cles by ^{58}Ni and ^{58}Fe can be well represented by an optical model, but only the potential at distances greater than about 6–7 F is well determined. The real potential in this region can be obtained from relatively inextensive measurements, but to determine the imaginary-potential tail requires accurate measurements over a wide angular region showing considerable structure. It is unlikely that such data will yield useful information concerning nuclear sizes, but the possibility of obtaining information concerning nuclear densities at large radii cannot be excluded.

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Level Structure of ^{50}V†

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The reaction ^{49}Ti(^{3}He, d)^{50}V has been studied at 15-MeV incident energy with an over-all energy resolution better than 20 keV full width at half maximum. 54 states were observed in ^{50}V up to 6-MeV excitation, and corresponding deuteron angular distributions were measured in the angular interval 7 to 50°. Spectroscopic information has been extracted for 30 of the stronger or well-isolated transitions by means of a distorted-wave analysis of the differential cross sections. The results are compared with nuclear-model predictions.

I. INTRODUCTION

The present investigation of the ^{49}Ti(^{3}He, d)^{50}V reaction forms part of a systematic study of proton states in the $Z = 23$ nuclei excited by means of the (^{3}He, d) reaction. The results for ^{47}V, ^{49}V, and ^{51}V have already been published.[1-3]

Definite spin and parity assignments are known[4] only for the ground state of ^{50}V. Level energies have been rather well established up to 3.7-MeV excitation from studies of the ^{50}Ti($p, n\gamma$),[5] ^{50}Ti(^{3}He, t),[6] ^{50}V(p, p'),[7] ^{50}V(d, d'),[8] ^{51}V(p, d),[9] and ^{52}Cr(d, α)[7] reactions, but tentative spins and parities have been obtained for very few of them. No previous study of the ^{49}Ti(^{3}He, d) reaction has been reported.

II. EXPERIMENTAL TECHNIQUES AND RESULTS

A self-supporting titanium metal foil, enriched to 76% in ^{49}Ti, was bombarded with 15-MeV ^{3}He^{++}

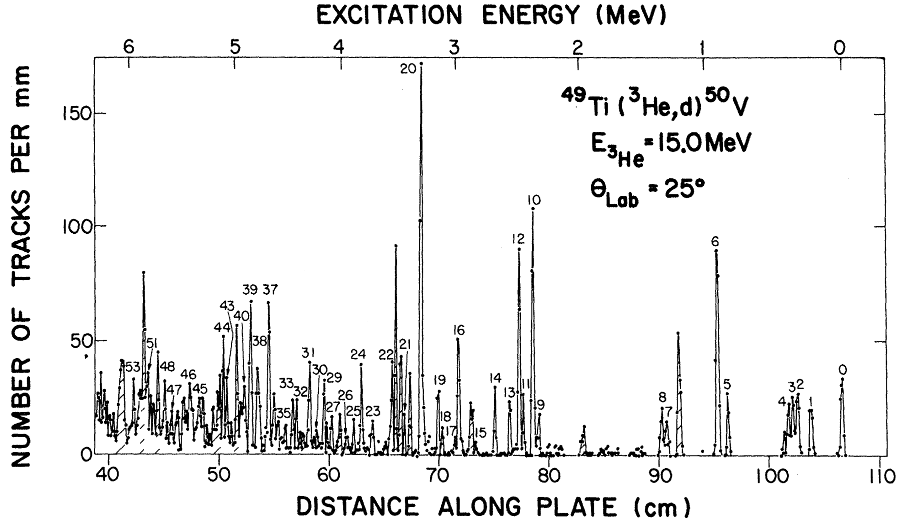

FIG. 1. Deuteron spectrum from the $^{49}Ti(^{3}He, d)^{50}V$ reaction at 15.0-MeV incident energy and $\theta_{lab} = 25°$. Groups labeled numerically correspond to state in ^{50}V, and those shown cross-hatched correspond to ^{49}V states (Ref. 2). The latter arise from approximately 20% of ^{48}Ti in the target.

ions from the University of Pennsylvania Tandem accelerator. The reaction deuterons were momentum-analyzed in a single-gap, broad-range spectrograph and detected in nuclear emulsions. In Fig. 1 is shown a deuteron spectrum measured at a lab angle of 25°. The energy resolution full width at half maximum is better than 20 keV. Groups corresponding to states in ^{50}V are labeled numerically, whereas those corresponding to states in ^{49}V, which originate from a significant amount of ^{48}Ti in the target, are shown cross-hatched. Identification of the latter groups was aided by the earlier study of the $^{48}Ti(^{3}He, d)$ reaction.[2] A total of 54 states in ^{50}V could be identified up to 6-MeV excitation, and the level energies listed in column three of Table I are the means of values obtained at three different angles. Excitation energies determined from the $^{50}V(p, p')$ reaction[7] are shown

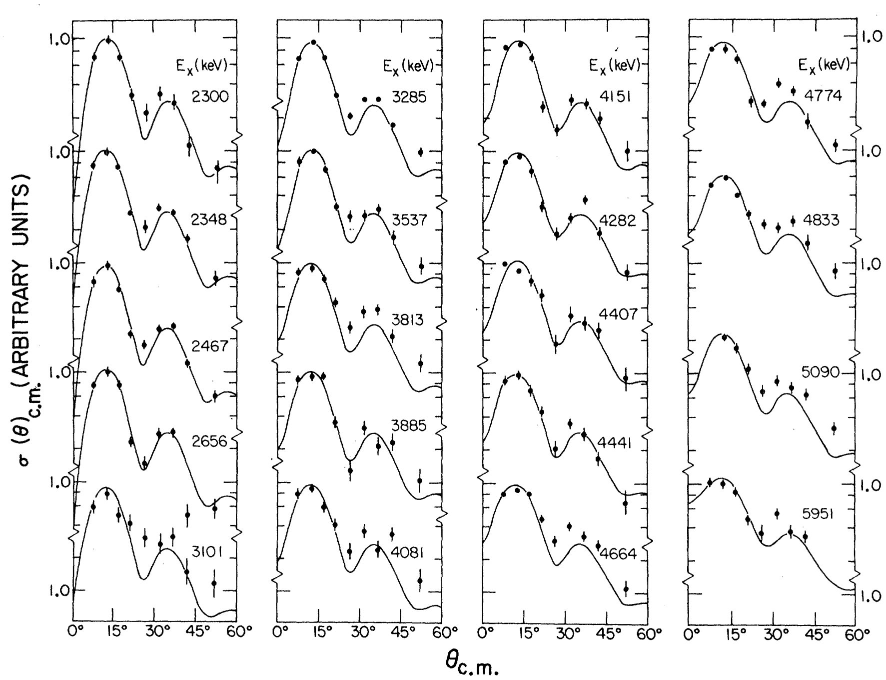

FIG. 2. Deuteron angular distributions having $l_p = 1$ character, observed in the $^{49}Ti(^{3}He, d)^{50}V$ reaction. The cross-section scales are arbitrary, absolute peak cross sections being given in Table I. The curves are from distorted-wave calculations (see text and Table II).

TABLE I. Results from the $^{49}Ti(^{3}He, d)^{50}V$ reaction.

Deuteron group	E_x (keV) Ref. a	Present[b]	J^{π} Ref. c	$d\sigma/d\omega$[d] (mb/sr)	l_p	Spectroscopic strength[e]
0	0	0	6^+	0.41	3	1.10
1	228	226	(5^+)	0.40	1+3	0.05+0.43
2	321	318	(4^+)	0.34	3	0.86
3	358	350	(3^+)	0.28	1+3	0.02+0.63
4	390	388	(2^+)	0.24	3	0.57
5	838	836		0.23		
6	910	911	(7^+)	0.92	3	2.06
7	1305	1307		0.17	1+3	0.01+0.21
8	1330	1347		0.37	1+3	0.04+0.24
	1404					
	1519					
	1561					
	1701					
	1725					
	1753					
	1760					
	1808					
	1884					
	1937					
	1957					
	2038					
	2112					
	2133					
	2162					
9	2314	2300		0.60	1	0.07
10	2342	2348		3.95	1	0.46
	2399					
11	2422	2427		0.49	0	0.04
12	2456	2467		3.48	1	0.37
	2481					
	2492					
	2512					
13	2533	2542		0.45	0	0.04
	2600					
14	2655	2656		1.20	1	0.13
	2738					
	2763					
	2792					
15	2815	2814		0.04		
	2828					
	2849					
	2931					
	2958					
16	2965	2966		1.00	1+3	0.09+0.40
17	2992[f]	2996		0.05		
	3011					
18	3099	3101		0.22	1	0.02
	3111					
19	3142	3140		0.20		
	3169					
	3202[f]					
	3274					
20	3297[f]	3285[f]		5.41	1	0.51
	3312					
21	3433	3434		0.07		
	3482					
22		3537		1.31	1	0.12
	3566					

TABLE I (*Continued*)

Deuteron group	E_x (keV) Ref. a	Present[b]	J^π Ref. c	$d\sigma/d\omega$ [d] (mb/sr)	l_p	Spectroscopic strength[e]
	3671					
23	3700	3713		0.16		
	3722					
	3730					
	3749					
24		3813		0.85	1	0.08
25		3885		0.51	1	0.05
26		3939		0.09		
27		4081		0.41	1	0.04
28		4124		0.08		
29		4151		1.25	1	0.11
30		4213		0.08		
31		4282		1.34	1	0.12
32		4407		0.48	1	0.05
33		4441		0.72	1	0.06
34		4513		0.10		
35		4581		0.10		
36		4602		0.14		
37		4664		1.25	1	0.11
38		4774		1.18	1	0.11
39		4833[f]		1.38	1	0.12
40		4898		0.23		
41		4928		0.17		
42		5018		0.08		
43		5058		0.20		
44		5090		0.88	1	0.08
45		5326		0.27		
46		5409		0.28		
47		5531		0.14		
48		5645		0.24		
49		5755		0.16		
50		5786		0.15		
51		5820		0.23		
52		5893		0.26		
53		5951		0.64	1	0.06

[a]Energy levels are from Ref. 7.

[b]From 0 to 3 MeV the uncertainty in the excitation energy (E_x) is ±10 keV. Between 3 and 6 MeV the uncertainty is ±15 keV.

[c]The spin-parity assignments are from Ref. 5, except for the 911-keV state, which is based on the present work.

[d]In the case of states for which l_p values are assigned, the cross sections are the values at the peaks of the angular distributions (or at 7° for $l_p=0$ transitions) and have an estimated uncertainty of ±15%. For other states, the cross sections were measured at 25° and have an uncertainty of ±25%.

[e]The strength is $(2J_f+1)S(j)/(2J_i+1)$ where j is the spin of the transferred proton and $S(j)$ the spectroscopic factor. If the isospin is introduced, then $S(j)=\langle\frac{5}{2}\frac{5}{2}\frac{1}{2}-\frac{1}{2}|T2\rangle^2 S(j,T)$. The strengths were determined assuming $j=l+\frac{1}{2}$.

[f]Probable doublet.

for comparison in column two of the Table. It is evident that many states have been missed in the present experiment, presumably owing to a greater selectivity of the (^{3}He,d) than the (p,p') reaction. The spin-parity assignments shown in column four are from the ^{50}Ti($p,n\gamma$) study by Blasi *et al.*[5] except for the 911-keV state which is based on the present work.

Angular distributions were extracted for 31 of the more intense or well-isolated deuteron groups. Of these, only the distribution for the 836-keV state appeared untypical of a stripping transition. The remaining distributions are shown in Figs. 2 to 4 and cover the angular range 7 to 50° in the lab system. The cross-section units are arbitrary. An absolute cross-section scale was later established in a separate experiment, using a similar titanium target foil. The target thickness was de-

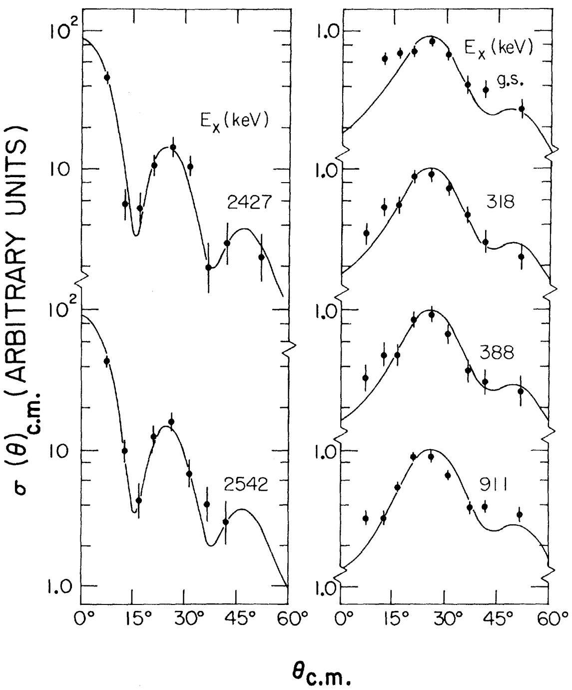

FIG. 3. Deuteron angular distributions having $l_p=0$ character (left of figure) and $l_p=3$ character (right of figure). See also caption to Fig. 2.

duced from an elastic scattering measurement of 6-MeV $^3He^{++}$ ions, and the (^{3}He, d) study was repeated at two angles under identical conditions to the earlier measurement for normalization purposes. The differential cross sections so determined are given in column five of Table I. For the stripping transitions, they are the values at the peaks of the distributions and have an estimated uncertainty of 15%.

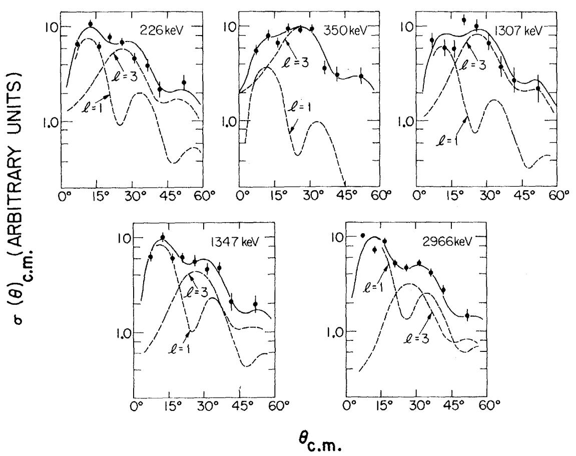

FIG. 4. Deuteron angular distributions of mixed $l_p=1+3$ character. See also caption to Fig. 2.

III. DISTORTED-WAVE ANALYSIS

The angular distributions were compared with distorted-wave calculations using the code JULIE.[10] A Woods-Saxon potential was used for the proton bound state as well as for the ingoing and outgoing channels. The potential also included a spin-orbit coupling term in the bound state and deuteron channel, and the calculations were made without the application of a lower cutoff on the radial integrals. The evaluations of the bound-state wave functions were made assuming proton capture into the $1f_{7/2}$, $2p_{3/2}$, and $2s_{1/2}$ orbits for $l_p=3$, 1, and 0 transitions, respectively. The optical-model parameters used in the calculations are given in Table II. These are identical to those used in our previous studies of the vanadium isotopes,[2] except that the volume absorption term in the incident channel was decreased from $W=25.72$ to 15.0 MeV.[11]

The spectroscopic strengths $(2J_f+1)S(j)/(2J_i+1)$ where j, J_i, and J_f are the angular momenta of the transferred proton, target nucleus, and final state, respectively, and $S(j)$ is the spectroscopic factor, were extracted using the relation

$$\left(\frac{d\sigma}{d\Omega}\right)_{\rm expt}=4.4\,\frac{(2J_f+1)}{(2J_i+1)}\,S(j)\,\sigma_{\rm DW}\ ,$$

as suggested by Bassel.[12] The strengths and corresponding l_p values are listed in Table I.

IV. DISCUSSION

Of the 28 transitions which are observed to proceed with odd orbital angular momentum transfer, 4 appear to proceed with pure $l_p=3$, 19 with pure $l_p=1$ and 5 with mixed $l_p=1+3$. The $l_p=3$ transitions can lead to states with J^π ranging from 0^+ to 7^+, whereas the transitions with $l_p=1$ or $1+3$ restrict the possible values to the range 2^+ to 5^+.

Two $l_p=0$ transitions are also observed, leading to 3^- or 4^- states at 2.43 and 2.54 MeV. In view of the low excitation energies, these presumably correspond to $2s_{1/2}$ hole states, thus implying an admixture of $[(1f_{7/2})^{11}(2s_{1/2})^{-2}]_{J=7/2}$ configuration in the ^{49}Ti ground state. The strengths of the transitions indicate that the $2s_{1/2}$ proton shell is approximately 96% filled in ^{49}Ti. No $l_p=2$ transitions are observed in this study below 6-MeV excitation.

The total spectroscopic strength for $l_p=3$ transitions, obtained by summing over the individual strengths listed in Table I, is 6.50.[13] Of this, 94% lies below 1.5-MeV excitation, which probably corresponds only to the $1f_{7/2}$ ($T=2$) component of the total $1f$ strength. The expected value[14] for this component, taking into account the $2s_{1/2}$ hole contribution to the ^{49}Ti ground state, is 5.75. The agreement is well within the uncertainties of the

TABLE II. Optical-model parameters. The potentials for ^{3}He and d were of the form

$$V(r) = -V(1+e^x)^{-1} - i\left(W - W'\frac{d}{dx'}\right)(1+e^{x'})^{-1} + V_{so}\left(\frac{\hbar}{M_\pi c}\right)^2 \frac{1}{r}\frac{d}{dr}(1+e^x)^{-1}\vec{1}\cdot\vec{\sigma} + V_C(r, r_C),$$

with $x = (r - r_0 A^{1/3})/a$, $x' = (r - r_0' A^{1/3})/a'$, and $r_C = r_{0C} A^{1/3}$. V_C is the Coulomb potential. V, W, W', and V_{so} are given in MeV and r_{0C}, r_0, r_0', a, and a' are in fm. The values are from Ref. 11.

Particle	V	r_0	a	W	W'	r_0'	a'	V_{so}	r_{0C}
^{3}He	177.8	1.14	0.723	15.0	0	1.548	0.80	0	1.40
d	112	0.974	0.912	0	73.2	1.439	0.60	6	1.30
p	a	1.20	0.65	0	0	0	0	a	1.25

[a]The spin-orbit part of the potential was proportional to V and its strength was 25 times the Thomas value. The depth V was chosen so that the proton received a binding energy equal to $Q(^3\text{He}, d) + 5.49$ MeV.

cross-section estimates and distorted-wave analysis, and furthermore indicates that essentially the full strength of this component is seen.

The total observed $2p$ strength is 2.9 compared with a possible maximum of 6. However, below 6-MeV excitation only the $T = 2$ component of the $2p$ strength is expected, which should sum[14] to 5. It is probable that some of the missing strength is distributed among the weaker transitions in this

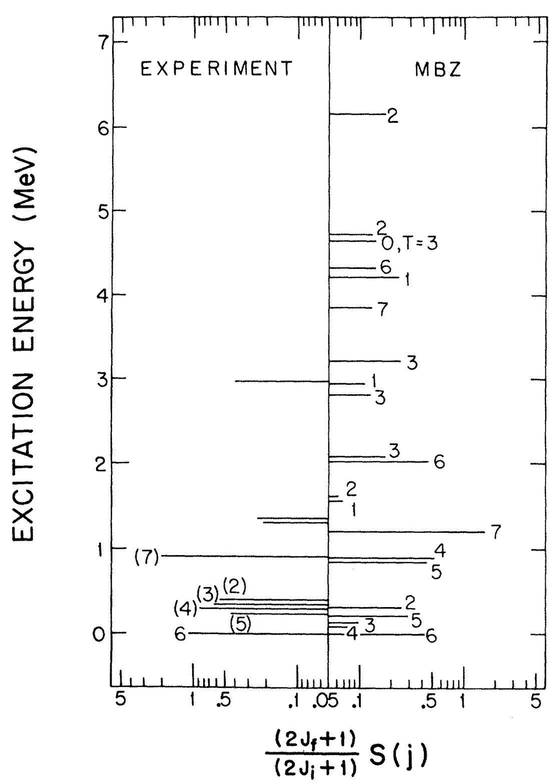

FIG. 5. Comparison of experimental $1f_{7/2}$ level scheme and spectroscopic strengths with the predictions of the MBZ model (Ref. 16) for ^{50}V. Levels predicted with less than 0.05 strength have been omitted from the MBZ level scheme. The experimental spin-parity assignments are from Table I.

region for which angular distributions could not be extracted, and also a significant part of the $2p_{1/2}$ component probably lies above 6 MeV.

The $T = 3$ analog to the 0^+ ground state of ^{50}Ti is expected at around 4.8 MeV in ^{50}V. However, no $l_p = 3$ transition is observed in this region, and since only 3% of the total $1f$ strength should go to the $T = 3$ component, it is unlikely that such a weak transition could be observed in the present study. Braid, Meyer-Schutzmeister, and Borlin,[15] in a study of the ^{51}V(^{3}He, α) reaction, have reported observing the ground-state analog at 4.83-MeV excitation. In the present work, a level at 4833 keV is observed to be strongly excited with $l_p = 1$. However, this state appears at some angles to have almost twice the width of adjacent states, suggestive of a doublet at this excitation.

McCullen, Bayman, and Zamick[16] (MBZ) and Ginocchio[17] have calculated the low-lying level scheme of ^{50}V within the framework of a pure $(1f_{7/2})^{10}$ configuration. The observed $1f_{7/2}$ levels and measured spectroscopic strengths are compared with the MBZ predictions up to 7-MeV excitation in Fig. 5. For clarity, levels with strengths less than 0.05 have been omitted from the theoretical level scheme. Although many more levels are predicted than observed, many of these still have spectroscopic strengths less than 0.15, which, below 3-MeV excitation, is about the lower limit of what could be readily observed in the present study. Above 3 MeV, this experimental limit is closer to 0.2, which may account for the apparent absence of $1f_{7/2}$ transitions in this region, the predicted levels mostly having less than this strength.

The strongest transition is predicted to occur to a 7^+ level at about 1.2 MeV. Experimentally, the level at 911 keV, which is excited by a pure $l_p = 3$ transition, is an excellent candidate for the 7^+ state, having nearly the expected strength and at least twice the strength of any other state excited with $l_p = 3$. The doublet observed around 1.3 MeV may correspond to that predicted at about 0.9 MeV.

Both states are excited with an $l_p = 1$ component as well as $l_p = 3$, which is consistent with the expected spins and parities of 4^+ and 5^+. One 1^+ and two 3^+ levels are predicted around 3-MeV excitation. Only one $l_p = 3$ transition is observed in this region, however, and this occurs to the state at 2966 keV. Since this is also excited by an $l_p = 1$ component, the possibility of 1^+ can be excluded.

Within the framework of the pure $(1f_{7/2})^n$ model, ^{50}V and ^{46}Sc should have identical level schemes, since they are particle-hole conjugates of one another. A detailed comparison is not yet fruitful, however, in view of the lack of definite J^π assignments for excited states in both cases. It may be noted, however, that disagreement occurs even for the ground states, since ^{46}Sc is 4^+,[4] whereas ^{50}V is 6^+.

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