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Intermediate Coupling in the $N = 29$ and $Z = 29$ Nuclei*†

D. Larner
The University of Kansas, Lawrence, Kansas 66044
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Levels are calculated for the $N=29$ and $Z=29$ nuclei, Ti^{51}, Cr^{53}, Fe^{55}, $Cu^{61,63,65}$. The model used is the intermediate-coupling model with quasivibrational core, as developed by True and Thankappan. A pattern search routine is used to fit the five parameters to the levels and stripping strengths. Except for the case of Cr^{53}, levels and strengths are fitted fairly well. In particular, in Fe^{55} we identify the states at 1.917 and 2.051 MeV as $\frac{3}{2}^-$ and $\frac{1}{2}^-$ states, respectively. $B(E2)$ values for transitions to the ground state are calculated, and the comparison with data for Cu^{63} and Cu^{65} shows good agreement. However, there are no experimental $B(E2)$ data available for the other nuclei.

I. INTRODUCTION

Certain regions of the chart of nuclei are well treated by specific theoretical models. For $155 \leq A \leq 185$, the collective model works very well. For stable doubly magic nuclei, and nuclei near them, the shell model is effective. But there have been extensive calculations done in the vicinity of nickel, in the p-f shell, and none has been strikingly successful. Bouten and van Leuven[1] have applied the unified model to the copper isotopes $Cu^{59,61,63,65}$. Ramavataram[2] has applied the unified model to the $N=29$ nuclei, Ti^{51}, Cr^{53}, Fe^{55}. Cohen *et al.*[3] and Auerbach[4] have used the shell model for calculation of the structure of the Ni isotopes, and Maxwell and Parkinson,[5] Ohnuma,[6] and Vervier[7] have made shell-model calculations for Ti^{51}, Cr^{53}, and Fe^{55}. Beres[8] has performed a quasiparticle calculation for Cu^{63}.

The unified model uses a perfect vibrator as a core. Several investigators have used a core of less rigidly fixed form. Vervier[9] made a calculation for Cu^{65}, in which he used ground and first excited core states of unspecified nature. He considered only the $p_{3/2}$ orbit for the odd particle, and in his calculation avoided using any explicit force. Thankappan and True[10] (TT) greatly extended this model, and applied it to Cu^{63}. They use the $2p_{3/2}$, $2p_{1/2}$, and $1f_{5/2}$ orbits, and they use a force with dipole-dipole and quadrupole-quadrupole terms. The TT model accounted for both the single-particle stripping strengths of Cu^{63} and the $E2$ transition rates. That is, for both the single particle and the collective properties.

Since the TT calculation, new data have appeared on several nuclei. Blair[11] published his revised results on Cu^{63} and Cu^{65}, Pilt *et al.*[12] have reported on measurements in Cr^{53}, and Carola *et al.*[13] have reported work on Cr^{53}. It was decided then to perform a TT calculation for the $N=29$ and $Z=29$ nuclei for which data were available, since the prospect of accounting for both single-particle strengths and $E2$ rates was very attractive. The nucleus Cu^{59} was excluded, since the data for it are rather scanty.

The second section of the paper presents the essential formalism for the calculation. The third section presents the results and a discussion for each nucleus treated here. The concluding section presents a summary, and some speculation on the limits and possible extension of this work.

II. FORMALISM AND COMPUTATIONS

The general form of the coupling between the core and the particle is taken to be a sum of scalar products of terms of rank k. This has the form

$$H_{\rm int} = \sum_k T_c^{(k)} \cdot T_p^{(k)}, \qquad (1)$$

where $T_c^{(k)}$ acts only on the core, and $T_p^{(k)}$ only on the odd particle. The Hamiltonian is taken to be

$$H = H_c + H_p + H_{\rm int}, \qquad (2)$$

where H_c is the Hamiltonian describing the core, H_p the Hamiltonian describing the particle, and $H_{\rm int}$ is the coupling term.

The basis states are eigenfunctions of $H_c + H_p$, and may be written as $|JjIM\rangle$, where J is the spin of the core, j the spin of the particle, and these two angular momenta are coupled to I, the total angular momentum of the state, with a z projection of M. The eigenfunctions of H will, in general, be linear combinations of these states.

Only two states of the core, the 0^+ ground state and the 2^+ first excited state, will be considered here. The energy of the first excited core state, designated by $\hbar\omega$, will be taken from experiment. The orbits directly above the $f_{7/2}$ subshell are $1p_{3/2}$, $1p_{1/2}$, $1f_{5/2}$, $1g_{9/2}$. The $1g_{9/2}$ lies considerably higher than the first three and is of different parity from them, so it is ignored here.

The specific form assumed for $H_{\rm int}$ is

$$H_{\rm int} = -\xi\vec{J}\cdot\vec{j} - \eta Q_c\cdot Q_p, \qquad (3)$$

where $\vec{J}$ and $\vec{j}$ are the total angular momentum operators for the core and the particle, respectively, and the Q_c and Q_p, given below, are the quadrupole moment operators for the core and the particle. The parameters ξ and η are the strengths of the dipole and quadrupole interactions.

$$(Q_c)_\mu = \sum_i r_i^2 Y_{2\mu}(\theta_i, \varphi_i), \qquad (4a)$$

$$(Q_p)_\mu = r_p^2 Y_{2\mu}(\theta_p, \varphi_p). \qquad (4b)$$

The matrix element of the Hamiltonian between two basis states is

$$\begin{aligned}\langle J'j'IM|H|JjIM\rangle &= \delta_{JJ'}\delta_{jj'}\{E_J + E_j + \xi W(1jJI; jI) \\ &\quad\times[J(J+1)(2J+1)j(j+1)(2j+1)]^{1/2}\} \\ &\quad -\eta W(2j'JI;jI')\langle J'\|Q_c\|J\rangle\langle j'\|Q_p\|j\rangle, \qquad (5)\end{aligned}$$

where E_J and E_j are the eigenvalues of H_c and H_p, respectively. We use the definitions of Racah coefficients and reduced matrix elements found in de-Shalit and Talmi.[14] The parameter ν appears in the harmonic-oscillator wave functions used for the calculation of $\langle j'\|Q_p\|j\rangle$ and has the value

$$\nu = 41M/(\hbar^2 A^{1/3}).$$

Values of ν and $\hbar\omega$ for the nuclei studied here are shown in Table I.

In this calculation the core reduced matrix elements, $\langle J'\|Q_c\|J\rangle$, are taken as free parameters. Then the fitting parameters are these matrix elements, the strengths of the interactions, and the single-particle energies. These can be reduced to five parameters, $\chi_1 = \eta\langle 0\|Q_c\|2\rangle$, $\chi_2 = \eta\langle 2\|Q_c\|2\rangle$, ξ, $\epsilon_1 = \epsilon(p_{1/2}) - \epsilon(p_{3/2})$, $\epsilon_2 = \epsilon(f_{5/2}) - \epsilon(p_{3/2})$. Other details of the formalism may be found in the work of Thankappan and True[10] and De Pinho, Jeronymo, and Goldman.[15]

The search program used is a pattern search program.[16] This program is fundamentally different from programs such as the method of steepest descent. No first or second derivatives are calculated. The program increases and decreases each of the parameters in turn in an attempt to decrease the function being minimized. A successful attempt will leave the base point for the search unchanged, and produce a new current point. The direction and distance of the next move of the current point, after a minimization success, is determined by the vector distance from the base point to the current point. In this way, several successive successful attempts to minimize the function will build a preferred direction, in the parameter space, that the search will take.

TABLE I. The constants $\hbar\omega$ and ν for each nucleus.

A	Nucleus	Core	$\hbar\omega$ (MeV)	ν (F^{-2})
51	Ti^{51}	Ti^{50}	1.55	0.266 42
53	Cr^{53}	Cr^{52}	1.4336	0.263 03
55	Fe^{55}	Fe^{54}	1.409	0.259 80
61	Cu^{61}	Ni^{60}	1.3325	0.250 99
63	Cu^{63}	Ni^{62}	1.172	0.248 30
65	Cu^{65}	Ni^{64}	1.34	0.245 73

The model is fitted to both the energy levels and the single-particle strengths, since it is necessary to do this in order to obtain reasonable values for the parameters. Testing indicates that the final parameters are independent of the starting parameters, within reasonable limits, but the starting point will naturally affect the amount of computer time used.

III. RESULTS AND DISCUSSION

In this section we gather the results of the calculations and then discuss them. In Table I we show values of $\hbar\omega$ and ν used in the calculation. In Table II we show the final parameters obtained from the fitting process with each nucleus. The nuclei Ti^{51} and Cu^{63} each have two entries. For Ti^{51} I. denotes a fit with χ_2, the diagonal core reduced matrix element, held equal to zero. In this case the model of the core becomes very similar to a pure vibrator. The designation Ti^{51} II. stands for a free five-parameter fit. For Cu^{63} I. denotes a fit to the energy levels and strengths, and II. denotes a fit to energy levels and $B(E2)$ values.

Tables III through VIII compare the experimental results for the levels and strengths with the calculated values. Again, I. and II. have the same sig-

Table II. Parameters for nuclei calculated here. The notations I. and II. for Ti^{51} and Cu^{63} are explained in the text.

Nucleus	χ_1 (MeV F^{-2})	χ_2 (MeV F^{-2})	ξ (MeV)	ϵ_1 (MeV)	ϵ_2 (MeV)
Ti^{51} I.	0.271	0.0	0.108	1.750	1.788
Ti^{51} II.	0.273	−0.039	0.110	1.773	1.780
Cr^{53}	0.939	1.070	0.499	1.180	1.460
Fe^{55}	0.240	−0.464	0.105	0.863	1.178
Cu^{61}	0.475	0.485	0.279	0.800	1.253
Cu^{63} I.	0.485	0.480	0.249	1.158	1.310
Cu^{63} II.	0.485	0.473	0.259	1.118	1.348
Cu^{65}	0.520	0.560	0.259	1.298	1.600

TABLE III. Ti^{51}: levels (in MeV) and strengths. The I. and II. notations are explained in the text.

	I^{π}	$\frac{3}{2}^-$	$\frac{1}{2}^-$	$\frac{7}{2}^-$	$\frac{5}{2}^-$	$\frac{5}{2}^-$	$\frac{3}{2}^-$	$\frac{7}{2}^-$	$\frac{1}{2}^-$
Experimental[a]	levels	0.00	1.16	1.43	1.56	2.14	2.19	2.69	2.90
	strength	0.82	0.59	0.075	0.04	0.28	0.06	0.01	0.34
Calculated I.	levels	0.000	1.196	1.497	1.562	2.054	2.287		2.831
	strength	0.907	0.619		0.551	0.308	0.509		0.271
Calculated II.	levels	0.000	1.183	1.476	1.565	2.103	2.309		2.812
	strength	0.904	0.597		0.589	0.268	0.063		0.286

[a]See Ref. 31.

TABLE IV. Cr^{53}: levels (in MeV) and strengths.

	I^{π}	$\frac{3}{2}^-$	$\frac{1}{2}^-$	$\frac{7}{2}^-$	$\frac{7}{2}^-$	$\frac{7}{2}^-$	$\frac{3}{2}^-$
Experimental[a]	levels	0.000	0.565	1.008	1.285	1.537	2.324
	strength	0.55	0.36	0.25	0.05	0.016	0.24
	strength (×1.45)	0.80	0.52	0.36			
Calculated	levels	0.000	0.572	0.973	1.437	1.600	2.375
	strength	0.816	0.796	0.360	0.37		

[a]See Ref. 17.

TABLE V. Fe^{55}: levels (in MeV) and strengths.

	I^{π}	$\frac{3}{2}^-$	$\frac{1}{2}^-$	$\frac{5}{2}^-$	$\frac{7}{2}^-$	$\frac{7}{2}^-$	$\frac{3}{2}^-$	$\frac{1}{2}^-$	$\frac{5}{2}^-$
Experimental	levels[a]	0.000	0.412	0.930	1.316	1.408	1.917	2.051	2.144
	strength[b] (×0.7)	0.70	0.56	0.51					
Calculated	levels	0.000	0.353	0.989	1.202		1.863	1.984	2.292
	strength	0.809	0.559	0.607			0.164	0.280	0.329

[a]See Ref. 12. [b]See Ref. 2.

TABLE VI. Cu^{61}: levels (in MeV) and strengths.

	I^{π}	$\frac{3}{2}^-$	$\frac{1}{2}^-$	$\frac{5}{2}^-$	$\frac{7}{2}^-$	$\frac{5}{2}^-$		$\frac{3}{2}^-$	$\frac{1}{2}^-$
Experimental	levels[a]	0.000	0.476	0.971	1.311	1.395	1.661	1.935	2.090
	strength[b]	0.24	0.25	0.17		0.05		0.03	0.015
	strength (×3)	0.72	0.75	0.51		0.15		0.09	0.05
Calculated	levels	0.000	0.469	1.015	1.343	1.338		1.988	
	strength	0.857	0.821	0.587		0.171		0.008	

[a]See Ref. 32. [b]See Ref. 20.

TABLE VII. Cu^{63}: levels (in MeV) and strengths. The I. and II. notation is explained in the text.

	I^{π}	$\frac{3}{2}^-$	$\frac{1}{2}^-$	$\frac{5}{2}^-$	$\frac{7}{2}^-$	$\frac{5}{2}^-$			$\frac{3}{2}^-$
Experimental	levels[a]	0.000	0.669	0.962	1.327	1.412	1.547	1.862	2.06
	strength[b]	0.66	0.70	0.33	0.057	0.45			0.23
Calculated I.	levels	0.000	0.693	0.983	1.343	1.392			1.978
	strength	0.836	0.772	0.412		0.353			0.024
Calculated II.	levels	0.000	0.670	1.008	1.315	1.402			2.000
	strength	0.837	0.778	0.398		0.360			0.021

[a]See Ref. 33. [b]See Ref. 11.

TABLE VIII. Cu^{65}: levels (in MeV) and strengths.

	I_{π}	$\frac{3}{2}^-$	$\frac{1}{2}^-$	$\frac{5}{2}^-$	$\frac{7}{2}^-$	$\frac{5}{2}^-$	$\frac{1}{2}^-$	$\frac{5}{2}^-$
Experimental	levels[a]	0.000	0.770	1.115	1.481	1.623	1.725	2.10
	strength[b]	0.79	0.75	0.26	0.054	0.57	0.032	0.073
Calculated	levels	0.000	0.810	1.116	1.539	1.595		2.088
	strength	0.843	0.781	0.243		0.529		0.026

[a]See Refs. 34 and 11. [b]See Ref. 11.

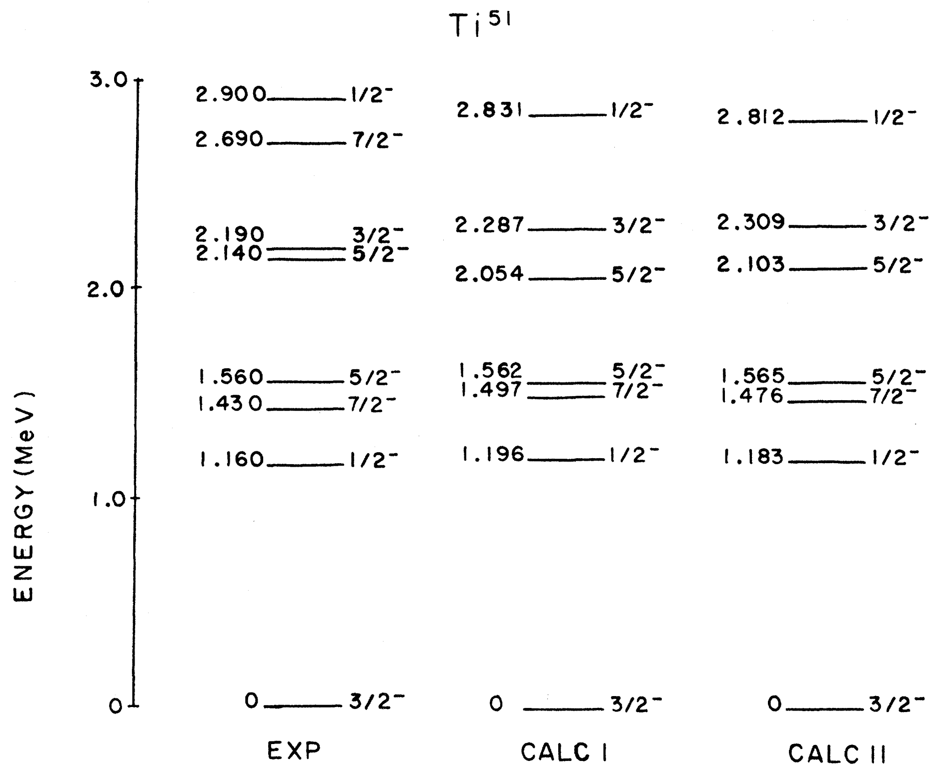

FIG. 1. Experimental and theoretical levels for Ti^{51}. The experimental levels are taken from Ref. 31. The theoretical levels I. are calculated with $\chi_2=0$. The levels II. are calculated with no restrictions on χ_2. The parameters used for I. are $\chi_1=0.271$ MeV F^{-2}, $\chi_2=0$, $\xi=0.108$ MeV, $\epsilon_1=1.750$ MeV, $\epsilon_2=1.788$ MeV. The parameters used in calculation II. are $\chi_1=0.273$ MeV F^{-2}, $\chi_2=-0.039$ MeV F^{-2}, $\xi=0.110$ MeV, $\epsilon_1=1.1733$ MeV, $\epsilon_2=1.780$ MeV.

nificance as above for Ti^{51} and Cu^{63}.

Figures 1 to 6 show experimental and calculated spectra for the given nuclei. And finally, Tables IX and X show calculated absolute and relative values for the $B(E2)$'s of transitions to the ground state. R is the ratio of the $B(E2)$'s to the $2^+ \to 0^+$ $B(E2)$ of the core nucleus. These results are more fully discussed later.

A. Ti^{51}

In general, the calculated levels and strengths for this nucleus are rather close to the experimental values. The results are shown in Fig. 1. In the several attempts at fitting this nucleus, the second $\frac{3}{2}^-$ state falls somewhat above the experimental level, and the second calculated $\frac{1}{2}^-$ is usually a bit low. In the fits shown, the second $\frac{7}{2}^-$ is calculated at about 3.6 MeV, and inclusion of this state in the fit considerably worsens the agreement of the other states. The diagonal reduced matrix element of the core, χ_2, will go negative in a free fit, indicating a slightly oblate shape for the first 2^+ state in Ti^{50}. This is not a very pronounced tendency, since holding $\chi_2 = 0$ does not change the fit very much, and the largest absolute value of χ_2 encountered in the fitting is -0.095 MeV F^{-2}.

The fit to the strengths is, on the whole, rather good. The model predicts no strength for the first $\frac{7}{2}$ state, and the experimental value is small. The ground-state strength is somewhat large. But five strengths are fitted fairly closely. This makes the first $\frac{5}{2}$ strength stand out. The experimental and theoretical strengths are 0.04, and 0.551, or 0.589, respectively. This suggests that the calculation is rather far from the true nature of this state, and this is interesting in light of the general agreement present in this nucleus.

TABLE IX. Calculated values of $B(E2)$ for transitions to the ground state for Ti^{51}, Cr^{53}, Fe^{55}. E_i and I_i are the experimental energy and spin of the initial state. The I. and II. notation for Ti^{51}, and the meaning of R, are explained in the text.

Nucleus	E_i	I_i	$B(E2)$ (e^2 F^4)	R
Ti^{51} I.	1.16	1/2	109.1	2.272
	1.43	7/2	58.86	1.226
	1.56	5/2	37.57	0.783
	2.14	5/2	14.33	0.298
Ti^{51} II.	1.16	1/2	112.8	2.349
	1.43	7/2	60.03	1.251
	1.56	5/2	34.10	0.711
	2.14	5/2	15.68	0.327
Cr^{53}	0.565	1/2	117.4	1.140
	1.008	5/2	107.9	1.048
	1.537	7/2	78.11	0.758
Fe^{55}	0.412	1/2	294.0	2.883
	0.930	5/2	30.86	0.303
	1.316	7/2	153.8	1.508
	1.917	3/2	38.42	0.377
	2.051	1/2	30.74	0.301
	2.144	5/2	13.58	0.133

TABLE X. Calculated values of $B(E2)$ for transitions to the ground state for $Cu^{61,63,65}$. E_i and I_i are the experimental energy and spin of the initial state, and the meaning of R is explained in the text. The Cu^{63} values are computed for case I.

Nucleus	E_i	I_i	$B(E2)$ (e^2 F^4)	R
Cu^{61}	0.476	1/2	168.0	0.994
	0.971	5/2	115.5	0.683
	1.311	7/2	150.9	0.893
	1.395	5/2	81.20	0.480
Cu^{63}	0.669	1/2	166.6	0.952
	0.962	5/2	169.1	0.966
	1.327	7/2	151.2	0.864
	1.412	5/2	49.44	0.283
Cu^{65}	0.770	1/2	158.7	0.907
	1.115	5/2	209.7	1.198
	1.481	7/2	149.5	0.854
	1.623	5/2	16.70	0.095

B. Cr^{53}

The fit to the levels and strengths for Cr^{53} is not as good as the fits for the other nuclei. The two reduced core matrix elements, χ_1 and χ_2, are both rather large. The quality of fit, a χ^2 for the levels and strengths used in the fit, is several times that found for the other nuclei presented here. Both the 1.285- and 1.537-MeV states are $\frac{7}{2}^-$ states. The model can produce only one $\frac{7}{2}^-$ state in this vicinity. The 1.537-MeV state should not appear in this calculation. It appears strongly in (p,d) work[17] and is very likely a $\frac{7}{2}^-$ hole state. For most reasonable parameters, the model will put a second $\frac{5}{2}^-$ state somewhere between 1.5 and 2.2 MeV. In the Cr^{53} calculation we usually find this state around 1.5 MeV, but it cannot be identified with either of the states in that vicinity.

The state at 1.971 MeV has been seen in stripping[18] and angular correlation work.[13] The result has been that no definite spin has been assigned, so we conclude that this state has a rather complicated structure. Earlier calculations by Maxwell and Parkinson[5] and Vervier,[7] however, have identified this state as a $\frac{5}{2}^-$ state.[19]

C. Fe^{55}

The strengths and levels for this nucleus are fitted

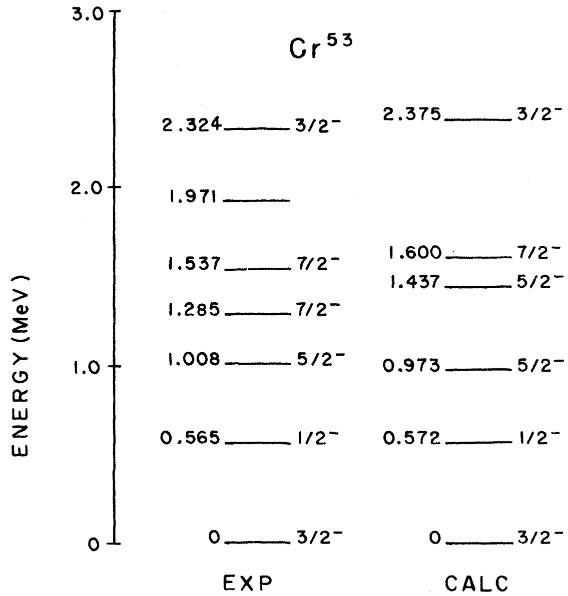

FIG. 2. Experimental and theoretical levels for Cr^{53}. The experimental levels are taken from Ref. 17. The parameters used in this calculation are $\chi_1=0.939$ MeV F^{-2}, $\chi_2=1.070$ MeV F^{-2}, $\xi=0.499$ MeV, $\epsilon_1=1.180$ MeV, $\epsilon_2=1.460$ MeV.

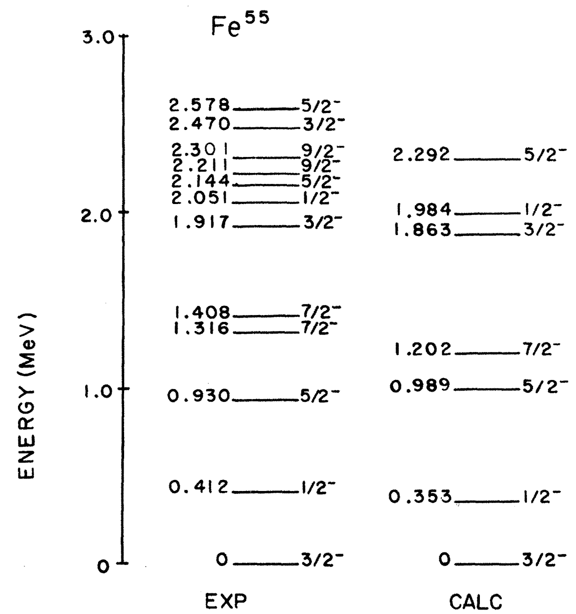

FIG. 3. Experimental and theoretical levels for Fe^{55}: for the experimental levels (see Ref. 12). The parameters used in this calculation are $\chi_1=0.240$ MeV F^{-2}, $\chi_2=-0.464$ MeV F^{-2}, $\xi=0.105$ MeV, $\epsilon_1=0.863$ MeV, $\epsilon_2=1.178$ MeV.

well, although not as well as Ti^{51}. The results are shown in Fig. 3. Pilt *et al.*[12] find two $\frac{7}{2}^-$ states at 1.316 and 1.408 MeV. The calculation can account for only one of them. Pilt *et al.* also find two $\frac{9}{2}^-$ states at 2.211 and 2.301 MeV. We calculate the one $\frac{9}{2}^-$ state in the model at 2.44 MeV, so it is likely that this corresponds to the experimental state at 2.301 MeV. The $\frac{7}{2}^-$ state at 1.408 MeV is seen strongly in (p,d) work. This suggests that this is primarily a $\frac{7}{2}$ hole coupled to the ground state of the core. The $\frac{9}{2}^-$ state at 2.211 MeV decays strongly to the 1.408-MeV state, so it is likely that this state also involves a $\frac{7}{2}^-$ hole coupled to the core. Thus, it is reasonable that neither the $\frac{7}{2}^-$ state at 1.408 MeV, nor the $\frac{9}{2}^-$ state at 2.211 MeV should show up in the present calculation.

For both Ti^{51} and Fe^{55} the parameters χ_1 and ξ are always close to 0.26 and 0.10, respectively, though the parameters are determined by a free fit, so no constraint is put upon them. The cores of these nuclei are Ti^{50} and Fe^{54}. These have a closed shell for neutrons, $N=28$, and conjugate configurations for protons, $(\pi f_{7/2})^2$ for Ti^{51} and $(\pi f_{7/2})^{-2}$ for Fe^{54}. Thus, it is reasonable that the parameters obtained for these two nuclei should be similar. Further, Cr^{53} has a core of Cr^{52}, and a core configuration of $(\pi f_{7/2})^4$, so it is clear that the parameters for Cr^{53} will be quite different from those for Ti^{51} and Fe^{55}.

D. Cu^{61}, Cu^{63}, Cu^{65}

Before commenting on the copper results, a word should be said about the experimental single-particle strengths. These must be extracted from the experimental data by using a distorted-wave Born-approximation (DWBA) code. The DWBA theory accounts well for the relative values of the strengths, but there is also a normalization factor in the distorted-wave theory. Because of the uncertainty in this normalization factor, we feel that the essential experimental result is the relative strengths and not the absolute magnitudes. Thus, in some cases, all the experimental strengths have been multiplied by a constant factor in order to bring them more nearly into line with other experimental results, or the general trend of the calculation, so some of the experimental stripping strengths are listed as (×3), (×1.45), or (× 0.7).

In general, the single-particle strengths and the level energies for the copper isotopes are fit very well. The level energies are shown in Figs. 4–6. The spacing between the calculated second $\frac{5}{2}^-$ and first $\frac{7}{2}^-$ is consistently too small. The predicted single-particle strength of the ground state is usually too large. Also, the strength of the second $\frac{3}{2}$ state is consistently predicted to be very small,

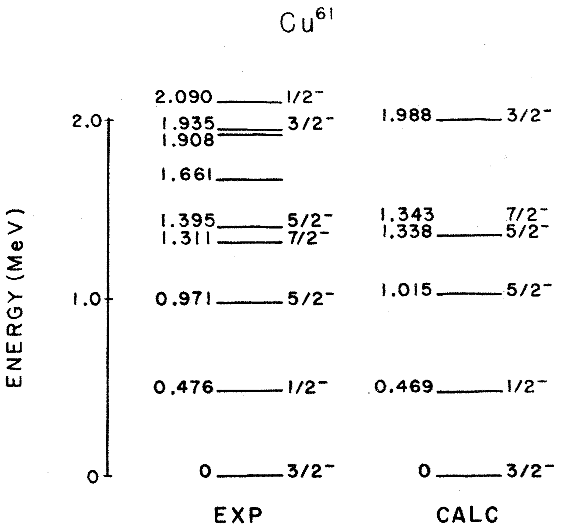

FIG. 4. Experimental and theoretical levels for Cu^{61}: for the experimental levels (see Ref. 32). The parameters used in this calculation are $\chi_1 = 0.475$ MeV F^{-2}, $\chi_2 = 0.485$ MeV F^{-2}, $\xi = 0.279$ MeV, $\epsilon_1 = 0.800$ MeV, $\epsilon_2 = 1.253$ MeV.

and only in Cu^{65} is this in accord with the experimental results.

There are several experimentally found states which do not show up in the calculation, and this deserves comment. In Cu^{61} the states at 1.661 and 1.908 MeV are not accounted for by the calculation. Pullen and Rosner[20] and Blair[21] have investigated the $Ni^{60}(He^3, d)Cu^{61}$ reaction, and neither sees the states at 1.661 and 1.908 MeV. This failure of the stripping reaction to find these states makes it very unlikely that a core-plus-particle model could account for them.

In Cu^{63} there are states at 1.547 and 1.862 MeV which the calculation does not account for. The (p, p') work has not been able to assign a spin to the 1.547-MeV state.[23] This state is seen weakly in pickup[24] and not at all in stripping.[11] Evidently, this state is considerably more complex than a core plus a single particle or a hole. The 1.862-MeV state shows up strongly in pickup[24] and has an $l = 3$ angular distribution. It would seem that the 1.862-MeV state is a $\frac{7}{2}$ hole, and so would be unlikely to show up in the present calculation.

In Cu^{65} the state at 1.725 MeV is missing from the theoretical results. This state has no spin assigned to it from the inelastic proton scattering work.[23] It is not seen in pickup,[24] and is seen only weakly in stripping.[11] So this state would appear to be a more complex excitation than this calculation can account for.

The nucleus Cu^{63} deserves special mention. Here, data are available on $B(E2)$'s for low-lying states to the ground state. Table II shows the parameters resulting from two fits for Cu^{63}. The sixth line shows the parameters for a fit to the energy levels and the stripping strengths. The seventh line shows the parameters for a fit to the energy levels and the $E2$ transition rates. It is in-

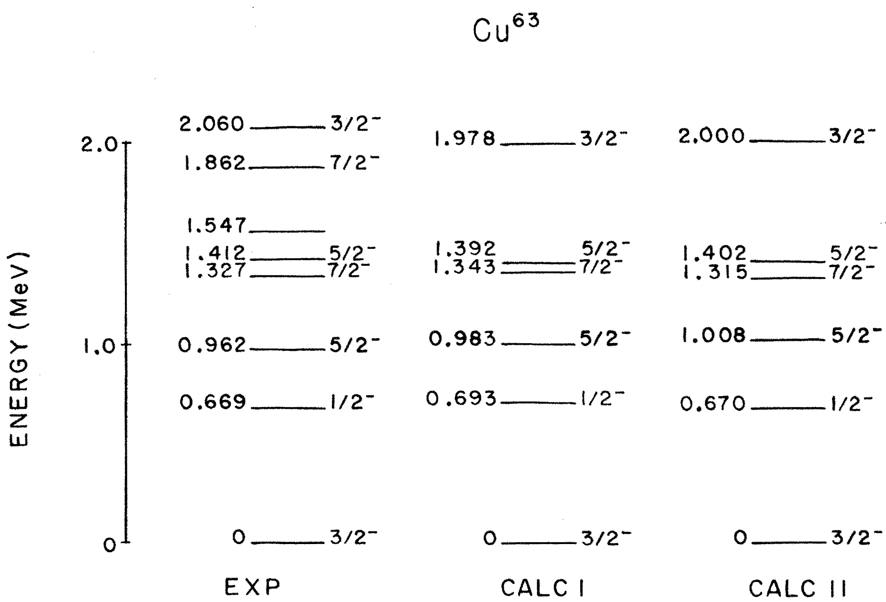

FIG. 5. Experimental and theoretical levels for Cu^{63}: for the experimental levels (see Ref. 33). The theoretical levels in calculation I. are fitted to the experimental energy levels and single-particle strengths. The parameters used for I. are $\chi_1 = 0.485$ MeV F^{-2}, $\chi_2 = 0.480$ MeV F^{-2}, $\xi = 0.249$ MeV, $\epsilon_1 = 1.158$ MeV, $\epsilon_2 = 1.310$ MeV. The levels II. are fitted to the energy levels and $B(E2)$ values. The parameters used for II. are $\chi_1 = 0.485$ MeV F^{-2}, $\chi_2 = 0.473$ MeV F^{-2}, $\xi = 0.259$ MeV, $\epsilon_1 = 1.118$ MeV, $\epsilon_2 = 1.348$ MeV.

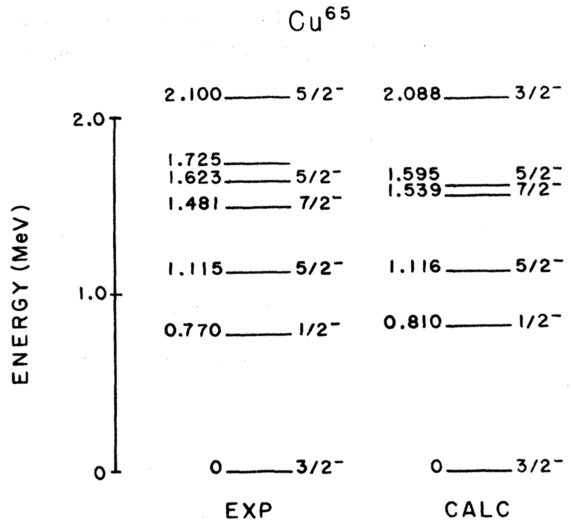

FIG. 6. Experimental and theoretical levels for Cu^{65}: for the experimental levels (see Ref. 34). The parameters used in this calculation are $\chi_1=0.520$ MeV F^{-2}, $\chi_2=0.560$ MeV F^{-2}, $\xi=0.259$ MeV, $\epsilon_1=1.298$ MeV, $\epsilon_2=1.600$ MeV.

teresting that the parameters for this case agree rather well with the parameters found in the first case.

E. $B(E2)$ Values

In order to calculate the $E2$ transition rates in the nuclei studied, it is necessary in the present model to know the $B(E2)$ value from the first 2^+ state of the core to the core's ground state. This is usually denoted as the $B(E2\text{: } 2^+\to 0^+)$. In this study we need the number $B(E2\text{: } 2^+\to 0^+)$ for the nuclei Ti^{50}, Cr^{52}, Fe^{54}, Ni^{60}, Ni^{62}, Ni^{64}. Simpson *et al.*[25] give values of this $B(E2)$ for Ti^{50}, Cr^{52}, and Fe^{54} with an error of 5–10%. Their values for Ti^{50} and Fe^{54} have been used in the calculation.

Bellicard[26] gives a value of 103, in units of $e^2\,F^4$, for this $B(E2)$ for Cr^{52}. This value is in substantial agreement with values of 96 ± 4, given by Simpson *et al.*,[25] 120 ± 30 given by Adams,[27] and 124 ± 36 given by Lemberg.[28] However, Meriwether *et al.*[29] give a value of 57.9. We used the value of Bellicard for our calculations. The $B(E2\text{: } 2^+\to 0^+)$ value is readily available for Ni^{60} and Ni^{62}, and we have chosen the values of Duguay *et al.*,[30] 169 for Ni^{60} and 175 for Ni^{62}, in units of $e^2\,F^4$. In view of the similarity between Cu^{63} and Cu^{65}, we have assumed that the relevant $B(E2)$ for Ni^{64} has the same value as that for Ni^{62}. This is confirmed by Lemberg's value of 174, in units of $e^2\,F^4$, for this Ni^{64} $B(E2)$ value. We summarize these $B(E2)$ values in Table XI.

The results of the calculations are given in Tables IX and X. The result of the $B(E2)$ calculation for Cu^{63} agrees well with experiment. This result has been noted elsewhere.[10] The results for Cu^{65} are quite similar, which is not surprising. The parameter R, for a given state in Cr^{53}, is given by

$$B(Cr^{53}, E2\text{: excited state} \to \text{ground state})/B(Cr^{52}, E2\text{: } 2^+\to 0^+)\,.$$

For the transitions from the $(\frac{1}{2})_1$, $(\frac{5}{2})_1$, $(\frac{7}{2})_1$ states, respectively, the experimental values of R given by Meriwether *et al.* for Cr^{53} are 3.074, 0.604, 1.192. It is interesting to note that our values of R for Cr^{53} are roughly the same as for Cu^{63} and Cu^{65}, but the values of R for Ti^{51} and Fe^{55} compare quite closely with Meriwether's values for Cr^{53}.

TABLE XI. $B(E2\text{: } 2^+\to 0^+)$ values for the core nuclei.

Nucleus	$B(E2)$ ($e^2\,F^4$)	Reference
Ti^{50}	48	16
Cr^{52}	103	17
Fe^{54}	102	16
Ni^{60}	169	21
Ni^{62}	175	21
Ni^{64}	174	17

IV. SUMMARY

For all of the nuclei studied here, with the one exception of Cr^{53}, the levels and energies are fitted quite well. In the better cases, 10 or 11 levels and strengths are fit by varying five parameters. In some cases single-particle strengths are calculated, which have yet to be checked by experiment. We also calculate $B(E2)$ values for transitions to the ground state. In Cu^{63} there is good agreement with experiment, and in the other nuclei, measurement of these $B(E2)$'s would constitute an interesting test of the model.

The present model works significantly better than the intermediate-coupling model with pure vibrator core. This is apparently because we allow the parameter χ_2 to become not only nonzero, but relatively large, and this becomes essentially the "collective" part of the model. We are limited by having only one excited core state. This should not have a large effect on states lying below the energy of the second excited core state. Presumably, states resulting from single-particle states coupling to the second excited core state will lie near or above the energy of this core state. We do miss several states under 2 MeV. These states have been discussed in Sec. III, and it was shown there that they are probably hole states, or states considerably more complex than a core plus a particle.

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