It can be seen in Figs. 2, 3, and 4 that the coupled-channel theory predicts the changes in cross section between Sm^{154} and Sm^{148} in close agreement with experiment solely on the basis of changes in the deformation parameters. A similar observation was made by Glendenning, Hendrie, and Jarvis[9] in an analysis of the previously referred to α-particle scattering results.[5,6]

DISCUSSION

The above analysis indicates that in the case of 16-MeV protons scattering from Sm^{154} and Sm^{148} a value of β_2 less than the value obtained from Coulomb excitation studies seems to be called for. Some possible reasons for such a discrepancy have been pointed out in Ref. 6 and by Bromley and Weneser.[10] Coulomb excitation measures only the distribution of nuclear charge. The distribution of nuclear mass which gives rise to the nuclear potential need not be the same as that of nuclear charge. Also, the shape of the optical model and of the nucleus may not be rigidly connected. In this particular case the reaction is nearly adiabatic and the incoming particle sees the target nucleus as a static deformed shape. Thus, the reaction should be relatively independent of nuclear dynamics.

Finally, a more exhaustive parameter search together with inclusion of a β_6 term should quantitatively improve the over-all fit, and especially the 4(+) fit.

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Relation Between the $2I+1$ Rule and Channel Correlation

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The fluctuation cross section is derived as a function of the reduced-width correlation coefficient and also as a function of the channel cross correlation, with the resulting indication that the fluctuation cross section depends strongly on the channel cross correlation. The total average cross section is shown to be proportional to the quantity $2I+1$ only when the total level width is sufficiently large. Finally, the author suggests that only for experiments in the region of very large Γ_μ should the total cross sections be compared with the $2I+1$ rule. In order to get the above condition, low energy with a high Q value are highly desirable.

This is one of a series of discussions of the $2I+1$ rule which indicates that the average total cross section is proportional to the quantity $2I+1$, where the spin I is that of the residual nucleus in a compound-nuclear reaction.

Recently,[1] four conditions for improving the $2I$

+1-rule proportionality were presented by the author. They are: (a) A large number of compound states should be excited with the spin J of compound states larger than the spin I of the final states; (b) the energy of outgoing particles should be large enough to ensure that the effect of the barrier penetration does not significantly suppress any possible l value; (c) the spin cutoff parameter σ^2 of compound nuclei should be small; and (d) the total level width Γ_μ should be large. In this communication, I would also emphasize the importance of the condition (d) by discussing the channel correlation effect.

Moldauer[2] has derived the fluctuation cross section as

$$\sigma_{c'c} = \pi \lambda\!\!\!^{-}{}_c^2 \left(\frac{2\pi}{D} \left\langle \frac{|\gamma_{\mu c}|^2 |\gamma_{\mu c'}|^2}{\Gamma_\mu} \right\rangle_\mu - I_{ij} \right), \tag{1}$$

where I_{ij} is a resonance interference term and is expressed as

$$I_{ij} = \frac{2\pi^2}{D^2} \left[|\langle \gamma_{\mu i} \gamma_{\mu j} \rangle_\mu|^2 - \left\langle \gamma_{\mu i} \gamma_{\mu j} \gamma_{\nu i}{}^* \gamma_{\nu j}{}^* \Phi_0 \left(\frac{\Gamma_\mu + \Gamma_\nu}{2D} \right) \right\rangle_{\mu \neq \nu} \right]. \tag{2}$$

Where i and j express any two channels that are specified by the usual coupling scheme ($\alpha SlJM$), and c and c' are the entrance channel and exit channel, respectively. The quantities γ_μ and γ_ν are single-channel reduced-width amplitudes of level μ and ν, and $|\gamma_{\mu c}{}^2| = \Gamma_{\mu c}$. Γ_μ is the total level width, D is the average level spacing.

Equation (1) can be immediately transformed to

$$\sigma_{c'c} = \pi \lambda\!\!\!^{-}{}_c^2 \left(\frac{T_c T_{c'}}{\sum_{c''} T_{c''}} F - I_{ij} \right), \tag{3}$$

where T_c and $T_{c'}$ are the transmission coefficients of channel c and c', respectively. And F[1] is the width-fluctuation correction factor.

I discussed the first term of Eq. (3) in Ref. 1 assuming that I_{ij} is negligible and concluded that if the total level width is large enough, the $2I+1$ rule may be improved.

Right now, I will discuss Eq. (2), the interference term I_{ij}. Considering the fact that in the presence of several channels the sum of two total widths, μ and ν, fluctuates very little and furthermore that Φ_0 is a slowly varying function[3] of its argument, we may ordinarily approximate Eq. (2) by

$$I_{ij} = \frac{2\pi^2}{D^2} |\langle \gamma_{\mu i} \gamma_{\mu j} \rangle_\mu|^2 \left[1 - \Phi_0 \left(\frac{\Gamma_\mu}{D} \right) \right], \tag{4}$$

where the function $\Phi_0(\Gamma_\mu/D)$ is evaluated with Dyson's[3] two-level correlation function. According to the assumption of Krieger and Porter[4] and Ullah,[5,6] the reduced-width amplitude distribution $P_{ijk\ldots}$ may be expressed as

$$P_{ijk\ldots} = \prod_\mu \frac{|M|^{1/2}}{(2)^{m/2}} \exp[-\tfrac{1}{2}(\gamma_\mu, M\gamma_\mu)], \tag{5}$$

where M is an $m \times m$ real symmetric positive-definite matrix. It is easy to show from Eq. (5) that

$$\langle \gamma_{\mu i} \gamma_{\mu j} \rangle_\mu = \frac{1}{2|M|} \frac{\partial}{\partial M_{ij}} |M|, \tag{6}$$

and

$$\langle \gamma_{\mu i}{}^2 \rangle_\mu = \frac{1}{|M|} \frac{\partial}{\partial M_{ii}} |M|. \tag{7}$$

For the case of only two channels, i and j,

$$P_{ij} = \prod_\mu \frac{|M|^{1/2}}{2\pi} \exp[-\tfrac{1}{2}(M_{ii}\gamma_{\mu i}{}^2 + M_{jj}\gamma_{\mu j}{}^2 + 2M_{ij}\gamma_{\mu i}\gamma_{\mu j})], \tag{8}$$

$$\langle \gamma_{\mu i}{}^2 \rangle_\mu = \frac{M_{jj}}{M_{ii}M_{jj} - M^2{}_{ij}}, \quad \langle \gamma_{\mu j}{}^2 \rangle_\mu = \frac{M_{ii}}{M_{ii}M_{jj} - M^2{}_{ij}}, \tag{9}$$

and

$$\langle \gamma_{\mu i} \gamma_{\mu j} \rangle_\mu = \frac{-M_{ij}}{M_{ii}M_{jj} - M^2{}_{ij}}, \tag{10}$$

where

$$M = \begin{Vmatrix} M_{ii} M_{ij} \\ M_{ij} M_{jj} \end{Vmatrix},$$

so that the channel correlation coefficient C_{ij} becomes

$$C_{ij} \equiv \frac{\langle \gamma_{\mu i} \gamma_{\mu j} \rangle_\mu}{(\langle \gamma_{\mu i}{}^2 \rangle_\mu \langle \gamma_{\mu j}{}^2 \rangle_\mu)^{1/2}}, \quad = \frac{-M_{ij}}{(M_{ii}M_{jj})^{1/2}}, \tag{11}$$

and the reduced-width correlation coefficient is easily found to be

$$\frac{\langle \gamma_{\mu i}{}^2 \gamma_{\mu j}{}^2 \rangle_\mu - \langle \gamma_{\mu i}{}^2 \rangle_\mu \langle \gamma_{\mu j}{}^2 \rangle_\mu}{[(\langle \gamma_{\mu i}{}^4 \rangle_\mu - \langle \gamma_{\mu i}{}^2 \rangle_\mu{}^2)(\langle \gamma_{\mu j}{}^4 \rangle_\mu - \langle \gamma_{\mu j}{}^2 \rangle_\mu{}^2)]^{1/2}} = C^2{}_{ij}$$

$$= C^2{}_{ij} = \frac{C_{ij}(0)}{[C_{ii}(0) C_{jj}(0)]^{1/2}}, \tag{12}$$

where $C_{ij}(0)$, $C_{ii}(0)$, and $C_{jj}(0)$ express the channel cross correlation, the autocross correlation of channel i, and the autocross correlation of channel j, respectively. They are

$$C_{ij}(0) = \left\langle \left(\frac{\sigma_i(E)}{\langle \sigma_i(E) \rangle} - 1 \right) \left(\frac{\sigma_j(E)}{\langle \sigma_j(E) \rangle} - 1 \right) \right\rangle,$$

$$C_{ii}(0) = \left\langle \left(\frac{\sigma_i(E)}{\langle \sigma_i(E) \rangle} - 1 \right)^2 \right\rangle,$$

and

$$C_{jj}(0)=\left\langle\left(\frac{\sigma_j(E)}{\langle\sigma_j(E)\rangle}-1\right)^2\right\rangle.$$

It should be noted from Eqs. (9) and (10), that unless M is diagonal, $\langle\gamma_{\mu i}{}^2\rangle_\mu$, $\langle\gamma_{\mu j}{}^2\rangle_\mu$, and $\langle\gamma_{\mu i}\gamma_{\mu j}\rangle_\mu$ are not independent with respect to channels.

Using the Eqs. (10) and (11), it is easy to get

$$|\langle\gamma_{\mu i}\gamma_{\mu j}\rangle_\mu|^2=\left[\frac{C^2{}_{ij}}{M_{ij}(1-C^2{}_{ij})}\right]^2. \quad (13)$$

Substituting Eq. (13) into Eq. (4), I_{ij} becomes

$$I_{ij}=\frac{2\pi^2}{D^2}\left[\frac{C^2{}_{ij}}{M_{ij}(1-C^2{}_{ij})}\right]^2\left[1-\Phi_0\left(\frac{\Gamma_\mu}{D}\right)\right]. \quad (14)$$

When M is diagonal $M_{ij}=0$ and $C_{ij}=0$ too, so $C^2{}_{ij}$ also equals zero. Then $I_{ij}=0$, so that Eq. (3) becomes

$$\sigma_{c'c}=\pi\lambda\!\!\!^{-}{}_c{}^2\left(\frac{T_cT_{c'}}{\sum_{c''}T_{c''}}F\right). \quad (15)$$

It is this equation that I discussed in Ref. 1. When M is not a diagonal matrix, and if M_{ij} approximately equals $(M_{ii}M_{jj})^{1/2}$, then C_{ij} is approximately equal to 1, and $C^2{}_{ij}$ must also be approximately equal to 1. The matrix elements I_{ij} become ∞, so they should not be neglected in Eq. (3) when one is discussing the $2I+1$ rule. Next we consider Eq. (12)

$$C^2{}_{ij}=\frac{C_{ij}(0)}{[C_{ii}(0)C_{jj}(0)]^{1/2}}.$$

For a wide range of angles (~±40°) about 90°,[7]

$$C_{ii}(0)=[\tfrac{1}{2}(2I_c+1)(2i_c+1)(2I_i+1)(2i_{c'}+1)]^{-1},$$

$$C_{jj}(0)=[\tfrac{1}{2}(2I_c+1)(2i_c+1)(2I_j+1)(2i_{c'}+1)]^{-1},$$

then

$$C^2{}_{ij}=\tfrac{1}{2}(2I_c+1)(2i_c+1)(2i_{c'}+1)(2I_i+1)^{1/2}\times(2I_j+1)^{1/2}C_{ij}(0).$$

Substituting the above equation into the Eq. (14), I_{ij} becomes

$$I_{ij}=\frac{2\pi^2}{D^2}\left\{\frac{(2I_c+1)^2(2i_c+1)^2(2i_{c'}+1)^2(2I_i+1)(2I_j+1)C^2{}_{ij}(0)}{4M_{ij}{}^2[1-\frac{1}{2}(2I_c+1)(2i_c+1)(2i_{c'}+1)(2I_i+1)^{1/2}(2I_j+1)^{1/2}C_{ij}(0)]^2}\left[1-\Phi_0\left(\frac{\Gamma_\mu}{D}\right)\right]\right\}, \quad (16)$$

where I_c, i_c, $i_{c'}$ are spins of target, incident particle, and outgoing particle, and I_i, I_j are spins of final states of the residual nucleus. It is easy to understand that I_{ij} is not proportional to $2I+1$ of either final state i or j, so therefore the total cross section $\sigma_{c'c}$ given by Eq. (3) could not be proportional to $2I+1$ of either final state i or j. In any case, the proportionality could be saved by $\Phi_0(\Gamma_\mu/D)$ when Γ_μ is very large. Because $\Phi_0(\Gamma_\mu/D)$ will be approximately equal to 1 when Γ_μ is very large, the term $[1-\Phi_0(\Gamma_\mu/D)]$ will approximately equals zero, and I_{ij} can then be neglected. According to the above discussion, we can conclude that one should determine whether or not the experiment proceeds by means of very large Γ_μ before comparing the cross section to the $2I+1$ rule. In order to satisfy the conditions of Hsu,[1] a high Q value and low incident-particle energy are highly desirable.

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