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α-Particle Trajectories in Ternary Fission

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The trajectories of the α particle and the two main fragments produced in ternary fission are calculated by computer on the basis of initial conditions determined by the statistical theory of fission. The angular and energy distributions of the three particles as functions of the mass ratio of the main fragments are deduced; they compare well with the experimental results of Fraenkel on spontaneous fission of Cf^{252}. Calculations are repeated with varied initial conditions; the final results are insensitive to the initial velocities of the fragments and the initial position of the α particle, but are very sensitive to the initial velocity of the α particle. Based on this sensitivity, the experimental angular distribution strongly supports the view of the statistical theory that the fission process is a slow process. The initial positions of the fragments are related to the prompt-neutron distribution; the present results on angular and energy distributions are consistent with experimental results of prompt-neutron distribution.

1. INTRODUCTION

Long-range α particles have been observed accompanying nuclear fission, the rate of this kind of ternary fission being less than 1% that of binary fission. The energy and angular distributions of the α particle, and their correlation with the two main fission fragments can be, and have been, measured experimentally; extensive results have been obtained by Fraenkel.[1] This information may be used to determine the nuclear configuration of the fissioning system at the scission point, because, after the scission point, the three particles move apart following largely the classical laws of motion, the outcome of which can be calculated accurately by high-speed computer. The nuclear configuration at the scission point, for which we have little previous experimental information, is a crucial point of consideration in the theory of fission. Various theories give different predictions; for example, the statistical theory predicts that the fission fragments at the scission point have little kinetic energy, of the order of 0.5 MeV,[2] whereas any dynamical theory would lead to a much greater kinetic-energy value. Once we have information to check the initial energy of the fission fragments at the moment of scission, the basic question in the fission theory, whether the fission process is slow (statistical) or fast (dynamical), can be resolved on an experimental basis.

To determine the nuclear configuration at the scission point, one has to establish the connection between the initial conditions of position and momentum of the fragments at the scission point and the final conditions of the three particles at "infinity"; some of the latter conditions can be experimentally determined. To establish this connection following the laws of motion of the three particles, the classical approximation of three mass points is usually adequate. Although the classical three-body problem has no exact solution, the evolution of the dynamical system can be calculated accurately by numerical computation. The α-particle-trajectory calculations have been carried out by a number of authors, including Halpern,[3] Geilikman and Khlebnikov,[4] Boneh, Fraenkel, and Nebenzahl,[5] Ertel,[6] and Fong,[7] Raisbeck and Thomas,[8] and Katase.[9]

Since the final conditions that can be experimentally determined are the energies and angles of the three particles, we do not have a complete set of position and momentum information to enable us to solve the equations of motion "backward" in time to determine the initial conditions. Therefore, most earlier authors tried various combinations of assumed initial conditions, and then determined the corresponding trajectories for comparison with experimental results in an attempt to select a set of initial conditions that will best account for the observed energy and angular distributions. This approach encounters the difficulty that the initial conditions are so numerous that even with the help of a high-speed computer, it is difficult to exhaust all possible combinations of the initial conditions. Usually, additional arguments are used to narrow down the choice of initial conditions.

Some of the initial conditions may be determined by reasonable assumptions, such as the two-dimensional approximation which is adopted by most authors and which is used in the present work. On the other hand, a few other initial conditions, such as the initial momentum, may require a more specific knowledge of the fission process for their determination. The statistical theory is in a position to supply the specific information required to determine the initial conditions for the trajectory calculation, and it is natural to ask what the resulting α-particle energy and angular distributions are and how they compare with experimental results. Ertel[6] was the first to carry out calculations along this line, and good agreement in angular distribution is obtained when the results are compared with experimental information.[7] The calculation also brings out the fact that the final conditions depend very sensitively on some specific initial condition, adding to the difficulty of reconstructing the initial condition from the final condition by trying out all possible combinations of the former. This sensitivity is helpful in experimental tests of the fission theories based on their predictions of the initial conditions at the scission point.

The present work improves Ertel's treatment by approximating the α particle by an extended sphere of radius $1.2\times4^{1/3}\times10^{-13}$ cm, instead of by a dimensionless point as in the previous work. When the α particle is introduced as a point placed at the scission point in the previous work, the total Coulomb potential energy of the system is increased by about 30 MeV. This would mean that the total kinetic energy of the fission products in ternary fission would be about 30 MeV higher than that in binary fission, whereas experimentally the two are about the same. This diffucilty arises because of the point-α-particle assumption, which is not realistic. An extended α particle is more realistic. The insertion of an extended sphere between two fission fragments in contact forces the two fragments to be separated farther apart, and thus reduces the Coulomb energy between them. In this way, the difficulty may be removed. Physically, the new assumption incorporates the idea that the α particle originates only when the binary fragments have greater then normal deformation, a point already made by Halpern.[3] Besides this major change, the present treatment also incorporates two additional improvements: (1) The small recoil forces the α particle exerts on the two main fragments are now included in the calculation. The change on the angular correlation is small, as expected (of the order of 2°), but is not negligible. (2) The numerical calculation is carried out to a much longer time of integration, increased from 2×10^{-20} to 10^{-13} sec. This does not noticeably change the results of the angular correlation, but improves the results of the energy values obtained.

2. TRAJECTORY CALCULATION

The masses, charges, and position vectors of the two main fission fragments and the α particle are designated by M_1, M_2, M_α; Q_1, Q_2, Q_α; and $\vec{r}_1$, $\vec{r}_2$, $\vec{r}_\alpha$, respectively. The equations of motion of the three particles are

$$M_1\frac{d^2\vec{r}_1}{dt^2}=\frac{Q_1Q_2}{|\vec{r}_1-\vec{r}_2|^3}(\vec{r}_1-\vec{r}_2)+\frac{Q_1Q_\alpha}{|\vec{r}_1-\vec{r}_\alpha|^3}(\vec{r}_1-\vec{r}_\alpha), \tag{1}$$

$$M_2\frac{d^2\vec{r}_2}{dt^2}=\frac{Q_2Q_1}{|\vec{r}_2-\vec{r}_1|^3}(\vec{r}_2-\vec{r}_1)+\frac{Q_2Q_\alpha}{|\vec{r}_2-\vec{r}_\alpha|^3}(\vec{r}_2-\vec{r}_\alpha), \tag{2}$$

$$M_\alpha\frac{d^2\vec{r}_\alpha}{dt^2}=\frac{Q_\alpha Q_1}{|\vec{r}_\alpha-\vec{r}_1|^3}(\vec{r}_\alpha-\vec{r}_1)+\frac{Q_\alpha Q_2}{|\vec{r}_\alpha-\vec{r}_2|^3}(\vec{r}_\alpha-\vec{r}_2). \tag{3}$$

The forces being central, we simplify the problem by a two-dimensional approximation. Only six coordinates are required, which are governed by six simultaneous differential equations.

The initial conditions of the six coordinates and the six corresponding velocity components are determined by the statistical theory, the details having been described previously.[6,7] Briefly, the statistical theory is applied to determine the most probable deformation shapes of the fission fragments at the scission point. This information has been obtained previously in connection with the prediction of the kinetic-energy and prompt-neutron distributions.[10] It can now be used to determine the initial positions of the charge centers of the main fragments. The α particle is assumed to be a sphere inserted between the two main fragments, as previously noted. Moreover, the statistical theory has previously determined the initial kinetic energy of the two main fragments to be 0.5 MeV,[2] as already mentioned. By an argument based on the equipartition of energy, we conclude that the initial kinetic energy of the α particle is about 0.5 MeV. The positions and velocities of the three particles can then be determined.

Once the initial conditions are given, the equations of motion may be integrated numerically for an infinitesimal time increment Δt, taken to be 10^{-23} sec. The position and velocity information at the end of this time increment may be used as the initial conditions for the integration of the equations of motion in the next time increment Δt, and this procedure may be repeated for one increment after another by the computer. Two hundred iterations are carried out with $\Delta t = 10^{-23}$ sec, after which Δt is changed to 10^{-22} sec, and another 100 iterations are carried out, after which the time interval is again increased by a factor of 10 and another 100 iterations are carried out, and so on, until a total of 1000 iterations are carried out, corresponding to a total time of integration of 10^{-13} sec. The results of the last 300 iterations do not change one another significantly as far as the velocity components are concerned; they may be considered as the value at infinity and may be compared with experiment. From the six velocity components, we determine the magnitude and direction of the three velocity vectors of the three particles. The magnitudes determine the kinetic energy distribution, and the directions determine the angular distribution. The calculation is repeated for a number of mass ratios of the main fragments; thus we determine the energy distribution and angular distribution as a function of the mass ratio of fission.

3. RESULTS

The results for the spontaneous fission of Cf^{252} are presented here. The trajectories of the three particles calculated at seven mass ratios ranging from 1 to 2 are shown in Figs. 1–7. Two successive points on a trajectory represent a time span of 2×10^{-22} sec; the trajectories depicted cover a total time of 2×10^{-21} sec, whereas the total time of integration is 10^{-13} sec. The angular correlation of the three particles as a function of the mass ratio is shown in Fig. 8. In the figure, the angle the α particle makes with a fragment (light or heavy) is plotted as a function of the mass number of the fragment and is shown by the solid curve, which is compared with the experimental results of Fraenkel[1] shown by dashed lines. The agreement is satisfactory. The calculated kinetic energy of the α particle as a function of the mass ratio of fission is shown in Fig. 9 by the solid curve, which is compared with the experimental values[1] shown by the dashed lines. The experimental values are said to be meaningful only in a relative sense because of limitations in experimental measurement. The agreement thus seems reasonable. The total kinetic energy of the two main fission fragments as a function of the mass ratio of fission is shown by the curves in Fig. 10, which are compared with the experimental results of Fraenkel[1] shown by horizontal lines. The solid curve and lines refer to ternary fission, whereas the dashed curve and lines refer to binary fission[10] which are included for reference. While comparing theory with experiment, attention should be directed to the change from binary to ternary fission. The theoretical change is in good agreement with the experimental change. The difference in the absolute magnitude between theory and experiment in ternary fission, even though small from a percentage point of view, is inherited from the early calculations in binary fission, and thus is no reflec-

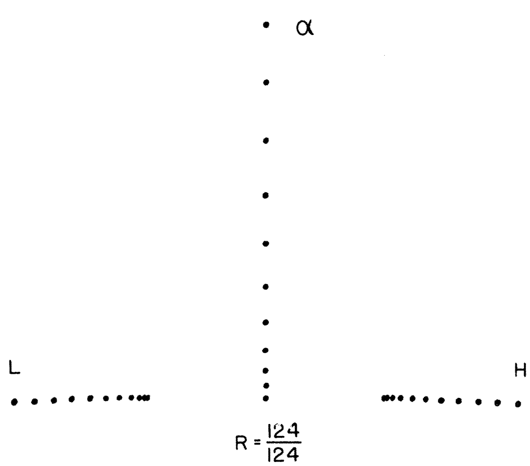

FIG. 1. Trajectories of the α particle and the main fragments for mass ratio 124:124.

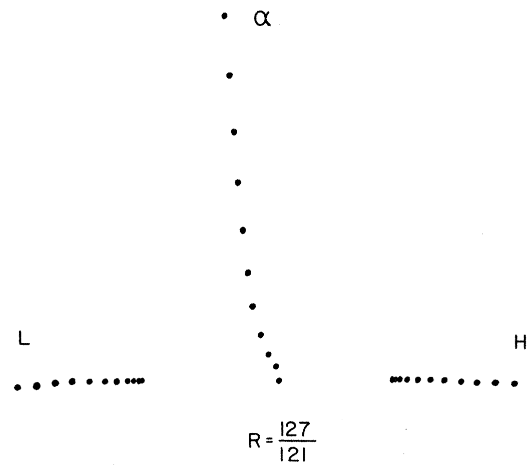

FIG. 2. Trajectories of the α particle and the main fragments for mass ratio 127:121.

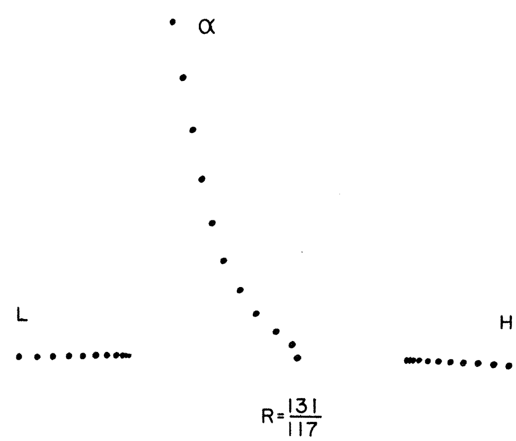

FIG. 3. Trajectories of the α particle and the main fragments for mass ratio 131:117.

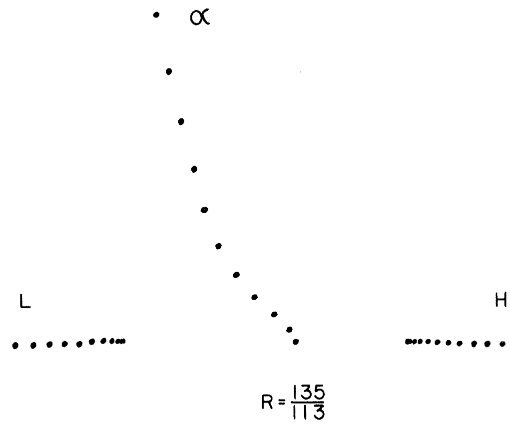

FIG. 4. Trajectories of the α particle and the main fragments for mass ratio 135:113.

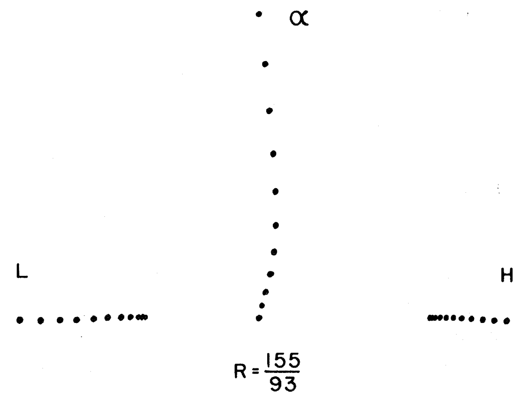

FIG. 6. Trajectories of the α particle and the main fragments for mass ratio 155:93.

tion on the validity of the present treatment on ternary fission. In other words, if we should start with a set of initial positions that gives better agreement in kinetic-energy distribution in binary fission, our present treatment on ternary fission would lead to better agreement in kinetic-energy distribution of the fragments.

4. ADDITIONAL CALCULATIONS

The complete calculation outlined above is repeated several times with several variations of the initial conditions to learn how sensitively a change in a particular initial condition affects the final results of angular and energy distributions.

First we change the initial velocities of the main fission fragments to zero and repeat the whole computation. The results are nearly indistinguishable from those of above, based on a total initial kinetic energy of 0.5 MeV for the two fragments. The final directions of the two main fragments are changed merely by 0.1° uniformly for all mass ratios; only in the far-asymmetric fission region does the final direction of the α particle show a slight change toward the center, with a magnitude less than 1°. For all mass ratios the change of the fragment kinetic energy is a decrease of the order of 1 MeV and the change of the α-particle kinetic energy is an increase of the order of 1 MeV. Thus within the change of the order of 1 MeV of the initial kinetic energy of the fragments, no significant change of the final results is expected. Part of this conclusion has been obtained earlier by Ertel.[6]

Next, we consider the effect of changing the initial velocity of the α particle. Ertel[6] has shown that the angular correlation curve changes drastically when the initial kinetic energy of the α particle, $E_{\alpha 0}$, is changed from 0.5 to 2 MeV. On the other hand, a change of $E_{\alpha 0}$ from 2 to 8 MeV does not change the angular correlation curve to any great extent. Therefore, within a certain energy range the angular distribution depends sensitively on $E_{\alpha 0}$. The reason for this sensitive dependence will be discussed later. To find out how the angular correlation curve changes for $E_{\alpha 0}$ below 0.5 MeV, we repeat the calculation with $E_{\alpha 0} = 0.125$

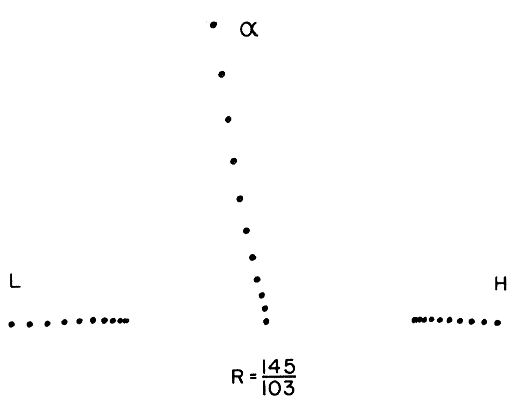

FIG. 5. Trajectories of the α particle and the main fragments for mass ratio 145:103.

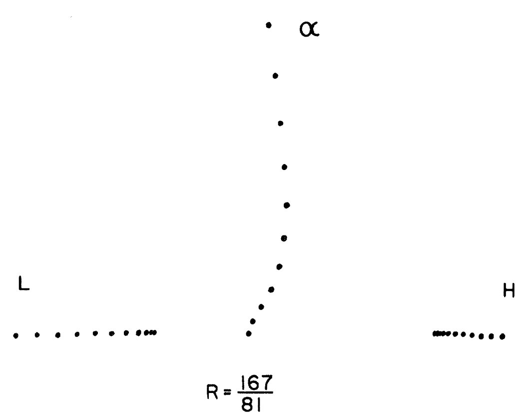

FIG. 7. Trajectories of the α particle and the main fragments for mass ratio 167:81.

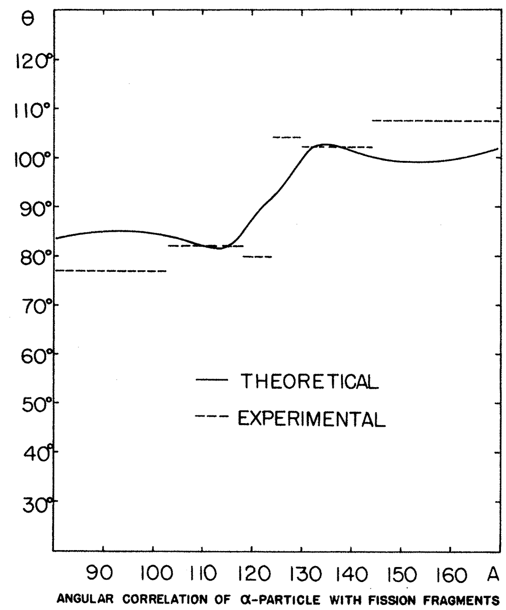

FIG. 8. The angle the α particle makes with a fragment (light or heavy) as a function of the mass number of the fragment, compared with experimental results of Fraenkel shown in horizontal bars.

MeV. In this case, the angular correlation curve becomes completely different from any of those obtained before: The angle between the α particle and the light fragment, θ_L, is nearly 90° for mass

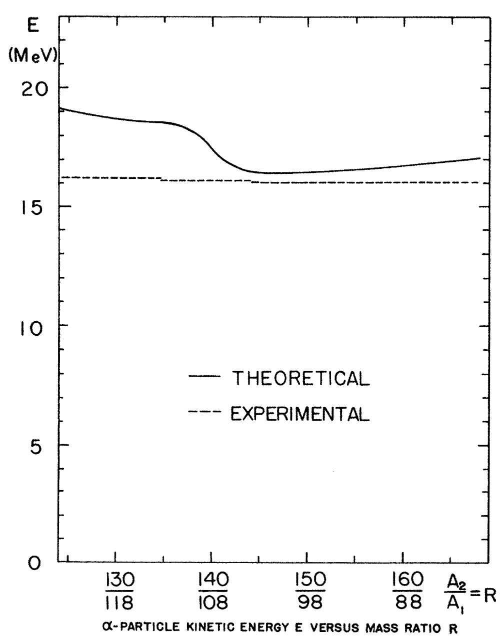

FIG. 9. The calculated kinetic energy of the α particle as a function of the mass ratio of the main fragments, compared with experimental results of Fraenkel shown by dashed lines.

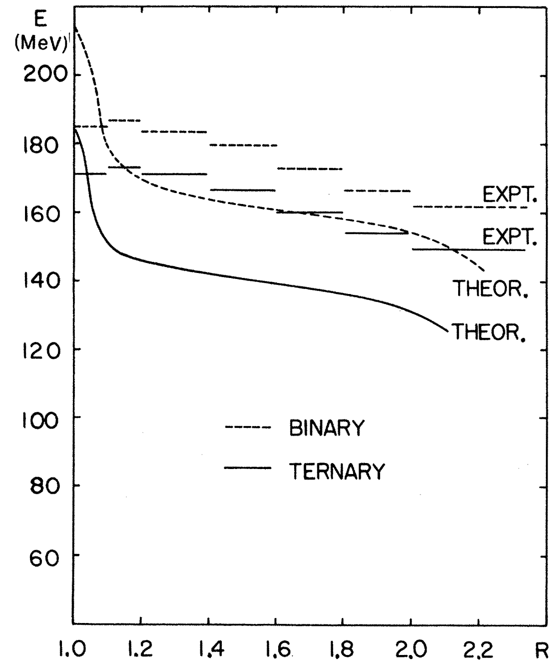

FIG. 10. The total kinetic energy of the two main fragments as a function of their mass ratio, theoretical curves compared with experimental results shown in horizontal lines. Ternary and binary fission are distinguished by solid and dashed lines and curves.

number from 124 to 113 and decreases to 71° monotonically when the mass number decreases to 81. Concerning the energy distribution with respect to the mass ratio, we find in this case that the α-particle final energy is uniformly reduced by 4 MeV, while the fragment kinetic energy increases by this amount.

Finally, we want to determine the effect of displacing the initial position of the α particle along the line (y axis) perpendicular to the fission axis – the line joining the centers of the two main fragments (x axis). In our classical approximation, the α particle is assigned exact position and momentum values. In reality, it should be more properly represented by a wave packet with position and momentum uncertainties Δy, Δp_y connected by the uncertainty relation. If the wave packet represents momentum values with a root-mean-square average corresponding to an energy of 0.5 MeV, the uncertainty relation calls for a position uncertainty of the order of 3 F. Thus we repeat the whole calculation with the α particle displaced in the y direction by 3 F. The results are that the α-particle final direction changes within a range of ±4°, such that the saw-tooth-shaped angular correlation curve becomes more accentuated – as the mass number of the light fragment decreases from 124 to 81, the angle θ_L decreases and later increases more sharply. Thus the introduction of this quantum-mechanical effect is not likely to change the angular correlation to any drastic extent, and will not change the conclusions previously arrived at. The kinetic energy of the α particle increases by about 3 MeV; the increase tends to smooth out the distribution curve in Fig. 9. The kinetic energy of the fragments decreases accordingly.

The initial positions of the charge centers of the main fragments are taken from those determined in a previous paper,[10] which explained the kinetic-energy, as well as the prompt-neutron distributions. Any drastic variation of these positions would be contrary to the known experimental facts for these distributions, and thus is not realistic. Therefore, we do not repeat the calculation with varied initial positions. On the other hand, Figs. 1–7 show that the characteristics of the trajectory are obviously determined by the closeness of the α particle to the initial position of the charge center of one fragment or the other.

5. DISCUSSIONS

The most remarkable feature of the experimental results is the saw-tooth–shaped angular-correlation curve. Curves of such a shape have been seen in other aspects of fission, such as in prompt-neutron distribution, and are usually traceable to local perturbing factors due to closed nuclear shells. It is natural to look into the shell effects to gain an understanding of the α-particle distributions.

The shell effects manifest themselves in the initial positions of the charge centers of the main fragments, as shown in Figs. 1–7. In Figs. 3 and 4 the heavy fragment is in the 82-neutron closed-shell region and so deforms only little, while the complementary light fragment deforms to a large extent; the result is that the initial position of the charge center of the heavy fragment is much closer to the α particle than that of the light fragment. The heavy fragment thus exerts a much stronger repelling force on the α particle than the light fragment, with the result that the α particle is emitted veering toward the light fragment. On the other hand, in the 50-neutron shell region the initial positions of the charge centers of the heavy and the light fragments are just the opposite as shown in Figs. 6 and 7, and one would suspect that the α particle to be emitted veering toward the heavy fragment. Indeed, this is the case in the earlier experimental results of Fraenkel and Thompson,[11] who explained this behavior by essentially the same argument based on experimental information of prompt-neutron distribution. However, the later experimental results of Fraenkel[1] contradicted the earlier ones – α particles in the mass ratio region corresponding to the 50-neutron shell are still emitted veering toward the light particle. This discrepancy between experiment and explanation is now resolved in the present calculation, as shown in Figs. 6 and 7. The α particles concerned are actually pushed toward the heavy fragment initially, as explained. But after a while the α particle has moved closer to the heavy fragment and the force from the latter becomes so great that it eventually pushes the α particle backward (reflection) and the α particle emerges veering toward the light fragment.

The occurrence of the above-mentioned reflection makes the correlation between the initial and final conditions of the trajectory subtle and complicated. Ertel's[6] calculations show that if $E_{\alpha 0}$ were taken to be 2 MeV or higher (up to 8 MeV), no reflection would take place. This is because of the fact that the α particle, having a higher initial velocity, now moves out of the accelerating field of the main fragments more quickly so that the x component of the electric field does not have enough time to cause a reflection. Without the reflection, the angular correlation would be like that of the earlier results of Fraenkel and Thompson, contradictory to the currently accepted experimental results. Therefore, accepting the current explanation of prompt-neutron distribution as due to shell effects on nuclear deformation, we can see little likelihood that $E_{\alpha 0}$ can have a value of more than 2 MeV. By a similar argument, for $E_{\alpha 0}$ below 0.5 MeV, we can expect reflection to take place in the 82-neutron shell region. Moreover, the number of reflections may be greater than 1, and may be different for the 82-neutron-shell and 50-neutron-shell regions, so that the calculated angular correlation curve may not be expected to agree with the experimental curve. Thus the sensitive dependence of the final condition on $E_{\alpha 0}$ leaves us little choice on the value of $E_{\alpha 0}$ other than in the neighborhood of 0.5 MeV. Therefore, the value predicted by the statistical theory receives strong experimental support.

The quantity $E_{\alpha 0}$ is crucial in the fission theory, because it is related to the problem of whether the fission process is fast or slow. A slow process calls for a statistical theory and a fast process necessitates a dynamical theory. The experimental evidence discussed here is thus strongly in favor of the statistical theory.

Boneh, Fraenkel, and Nebenzahl used a value of 3 MeV for $E_{\alpha 0}$ in their trajectory calculations. This value is arrived at by extraneous arguments that are not conclusive. Since at this energy no reflection is expected, it is necessary to assume that in the 50-neutron-shell region the light-fragment charge center is sufficiently far from the α particle in the initial position in order to account for the fact that the emitted α particle veers toward the light fragment. This requires that the light fragment be deformed (elongated) to a large extent, contradicting evidence from the prompt-neutron emission. The use of a higher value of $E_{\alpha 0}$ corresponds mathematically to the use of a later time in our trajectory as the initial time at which the particles have already been accelerated for a while and thus have acquired additional kinetic energy. Indeed, their initial conditions for the main fragments are such that their positions are much farther apart than ours and their velocities are much larger. Thus the mathematical features of the two sets of calculations are consistent and the major difference is in the choice of the value of $E_{\alpha 0}$.

Raisbeck and Thomas[8] carried out three-particle trajectory calculations in fission and concluded that an initial kinetic energy $E_{\alpha 0}$ of 2 MeV fits their experimental data best. It is to be noted first of all that their primary concern is to fit the experimental energy spectrum of the α particle determined without regard to the mass ratio of fission (in this way they succeeded remarkably in establishing that the same mechanism is involved in all kinds of light-particle emission), whereas our emphasis is to study the variation of the emission angle with respect to the mass ratio of fission. Though the two calculations deal with different problems and cannot be compared, the discrepancy in the value of $E_{\alpha 0}$ is a point that should be resolved. In this connection, we note that in their calculation the initial position of the α particle is taken to be the point of minimum potential energy on the interfragment axis, which is closer to the light fragment. On the other hand, our initial position for the most probable mass ratio is closer to the heavy fragment because of the closing of the 82-neutron shell in the heavy fragment. Because of this large difference in initial position, the corresponding values of $E_{\alpha 0}$ are not expected to be the same. We believe our initial position is more realistic, because it correlates the experimental results of prompt-neutron distribution, which cannot be ignored in a complete theory of fission. Except for the initial conditions for the α particle, other conditions in the two sets of calculations are comparable. Their interfragment distance at the moment of scission (20.5 F) is very close to ours in binary fission. For ternary fission this distance is increased in our calculation by the diameter of the α particle (3.8 F). This roughly corresponds to the increase in the distance in their calculation owing to their use of a later emission time for the α particle (of the order of 10^{-21} sec). Again, the mathematical features of the two sets of calculations are consistent, and the main difference is in the choice of the initial condition for the α particle.

Thus the reason that Boneh, Fraenkel, and Nebenzahl, and Raisbeck and Thomas obtained "best-fit" values of $E_{\alpha 0}$ different from ours is largely that they chose an initial position of the α particle different from ours. Their choices do not corroborate with the known information of prompt-neutron distribution and therefore their values of $E_{\alpha 0}$ cannot be regarded as conclusive.

Geilikman and Khlebnikov[4] assumed a maximum initial kinetic energy for the α particle of 1 MeV in trajectory calculations to fit the experimental distribution. Their value of $E_{\alpha 0}$ is much smaller than those of the above two groups, but is fairly close to the value we used here.

Katase[9] carried out extensive trajectory calculations by varying many of the parameters representing initial conditions over a fairly wide range, and determined functional correlation between the initial and final conditions. Because there are numerous combinations of possible values of initial parameters, it is difficult to exhaust all possibilities. It so happened that our initial conditions determined by the statistical theory were not covered by his calculations, in spite of the extensiveness of his coverage (Figs. 8–10 of Ref. 9). Therefore, the most interesting feature of our work, the reflection, does not appear in his results. His calculation of the α-particle energy spectrum at 90° does not include the effect of the dispersion of the mass ratio R and the dispersion of the total kinetic energy E_T. Furthermore, his "best-fit" distribution of interfragment distance calls for an excessive amount of fragment deformation, and therefore an excessive number of prompt neutrons. Moreover, the neutron distribution so deduced is not correlated with total kinetic energy, whereas experimentally there is definitely a negative correlation. Therefore, the conclusions derived from his results are not expected to be in agreement with ours.

Halpern[3] estimated theoretically the value of $E_{\alpha 0}$ to be 4.4 MeV, on the basis of the volume of the neck at scission. To fit experimental data on this basis, it is necessary to assume a scission configuration in which the main fragments have gained at least half the final speed. This implies excessively large deformation, and the same difficulty mentioned above occurs. All calculations with large values of $E_{\alpha 0}$ involve large initial interfragment distance D and thus have the same difficulty. This correlation of $E_{\alpha 0}$ with D is understandable from our calculation; it roughly corresponds to taking a later time in our calculation as the initial time. Thus their results do not contradict ours. On the other hand, when a later time is taken as the initial time, other information about the scission configuration is distorted, such as the initial position of the α particle and the deformation shapes of the main fragments. It is here that the major differences between their results and ours lie and their difficulties originate.

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PHYSICAL REVIEW C VOLUME 2, NUMBER 2 AUGUST 1970

Nuclear Matrix Elements in the Double Beta Decay $Te^{130} \to Xe^{130}$ †

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The nuclear matrix element for the double β decay $Te^{130} \to Xe^{130}$ is calculated with a specific Gamow-Teller force. A reasonable strength of this force is unable to give enough reduction in the rate to agree with the experimental half-life. The amount of isospin violation in the wave function owing to the difference of the neutron and proton single-particle energies is calculated. The number and isospin problems are discussed in connection with the use of the random-phase approximation in a β-decay calculation. It is shown that the β-decay operator has no spurious terms due to number or isospin dispersion.

I. INTRODUCTION

Double β-decay rates can give information about weak-interaction theory and nuclear structure. Primakoff and Rosen[1] have estimated the rates of several double β decays and have discussed the implications of the results on weak-interaction theory. The first actual measurement of a double-β-decay half-life has been obtained by Kirsten *et al.*[2] who detected excess Xe^{130} in a native Te^{130} mineral sample by mass-spectrographic techniques. Compared with the theoretical estimate,[1] the half-life of $Te^{130} \to Xe^{130}$,

$$T_{1/2} = 10^{21.34\pm0.12}\ \text{yr},$$

is in agreement with (but does not prove) the usually assumed theories of lepton conservation and the distinguishability of the antineutrino and the neutrino.

Primakoff and Rosen have estimated the nuclear matrix elements involved in double β decay as

$$|\langle\psi_f|\sum_{nm}\tau_n^-\tau_m^-|\psi_i\rangle|^2 \approx 0.01,$$

$$|\langle\psi_f|\sum_{nm}\tau_n^-\tau_m^-\vec{\sigma}_n\cdot\vec{\sigma}_m|\psi_i\rangle|^2 \approx 0.01.$$

Here, τ_n^- and $\vec{\sigma}_n$ are the isospin lowering operator and spin operator of the nth nucleon. The sum over intermediate states has been performed by closure, the energy denominators having been set equal to an average value. With the new mass-excess data[3] for the mass difference between Te^{130} and Xe^{130}, 2.509 ± 0.019 MeV, Primakoff and Rosen's estimate for the half-life of the two-neutrino decay of Te^{130} is

$$T_{1/2} = 10^{22.1\pm2}\ \text{yr}.$$

It is the purpose of this note to see if the above