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PHYSICAL REVIEW C VOLUME 2, NUMBER 1 JULY 1970

Relationship Between the Triton Energy and the Neutron-Deuteron Doublet Scattering Length*

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Phillips found a linear relation between the triton energy E_t and the doublet n-d scattering length ^2a for a variety of separable tensor potentials. We have treated many more separable potentials, and also the Tabakin and Mongan rank-two separable potentials. The linear relation holds well for two-body potentials that fit the energies of the two-body triplet bound and singlet antibound states.

I. INTRODUCTION

In this paper we present a number of calculations of the triton energy (E_t) and the doublet n-d scattering length (^2a) supporting the linear relation between these two quantities found by Phillips.[1] Phillips used Yamaguchi[2] singlet and triplet two-body potentials, and obtained different linearly related values of E_t and ^2a when he varied either the singlet effective range or the deuteron percent D state (P_D). He kept constant both the deuteron energy and the energy of the singlet antibound state. Karchenko, Petrov, and Storozhenko[3] varied the exponent n in modified singlet and central triplet form factors: $g(p)=(p^2+\beta^2)^{-n}$. The value $n=1$ gives the Yamaguchi shape used by Phillips; the values $n=2$ and 3 give additional points on the Phillips line, shown in Fig. 1. We also show calculated values using a central spin-dependent separable potential.[3]

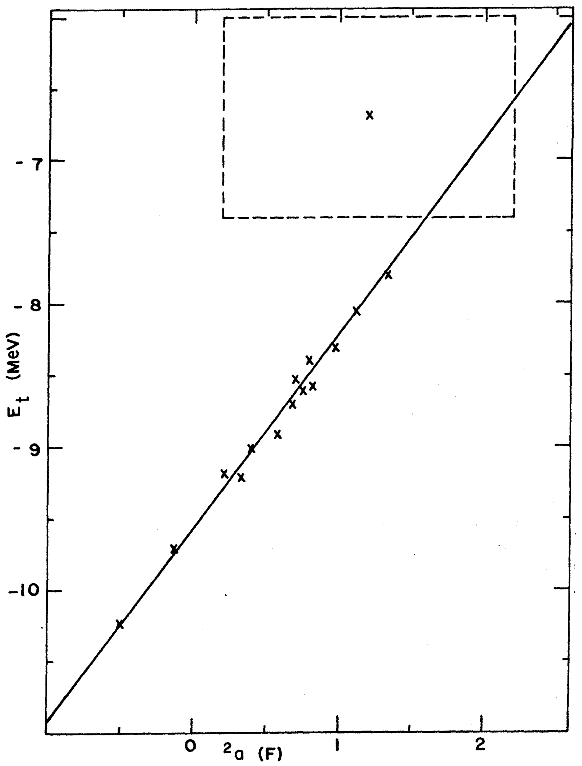

FIG. 1. Published values of the triton energy E_t plotted against the doublet n-d scattering length ^2a. For separable potentials see Phillips (Ref. 1) and Kharchenko, Petrov, and Storozhenko (Ref. 3). The x in a large box shows the value and errors of Delves *et al.* (Ref. 6) for the Hamada-Johnson potential. The line shown is due to Phillips (Ref. 1) and Delves and Phillips (Ref. 4).

Delves and Phillips[4] discuss several questions related to this line: (i) Does the line fit experimental values of E_t and ^2a? (ii) Do calculations with a local potential give values that lie on the line? (iii) Should the line hold, in general, for two-body separable potentials? First, the line agrees with the experimental values $E_t = -8.48$ MeV and the old Set A, $^2a = 0.7 \pm 0.3$ F. If the new set A′, $^2a = 0.15 \pm 0.05$ F, is as accurate as written, we would need some method to get off the line: either those indicated by questions (ii) or (iii), or alternatively by the use of three-body forces,[5] or by relativistic effects. Second, the calculation by Delves *et al.*[6] for the Hamada-Johnston potential involves relatively large errors; this is indicated by the box surrounding the x in Fig. 1. The box is not inconsistent with the line; and, in fact, we have some evidence[7] that a separable approximation may be quite good for a local potential with a soft core. (By "quite good" we mean errors of order 0.1 MeV in E_t and of order 0.1 F in ^2a.)

This paper is concerned with the third question. We answer in the affirmative, by giving many more numerical examples, including higher-rank separable potentials.

Note added in proof: See "Three-Nucleon Calculations with Realistic Forces" by Malfliet and Tjon.[7a] Their results for Reid's potential fall on Phillips's line.

II. POTENTIALS AND RESULTS

We first consider the unitary-pole-approximation (UPA) tensor potentials discussed earlier.[8] We developed several UPA form factors: Yamaguchi, modified Hulthén,[8] UPA for Tabakin,[9] UPA for Schrenk-Mitra,[10] and more recently[11] the UPA for the Reid singlet soft-core potential.[12] We also consider the new Tabakin[13] singlet form factor. As before, we vary deuteron percent D state (P_D) in the range $0.78 \leq P_D \leq 7.0\%$. Results for the energy E_t were published earlier.[8,11] We include these older results in Table I along with present results for the doublet scattering length ^2a calculated by numerical methods similar to those used[8] in the calculation of E_t. All potentials in Table I use Yamaguchi's shape for the tensor form factor $T(p)$, and fit the same low-energy properties of the spin singlet and spin triplet systems: the singlet effective range and scattering length, and the triplet deuteron energy, quadrupole moment, and scattering length.

We next consider higher-rank tensor separable potentials, introduced by Tabakin[9] and recently used by Mongan[14,15] to fit phase parameters for nucleon-nucleon scattering. The singlet potential is changed from $S(p)S(k)$ to $g(p)g(k)-h(p)h(p)$. The triplet (central and tensor) potential has Tabakin's form

$$V(p,k) = \sum_{JSMLL'} \mathcal{Y}^M_{LJS}(\hat{p}) V^{SJ}_{LL'}(p,k) \mathcal{Y}^M_{L'SJ}(\hat{k}) \qquad (1)$$

with $V_{LL'}(p,k) = g_L(p) g_{L'}(k)$. Here $\mathcal{Y}^M_{LJS}$ is a spin spherical harmonic, and $\hat{p}$ and $\hat{k}$ are unit vectors. The sum goes over L or $L' = 0$ or 2, respectively.

Stagat[16] and Harms[17] present the coupled integral equations resulting from using potentials of form (1) in the Faddeev equations. The numerical methods used to solve these six coupled one-dimensional integral equations are an extension of those used earlier[8] for three coupled integral equations. We have tested the adequacy of our numerical work by varying the resulting matrix problem from 24×24 to 84×84, and obtain the same energy eigenvalue for the bound state (to three significant figures) for a 36×36, 48×48, 60×60, 72×72, and 84×84 matrix problem. Usually we solved a 60 ×60 matrix.

Our results for E_t and ^2a for various potentials of form (1) are presented in Table II and illustrated in Fig. 2. The straight line in the figure is Phillips's line drawn in Fig. 1.

The first row gives our results for Tabakin's original potential.[9] The trinucleon energy is −7.02 MeV, and the point to be shown in Fig. 2 misses the Phillips line by so much that it also misses the page, so it is not illustrated. Presumably the reason for this anomalous behavior is that Tabakin's potential binds the deuteron[18] by only 1.18 MeV.

We next present results for Schrenk-Mitra's potential[10] *G*1. Here two terms are used in the singlet potential, to include singlet repulsion; but h_L is taken to be zero. Since our values for E_t and ^2a given in Table II are substantially different from those found by Schrenk and Mitra, we checked our work by performing another independent numerical calculation that agreed with our first result.

We calculate E_t and ^2a for five different Mongan potentials,[14,15] using different shapes and different values of parameters. All five Mongan potentials used give reasonable fits to nucleon-nucleon phase parameters up to 300 MeV, and also fit the deuteron energy and quadrupole moment. However, as shown in Table II, they give surprisingly low values for the deuteron percent *D* state, P_D varying from 0.7 to 1.4%. Three of the five Mongan potentials give results falling right on the line. The Mongan 68 case II,[14] marked d, misses the line by 0.2 F in scattering length (or 0.3 MeV in energy); while the 69 case I,[15] marked e, misses by 0.45 F in scattering length (or 0.6 MeV in energy). [The latter deviation might be blamed on

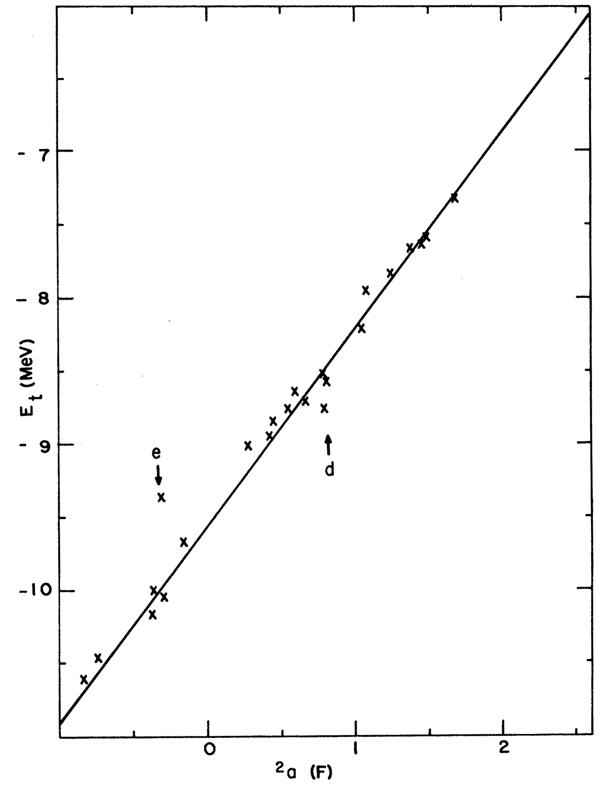

FIG. 2. Our values of the trinucleon energy E_t plotted against the doublet n-d scattering length ^2a, for the potentials of Tables I and II. The Phillips line is copied from Fig. 1. The points d and e that are relatively far off the line are for Mongan potentials denoted by these footnotes in Table II.

TABLE I. Dependence of the three-body energy E_t and of the doublet *n*-*d* scattering length ^2a on the shapes of the singlet form factor $S(p)$, the triplet central form factor $C(p)$, and on the deuteron percent *D* state P_D.

Singlet $S(p)$	Triplet central $C(p)$	P_D (%)	E_t (MeV)	^2a (F)
Yamaguchi	Yamaguchi	0.78	−10.60	−0.84
Yamaguchi	Yamaguchi	1.0	−10.45	−0.76
Yamaguchi	Yamaguchi	2.0	−9.97	−0.40
Yamaguchi	Yamaguchi	4.0	−9.01	0.28
Yamaguchi	Yamaguchi	7.0	−7.94	1.08
UPA for Tabakin[a]	Yamaguchi	4.0	−8.76	0.54
UPA for Schrenk-Mitra[a]	Yamaguchi	4.0	−8.64	0.59
New Tabakin[b]	Yamaguchi	4.0	−7.30	1.68
Yamaguchi	Modified Hulthén[a]	4.0	−8.69	0.65
UPA for Tabakin[a]	Modified Hulthén[a]	4.0	−8.55	0.80
UPA for Tabakin[a]	Yamaguchi	7.0	−7.83	1.23
Yamaguchi	Modified Hulthén[a]	7.0	−7.67	1.37
UPA for Tabakin[a]	Modified Hulthén[a]	7.0	−7.65	1.45
UPA for Reid[c]	Modified Hulthén[a]	4.0	−8.19	1.04
UPA for Reid[c]	Modified Hulthén[a]	7.0	−7.59	1.48

[a] See Ref. 8.
[b] See Ref. 13.
[c] See Ref. 11.

TABLE II. Dependence of the three-body energy E_t and of the doublet n-d scattering length ^2a on the singlet and triplet potentials used.

Singlet shapes	Triplet shapes	P_D (%)	E_t (MeV)	^2a (F)
Tabakin[a]	Tabakin[a]	3.2	−7.02	4.1
Schrenk-Mitra[b]	Yamaguchi[b]	4.0	−8.51	0.77
Mongan 68, case I[c]	Mongan 68, I SR[c]	1.0	−8.94	0.42
Mongan 68, case II[d]	Mongan 68, case II[d]	1.1	−8.76	0.79
Mongan 69, case I[e]	Mongan 69, case I[e]	0.7	−9.36	−0.30
Mongan 69, case II[f]	Mongan 69, case II[f]	1.1	−10.16	−0.38
Mongan 69, case IV[g]	Mongan 69, case IV[g]	1.4	−10.04	−0.30
Mongan 69, case II[f]	Modified Tabakin[h]	2.0	−9.67	−0.17
Mongan 69, case II[f]	Modified Tabakin[h]	4.0	−8.85	0.44

[a] Tabakin, Ref. 9.
[b] Schrenk and Mitra, Ref. 10, potential $G1$.
[c] Mongan, Ref. 14, Tables I and IV (Special Repulsion).
[d] Mongan, Ref. 14, Tables II and V.
[e] Mongan, Ref. 15, Tables I and V.
[f] Mongan, Ref. 15, Tables II and VI.
[g] Mongan, Ref. 15, Tables IV and VIII.
[h] Brady, Ref. 19.

the poor triplet scattering length (5.65 F) and poor triplet effective range (2.04 F) for this particular Mongan potential.]

We avoid the low values of P_D for Mongan's potentials by using triplet potentials of Tabakin's original form, but with parameters adjusted[19] to fit the deuteron binding energy, to give P_D values of 2 and 4%, respectively, and to fit phase parameters. These potentials do not fit the phase parameters as well as Mongan's potentials do, since we did not make a thorough search to improve the fit. These modified Tabakin triplet potentials, combined with Mongan's 69 case II singlet potentials,[15] give E_t and ^2a values on the Phillips line, as shown in Fig. 2.

III. DISCUSSION

We have shown that Phillips's linear relation between E_t and ^2a holds surprisingly well for a large variety of rank-one and rank-two separable potentials. We have extended Phillips's range of variation of P_D; we have treated different singlet and central triplet form-factor shapes besides those treated by Karchenko, and we have varied the tensor form-factor shape and treated rank-two potentials of Tabakin and Mongan. Most results fall within 0.1 F of the scattering length given by the line, though two cases miss by 0.2 and 0.45 F, respectively.

The mathematical reason for this linear relation between E_t and ^2a is uncertain at present.[20] Phillips's earlier conclusion still holds; i.e., one cannot fit both E_t and the new value A′ of the doublet scattering length. One[5] of us has introduced phenomenological three-body forces to take us off the Phillips line, and reproduce both E_t and the A′ value of ^2a.

Considering the present uncertainties in our knowledge of three-body forces, and of relativistic effects, and in view of the discrepancy between Sets A and A′, we cannot at present reach any firm conclusions that one or another specified separable two-body potential does (or does not) provide a satisfactory fit to the experimental properties of the three-nucleon system. (We note parenthetically that the neglected forces in other two-body states, $^1P_1$, $^3P_{0,1,2}$, $^1D_1$, contribute[21] only on the order of 0.01 MeV to E_t.) It is encouraging that "reasonable" two-body forces, such as Mongan's 69 case II singlet potential[15] combined with a modified Tabakin triplet potential[19] (4% D state) give a trinucleon energy within $\frac{1}{2}$ MeV of the experimental value, and give a doublet scattering length within $\frac{1}{3}$ F of either experimental value (either Set A or Set A′).

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PHYSICAL REVIEW C VOLUME 2, NUMBER 1 JULY 1970

$^{12}C(\alpha,\gamma)^{16}O$ Capture Cross Section Below 3.2 MeV[†]

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The $^{12}C(\alpha,\gamma)^{16}O$ capture cross section was measured for α-particle energies between 1.86 MeV ($\sigma = 1.1 \pm 0.4$ nb) and 3.11 MeV ($\sigma = 29 \pm 4$ nb) using a pulsed $^4He^+$ beam from the ORNL 6-MV Van de Graaff accelerator and a 23- by 30-cm NaI(Tl) crystal viewed by six matched, bialkali photomultiplier tubes. An upper limit was obtained for the capture cross section at $E_\alpha = 1.6$ MeV. The over-all time resolution (full width at half maximum) of the system for 8–10-MeV pulses due to γ rays is 2.7 nsec. Enriched (99.94%) ^{12}C targets ranging in thickness from 98 to 178 $\mu g/cm^2$ were used. Pulses resulting from fast neutrons [from the $^{13}C(\alpha,n)^{16}O$ reaction] interacting with the NaI(Tl) crystal were further separated from true γ pulses through the use of a new technique based on rise-time distribution differences of the respective neutron- and γ-ray-produced pulses. The face of the crystal (shielded with a 10.2 cm thickness of ^6LiH) was 12.2 cm from the target. The astrophysical significance of this reaction in the helium-burning sequence of stellar nucleosynthesis is also discussed.

INTRODUCTION

The $^{12}C(\alpha,\gamma)^{16}O$ reaction is a major link in the "helium-burning" sequence of events believed to occur during stellar evolution.[1] The formation of ^{16}O in stellar interiors by radiative α capture is thought to be strongly influenced by the 7.12-MeV (1^-) level in ^{16}O, which is 40 keV below the $^{12}C + {}^4He$ threshold.[2] While the γ-ray width of this state has been measured,[3] the dimensionless reduced α width θ_α^2, has only been indirectly inferred by Loebenstein *et al.*[4] through the $^6Li(^{12}C, d)^{16}O^*$ reaction. They give the limits 0.06–0.14 for the extracted θ_α^2 for the 7.12-MeV state in ^{16}O. Tombrello[5] has calculated the interference effects between the 1^- level at 7.12 and the 1^- state at 9.59-MeV excitation in ^{16}O, and found that the $^{12}C(\alpha,\gamma)^{16}O$ capture cross section in the region between these "resonances" will be affected. It is proposed that with a sufficiently accurate measurement of the cross section it would be possible to determine θ_α^2 for the 7.12-MeV state. Larson and Spear[6] found a value of 36 nb for the capture cross section at $E_\alpha = 3.2$ MeV using enriched ^{12}C targets, while Adams *et al.*[7] using time-of-flight methods and natural-carbon targets presented a preliminary value of 10 nb at about $E_\alpha = 2.7$ MeV. One of us (RJJ)[8] measured the cross section using en-