Phenomenological Analysis of Quasirotational Spectra and Possible Evidence for Higher-Phonon States*

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We have developed a semiempirical description of the ground-state band of even quasirotational nuclei, based on the picture of anharmonic vibrations, which agrees surprisingly well with experiment. Comparison with previous empirical and semiempirical formulas is presented.

I. INTRODUCTION

Even nuclei in the mass region above $A = 50$ can be classified roughly according to the ratio of the excitation energy of the first excited 4^+ state to the excitation energy of the first excited 2^+ state. This parameter,

$$R_4 = E(I=4)/E(I=2), \tag{1}$$

ranges from the value $\frac{10}{3}$ corresponding to the ideal rotational limit down to the value 2, describing an ideal quadrupole vibrator.[1]

In the regions of strongly deformed nuclei, where one is not far from the rigid-rotator limit, there is both experimental and theoretical sanction for representing the excitation energy by a power series in $I(I+1)$,

$$E_I = [I(I+1)/2\mathcal{I}] - BI^2(I+1)^2 + \cdots . \tag{2}$$

Here we consider only the empirical basis for (2), and refer to the literature for recent attempts to compute the parameters contained therein.[2]

For the most strongly deformed nuclei, we have $2\mathcal{I}B \sim 10^{-3}$. Therefore for $I = 10$, the second term is only 10% of the first. Moreover, from a strictly mathematical point of view, assuming that it maintains its alternating character the series still converges for the highest I observed ($I \sim 20$), but for these large values of angular momentum, many terms would be needed for an accurate representation of the data. By contrast, at the edge of the deformed region, we find typically $2\mathcal{I}B \sim 10^{-2}$. Thus *mathematical* convergence of the series fails at $I \sim 10$, and we must seek alternative representations of the data here and *a fortiori* in the transition region between deformed and spherical nuclei.

A power-series expansion with improved convergence properties was first suggested by Harris,[3] as an extension of cranking ideas. In the Harris formulation, both the excitation energies and the angular momenta are given in terms of related power series in the intermediary variable ω, the "angular velocity,"

$$\begin{aligned} E_I &= \tfrac{1}{2}\omega^2[\mathcal{I}_0 + 3C\omega^2 + 5D\omega^4 + 7F\omega^6 + \cdots], \\ [I(I+1)]^{1/2} &= \omega[\mathcal{I}_0 + 2C\omega^2 + 3D\omega^4 + 4F\omega^6 + \cdots]. \end{aligned} \tag{3}$$

Even the two-parameter approximation to (3) yields excellent fits throughout the rotational region, to which Harris restricted his analysis.

More recently the data have been analyzed by Mariscotti, Scharff-Goldhaber, and Buck[4] by means of a vibrating-potential model with a variable-moment-of-inertia (VMI model). The two-parameter form of this proposal, represented by the equations

$$\begin{aligned} E_I &= [I(I+1)/2\mathcal{I}_I] + \tfrac{1}{2}C_0(\mathcal{I}_I - \mathcal{I}_0)^2, \\ (\partial E_I/\partial \mathcal{I})\,|_I &= C_0(\mathcal{I}_I - \mathcal{I}_0) - [I(I+1)/2\mathcal{I}_I^2] = 0, \end{aligned} \tag{4}$$

turns out to be exactly equivalent to the two-parameter approximation to the Harris equations. Excellent fits in the rotational region, here defined as $3 \leq R_4 \leq \frac{10}{3}$, are thereby guaranteed, but surprisingly enough the formulation (3) or (4) continues to yield fits to the data in the transition region, $2.40 \leq R_4 \leq 3.00$, only slightly less accurate than in the rotational region. Remarkably, one continues to obtain fits even into the good vibrational region ($2 \leq R_4 \leq 2.4$) where the moment-of-inertia parameter $\mathcal{I}_0$ turns out to be negative below $R_4 = 2.23$.

We believe that the explanation for this startling success is that the approximate forms of (3) or (4) represent partial summation of the series (2) which defines analytical continuations beyond the good rotational region. We have shown elsewhere[5] that (2) and (3) are rigorously equivalent where both converge and that (3) and a rational extension of (4) are also equivalent. The improved convergence can be understood qualitatively from (3), from which it is easily established that the parameter ω increases with $[I(I+1)]^{1/2}$ more slowly than linearly.

We were led to the present investigation in attempting to answer the question: Is there any evi-

dence for higher-phonon states among vibrational nuclei? It occurred to us then to question the predisposition to accept the sequence of states $I=0, 2, 4, \ldots$ found in the transition region as quasirotational spectra. Order and angular momentum are not sufficient criteria, since the Nth quadrupole phonon multiplet always has a unique state, with $I=2N$ yielding the same over-all sequence. Setting out from the vibration end, we have derived a general formula based on the assumption of anharmonic vibration, which contains the ideal vibrational limit as a special case, gives a fit of *comparable* accuracy to the VMI model in the transition region, and continues to fit well into the rotational region (though it now fails the VMI model in this regard, just as the latter is inferior to our representation in the vibrational region). The new representation, a polynomial in I rather than in $I(I+1)$, had been suggested previously on purely empirical grounds by Ejiri *et al.*[6]

In Sec. II, we describe the model of anharmonic vibrations at the base of our analysis and derive a formula not only for the excitation energies but also for other observables such as the quadrupole operator. The comparison with experiment and the other phenomenological analyses mentioned above is described in Sec. III.

Though we shall argue finally that there is at the moment no experimental reason for preferring a rotational model over a vibrational one in the transition region (or *vice versa*), the skeptical reader may well ask why at most one member of each phonon multiplet survives as a vibrational state. A possible basis for such a situation is that the state with $I=2N$ always represents, in cases of interest, the *ground* state of the given angular momentum. The number of states of that angular momentum in its vicinity is then much lower than for the other members of the same phonon multiplet, and therefore the chances of purity are higher. We have examined such evidence as exists, and found that in general the $B(E2)$ value is closer to the harmonic limit of 2 for the transition $4(N=2)\rightarrow 2(N=1)$ than for either $2(N=2)\rightarrow 2(N=1)$ or $0(N=2)\rightarrow 2(N=1)$, in agreement with our conjecture.

II. MODEL OF ANHARMONIC VIBRATIONS

The description of quadrupole vibrations in nuclei in terms of phonons carrying angular momentum two gives the level scheme[7]

N = number of phonons	I = spins of states
0	0
1	2
2	0, 2, 4
3	0, 2, 3, 4, 6
4	$0, 2^2, 4^2, 5, 6, 8$.

We will follow the hypothesis that the states of highest angular momentum for each phonon number go over (more or less smoothly) into the members of the ground-state band, as one moves into the region of increased stability of the quadrupole shape. Similarly the 0^+ and 2^+ states of the two-phonon triplet are expected to provide the band heads for the β and γ bands, respectively. In order to accommodate the growing deviation from the picture of a harmonic vibration, we will introduce anharmonic terms.

The basis for such a description has already been pointed out by Kerman and collaborators.[8] One assumes that the eigenstates of the exact Hamiltonian can be represented in the form

$$|N, IM\rangle = \frac{1}{\sqrt{N!}}\,[B^\dagger_{2m_1}\cdots B^\dagger_{2m_N}]_{IM}\,|0\rangle\,, \tag{5}$$

with

$$[B_{2m}, B^\dagger_{2m'}] = \delta_{mm'}\,, \tag{6}$$

and the square brackets $[\ ]_{IM}$ represent angular momentum coupling. The expansion of the Hamiltonian in terms of the boson operators $B^\dagger_{2m}$ and B_{2m} should then be diagonal with respect to the states (5), but it will contain anharmonic terms. Up to terms of fourth order in B and $B^\dagger$ we then have

$$H_4 = b[B^\dagger_{2m_1}B_{2m_2}]_{00} + \sum_I c_I\,[[B^\dagger_{2m_1}B^\dagger_{2m_2}]_{IM}[B_{2m_3}B_{2m_4}]_{I-M}]_{00}\,. \tag{7}$$

These equations define the "physical" boson. If there is any resemblance to a vibrational picture, the expansion in the Hamiltonian (7) should converge well and the coefficients of the anharmonic term c_0, c_2, c_4 should be small compared to the coefficient of the harmonic term b.

The next-order term in the expansion would be of the form

$$H'_6 = \sum_I d_I\,[[B^\dagger_{2m_1}B^\dagger_{2m_2}B^\dagger_{2m_3}]_{IM}[B_{2m_4}B_{2m_5}B_{2m_6}]_{I,-M}]_{00}\,. \tag{8}$$

Operators representing other observables can be expanded in a similar fashion. For the quadrupole operator, one would write to a corresponding order

$$Q_{2q} = a_1[B^\dagger_{2q} + (-)^q B_{2-q}] + \sum_I a_{2I}\{[B^\dagger_{2m_1}[B_{2m_2}B_{2m_3}]_{IM}]_{2q} + [[B^\dagger_{2m_1}B^\dagger_{2m_2}]_{IM}B_{2m_3}]_{2q}\} + a_3[B^\dagger_{2m_1}B_{2m_2}]_{2q} + a_4\{[B_{2m_1}B_{2m_2}]_{2q} + [B^\dagger_{2m_1}B^\dagger_{2m_2}]_{2q}\}\,. \tag{9}$$

In this expression the harmonic limit is represented by the first term. This term corresponds to a

transition with a change of one in the phonon number. The second term allows for deviations from the harmonic limit, but again involves $\Delta N = \pm 1$. The third term gives rise to a static quadrupole moment for the states and the last term allows for crossover transitions involving $\Delta N = \pm 2$, e.g., the ground state to 2^+, $N = 2$ transition.

Within the approximations outlined above, all the parameters in the expressions (7) and (9) can be fitted by considering the experimental information on the five lowest states 0, 2, 0′, 2′, 4 of an even nucleus. If this information is complete, one then obtains predictions for the properties of the higher-phonon states, as, e.g., for the three-phonon states in Ref. 8.

We are interested in a different aspect of the model. As implied in the introduction, our specific aim is to check to what degree the states of highest angular momentum in each N-phonon multiplet describe the appearance of quasirotational bands. Aside from our inherent right to ask such a question, we have described, at the end of the introduction, our reasons for believing that such a query may not be *a priori* nonsense.

For the states with $I = 2N$, the angular momentum couplings are at their simplest for calculational purposes, since one can always use completely stretched angular momentum configurations. For the excitation energies of these states, we obtain from (5) and (7)

$$E_I = E(N,\ I = 2N) = (\tfrac{1}{2}b/\sqrt{5})I + \tfrac{1}{12}c_4 I(I-2)\,. \tag{10}$$

The sixth-order term of Eq. (8) gives the contribution

$$\Delta E_I = (\tfrac{1}{48}d_6/\sqrt{13})I(I-2)(I-4)\,. \tag{11}$$

From the quadrupole operator (9), one obtains for the reduced transition rates

$$B(E2;\ I+2 \to I) = \tfrac{1}{2}(I+2)\,a_1 + \tfrac{1}{6}\sqrt{5}\,a_{24}I^2\,, \tag{12}$$

and for the quadrupole moment

$$Q_2(I) = a_3 \begin{bmatrix} 2 & 2 & 2 \\ 2 & 0 & 2 \end{bmatrix} \tfrac{1}{2}I\,. \tag{13}$$

The square bracket in (13) represents a Clebsch-Gordan coefficient.

III. COMPARISON WITH EXPERIMENT AND OTHER PHENOMENOLOGICAL RESULTS

The two-parameter formula (10) can be rewritten in several suggestive ways which make contact with experiment. For example, with a slight rearrangement, one obtains the form

$$E_I = aI + kI(I+1)\,. \tag{14}$$

The new parameters are

$$a = \tfrac{1}{2}b/\sqrt{5} - \tfrac{1}{4}c_4\,, \quad k = \tfrac{1}{12}c_4\,. \tag{15}$$

Remarkably, this energy rule has been proposed previously on purely empirical grounds as a best over-all description of vibrational states in spherical nuclei, quasirotational bands in transition nuclei, as well as of rotational bands in deformed nuclei by Ejiri *et al.*[6]

For a second comparison with experimental results, we express the ratios $R_I = E_I/E_2$ as a function of R_4,

$$R_I = \tfrac{1}{8}I(I-2)R_4 - \tfrac{1}{4}I(I-4)\,. \tag{16}$$

It was noticed by Mallman[9] that the experimental ratios R_6 and R_8 plotted versus R_4 lie on two universal curves, which are nearly straight lines. This is irrespective of the structure of the states involved. In Fig. 1 we compare the straight line corresponding to (16) for $I = 6$ [curve (a)] with the experimental points. It follows the experimental points from the vibrational region through the transition region into the rotational region. Curve (c) results from the VMI model. The natural range for this model is from $R_4 = 2.23$ describing a very soft nucleus to $R_4 = \frac{10}{3}$ describing a rigid system. This curve clearly gives the best description of the nuclei in the rotational region. The experi-

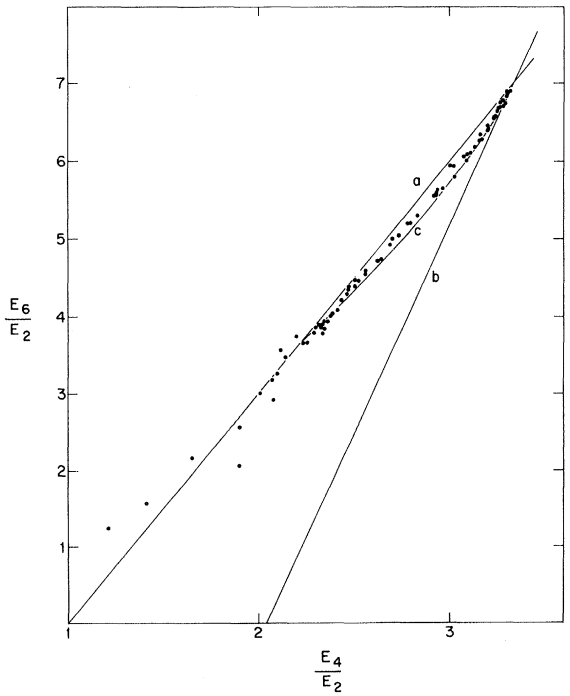

FIG. 1. Comparison of the experimental energy ratio R_6 as a function of R_4 with the predictions of the anharmonic vibrational model (a), rotation-vibration-coupling model (b), and the variable-moment-of-inertia model (c).

TABLE I. Comparison of calculated and experimental energies for representative nuclei in the region $2 \leq R_4 \leq \frac{10}{3}$. For each isotope we list in row (1), experimental energies; row (2), two-parameter anharmonic vibrational model; row (3), three-parameter vibrational model; row (4), VMI model. The last column gives the parameters of the vibrational model.

Isotope	R_4	Energy of the state (in MeV) 2^+	4^+	6^+	8^+	10^+	12^+	14^+	16^+	Parameters (MeV)
^{118}Te	2.008	0.605	1.215	1.832						$b=1.3530$
				1.830	2.450	3.075				$c_4=0.0075$
					2.458	3.095				$d_6=0.0072$
				(1.901)	(2.648)	(3.448)				
^{120}Te	2.073	0.562	1.165	1.782						$b=1.2570$
				1.809	2.494	3.220				$c_4=0.0615$
					2.386	2.950				$d_6=-0.0973$
				(1.849)	(2.598)	(3.402)				
^{96}Mo	2.116	0.770	1.629	2.750						$b=1.722$
				2.577	3.614	4.740				$c_4=0.134$
					4.306	6.470				$d_6=0.624$
				(2.609)	(3.685)	(4.843)				
^{148}Sm	2.143	0.551	1.181	1.908						$b=1.232$
				1.890	2.678	3.545				$c_4=0.119$
					2.750	3.725				$d_6=0.065$
				(1.902)	(2.697)	(3.552)				
^{154}Dy	2.232	0.335	0.747	1.224	1.748	2.306				$b=0.748$
				1.237	1.804	2.450	3.172	3.973		$c_4=0.116$
					1.754	2.325	2.922	3.535		$d_6=-0.045$
				1.225	1.755	2.328	2.939	3.583		
^{102}Pd	2.292	0.558	1.279	2.111						$b=1.248$
				2.163	3.210	4.420				$c_4=0.245$
					3.002	3.900				$d_6=-0.188$
				2.123	3.062	4.080				
^{156}Er	2.315	0.344	0.797	1.341	1.959					$b=0.770$
				1.359	2.029	2.807	3.694			$c_4=0.163$
					1.956	2.625	3.330			$d_6=-0.066$
				1.329	1.922	2.566	3.254			
^{106}Cd	2.362	0.633	1.495	2.495	3.504					$b=1.415$
				2.586	3.906	5.455	7.233			$c_4=0.344$
					3.542	4.545	5.413			$d_6=-0.328$
				2.514	3.655	4.897	6.226			
^{126}Xe	2.436	0.390	0.950	1.645	2.445					$b=0.872$
				1.680	2.580	3.650	4.890			$c_4=0.255$
					2.440	3.300	4.190			$d_6=-0.126$
				1.620	2.375	3.199	4.083			
^{120}Xe	2.469	0.322	0.794	1.396	2.097	2.870				$b=0.720$
				1.418	2.192	3.117	4.193	5.419		$c_4=0.226$
					2.105	2.899	4.757	4.656		$d_6=-0.079$
				1.363	2.005	2.707	3.462	4.263		
^{190}Pt	2.510	0.292	0.733	1.283	1.903	2.636				$b=0.653$
				1.323	2.062	2.950	3.987	5.173		$c_4=0.224$
					1.902	2.550	3.187	3.773		$d_6=-0.144$
				1.267	1.873	2.537	3.251	4.010		
^{186}Pt	2.562	0.191	0.490	0.877	1.341	1.856	2.407			$b=0.427$
				0.896	1.409	2.030	2.758	3.593	4.536	$c_4=0.161$
					1.334	1.843	2.384	2.939	3.489	$d_6=-0.067$
				0.855	1.270	1.727	2.220	2.743	3.295	

TABLE I (*Continued*)

Isotope	R_4	Energy of the state (in MeV) 2^+	4^+	6^+	8^+	10^+	12^+	14^+	16^+	Parameters (MeV)
^{160}Yb	2.627	0.243	0.638	1.147	1.736					$b=0.543$
				1.186	1.886	2.738	3.743			$c_4=0.228$
					1.731	2.350	2.967			$d_6=-0.140$
				1.128	1.688	2.306	2.974			
^{184}Pt	2.682	0.162	0.435	0.797	1.229	1.705	2.201	2.723	3.726	$b=0.363$
				0.818	1.312	1.917	2.632	2.457	4.394	$c_4=0.166$
					1.229	1.709	2.216	2.729	3.229	$d_6=-0.075$
				0.776	1.169	1.604	2.075	2.577	3.108	
^{182}Pt	2.708	0.154	0.416	0.771	1.202	1.695	2.238			$b=0.344$
				0.788	1.268	1.857	2.554	3.361	4.276	$c_4=0.163$
					1.203	1.696	2.232	2.797	3.374	$d_6=-0.058$
				0.746	1.127	1.550	2.008	2.497	3.015	
^{126}Ba	2.779	0.256	0.712	1.333	2.090	2.919				$b=0.573$
				1.367	2.221	3.275	4.528	5.980		$c_4=0.299$
					2.087	2.940	3.858	4.808		$d_6=-0.121$
				1.293	1.969	2.723	3.542	4.420		
^{130}Ce	2.797	0.254	0.711	1.324	2.053					$b=0.568$
				1.370	2.231	3.296	4.562			$c_4=0.304$
					2.049	2.839	3.648			$d_6=-0.165$
				1.296	1.978	2.739	3.568			
^{124}Ba	2.835	0.230	0.651	1.223	1.857					$b=0.513$
				1.263	2.068	3.064	4.251			$c_4=0.287$
					1.906	2.661	3.445			$d_6=-0.145$
				1.195	1.833	2.546	3.324			
^{128}Ce	2.930	0.207	0.607	1.158	1.820	2.573				$b=0.464$
				1.200	1.985	2.964	4.134	5.498		$c_4=0.289$
					1.817	2.542	3.290	4.021		$d_6=-0.152$
				1.137	1.765	2.473	3.250	4.086		
^{166}Hf	2.966	0.159	0.471	0.898	1.407	1.971	2.565	3.178		$b=0.355$
				0.936	1.555	2.327	3.252	4.330	5.562	$c_4=0.230$
					1.401	1.943	2.484	2.986	3.412	$d_6=-0.139$
				0.888	1.385	1.949	2.567	3.235	3.947	
^{152}Sm	3.009	0.122	0.366	0.712	1.122	1.615				$b=0.272$
				0.734	1.224	1.837	2.573	3.432		$c_4=0.184$
					1.137	1.619	2.136	2.667		$d_6=-0.079$
				0.698	1.096	1.548	2.046	2.586		
^{188}Os	3.083	0.155	0.478	0.940	1.514	2.170				$b=0.347$
				0.969	1.627	2.454	3.449	4.611		$c_4=0.252$
					1.512	2.165	2.871	3.599		$d_6=-0.104$
				0.926	1.472	2.098	2.793	3.549		
^{168}Hf	3.107	0.124	0.385	0.756	1.212	1.734	2.304	2.910		$b=0.277$
				0.783	1.319	1.992	2.801	3.749	4.833	$c_4=0.206$
					1.210	1.720	2.257	2.797	3.310	$d_6=-0.098$
				0.750	1.198	1.714	2.287	2.912	3.582	
^{186}Os	3.163	0.137	0.434	0.869	1.421	2.068				$b=0.307$
				0.890	1.506	2.281	3.216	4.310		$c_4=0.239$
					1.420	2.067	2.788	3.561		$d_6=-0.077$
				0.858	1.386	1.999	2.687	3.439		
^{170}Hf	3.206	0.100	0.321	0.641	1.041	1.503	2.013	2.564	3.147	$b=0.224$
				0.662	1.124	1.706	2.409	3.233	4.177	$c_4=0.181$
					1.041	1.499	1.995	2.508	3.018	$d_6=-0.075$
				0.642	1.048	1.524	2.062	2.655	3.297	

TABLE I *(Continued)*

Isotope	R_4	Energy of the state (in MeV) 2^+	4^+	6^+	8^+	10^+	12^+	14^+	16^+	Parameters (MeV)
^{172}Hf	3.258	0.095	0.308	0.627	1.036	1.519	2.063	2.651	3.273	$b=0.211$
				0.640	1.091	1.662	2.351	3.158	4.085	$c_4=0.178$
					1.039	1.530	2.087	2.696	3.346	$d_6=-0.048$
				0.627	1.039	1.532	2.097	2.725	3.410	
^{172}Yb	3.308	0.079	0.260	0.540	0.910	1.352				$b=0.176$
				0.545	0.932	1.423	2.016	2.712		$c_4=0.154$
					0.913	1.375	1.920	2.544		$d_6=-0.017$
				0.540	0.913	1.372	1.911	2.525		
^{244}Cm	3.317	0.043	0.142	0.296	0.502					$b=0.096$
				0.298	0.511	0.780	1.105			$c_4=0.085$
					0.502	0.758	1.061			$d_6=-0.0079$
				0.297	0.504	0.761	1.066			

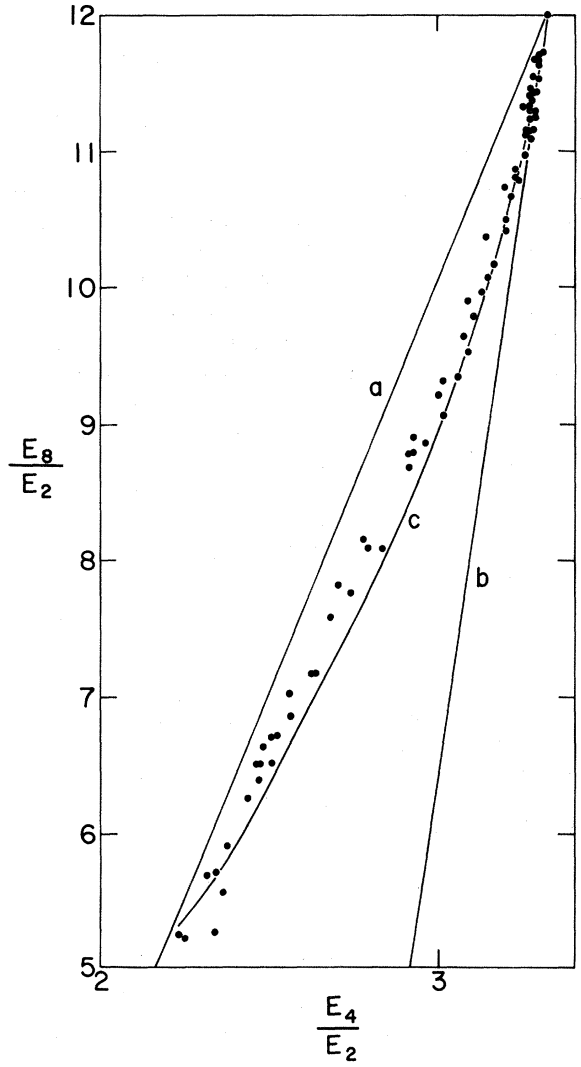

FIG. 2. Comparison of the experimental energy ratio R_8 as a function of R_4 with the same cases as in Fig. 1.

mental points in the transition region tend to lie between the curves (a) and (c).

By contrast, the curve (b), obtained from the straightforward rotational two-parameter formula (2), fails outside the ideal rotational region. The similar situation for $I=8$ is illustrated in Fig. 2.

For a more detailed picture, we compare in Table I the energy levels of representative nuclei covering the range $2 \leq R_4 \leq \frac{10}{3}$ with the two-parameter formula (10), the three-parameter formula obtained by including Eq. (11), and the two-parameter VMI model. The parameters in each case are fitted by adjustment to the required number of lowest energy levels rather than by a least-squares procedure. For the cases with $R_4 < 2.23$, the VMI model requires a negative value of g_0. We have marked these cases by bracketing the results. We concentrate our discussion on the parameters b, c_4, and d_6 extracted from the fitting procedure.

In the vibrational region and the beginning of the transition region, we find typically

$$|b| > |c_4| \gtrsim |d_6| . \tag{17}$$

The first term in the expansion dominates the series up to fairly large values of I [note the additional numerical factors in (10) and (11)], but the second and third terms are in competition.

In the rotational region and the end of the transition region, we always have

$$b \gtrsim |c_4| > |d_6| . \tag{18}$$

The first two terms are now in competition, but the third term is definitely smaller.

Over-all there is a good fit of the power series in I, originally derived from the model of anharmonic vibrations, over the entire range of R_4 and certainly well beyond any *a priori* expectations.

For the Harris and VMI formulations, the pattern

TABLE II. Comparison of the polynomial fit to the VMI model with the anharmonic vibrational model (AVM). The order of the polynomial used to fit the VMI energies does not effect the first two terms significantly.

Isotope	R_4	Model	Coefficient of the term proportional to (in MeV). I	I^2	I^3	I^4
^{118}Te	2.0083	AVM	0.3016	0.0004	0.0000	
		VMI	0.3527	−0.0517	0.0185	-0.32×10^{-2}
^{154}Dy	2.2318	AVM	0.1459	0.0112	−0.0002	
		VMI	0.1497	0.0068	0.0016	-0.39×10^{-3}
^{126}Xe	2.4359	AVM	0.1466	0.0255	−0.0007	
		VMI	0.1329	0.0387	−0.0048	0.57×10^{-3}
^{186}Pt	2.5620	AMV	0.0656	0.0157	−0.0003	
		VMI	0.0545	0.0263	−0.0037	0.46×10^{-3}
^{126}Ba	2.7786	AVM	0.0726	0.0290	−0.0006	
		VMI	0.0541	0.0464	−0.0059	0.70×10^{-3}
^{152}Sm	3.0087	AVM	0.0265	0.0180	−0.0004	
		VMI	0.0209	0.0231	−0.0018	0.13×10^{-3}
^{170}Hf	3.2060	AVM	0.0165	0.0175	−0.0004	
		VMI	0.0169	0.0171	−0.0002	-0.61×10^{-4}
^{244}Cm	3.3170	AVM	0.0070	0.0072	−0.000 046	
		VMI	0.0072	0.0072	−0.000 005	-0.26×10^{-5}

of convergence is reversed. For $I=2$, we find for the ratio $|3C\omega^2/\mathcal{I}_0|$ a value of about 3 for a vibrational nucleus like ^{108}Pd, but a small value of 0.0064 for a rotational nucleus like ^{248}Cm. This shows that the power series in ω is dominated by one term in the rotational region, but the first two terms compete in the vibrational region. The acceptable fit of the energies of the 6^+ states in the vibrational region would again indicate, however, that the series converges fairly well even if $3C\omega^2$ is comparable in magnitude to $\mathcal{I}_0$. This was again not to be anticipated on *a priori* physical grounds.

The comparable quality of the fits for the VMI and the anharmonic models can be stressed by expanding the VMI results in a polynomial in I. This comparison is carried out in Table II. One finds that the coefficients of the first two (three) terms are very close to the ones obtained in the anharmonic model. The coefficients of the higher-order terms are small. Nevertheless the concordance is not perfect and cannot be, as is quite clear from the trends of Figs. 1 and 2. It would appear that we need a formulation with more than two parameters to obtain a precision fit in the transition region. Certainly there is not much to choose between the models in this region on the basis of energetics alone. At the moment the information on $B(E2)$ values is too skimpy to be of help.

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[1]We do not consider magic nuclei, as, e.g., the Ni isotopes.

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