Nuclear Matrix Elements of the 437-keV Beta Transition in the Decay of Sb^{125}

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(Received 7 January 1970)

The energy dependance of the 437-keV β–176-keV γ directional correlation is measured with a conventional slow-fast scintillation assembly. The results are in good agreement with those of Dubard *et al.* The matrix-element parameters of this β transition are determined using the present results and the allowed shape of the β spectrum. The matrix-element parameters indicate that an algebraic cancellation among rank-1 matrix elements is responsible for the retardation of the 437-keV β transition. The ratio of the matrix elements $\int \vec{\alpha}$ and $\int i\vec{r}$ is not in agreement with the value predicted on the basis of conserved-vector-current theory.

I. INTRODUCTION

The decay scheme of Sb^{125} is well investigated[1] and is shown in Fig. 1. The 437-keV β transition in the decay of Sb^{125} is a nonunique first-forbidden transition with a high $\log ft$ value (9.3) and follows the spin sequence $(\frac{7}{2}^+) \xrightarrow{\beta} (\frac{9}{2}^-)$. The energy dependence of the 437-keV β–176-keV γ directional correlation was studied by Dubard *et al.*,[2] and the β-γ anisotropy varies from 10 to 30% in the β-particle energy range 250 to 400 keV. The high $\log ft$ value and the large β-γ anisotropy suggest a deviation of the 437-keV β transition from the ξ approximation.[3] However, Palmer, Arya, and Walls[4] recently measured the spectrum shape of this β transition to be statistical, which is consistent with the requirements of the ξ approximation. Also in the earlier works,[2,4] the matrix elements governing this β transition are not determined.

The present work is undertaken to investigate the matrix elements governing the 437-keV β transition in Sb^{125} employing the electron radial-wave-function parameters of Bhalla and Rose[5] and taking into account the finite-nuclear-size effects. The energy dependence of the 437-keV β–176-keV γ directional correlation is remeasured, and the present data on directional correlation are combined with the statistical shape of the 437-keV β transition to extract the nuclear matrix elements. The procedure adopted for the extraction of nuclear matrix elements is due to Simms.[6,7] Theoretical ratios of matrix-element parameters are also estimated on the basis of single-particle and core-excitation models and compared with the experimental ratios. From the experimental values of the matrix elements, the validity of the conserved-vector-current theory is tested.

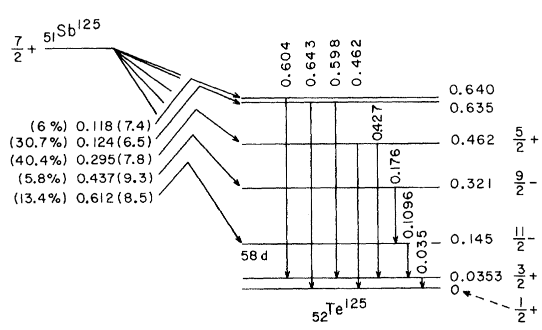

FIG. 1. Decay scheme of Sb^{125}.

II. EXPERIMENTAL PROCEDURE AND RESULTS

The source Sb^{125} was obtained in liquid form as $SbCl_3$ and SbOCl in HCl solution. The experimental setup used in the present work is a conventional slow-fast scintillation assembly. The source preparation, source centering, experimental details, and the treatment of the data were described in our earlier works.[8,9]

Results of Integral-Correlation Experiment

The singles γ spectrum is shown in Fig. 2. In the 437-keV β–176-keV γ cascade in the decay of Sb^{125}, the 176-keV γ photopeak is accepted in a window of 4 V in integral- and differential-correlation measurements. For integral-correlation measurements the β particles of energy 250 keV and above were accepted in the β channel. As described earlier,[8,9] the integral-correlation data were collected, and an application of Rose's method of least squares[10] yields the angular-correlation function to be

$$W_{\beta\gamma}(\theta) = 1 + (0.142 \pm 0.011) P_2(\cos\theta) + (0.008 \pm 0.016) P_4(\cos\theta). \qquad (1)$$

From this it can be seen that the coefficient ϵ_4 is

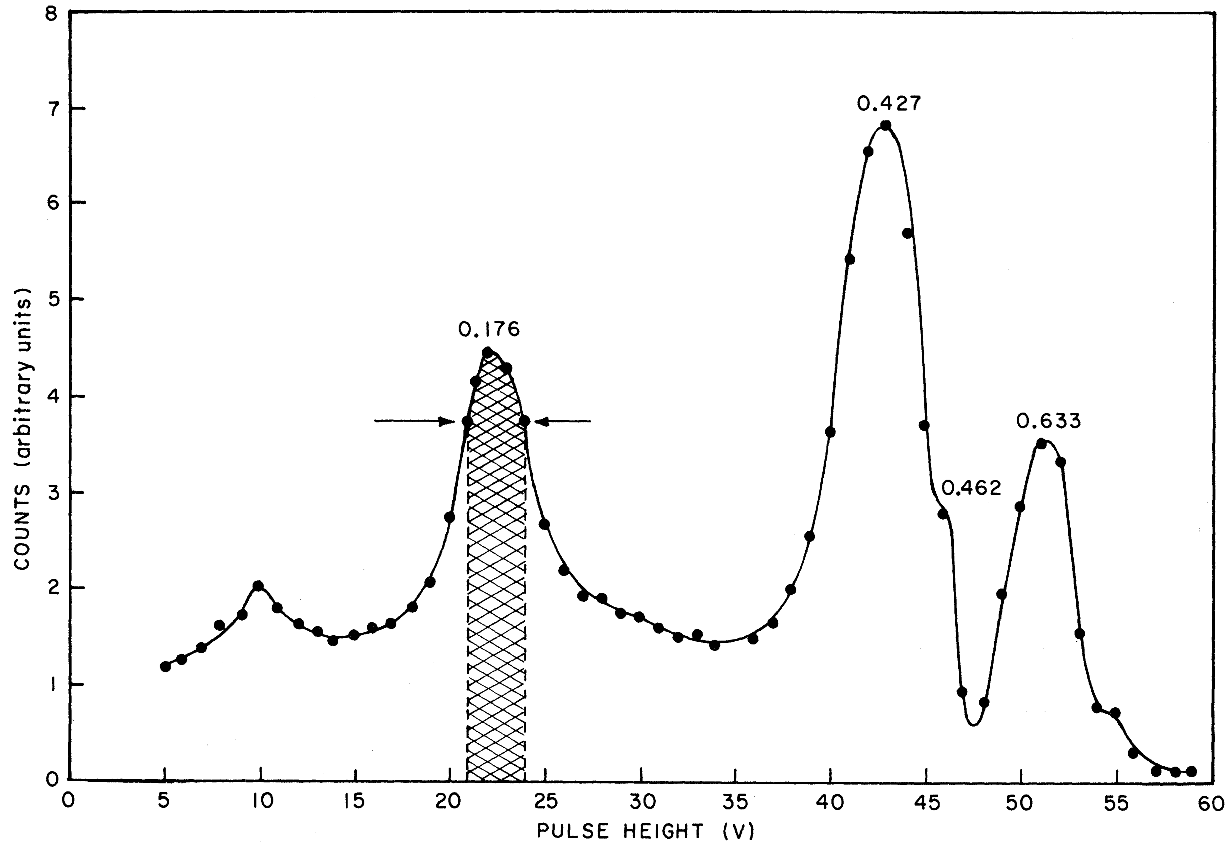

FIG. 2. Singles γ spectrum of Sb^{125}.

zero within experimental errors, establishing the first-forbidden nature of this β transition. Figure 3 represents a least-squares fit to integral-correlation data.

Results of Differential-Correlation Experiment

The data were collected at four β energies in steps of 50 keV ranging from 250 to 400 keV as described in Ref. 8. There might be contributions to the β-γ coincidences at the 250-keV β energy setting from the inner β groups of end-point energy 295 keV and the Compton scattering of the following γ rays of energy 427 and 462 keV. This contribution was estimated in a separate β-γ coincidence experiment. The Compton background that comes from the high-energy 427- and 462-keV γ rays falling in the window set for the 176-keV γ photopeak can be taken as approximately equal to the Compton background at 250 keV with the same window. The base level in the γ channel was set at 250 keV with the same window and with the other conditions remaining the same, β-γ coincidences were collected. The Compton background was estimated to be of the order of 10% of the observed coincidences. The β-γ coincidences at 250 keV were corrected for this. The directional-correlation data, after correcting for the geometry of both the detectors and the finite resolution of the β spectrometer, are given in Table I in their final form. The present results are in good agreement with those of Dubard, Wyly, and Braden.[2]

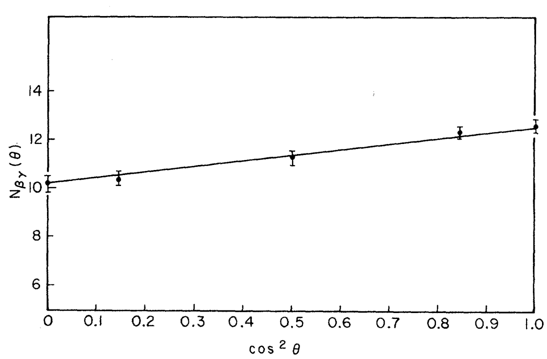

FIG. 3. Integral-correlation results.

It may be noted that the lifetime of the intermediate state (the 321-keV state), involved in the present β-γ cascade, is about 0.8 nsec. This may attenuate the observed β-γ angular correlation. However, it is theoretically shown that the

TABLE I. Directional-correlation data for the 437-keV β transition in Sb^{125}.

Energy (mc^2 units) W	Anisotropy A	Correlation coefficient ϵ	Reduced correlation coefficient $\epsilon' = \epsilon W/p^2$
1.4893	0.139 ± 0.009	0.095 ± 0.006	0.117 ± 0.007
1.5870	0.226 ± 0.013	0.151 ± 0.007	0.158 ± 0.007
1.6850	0.295 ± 0.013	0.194 ± 0.007	0.178 ± 0.006
1.7829	0.385 ± 0.017	0.245 ± 0.009	0.200 ± 0.008

perturbation of the angular correlation by the quadrupole coupling in the 321-keV intermediate state is negligibly small.[11] This is also experimentally confirmed by the measurements of Vikram Singh *et al.*[12] Hence, in the present measurements the attenuation of the angular correlation due to the finite lifetime of the intermediate state is neglected.

III. MATRIX-ELEMENT ANALYSIS

The present values of β-γ directional correlation are combined with the allowed shape of the 437-keV β transition[4] for the determination of the matrix-element parameters. The present β transition involves unit spin change and is governed by four matrix elements. The matrix-element parameters Y, x, u, and z in Kotani's notation[3] are given by

$$z = C_A \int i\, B_{ij}, \qquad y = -C_V \int \vec{\alpha},$$

$$x = -C_V \int i\vec{r}, \qquad u = C_A \int \vec{\sigma} \times \vec{r}, \tag{2}$$

and

$$Y = \xi' y - \xi(x+u),$$

where $\xi = \alpha Z/2R$, ξ' distinguishes the relativistic matrix elements from the nonrelativistic ones, and $\xi' \approx \xi$.

The parameters of the electron radial wave functions that would occur in the expressions for $\epsilon(W)$ and $C(W)$ were determined using the Bhalla and Rose tables.[5] The formulas used for the extraction of the matrix-element parameters were those due to Simms *et al.*[6,7] The analysis was done using a CDC 3600 computer. In the first instance a coarse search was made for x, u, and Y, in the ranges -15 to $+15$, in steps of 0.5 to know the approximate range of each parameter. Afterwards a fine search was made in steps of 0.01 over the ranges fixed by the initial coarse search. In the coarse search the values of $\epsilon(W)$ at only two energies, namely, 250 and 350 keV are used, and the ratio $C(350)/C(300)$ is taken as 1 ± 0.05. In the fine search the values of $\epsilon(W)$ at all the four β energies were used. The computer was instructed to print out only those sets of matrix-element parameters yielding values of $\epsilon(W)$ and $C(W)$ within the limits of experimental errors. Thus, finally, the following ranges of x, u, and Y consistent with the experimental data are derived:

$$\begin{aligned} z &= 1, \\ -2.26 \leq Y &\leq -1.95, \\ 1.84 \leq x &\leq 2.05, \\ 4.65 \leq u &\leq 5.65. \end{aligned} \tag{3}$$

As the fine search is done in steps of 0.01, there are many sets of satisfactory solutions in the above ranges. All the sets gave similar energy dependence, and it is not possible to single out a unique set of solutions. However, four typical sets of matrix-element parameters with the least χ^2 value (6.5) are picked up and tabulated in Table II, where χ^2 is defined as

$$\chi^2 = \sum_{i=1}^{4} \left[\frac{\epsilon(W_i)_{\text{th}} - \epsilon(W_i)_{\text{exp}}}{\Delta\epsilon(W_i)_{\text{exp}}}\right]^2 + \sum_{i=1}^{4} \left[\frac{c_r(W_i)_{\text{th}} - c_r(W_i)_{\text{exp}}}{\Delta c_r(W_i)_{\text{exp}}}\right]^2,$$

where Δc_r and $\Delta\epsilon$ stand for the experimental errors and $C_r(W_i) = C(W_i)/C(W_n)$. In this case $W_n = 300$ keV, $C_r(W_i)_{\text{exp}}$ is taken as 1, and $\Delta C_r(W_i)_{\text{exp}}$ is taken as 0.05.

The estimated energy dependence of $\epsilon(W)$, $\epsilon'(W)$, and $C(W)$ for the first set of matrix elements in Table II are shown in Figs. 4, 5, and 6, respectively. In Figs. 4 and 5 the experimental values are also shown. The estimated energy dependence with the other Sets (b), (c), and (d) is very nearly the same as for the Set (a) and is practically indistinguishable. From Fig. 6, it can be seen that

TABLE II. Typical sets of matrix-element parameters.

	x	u	Y	z
(a)	1.86	5.01	−1.95	1.00
(b)	1.87	5.03	−1.96	1.00
(c)	1.87	5.07	−1.97	1.00
(d)	1.88	5.09	−1.98	1.00

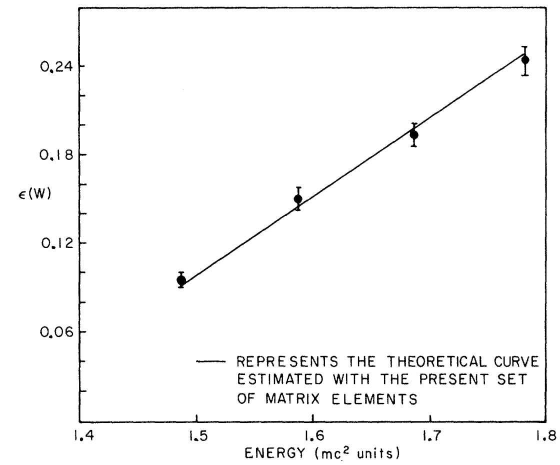

FIG. 4. Correlation coefficient versus β energy.

the predicted shape is statistical but for a small deviation of nearly 6% at 250 keV. From Figs. 4 and 5 there is good agreement between the predicted and experimental values of $\epsilon(W)$ and $\epsilon'(W)$, respectively.

The absolute values of the matrix elements can be estimated with a knowledge of the logft value. Using the experimental logft value and the present sets of matrix-element parameters (Table II), the absolute values of the matrix elements are obtained. In order to have a significant comparison of the matrix elements, the matrix elements containing $\vec{r}$ are divided by the nuclear radius, R. For Sb^{125}, $R=0.0156$ natural units. The normalized matrix elements are given in Table III.

IV. RATIOS OF THE MATRIX ELEMENTS ON THE BASIS OF SINGLE-PARTICLE AND CORE-EXCITATION MODELS

Rose and Osborn[13] have furnished expressions for the reduced matrix elements of the β-decay operator in the jj-coupling shell model for one-

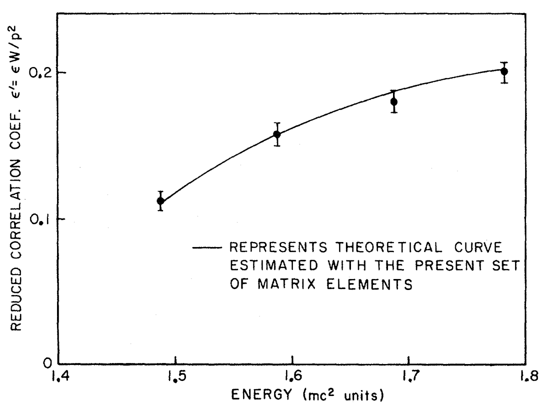

FIG. 5. Reduced correlation coefficient versus β energy.

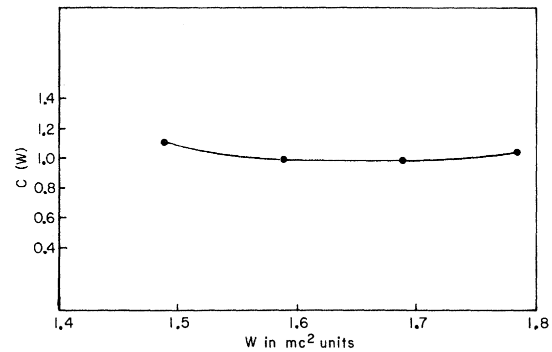

FIG. 6. Theoretical energy dependence of the β-spectrum shape.

and two-particle configurations. In the notation of Rose and Osborn, it can be easily derived that

$$\frac{x}{u}=-\frac{C_V}{C_A}\frac{1}{\sqrt{2}}\frac{\langle f\,||\,T_{110}\,||\,i\rangle}{\langle f\,||\,T_{111}\,||\,i\rangle},$$

$$\frac{z}{u}=\frac{1}{\sqrt{2}}\frac{\langle f\,||\,T_{211}\,||\,i\rangle}{\langle f\,||\,T_{111}\,||\,i\rangle}. \qquad (4)$$

The single-particle estimates of the ratios of the reduced matrix elements are tabulated.[14] Sb^{125} is a nondeformed odd-proton nucleus, the last neutron and proton belonging to the same major shell. Assuming the initial and final states involved in the 437-keV β decay of Sb^{125} to be pure single-particle states, namely, $g_{7/2}$ and $h_{9/2}$, the ratios of the single-particle matrix-element parameters are

$$x/z=-0.66 \text{ and } u/z=0.786. \qquad (5)$$

These single-particle estimates of the ratios of the matrix-element parameters are not in agreement with the experimental values.

Thus the information given by the nuclear parameters in the present case does not support characterizing the 321-keV level of Te^{125} as a pure single-particle state. This conclusion is also supported by the observed electromagnetic transition probabilities of the 176-keV γ ray between the states $|\frac{9}{2}^-\rangle$ and $|\frac{11}{2}^-\rangle$ in Te^{125}. Inamura[15] observed that the experimental $B(E2)$ values were greatly enhanced, and the experimental $B(M1)$ values were greatly hindred over the single-particle estimates. This shows that the 321-keV level of Te^{125} is of collective nature. It is also interesting to note that the observed $B(E2)$ value[15] of the 176-keV γ transition is very nearly the same as the $B(E2)$ value[1] of the 608-keV γ transition proceeding from the first 2^+ state to the 0^+ ground state in Te^{124}:

TABLE III. Absolute values of the matrix elements.

$\frac{1}{R}\int i\vec{r}$	$\frac{1}{R}\int \vec{\sigma}\times\vec{r}$	$\int \vec{\alpha}$	$\frac{1}{R}\int iB_{ij}$
0.0307 ±0.0024	0.0793 ±0.0064	0.0205 ±0.0016	0.0165 ±0.0013
0.0309 ±0.0024	0.0831 ±0.0065	0.0211 ±0.0016	0.0165 ±0.0013
0.0309 ±0.0024	0.0837 ±0.0065	0.0212 ±0.0017	0.0165 ±0.0013
0.0310 ±0.0024	0.0840 ±0.0065	0.0213 ±0.0017	0.0165 ±0.0013

$$B(E2, 2^+ \to 0^+)_{608\text{ keV}} = 0.077 \pm 0.01$$

in units of $e^2 \times 10^{-48}\text{cm}^4$, and

$$B(E2, \tfrac{9}{2}^- \to \tfrac{11}{2}^-)_{176\text{ keV}} = 0.093 \pm 0.009$$

in units of $e^2 \times 10^{-48}\text{cm}^4$. This suggests that the 321-keV level in Te^{125} can best be described by the core-excitation model proposed by de-Shalit.[16] The $|\frac{11}{2}^-\rangle$ state in Te^{125} is an isomeric state with a half-life of 58 days. The $|\frac{9}{2}^-\rangle$ level can be interpreted as one of the members of the multiplet which results from the coupling of the odd neutron to the 2^+ state of the core. In the present case the 2^+ state of the core can be taken as the first 2^+ state (608-keV state) of the Te^{124} nucleus. However, such simple description does not account for the observed 176-keV $M1$ transition between the states $|\frac{9}{2}^-\rangle$ and $|\frac{11}{2}^-\rangle$. Thus the 321-keV level in Te^{125} can be written as, following the notation of Delabaye, Deutch, and Lipnik,[17]

$$|J, M\rangle = \alpha |h_j, J = j, M\rangle + \beta |h_{11/2}, J_c = 2; J, M\rangle, \tag{6}$$

where $\alpha = 0$ corresponds to pure de-Shalit type of excitation, and $\beta = 0$ corresponds to an extreme single-particle excitation.

The Sb^{125} ground state can be written in the lowest-seniority approximation

$$|J', M'\rangle = |g_{7/2};\ J' = \tfrac{7}{2}, M'\rangle. \tag{7}$$

By using angular momentum decoupling rules,[18] the reduced matrix elements of the 437-keV β transition are composed of only one single-particle matrix element

$$\langle J = \tfrac{9}{2} || T_{KL\gamma} || J' = \tfrac{7}{2}\rangle = \alpha\, a_k \,\langle h_{9/2} || T_{KL\gamma} || g_{7/2}\rangle, \tag{8}$$

where a_k are the angular momentum decoupling coefficients. The nuclear parameters x/u and z/u of this β transition turned out to be the same in both the descriptions. Thus, in this simple case the nuclear parameters x/u and z/u do not permit a distinction between the single-particle excitation and core-excitation mechanisms [as described by the wave function (6)]. Similar conclusions were found[17,19] in the decays of Ag^{111} and Mo^{99}. However, it may be noted that in this case a pure de-Shalit type of excitation predicts all the three reduced matrix elements to be zero. When the 321-keV level is described by the wave function (6), the values of the reduced matrix elements depend upon the coefficient α, the single-particle contribution in the wave function (6).

V. DISCUSSION

The validity of the ξ approximation requires that $|Y| \gg |x| \sim |u| \sim |z|$. The present values of the matrix-element parameters do not satisfy the above relationship, thereby establishing the failure of the ξ approximation in this case. The failure of the ξ approximation can be accounted for by a selection-rule effect or a cancellation effect. The K selection rule cannot be applied, as Sb^{125} does not fall in the deformed region. The j selection rule is applicable, as the last neutron and the last proton belong to the same major shell. However the single-particle matrix-element ratios (discussed earlier) are not in agreement with the experimental values. Also, the values of the matrix elements do not suggest the domination of the B_{ij} matrix element. The matrix-element parameters given in (3) satisfy the requirement $|Y| \gtrsim |x| \sim |u| \sim |z|$ thus indicating, as shown by Kotani,[3] that an algebraic cancellation among the nuclear matrix elements rather than a nuclear selection rule is responsible for the retardation of the 437-keV β transition of Sb^{125}.

Fujita[20] has suggested that the experimentally obtained matrix elements can be used as a test of the conserved-vector-current (CVC) theory of β decay. For Sb^{125} the CVC theory predicts the ratio $(1/\xi)(\int\vec{\alpha}/\int i\vec{r}) \equiv \Lambda$ to be 2.4. Considering all the sets of matrix elements in the ranges given in

(3) the following limits could be set for Λ:

$$3.4 \leq \Lambda \leq 4.6 .$$

Thus, the experimental value of Λ is much higher than Fujita's estimate. It should be noted that in setting the above limits for Λ, the higher-order matrix elements are neglected. Usually it is not possible to break up combinations of the type $x + ax'$, etc., where x' is the higher-order matrix element. If for the moment one assumes that $x' \approx x$ and $u' \approx u$, then Λ is estimated to be 25% smaller. Then the lower limit of the experimental value of Λ would be 2.65, which is even slightly higher than the Fujita's estimate. Thus, it can be seen that unless the first-order matrix elements are smaller than the corresponding higher-order matrix elements, the experimental value of Λ is not in agreement with Fujuta's estimate. There is no substantial reason to suspect that the higher-order matrix elements dominate the first-order matrix elements. Thus, if one assumes that the higher-order matrix elements are smaller than the first-order matrix elements, the disagreement between the experimental and theoretical values of Λ seems to support the criticism of Damgaard and Winther[21] that the approximation of neglecting the nondiagonal matrix elements in H_C, the operator for the Coulomb potential, is not valid. (The CVC-predicted value $\Lambda = 2.4$ implies this approximation.)

Thus, from the present work it may be concluded finally that the 437-keV β transition in Sb^{125} shows a deviation from the ξ approximation; an algebraic cancellation among rank-1 matrix elements may be held responsible for the deviation. The single-particle and core-excitation models do not explain the experimental values of the matrix elements, and the experimental values of the matrix elements seem to support the criticism of Damgaard *et al.* on CVC theory.

ACKNOWLEDGMENTS

One of the authors (V.S.R.) thanks the Council of Scientific and Industrial Research for awarding him a Senior Research Fellowship during the course of the present work.

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