$^{12}C + {}^{12}C$ Coulomb barrier resonances: Direct determination of resonance parameters from elastic scattering measurements

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$^{12}C + {}^{12}C$ elastic scattering angular distributions have been measured between $E_{c.m.} = 6.50$ and 7.05 MeV at intervals of 0.05 MeV. Resonances were observed in these yields at $E_{c.m.} = 6.64$ and 6.83 MeV. A phase shift analysis of these data was carried out which resulted in assignments of $J^{\pi} = 2^+$ ($E_{c.m.} = 6.64$ MeV) and $J^{\pi} = 4^+$ ($E_{c.m.} = 6.83$ MeV) for the resonances. Carbon partial widths were determined by parametrizing the deduced phase shifts in terms of isolated Breit-Wigner resonances. The results, $\Gamma_c = 29^{+6}_{-9}$ keV at $E_{c.m.} = 6.64$ MeV and $\Gamma_c = 11 \pm 3$ keV at $E_{c.m.} = 6.83$ MeV, demonstrate unambiguously the quasi-molecular nature of the resonances.

[NUCLEAR REACTIONS $^{12}C(^{12}C, ^{12}C)^{12}C$, $E = 13.0$–14.1 MeV; measured $\sigma(E;\theta)$; kinematic coincidence. Deduced phase shift parameters; deduced spins and carbon widths for resonances.]

INTRODUCTION

As a result of recent experimental activity, a considerable body of data is now available concerning the Coulomb barrier resonances[1–5] observed in the $^{12}C + {}^{12}C$ system. Many new resonances have been identified, and spins have been measured for most of these.[6–12] However, the key information which is needed to establish the detailed nature of the resonances—partial widths for decay of the resonances into the various open channels—remains incomplete. Carbon partial widths have been estimated for many of the resonances through the study of reaction channels[2,6] and total reaction cross sections,[11,13] and these measurements demonstrate the particular importance of the carbon-carbon configuration in the structure of the resonances. It is important to confirm the quantitative accuracy of these estimates, particularly since partial widths for other channels are most conveniently studied through rearrangement reactions involving the $^{12}C + {}^{12}C$ entrance channel. Resonances observed in such reaction studies involve products of the carbon and reaction channel partial widths, and thus an accurate knowledge of the former widths is required before the latter can be extracted.

The purpose of the present work is to measure and analyze $^{12}C + {}^{12}C$ elastic scattering cross sections with sufficient precision to enable for the first time the *direct* extraction of carbon partial widths, and thereby to calibrate the indirect estimates. The bombarding energy region $6.5 \leq E_{c.m.} \leq 7.0$ MeV was chosen for detailed study for several reasons. Resonances have been observed previously in this interval in studies of the $^{12}C + {}^{12}C$ total reaction cross section,[11] of the $^{12}C(^{12}C, {}^{8}Be)^{16}O$ (Ref. 10) and $^{12}C(^{12}C, \alpha)^{20}Ne$ (Refs. 8, 11) differential cross sections and in $^{12}C + {}^{12}C$ radiative capture.[12] Thus a direct determination of the resonant carbon widths also would make possible the determination of resonant widths for these additional, possibly important, channels.

As illustrated in Fig. 1, evidence of resonant behavior correlated with the above appears in the 90° $^{12}C + {}^{12}C$ elastic scattering data of Spinka and Winkler.[14] We have found in a preliminary analysis of the present data[15] that these resonances occur in weakly absorbed surface waves, so that the usual difficulties associated with resonance analyses in the presence of strong absorption do not arise. For this reason, and because only a few partial waves need be considered at the low energies of interest here, the phase shift analyses and data reported in the present paper result in an unambiguous characterization of the resonances.

THE EXPERIMENTAL METHOD

The momentum-analyzed ^{12}C beam for the experiment was obtained using the Yale MP tandem accelerator facility. To study the narrow resonances of interest, data were taken every 100 keV in the laboratory energy range 13.0 to 14.1 MeV. A thin (nominally 5 $\mu g/cm^2$) natural carbon foil was used for the target, yielding a total laboratory energy resolution of better than 40 keV. Data were measured simultaneously at five angles. Differential cross section measurements for elastic scattering were made for 37 angles in the interval $25° \leq \theta_{lab} \leq 55°$ at each energy. Changes in the

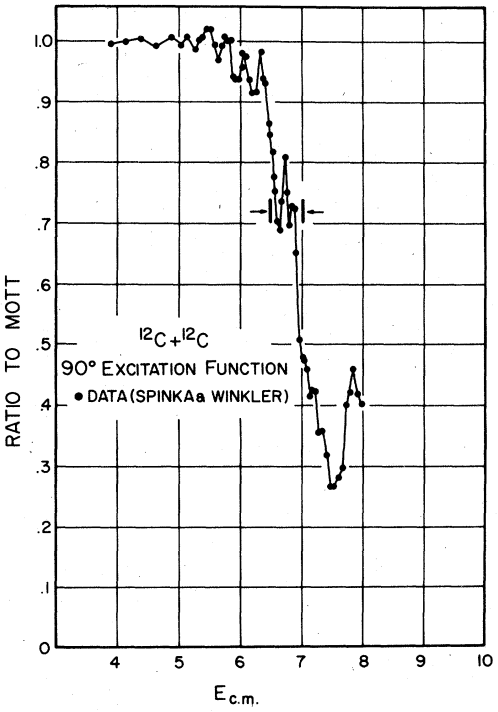

FIG. 1. $^{12}C+^{12}C$ 90° elastic excitation function as measured by Spinka and Winkler (Ref. 14). The energy region investigated in the present work is indicated. Comparison with the above excitation function provided the absolute normalization for our angular distribution data set.

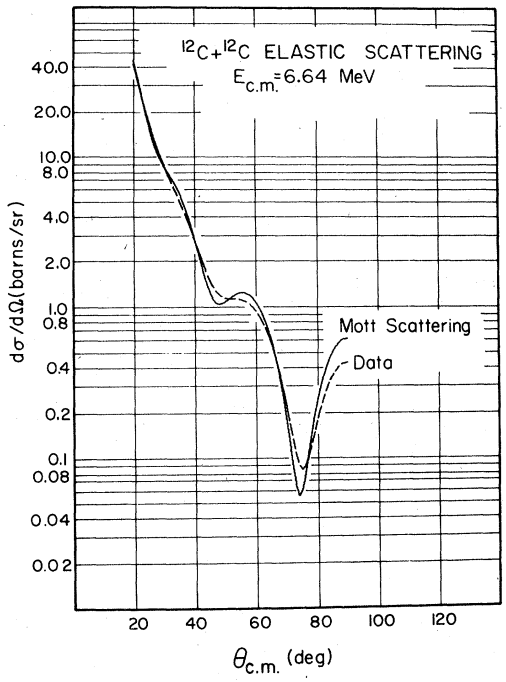

FIG. 2. Comparison of experimental angular distribution and Mott scattering at $E_{c.m.} = 6.64$ MeV. The dashed curve passes through the experimental data, which are not plotted here.

angular settings were accurate to 0.1° lab, while the absolute angle of a detector was determined to <0.25° lab. Particle identification was achieved through a standard kinematic coincidence technique in which pairs of detectors separated by 90° were operated in time coincidence. One detector from each pair was used to form a group of defining detectors.

The defining group had polar angle apertures of 0.3° and an average solid angle of 0.80×10^{-3} sr. Complete detection efficiency for elastics was ensured by choosing appropriately larger solid angles for the coincidence group. The spot size, angle of incidence, and angle of divergence of the beam at the target contributed no more than 0.15° to the angular resolution. Combining a small error due to multiple scattering, our estimated angular resolution for this experiment is $\delta\theta_{c.m.} \leq 0.5°$.

Relative cross sections were obtained by correcting the raw particle yields for integrated beam current, electronic dead time, carbon buildup on target, and detector solid angle normalization. After accounting for the energy loss suffered by the beam in the target, our 90° data were compared to the absolute 90° elastic cross sections mea-

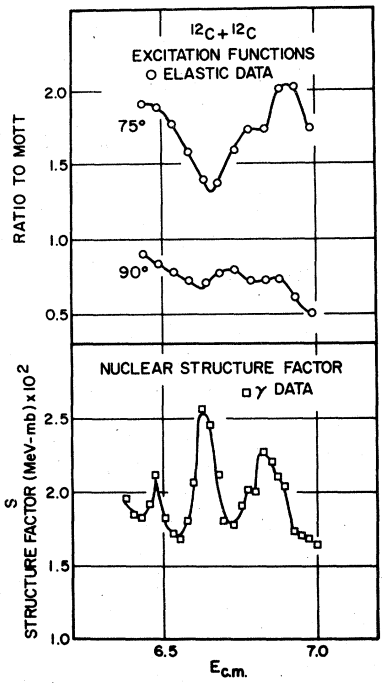

FIG. 3. Elastic and total reaction cross sections for $^{12}C+^{12}C$ as a function of energy. The total reaction cross section was measured using γ-ray techniques (Ref. 11) and is plotted here as S factor. The minima in the elastic yields correlate with the maxima in the reaction cross section.

sured by Spinka and Winkler[14] (Fig. 1). Since the agreement was good, absolute cross sections for our entire data set were obtained by applying a single normalization constant determined from the comparison. The absolute errors calculated for the data are on the order of 5–8%. This includes the error from all corrections applied and those due to angular resolution and possible ^{13}C contamination.

With this degree of accuracy we have found that the measured angular distributions deviate significantly from pure Coulomb scattering. Figure 2 illustrates a comparison of our data and the corresponding Mott differential cross section. The differences between the two are most exaggerated near 55°, 75°, and 90°. At $E_{c.m.}=6.64$ MeV the percent deviations at 75° and 90° are both on the order of 50%. Measured excitation functions for 75° and 90° angles are shown in Fig. 3. In general, for energies near the Coulomb barrier the greatest deviations from Mott occur near 75° where the symmetrized Coulomb amplitude takes on its minimum value. Results of another experiment conducted at Yale to measure the elastic scattering of $^{12}C+^{12}C$ with errors <5% show that there are measurable deviations near 75° at energies as low as 5.0 MeV (c.m).[16]

For comparison with the elastic data, the energy dependence of the total reaction cross section structure factor determined in measurements of γ-radiation yields[11] is also shown in Fig. 3. The striking correlations evident in Fig. 3 verify that the same resonances are reflected in both the elastic and reaction cross sections. We note also that the experimental widths in the elastic data are essentially identical to those observed in the γ-radiation yields.

ANALYSIS AND RESULTS

To extract information from the elastic data we have made use of the conventional phase shift analysis formalism. The elastic scattering amplitude may be decomposed as the sum of Coulomb and nuclear contributions for identical charged scalar particles with the nuclear amplitude expressed as a partial wave expansion. The symmetrized differential cross section is given as

$$\left(\frac{d\sigma}{d\Omega}\right)=\left(\frac{Z^2e^2}{4E_{c.m.}}\right)^2\left|\frac{\exp(-i\gamma\ln\sin^2\frac{1}{2}\theta)}{\sin^2\frac{1}{2}\theta}+\frac{\exp(-i\gamma\ln\cos^2\frac{1}{2}\theta)}{\cos^2\frac{1}{2}\theta}\right.$$
$$\left.+\frac{2}{i\gamma}\sum_{l}^{\text{even}}(2l+1)e^{2i(\sigma_l-\sigma_0)}\left[1-S_l(E)\right]P_l(\cos\theta)\right|^2, \tag{1}$$

where $\gamma=\alpha Z^2(\mu c^2/2E_{c.m.})^{1/2}$ is the Sommerfeld parameter in terms of the fine structure constant, α, and the reduced mass in energy units, μc^2. S_l is the matrix element for nuclear elastic scattering and σ_l is the Coulomb phase shift for partial wave l.

Values for $S_l(E)$ were determined by fitting the angular distribution for each energy with a curve calculated using Eq. (1). This was accomplished by varying the nuclear absorption η_l and the nuclear phase shift δ_l, which are defined such that $\eta_l=|S_l|$ and $S_l=\eta_l e^{2i\delta_l}$. For the energies measured we determined that there is no significant contribution to the nuclear amplitude from partial waves with $l>6$. For our fits to the data discussed here we have truncated the partial wave series at $l=6$ for a total of eight parameters. Although the angular distributions consist of at least 37 data points each, we observe that the set of parameters which gives the "best" fit is not truly unique if all parameters are allowed to vary freely. This can be understood by examining Fig. 2. As mentioned above there are basically three regions where the actual angular distribution differs from the pure Mott case. The data may be thought of as defining the value, slope, and second derivative at a point in each of these regions. If we account for the fact that the slope at 90° must be zero then the data considered in this way impose eight constraints. When we also consider that the data have finite errors and the fit is highly nonlinear in the parameters, the occurrence of ambiguities is not surprising.

It has been possible to remove these ambiguities by imposing additional constraints. A condition on the parameters that has proved to be very effective is the requirement that the absorption parameters reproduce the experimental reaction cross section. In terms of η_l the total reaction cross section is given by

$$\sigma_R=\frac{2\pi}{k^2}\sum_l(2l+1)(1-\eta_l^2).$$

To implement this constraint a term was added to the χ^2 to be minimized so that the magnitude of the experimental reaction cross section was fitted

simultaneously with the elastic data. Another constraint was that the parameters vary as smoothly as possible with energy.

Initial parameter values describing the background scattering were found by fitting the off-resonance data. In subsequent attempts to fit the data as a function of energy only the parameters of a single partial wave were permitted to vary. The remaining parameters were fixed at interpolated background values during the fit. In order to obtain good fits we have found that it is necessary and sufficient to allow the $L=2$ parameters to vary in the region of 6.6 MeV, while in the region of 6.8 MeV the $L=4$ parameters must be free. The $L=0$ partial wave does not contain enough cross section to account for the resonant yields, while variation of the $L=6$ phase shift parameters does not suffice to reproduce the measured angular distributions.

The results of the phase shift analysis are summarized in Figs. 4 and 5. Cross sections calculated from the deduced phase shifts are compared with the elastic angular distribution data in Fig. 4, while the phase shift parameters are plotted in the lower two panels of Fig. 5. The measured[11] and calculated total reaction cross sections are illustrated at the top of Fig. 5. Comparing the energies at which the reaction cross section resonates with the energy dependence of the deduced phase shift parameters demonstrates unambiguously that the $L=2$ partial wave resonates at $E_{c.m.}=6.64$ MeV, and that the $L=4$ partial wave is responsible for the $E_{c.m.}=6.83$ MeV resonance. Thus as reported previously[15] the phase shift analysis results in a determination of $J^{\pi}=2^{+}$ for the former resonance and $J^{\pi}=4^{+}$ for the latter.

A prominent $J^{\pi}=4^{+}$ resonance has been identified in the $^{12}C(^{12}C,\alpha)^{20}Ne$ reaction at $E_{c.m.}=6.87$ MeV by Voit *et al.*,[8] and it is probable that this structure corresponds to the resonance which we observe at 6.83 MeV. Wada *et al.*[10] have reported a $J^{\pi}=2^{+}$ structure with a total width of approximately 200 keV in the $^{12}C(^{12}C,^{8}Be)^{16}O$ reaction at $E_{c.m.}=6.65$ MeV. (In comparison the $J^{\pi}=2^{+}$, $E_{c.m.}=6.63$ MeV resonance observed in the present $^{12}C+^{12}C$ elastic scattering data and also in the total γ yields is significantly narrower—$\Gamma_{tot}\approx 100$ keV.) In the analysis of Ref. 10, a broad bump in the $^{12}C(^{12}C,^{8}Be)^{16}O$ yields centered at $E_{c.m.}=6.5$ MeV was decomposed into Breit-Wigner resonances located at 6.35 and 6.65 MeV, and it was concluded that the resulting resonance parameters are evidence for reduced widths in the $^{16}O+^{8}Be$ channel which are comparable to those observed in the $^{12}C+^{12}C$ channel. We note in connection with this intriguing possibility, however, the presence of an additional resonance near 6.47 MeV in our total reaction cross-section data (Fig. 5, top). This resonance may well contribute to the broad bump observed in the $^{12}C(^{12}C,^{8}Be)^{16}O$ yields, and including it in a reanalysis of the latter data may remove the Γ_{tot} discrepancy for the $J^{\pi}=2^{+}$ resonance near 6.64 MeV, and thus permit a more meaningful, direct comparison of the $^{12}C+^{12}C$ and $^{16}O+^{8}Be$ partial widths.

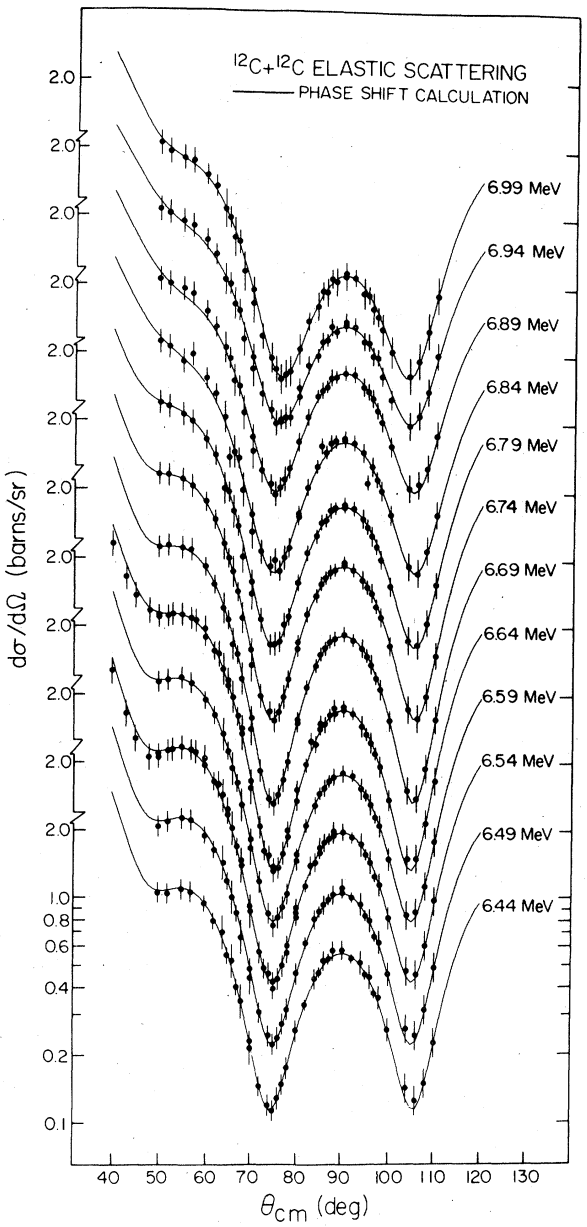

FIG. 4. Measured $^{12}C+^{12}C$ elastic scattering angular distributions. The curves are the result of the phase shift analysis described in the text.

Before taking up the subject of carbon partial widths for the resonances observed in the present

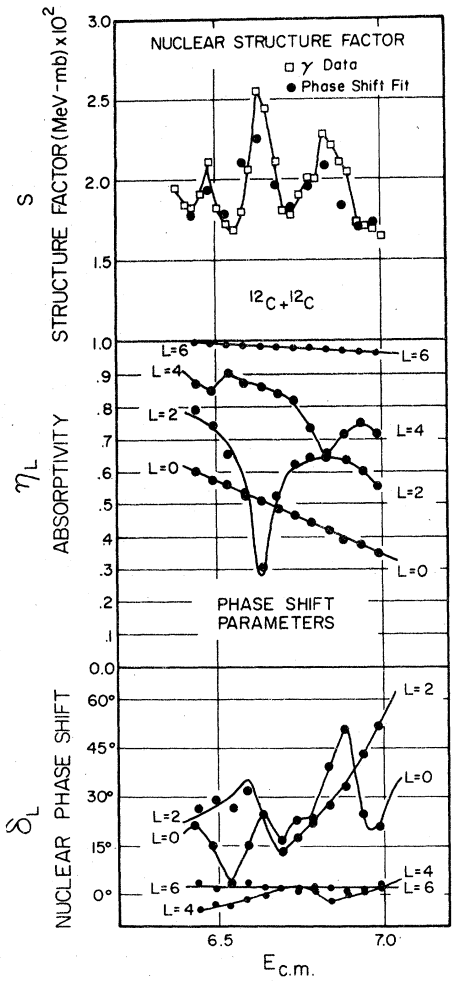

FIG. 5. The deduced phase shift parameters, η_l and δ_l, corresponding to the fitted curves of Fig. 4, are plotted as functions of energy in the lower two panels. The reaction cross section determined from these parameters is compared to experiment in the top panel.

study, we wish to mention another useful aspect of the phase shift analysis . Since the analysis is model independent to the extent that the phase shift parameters are unambiguously determined, it provides constraints on the interaction potentials used to model the physics of the scattering process. For example, if the resonant behavior is removed from the phase shift parameters, the resulting background scattering should be reproduced by the optical model. With this added constraint, optical potentials for various (e.g., neighboring) systems may be compared in the search for fundamental similarities and differences among these systems.

BREIT-WIGNER PARAMETRIZATION

Carbon partial widths for the resonances were obtained by assuming that the resonant contribution to the cross section can be described with the Breit-Wigner, one level approximation formula. If the contribution of a single resonant state is separated from the background, the scattering matrix S_l can be written as

$$S_l(E) = \langle S_l(E) \rangle \left[1 - e^{i\varphi} \frac{i\Gamma_{el}}{(E - E_0) + i\Gamma/2} \right], \qquad (2)$$

where $\langle S_l \rangle$ denotes the background scattering matrix and φ is a resonance mixing phase. In our analysis satisfactory agreement with experiment was obtained with $\varphi = 0$.

The fractional elastic partial width Γ_{el}/Γ was determined by comparing Eq. (2), at $E = E_0$, with the results of the phase shift analysis. The two solutions

$$\frac{\Gamma_{el}}{\Gamma} = \frac{1}{2} \left[1 \pm \frac{\eta_l}{\langle \eta_l \rangle} \right]$$

correspond to characteristically different phase shift behavior as the energy is varied over the resonance. The solution corresponding to $\Gamma_{el}/\Gamma > \frac{1}{2}$ ($< \frac{1}{2}$) requires that δ_l change by $\pi/2$ (0) from off-resonance to on-resonance. From Fig. 5, which shows the results of the phase shift analysis, we see that the observed resonances are characterized by the latter case. Using the values of the absorption parameters on resonance we then obtain values for Γ_{el}/Γ of $0.29^{+0.06}_{-0.09}$ at 6.64 MeV, and of 0.09 ± 0.02 at 6.83 MeV.

To complete the Breit-Wigner analysis we show that the S-matrix values given by the resonance expression, Eq. (2), reproduce the phase shift parameters and the elastic data for energies with $E \neq E_0$. In order to do this we have taken the background S-matrix parameters $\langle \eta_l \rangle$ and $\langle \delta_l \rangle$ to be linear functions of E over the restricted energy range considered in the present work. The comparison of fitted S-matrix values and the Breit-Wigner values for all energies measured is illustrated in the lower two panels of Fig. 6. The fitted values of the η's and δ's indeed are found characteristic of background plus resonant scattering. A direct comparison between cross sections calculated with the Breit-Wigner parametrization and yields measured at the most critical scattering angles is illustrated in the top panel of Fig. 6. Because of the agreement shown we conclude that the elastic data may be characterized by an isolated 2^+ resonance located at 6.64 MeV with $\Gamma_{el}/\Gamma = 29\%$ and an isolated 4^+ resonance at 6.83 with $\Gamma_{el}/\Gamma = 9\%$.

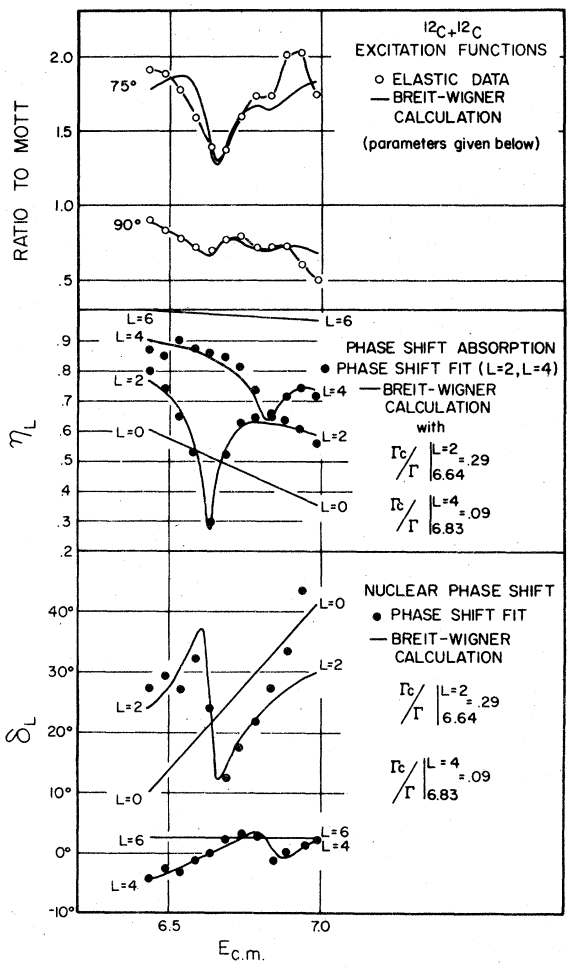

FIG. 6. Comparison of Breit-Wigner parametrization with the phase shift parametrization (lower two panels) and with elastic data measured at 75° and 90° (top panel).

DISCUSSION AND CONCLUSION

In order to assess the significance of the measured partial widths for carbon decay of the $E_{c.m.}=6.64$ and 6.83 MeV resonances, we compare these widths in Table I with published values[2] for two of the original Coulomb barrier resonances. We note that the latter structures (at $E_{c.m.}=5.6$ and 6.0 MeV), whose carbon partial widths were inferred through studies of the $^{12}C(^{12}C,\alpha)^{20}Ne$ reaction and found to be consistent with $^{12}C+^{12}C$ elastic scattering data,[2] are widely considered to represent classical examples of quasimolecular phenomena. All of the carbon widths listed in Table I are in fact of the same order of magnitude, with those associated with the higher energy resonances being a few times larger. This difference is not considered significant; it can be attributed to increasing penetrability with energy, as the analysis below indicates.

The four resonances listed in Table I all appear in a wide variety of exit channels, and thus cannot be interpreted as statistical phenomena. A quantitative measure of the nonstatistical nature of the resonances, which also provides perspective for the magnitudes of the measured carbon partial widths, results from consideration of a simple statistical compound nucleus model. Statistical carbon widths estimated using the standard relation[17]

$$\langle\Gamma_c\rangle_J=\frac{1-\langle\eta_l\rangle^2}{2\pi\rho_J(E_x)}$$

are listed for each of the four resonances under consideration in Table I. In the above expression $\rho_J(E_x)$ is the density of states in the compound nucleus (^{24}Mg) at the relevant excitation energy, while $\langle\eta_l\rangle$ is taken from the extrapolated energy-averaged, or background, behavior of the reflection coefficients determined in the preceding phase shift analysis. Values for ρ_J were calculated using a standard prescription[18] and are also included in Table I. For each of the resonances the ratios $\Gamma_c/\langle\Gamma_c\rangle$ are large, with the observed carbon widths being 20 to 40 times larger than the statistical

TABLE I. Summary of resonance parameters for structures observed at $E_{c.m.}=5.60$, 6.00, 6.64, and 6.83 MeV in the $^{12}C+^{12}C$ interaction, and comparison with predictions of a statistical compound nucleus model.

$E_{c.m.}$ (MeV)	J^π	Γ_{tot} (keV)	Γ_c (keV)	$\langle\eta_l\rangle$	$\rho_J(E_x)$[b] (MeV^{-1})	$\langle\Gamma_c\rangle$	$\Gamma_c/\langle\Gamma_c\rangle$
5.60[a]	2^+	130	10	0.95	70	0.22	46
6.00[a]	4^+	100	4	0.97	69	0.14	29
6.64	2^+	100	29^{+6}_{-9}	0.71	103	0.77	38
6.83	4^+	125	11 ± 3	0.80	96	0.59	19

[a] Resonance parameters from Ref. 2.

[b] $\rho_J(E_x)$ calculated using the level density expression of Lang (Ref. 18) with parameters $r_0=1.50$, $\beta=0$, $a/A=0.149$, $\Delta_p=5.13$ (see Ref. 19).

widths. The comparison with the compound nucleus estimates thus demonstrates the greatly enhanced carbon-carbon parentage of the resonances, and shows that all four structures may be properly termed quasimolecular.

In conclusion we have shown that precise measurements of the elastic scattering cross section, near the Coulomb barrier, can be analyzed by the phase shift method to provide both background phase shifts and resonance parameters. The former provide a quantitative constraint on the two-body optical potentials used to describe the average features of the interaction, while the latter yield detailed information concerning the resonances. Moreover, the carbon partial widths extracted from the phase shift analysis are consistent with values estimated[13] from the more easily measured total reaction cross section. This observation justifies the approximations invoked in the analysis of the latter data, and prepares the way for a systematic study of carbon widths of additional Coulomb barrier resonances along the lines indicated in Ref. 13. With carbon widths known to good accuracy, it then becomes possible to analyze resonant yields in individual reaction channels for reliable values of the corresponding partial widths. The resulting, more complete, set of resonance parameters will provide for the first time a detailed picture of the structures of the Coulomb barrier resonances.

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