\Omega d\epsilon}\bigg|_{\text{c.m.}} = \frac{1}{p'}\,\frac{d^2\sigma'}{d\Omega' d\epsilon'}\bigg|_{\text{lab}}.$$

T is the effective nuclear temperature and the curves plotted in Fig. 3 show the quantity R deduced using the parameter $T(\epsilon > 20)$ given in Table I.

It must be emphasized that the *effective* temperature we are discussing is, in the first place, a characteristic slope associated with the data rather than any fundamental measure of average kinetic energy, averaged over time, in either the entire nucleus or a localized region. The data, with its exponential dependence on the outoutgoing, invites the introduction of such a description. This description, however, has significance primarily as a representation of pre-compound or pre-equilibrium processes.

In Fig. 3 we have used $T(\epsilon > 20)$ to indicate the range of validity of describing the data by a single parameter T. If such a description were valid, then R should be constant with outgoing energy. The marked deviation of R at $\epsilon \leq 20$ MeV shows first of all why we have used $\epsilon = 20$ MeV to characterize the two temperature ranges, and second, the fact that R increases shows $T(\epsilon < 20)$ must be, within this description, less than $T(\epsilon > 20)$.

The quantity T is usually interpreted as the temperature of the residual nucleus and is related to the entropy and level density of that nucleus. The values found for $T(\epsilon < 20)$ are quite consistent with a description of the interaction in which a major fraction of the incident projectile momentum and energy are absorbed by the target nucleus which then "evaporates" the observed particle.[8] Calculation shows multiple charged particle emissions are not too important.

The higher values of $T(\epsilon > 20)$ suggest that the projectile energy is not shared among all the target nucleons within the framework of the usual equation of state.[14] Alternatively one might also use this higher temperature to quantify incomplete thermalization or a vestige of direct and/or pre-

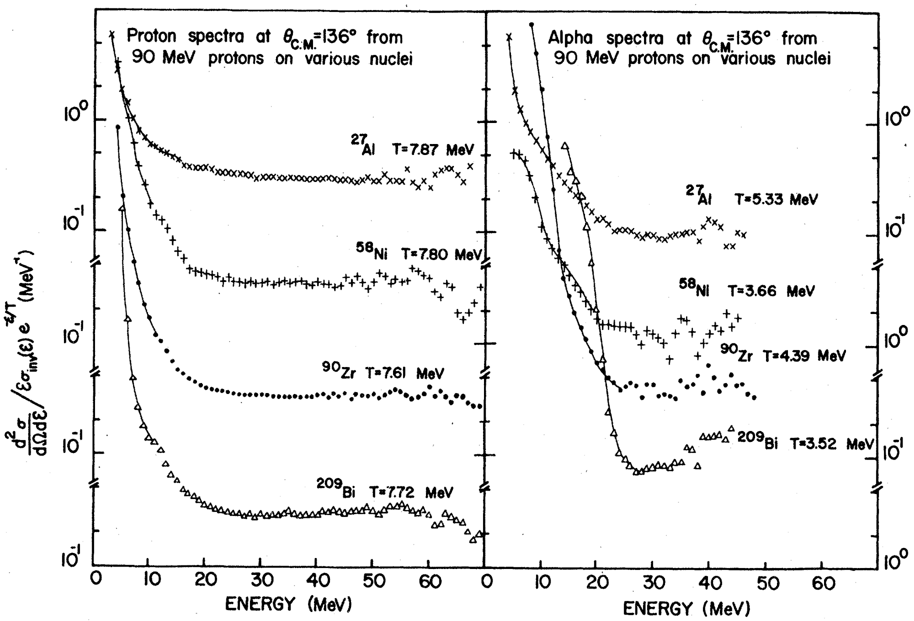

FIG. 3. The quantity R as defined in the text for the 136° data in the c.m. system.

compound emission.[12] These latter processes have been interpreted thermodynamically by Weiner and Westrom[15] in terms of a hot spot.[16,17] They have not as yet applied their results to inclusive experiments such as those of Refs. 6 and 7.

Figure 4 shows the temperature as deduced from that part of the proton spectrum corresponding to outgoing proton and α particle energies greater than 20 MeV ($T>20$). The incident energies were 90 MeV for protons and the targets are indicated. This figure shows that for angles greater than 90° $T(\epsilon>20)$ is essentially constant with both target and angle. Figure 3 of the paper by Weiner and Weström[15] shows that the effective temperature deduced in the hot-spot model when used to analyze the evaporation following a first fast collision, shows an angular behavior very similar to that shown in Fig. 4. It should be realized that the angles are different in that the angle in Fig. 4 is the c.m. angle of the observed particle and not the angle relative to the hot spot (unless the hot spot were at 180°, which is not unreasonable).

The results shown in Fig. 4 are also similar to those shown in Ref. 17, except that the temperatures found by Nomura *et al.*[17] are generally much smaller (the range they find for T is ~1–4 MeV, whereas our results are from ~3–15 MeV for emitted α particles). Both results show constancy of the deduced temperature for angles greater than 90°. Figure 4(b) shows the outgoing α particle deduced temperatures and is seen to be very similar to the results of Nomura *et al.*,[17] except for a

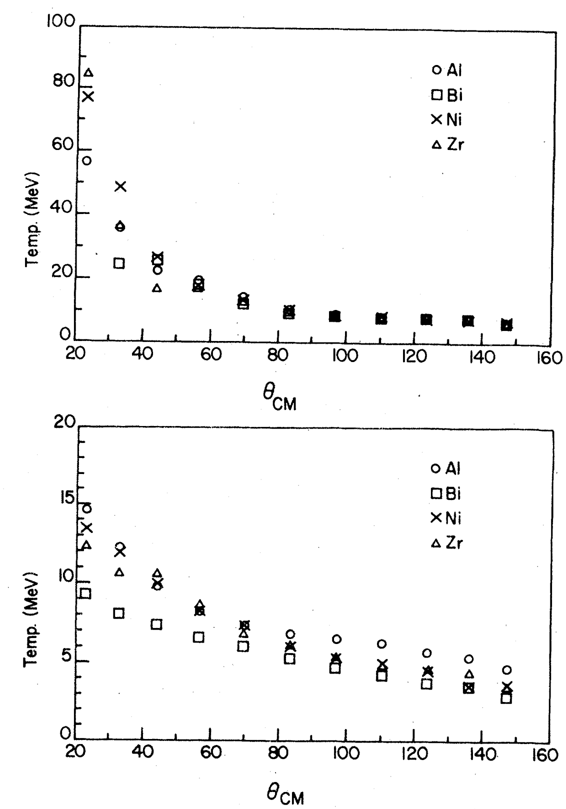

FIG. 4. (a) The effective temperature as a function of angle for outgoing *protons* deduced from the data of Ref. 7 for outgoing particle energies greater than 20 MeV. (b) Same as (a) for α particles. Note the factor of 5 change of scale for the ordinate.

factor of ~3 greater values.

For both outgoing protons and α particles the deduced temperatures for ^{209}Bi are systematically lower than the other nuclei. In fact, for protons the forward angle temperatures increase with A, except for ^{209}Bi. This is almost certainly an effect of the much higher Coulomb barrier, and this supposition could be checked by studying a nucleus intermediate between ^{90}Zr and ^{209}Bi.

If the analysis of these data in terms of a hot spot is at all valid, it suggests an effective particle emission time of order (0.1–0.2) τ_R where τ_R is the relaxation time. Reference 15 gives $10^3\tau_0 > \tau_R > 10\tau_0$, and in Fig. 3 of Ref. 15 the value $\tau_R = 20\tau_0$ is used. This in turn suggests the time scale for the pre-compound process is of order 2–4 times the single particle transit times, a value consistent with the idea of a localized hot spot.

As a conclusion we suggest that the hot spot model may be a possible alternative description for quasi-two-body scaling. This may be a fortuitous accident at the relatively low energies of the these experiments, which may arise because the low energy tends to require a "coherent recoil" mechanism.[1] However, it does afford a simple and semiquantitative explanation for the observed values of the slope parameter. We cannot exclude a fraction of the observed particles at large angles coming from a QTBS instead of evaporation.

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