Core-Plus-Particle Model and the Low-Lying Levels of Cr^{53}

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Experimental energies, spectroscopic factors, and electromagnetic data associated with the low-lying levels of Cr^{53} are fitted by means of a model in which the basis states are taken to be single-particle orbits vector-coupled to states of the Cr^{52} core. Unknown properties of the core states are treated as parameters. A least-squares minimization procedure is used to determine optimum fits to the data. Although most of the low-energy data can be fitted quite successfully, the spectroscopic factors and quadrupole moment are not well reproduced. The observed levels appear to be more highly mixed than is suggested by the model. It is also found to be necessary to introduce an effective charge in order to reproduce the $B(E2)$ data.

I. INTRODUCTION

The description of an odd-A nucleus in terms of a core-plus-particle model as discussed by de-Shalit[1] and used by Thankappan and True[2] appears at first sight to be both conceptually simple and yet capable of considerable generality. In this model, the states of the odd-A nucleus are constructed as linear combinations of the states of the underlying even-even core nucleus vector coupled to the states of a single nucleon moving "outside" the core, the amplitudes being determined by diagonalizing a suitably chosen Hamiltonian. Were it not for the requirement that the total wave function be antisymmetric, such a description would always be possible, at least in principle. Although it is possible to reformulate the model in such a manner that the requirements of antisymmetry are met, as Thankappan[3] has done, the result is a formalism of such complexity that the original attractive simplicity of the model is lost. Fortunately, however, the special case in which the odd nucleon is moving relative to a core whose low-lying states are characterized by excitations among nucleons of opposite type may be treated, to good approximation, without the encumbrances of the more general formalism. If, in addition, the low-lying states of the core nucleus are fairly well separated in energy, one may hope to achieve a useful description of the low states of the odd-A nucleus by retaining only a few of the lowest states of the core. Ambiguities remain in the choice of the model Hamiltonian and also, possibly, in other properties associated with the core states.

A somewhat specialized version of the core-plus-particle model was used previously[2] to discuss the properties of the low-lying states of Cu^{63}. The model appeared to be capable of correlating a large amount of low-energy data within a simple framework. In the present paper, we report the results obtained when the same model is applied to the nucleus Cr^{53}. The suggestion that such a calculation might be of interest for Cr^{53} has already appeared in the literature by Whitten[4] and by Rao *et al.*[5] Potentially, the model is capable of acheiving a simultaneous fit to the low-lying energy spectrum and the associated electromagnetic moments and transition-rate data in Cr^{53}, together with a number of spectroscopic factors observed in the $Cr^{52}(d,p)Cr^{53}$ and $Cr^{53}(p,d)Cr^{52}$ reactions. As we shall see, the Cr^{53} results are not as conclusive as in the Cu^{63} case.

II. FORMULATION

The particular form of the core-plus-particle model which we shall employ has been fully described elsewhere.[2] Thus we include here only sufficient detail to make the meaning of the model parameters clear.

A basis state of the model with total spin J and projection M has the form implied by the notation $|J_c, j_p, JM\rangle$, where J_c stands for the quantum numbers of the core state, and j_p similarly represents the quantum numbers of the particle state, a neutron in this case. The model Hamiltonian may be written quite generally as

$$H = H_c + H_p + H_{\text{int}}, \tag{1}$$

where H_c is the core Hamiltonian and H_p the particle Hamiltonian, both of which are diagonal in the model basis states.

The spectra of H_c and H_p may be partly obtained from experiment. Figure 1 shows the observed spectrum[6] of the core nucleus Cr^{52} up to an excitation energy of 3 MeV. The ground and first excited states are seen to be fairly well separated in energy from the more highly excited states. Provided that the interaction between the core and particle is not too large, it should be possible to represent the core components of the lowest states in Cr^{53} using only the lowest two core states in Cr^{52}. These are the only two core states retained in most of the present work, though we shall have occasion, later on, to consider some of the modifications implied by the presence of more highly excited states of the core. Again, on the basis of the shell model, the $f_{7/2}$ neutron shell is just filled in Cr^{52}. Thus we expect that, at least for the low-lying states of Cr^{53}, the additional neutron will be confined to the $2p_{3/2}$, $1f_{5/2}$, and $2p_{1/2}$ orbits. The situation is very similar to that encountered in Cu^{63} except that, in the latter case, it was an odd proton which could occupy these orbits.

Unfortunately, we are unable to obtain from experiment an unambiguous result for the relative energies of the single-particle orbits. For a number of preliminary calculations, we took the relevant energy differences to be $E_{5/2}-E_{3/2}=1.2$ MeV and $E_{1/2}-E_{3/2}=1.35$ MeV. In making this choice, we were guided partly by the experimental observations of Cohen[7] and by the energies used in other theoretical calculations.[8-11] Preliminary graphical surveys of the model parameter space suggested that this particular choice of single-particle energies would be suitable. However, in recognition of the fact that the choice of these numbers is rather arbitrary, we have treated the single-particle energy differences as free parameters in all calculations reported here.

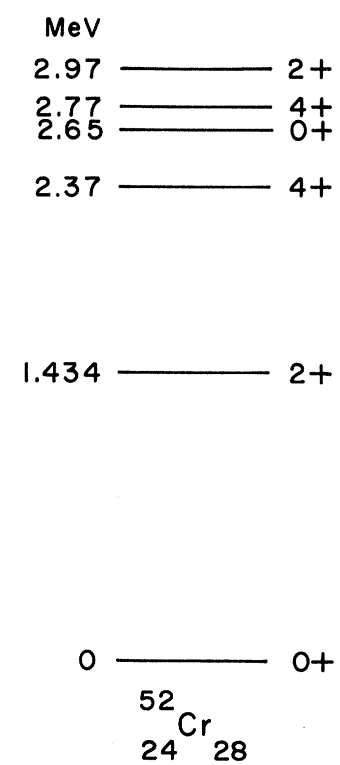

FIG. 1. Observed spectrum of Cr^{52} up to an excitation energy of 3 MeV. Spins and parities are shown opposite the energy levels.

The remaining part of the Hamiltonian, representing the interaction between the core and the particle, is given the form

$$H_{\rm int}=-\xi\vec{J}_c\cdot\vec{j}_p-\eta(Q_c^{(2)}\cdot Q_p^{(2)})\,. \tag{2}$$

Here $\vec{J}_c$ and $\vec{j}_p$ are the usual total angular momentum operators for the core and particle states, respectively, and $Q_c^{(2)}$ and $Q_p^{(2)}$ are the corresponding mass quadrupole moment operators. This particular form has been selected[2] to economize the number of terms included in $H_{\rm int}$, while retaining explicitly those terms which have appeared in the past[12] to be of significance.

The matrix elements of $H_{\rm int}$ taken between model basis states can be expressed in standard form as products of angular momentum recoupling coefficients and reduced matrix elements of the operators contained in $H_{\rm int}$. The reduced matrix elements of $Q_p^{(2)}$ were evaluated by using harmonic-oscillator wave functions with an oscillator parameter given by

$$\nu=41M/\hbar^2A^{1/3}=0.264\ {\rm F}^{-2}\,. \tag{3}$$

In the philosophy of the present model, it is assumed that the detailed structure of the core states is unknown. Thus, the reduced matrix elements in the subspace of the core (except for those of $\vec{J}_c$) must be introduced as parameters. Since our two core states have spins $J_c=0$ and $J_c=2$, it is sufficient to define the two quantities $\chi_1\equiv\eta\langle 0||Q_c^{(2)}||2\rangle$ and $\chi_2\equiv\eta\langle 2||Q_c^{(2)}||2\rangle$ which, together with ξ, form a set of three unknown parameters describing the matrix elements of $H_{\rm int}$.

The total Hamiltonian may be diagonalized among the model basis states to produce a theoretical energy spectrum and set of wave functions for the Cr^{53} nucleus. These wave functions may be used directly to provide additional information, such as predicted spectroscopic factors for the $Cr^{52}(d,p)$-Cr^{53} reaction. Electromagnetic moments and transition rates may also be obtained provided that appropriate operators are introduced. The operators for the magnetic and quadrupole moments appear in standard notation as[2]

$$\vec{\mu}=\mu_0(g_c\vec{J}_c+g_l\vec{l}_p+g_s\vec{s}_p) \tag{4}$$

and

$$Q_m=(\tfrac{16}{5}\pi)^{1/2}\Big[\sum_i e_i r_i^2 Y_m^{(2)}(\Omega_i)+e_p r_p^2 Y_m^{(2)}(\Omega_p)\Big], \tag{5}$$

respectively, where we have used e_p to represent the effective charge of the neutron moving outside the core. In these formulas, the first term acts only on the core and second and third terms act only on the particle. The quantity $\mu_0=e\hbar/2Mc$ is the usual nuclear magneton, and $g_c=Z/A$ is the gyromagnetic ratio for the core.

It is now convenient to introduce parameters

$$\beta_1\equiv\langle 0||\sum_i e_i r_i^2 Y_m^{(2)}(\Omega_i)||2\rangle$$

and

$$\beta_2\equiv\langle 2||\sum_i e_i r_i^2 Y_m^{(2)}(\Omega_i)||2\rangle$$

in analogy with the parameters χ_1 and χ_2. The parameter β_1 may be fixed by comparison with the observed matrix element for the decay of the 2+ state in Cr^{52}. We have taken $\beta_1/e=16.28$ F^2, which is consistent with $B(E2;\ 2+\to 0+)/e^2=53$ F^4 as found by Meriwether *et al.*[13] In Ref. 2, the parameter β_2 was fixed by assuming the relation

$$\beta_2/\beta_1=\chi_2/\chi_1. \tag{6}$$

This relation is true only if it is permissible to make the replacement

$$\sum_i e_i r_i^2 Y_m^{(2)}(\Omega_i)\to e_c\sum_i r_i^2 Y_m^{(2)}(\Omega_i)\equiv e_c(Q_c^{(2)})_m,$$

where e_c is an effective charge for the core. In the present work, we will not rely upon this assumption. However, even when β_2 is allowed to vary independently of χ_1 and χ_2, it was found to be necessary to give the odd neutron a nonvanishing effective charge, e_p, in order to reproduce the Cr^{53} data. In Ref. 2, it was also found to be necessary to renormalize the particle gyromagnetic ratios g_l and g_s in order to fit the $M1$ properties of Cu^{63}. Since the odd particle in Cr^{53} is a neutron, we may set $g_l=0$ and allow g_s alone to vary. Thus the calculation of electromagnetic data within the framework of the present model involves the three additional free parameters, β_2, e_p, and g_s.

When the present model is compared with the version employed previously,[2] it appears that we have introduced several new free parameters. Although the earlier work claims to have used only three free parameters, namely χ_1, χ_2, ξ, a careful reading reveals that the single-particle energies were adjusted somewhat from their initial values "in order to obtain better agreement with the observed energy levels of Cu^{63}". Actually, the only new free parameters introduced here are β_2 and e_p, both connected with the quadrupole moment operator. The effect of these additional parameters will be investigated further in Sec. V.

III. FITS TO THE LOW-ENERGY Cr^{53} DATA

A wealth of information on the energy levels of Cr^{53} is available.[4,5,14] In this section we shall describe certain fits to the Cr^{53} data, confining our attention to the lower end of the energy spectrum. Since the present model uses a highly truncated set of basis states, one expects the higher-lying model eigenstates to be fragmented by admixture with other states which have not been explicity included. Thus it is hard to determine *a priori* the expected location of the more highly excited model eigenstates from the (necessarily incomplete) experimental data. On the other hand, the inverse comparison – namely an attempt to understand the fragmentation of the predicted model states by direct comparison with the observed level scheme – is possible to some extent and will be taken up in Sec. IV.

Figure 2 shows the observed spectrum[4,5] of Cr^{53} up to an excitation energy of 2.7 MeV. The upward-going arrows indicate levels which are strongly excited in inelastic-scattering reactions.[4,5,13] Also

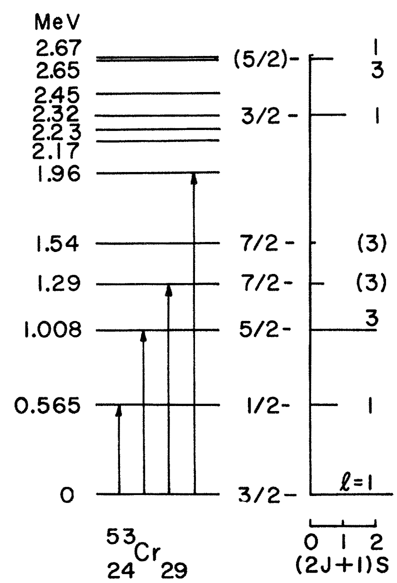

FIG. 2. Observed spectrum of Cr^{53} up to an excitation energy of 2.7 MeV. Known spins and parities are written opposite the corresponding energy levels. Excited levels populated strongly in inelastic scattering reactions are indicated by upward-going arrows. The spectroscopic factors $(2J+1)S$ observed in the $Cr^{52}(d,p)Cr^{53}$ reaction are shown on the right together with the assigned l values.

shown in the figure are the l values and spectroscopic factors observed[5,14] in the $Cr^{52}(d, p)Cr^{53}$ reaction. This information is supplemented with the measured $B(E2)$ values[13,15] for the decays of the first three excited levels to ground, with the observed spectroscopic factors[4] in the pickup reaction $Cr^{53}(p, d)Cr^{52}$ leading to the ground and first excited states in Cr^{52}, and with the known values[16] of the Cr^{53} ground-state magnetic and quadrupole moments.

Since we know in advance that our model cannot describe all the eigenstates of Cr^{53}, it is important to decide which of the observed levels might be expected to have a large overlap with the states which we include. We will then try to choose the parameters of the model to reproduce as closely as possible the positions and properties of these levels.

Two criteria have been employed to rationalize this choice. We have considered for inclusion (i) those levels in Cr^{53} which exhibit a large spectroscopic factor in the $Cr^{52}(d, p)Cr^{53}$ reaction, and (ii) those levels which appear to be excited collectively in inelastic scattering reactions. The first set of levels can be expected to have a large component of the model state $|J_c = 0^+, j_p = J, JM\rangle$; the second set may reasonably be expected to contain a large component of the model state $|J_c = 2+, j_p = \frac{3}{2}, JM\rangle$. With these criteria we are able to select 6 out of 11 excited states observed below 2.7 MeV in Cr^{53} as candidates for fitting the parameters. Levels which are not included by the above criteria are thought to be formed by single-particle states being coupled to more highly excited states of the core. In addition, the presence of the odd neutron in Cr^{53} encourages another excitation mode to appear, namely that in which an $f_{7/2}$ proton is excited into a higher orbit to form a pair with the extracore neutron. The $\frac{7}{2}-$ level at 1.54 MeV is believed[4,17] to have this type of structure. It is not unreasonable for the states built on the more highly excited core states (e.g. the 4+ state at 2.37 MeV in Cr^{52}) to begin at about 2-MeV excitation in Cr^{53}.

The level observed at 1.96 MeV in Cr^{53} poses an intriguing problem. It is observed strongly in inelastic scattering but does not exhibit a good stripping pattern in the $Cr^{52}(d, p)Cr^{53}$ reaction. One may explain these observations quite simply by supposing that this level is part of a multiplet built on the lowest 2+ state in Cr^{52}. If such an explanation is true, this level should be included in our fit. We shall see, however, that it will not be possible to make a definite identification of this state with one of the theoretically calculated states.

The spin of the level at 1.96 MeV is not known although the observations[17,18] that this level decays primarily to the ground state suggests that it has spin $J \leq \frac{7}{2}$. We have performed least-squares searches in the parameter space of our model assuming successively each one of the four possibilities, $\frac{1}{2}$, $\frac{3}{2}$, $\frac{5}{2}$, and $\frac{7}{2}$, for the spin of the 1.96-MeV level. Because none of the searches produced an entirely satisfactory fit to the available data, we have also performed a search in which the 1.96-MeV level was not included. The quantity Δ which was minimized in these least-squares searches is described in the Appendix. We expect that the results are insensitive to small changes in the definition of Δ.

Figure 3 shows the variation of Δ as a function of χ_2 in a series of least-squares fits to the energy spectrum of Fig. 2 supplemented with some of the available spectroscopic-factor information. For each value of χ_2, Δ is minimized with respect to

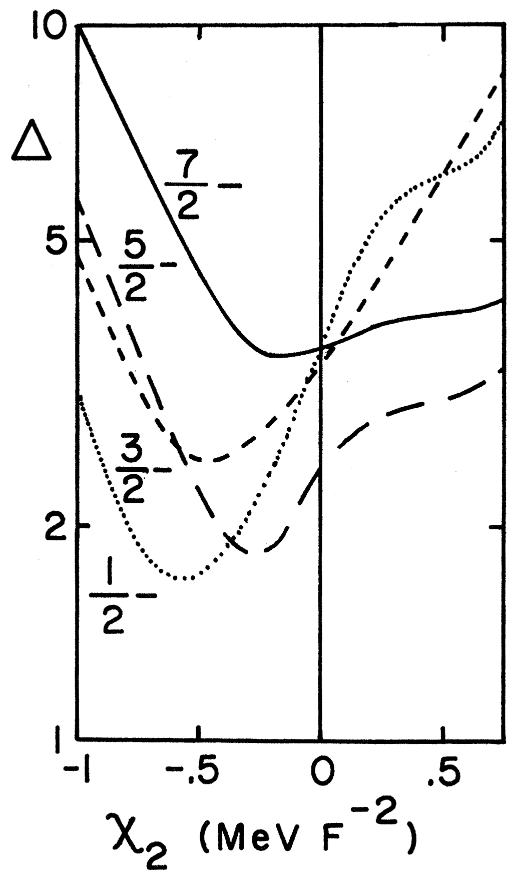

FIG. 3. Variation of the least-squares function Δ against the model parameter χ_2 using various trial values for the unknown spin of the level at 1.96 MeV in Cr^{53}. Each curve is marked with the spin value which was assumed in the calculation of Δ. At each point on any of the curves, the function Δ was minimized with respect to the parameters χ_1 and ξ and the two relative single-particle energies. Electromagnetic data were not included.

the parameters ξ, χ_1, $E_{5/2}-E_{3/2}$, and $E_{1/2}-E_{3/2}$. In each sequence of fits, the 1.96-MeV level was assigned a definite spin, which is used as a label in the figure. The figure indicates a definite tendency for χ_2 to prefer a negative value – which, of course, is here quite independent of the electromagnetic data. On the basis of these searches, the spins $J=\frac{1}{2}$ or $\frac{5}{2}$ are seen to be preferred over $J=\frac{3}{2}$ or $\frac{7}{2}$, since Δ can be smaller in the former cases. This may be readily understood in terms of the number of states contained in the model and the spins and position of those levels other than the 1.96-MeV level which are to be fitted. Thus, for example, the model contains only two $\frac{7}{2}-$ levels, and it is hard to bring them close together in energy and still maintain the fit to the remainder of the spectrum.

If we include the $B(E2)$ data, and perform a seven-parameter over-all fit, the values of the five parameters determined previously by the best fit to the energies and spectroscopic factors are reproduced with only insignificant changes, and we obtain the final values given in Table I. The eighth parameter, g_s, is fitted to the observed magnetic moment and has no effect on the values taken by the other seven parameters.

The energy spectra obtained with these sets of parameters are shown in Fig. 4. We see that the observed energies of the lowest-three excited levels are fairly well reproduced in each case, there being considerable more variation in the energies of the more highly excited levels. A notable feature of these calculations is the persistent appearance of a $\frac{9}{2}-$ level in the region of 2 MeV. This calculated level is possibly to be associated with the observed level at 2.23 MeV, for which a spin of $\frac{9}{2}-$ has been suggested by Carola.[17] Surprisingly enough, when the 1.96-MeV level is omitted from consideration, the fit to the energy spectrum appears to worsen rather than improve. The reason for this behavior is that the searching procedure is taken advantage of the additional freedom to improve the fit to the spectroscopic factors, at the expense of the fit to the energy spectrum. It is quite possible that too much weight has been given to the spectroscopic factors in this case, as the results[2] of Cr^{63} indicate that the spectroscopic factors are not explained very well with this simple model.

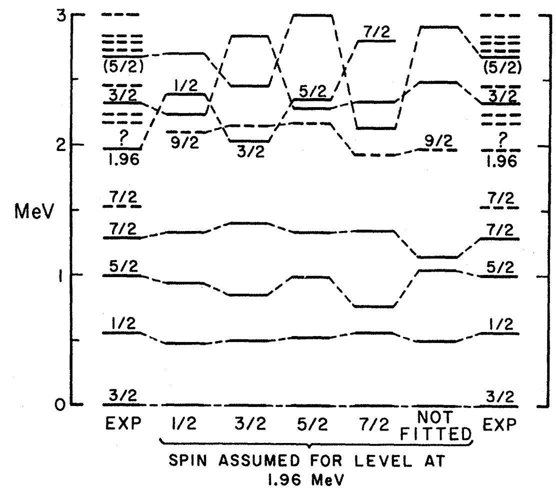

FIG. 4. Comparison of various theoretical spectra for Cr^{53} with experiment. The theoretical spectra are calculated using the parameters of Table I and are labeled in the same manner as the columns of that table. Theoretical levels which are fitted to observed ones are so indicated by the sloping dashed lines connecting levels in different columns. The spin is the same for all levels connected by such dashed lines with the exception of those fitted to the level at 1.96 MeV. The spin is marked individually on the latter levels and is the same as the column label. Empirical levels which are not explicitly fitted by the model are shown by dashed horizontal lines in the columns marked EXP. A theoretical $J=\frac{9}{2}-$ level is also indicated by means of a dashed line in the appropriate columns.

Sample sets of stripping spectroscopic factors are given in Table II. On the whole, the agreement between the calculated and empirical values is rather poor. For example, the predicted spec-

TABLE I. Parameter sets determined by least-squares fits to the Cr^{53} data as described in the text. The spin value assumed for the level at 1.96 MeV is used as a label for the first four columns. The final column gives the parameters obtained when the level at 1.96 MeV is ignored.

Parameters	$\frac{1}{2}$	$\frac{3}{2}$	$\frac{5}{2}$	$\frac{7}{2}$	Not fitted
χ_1 (MeV F^{-2})	0.327	0.287	0.294	0.289	0.360
χ_2 (MeV F^{-2})	−0.528	−0.422	−0.285	−0.151	−0.608
ξ (MeV)	0.114	0.072	0.120	0.124	0.189
$E_{5/2}-E_{3/2}$ (MeV)	1.08	0.947	1.13	0.851	1.29
$E_{1/2}-E_{3/2}$ (MeV)	1.60	1.51	1.09	1.03	1.43
β_2/e (F^2)	23.9	26.8	37.9	49.1	29.1
e_p/e	1.62	1.68	1.64	1.73	1.62
g_s	−1.24	−1.21	−1.17	−1.14	−1.27

TABLE II. Sample spectroscopic factors $(2J+1)S$ for the reaction $Cr^{52}(d,p)Cr^{53}$. The experimental numbers have been obtained by averaging the results given in Refs. 5 and 14. No l value was assigned to the angular distribution for stripping to the level at 1.96 MeV. Consequently, no empirical spectroscopic factor has been defined for this level. The labeling of the columns of calculated results is the same as Table I.

Energy (MeV)	Spin	Exp.	$\frac{1}{2}$	$\frac{3}{2}$	$\frac{5}{2}$	$\frac{7}{2}$	Not fitted
0	$\frac{3}{2}-$	2.5	3.1	3.3	3.3	3.4	3.1
0.565	$\frac{1}{2}-$	0.86	0.80	0.81	1.22	1.35	0.95
1.008	$\frac{5}{2}-$	2.0	4.0	4.4	4.2	4.7	3.5
1.96			0.35	0.47	0.65	0	···
2.32	$\frac{3}{2}-$	1.1	0.58	0.27	0.46	0.32	0.73
2.65	$(\frac{5}{2})-$	0.75	1.14	0.8	0.86	0.30	1.68

troscopic factor $(2J+1)S$ for the $\frac{5}{2}$ level at 1.008 MeV exceeds the empirically observed quantity by a factor of about 2. However, it is generally known that the extraction of absolute spectroscopic factors from empirical data is subject to a number of uncertainties. Thus, in performing our fits to the spectroscopic factors we have considered only the ratios between spectroscopic factors derived from transitions having the same l value. Moreover, because of the possibility that the level at 2.65 MeV contains only a fraction of the associated model eigenstate, we have not included the $l=3$ spectroscopic factors in the search. Despite this omission, the ratio $1.68/3.5=0.48$ derived from the search, which ignores the level at 1.96 MeV, is in fair agreement (and a little larger than) the empirical ratio of $0.75/2.0=0.38$. The values obtained for this ratio from the four columns in which the 1.96-MeV level is fitted are 0.29, 0.18, 0.21, and 0.06 for spin assignments $\frac{1}{2}$ to $\frac{7}{2}$, respectively. Of these, only the first is in approximate agreement with the empirical ratio. Similar consideration of other ratios between spectroscopic factors shows that, for example, the ratio S(0.565 MeV)/S(ground state) is fitted fairly well throughout, whereas the ratio S(2.32 MeV)/S (ground state) is rather badly underestimated by the model calculations.

Table III shows the empirical and calculated $B(E2)$ values for the decays to the ground state of some of the lower excited states of Cr^{53}. The agreement between the model results and experiment is quite good. Perhaps this agreement should not be too surprising, since we have effectively two free parameters, β_2 and e_p, with which to fit three pieces of data. The $B(E2)$ associated with the level at 0.565 MeV is consistently underestimated, whereas that associated with the level at 1.29 MeV is consistently overestimated. An exception to this statement is the search performed with the assumption that the 1.96-MeV level has spin $\frac{7}{2}-$ – but which also placed the corresponding calculated level at 2.8 MeV. The $B(E2)$ values for the decay of the 1.96-MeV level to the ground state are also shown in Table III. The transition is strong only for the spin assignments $J=\frac{1}{2}$ or $J=\frac{5}{2}$ to the 1.96-MeV level, the latter assignment being associated with the larger $B(E2)$ value.

TABLE III. $B(E2)$ values in units of $e^2\,F^4$ for the decay to the ground state of the lower excited states of Cr^{53}. The experimental values have been modified slightly (see Ref. 15) from those given in Ref. 13. The labeling of the columns of calculated results is the same as for Table I.

Energy (MeV)	Spin	Exp.	$\frac{1}{2}$	$\frac{3}{2}$	$\frac{5}{2}$	$\frac{7}{2}$	Not fitted
0.565	$\frac{1}{2}-$	153	133.	136.	146.	153.	135.
1.008	$\frac{5}{2}-$	34.3	34.6	34.8	35.0	34.1	34.9
1.29	$\frac{7}{2}-$	59.3	69.4	68.2	62.7	58.9	67.9
1.96			20.5	0.4	62.1	0.9	···

A potential additional item of data which might be taken into account in conjunction with the $B(E2)$ data is the measured value[16] of the Cr^{53} ground-state quadrupole moment, $Q=-0.03$ b. Early in this work it was found that any attempt to force a fit to the observed quadrupole moment by varying the parameters β_2 and e_p automatically destroyed the fit to the $B(E2)$ data. In the present searches we have assumed the accuracy of the $B(E2)$ data to be greater than that of the quadrupole moment measurement and have simply ignored the latter. All parameter sets listed in Table I lead to a value $Q=-0.15$ b, five times greater than that observed. We are unable to account for this difference.

Another interesting comparison which can be made is that provided by the intensity ratios for the decay of the 1.96-MeV level. These ratios are listed in Table IV and compared with the very recent experimental work of Carola.[17] The observation of a measurable branch to the $\frac{7}{2}$ state at 1.29 MeV serves to rule out the assignment of spin $\frac{1}{2}$ to the 1.96-MeV level. Of the other possible spin assignments, only the spin $J=\frac{5}{2}$ is consistent with the observed intensity ratios.

A number of facts have emerged during the above presentation, which tend to discredit the various possible assignments for the level at 1.96 MeV. Spin $J=\frac{1}{2}$ is disfavored experimentally, and the possibilities $J=\frac{3}{2}$ and $\frac{7}{2}$ are disfavored by the corresponding larger values of Δ, by the evident lack of collectivity in the $B(E2)$ matrix element for the decay to the ground state, and by the disagreement with the empirical branching ratios. A possible assignment of $J=\frac{5}{2}$ is favored by the

TABLE IV. Intensity ratios for the decay of the level at 1.96 MeV. Empirical data were obtained from Ref. 17. The labeling of the columns of calculated results is the same as for Table I.

Final state Energy (MeV)	Spin	Exp.	$\frac{1}{2}$	$\frac{3}{2}$	$\frac{5}{2}$	$\frac{7}{2}$
0	$\frac{3}{2}-$	80	38	34	71	16
0.565	$\frac{1}{2}-$	0	61	63	1	···
1.008	$\frac{5}{2}-$	0	1	3	3	82
1.29	$\frac{7}{2}-$	20	···	0	25	2

above tests. The main objections to this assignment are as follows: The 1.96-MeV level is predicted too high – at 2.35 MeV. The ratios of the spectroscopic factors in the $Cr^{52}(d,p)Cr^{53}$ reaction leading to the more highly excited $\frac{3}{2}$ and $\frac{5}{2}$ levels to those to the lowest levels are seriously underestimated. Last, but not least, the predicted spectroscopic factor to the 1.96-MeV level itself – even if renormalized to take into account the evident disparity between the predicted and observed spectroscopic factors – is found to be at least $(2J+1)S \approx 0.3$, despite the fact that the requirement $S=0$ was weighted heavily in the search. It is hard to understand how a level with such a large spectroscopic factor could be so poorly observed in the $Cr^{52}(d,p)Cr^{53}$ work. Similar objections apply also to the possibilities $J=\frac{1}{2}$ or $J=\frac{3}{2}$ for the 1.96-MeV level. Of all the possibilities considered above, the assignment $J=\frac{5}{2}$ for the 1.96-MeV level emerges as being the least disfavored.

It is of interest to note that a level having this same spin value appears in the region of 1.96 MeV in previous theoretical calculations[8-11] of the spectrum of Cr^{53}. In common with ours, these calculations also predict sizable spectroscopic factors for the (d,p) reaction leading to this level.

Because of the difficulties encountered in attempts to fit the level at 1.96 MeV directly into the framework of the present model, we briefly considered the possibility that one of the lower model states is fragmented by admixture with a state based on one or more of the higher states of the Cr^{52} core. If the additional state is postulated to have a component $|4+, p_{3/2}, JM\rangle$ the spin will be restricted to $J=\frac{5}{2}$ or $\frac{7}{2}$. We have already seen that it is hard to form a low-lying $\frac{5}{2}$ state from our model which would not be readily observed in the $Cr^{52}(d,p)Cr^{53}$ reaction. The same is true for a low-lying $J=\frac{3}{2}$ component fragmented by admixture with an additional state of the form $|0+$ (excited), $p_{3/2}, JM\rangle$. However, the possibility remains that the lowest $J=\frac{7}{2}$ eigenstate of the model may be split into two fragments, one of which is observed at 1.29 MeV and the other at 1.96 MeV. Although this notion appears in addition to provide a mechanism for diluting the predicted $B(E2)$ for the decay to the ground state of the lower $\frac{7}{2}$ state, it is found to be discredited by the predicted decay of the 1.96-MeV level. We discover a predicted 30% branch to the $\frac{5}{2}$ level at 1.008 MeV, which is in conflict with the experimental results.

IV. HIGHER-ENERGY REGION

As mentioned earlier, the available data on the energy levels of Cr^{53} extend to energies higher than those considered in connection with the parameter searches. A comparison between our calculated spectroscopic factors $(2J+1)S$ in the region between 2- and 5-MeV excitation and those observed[4,5] in the $Cr^{52}(d,p)Cr^{53}$ reaction is given in Fig. 5. Above 3 MeV, a number of weakly populated fragments are observed, together with two strongly populated levels at 3.63 MeV ($l=1$) and 4.66 MeV ($l=3$). Levels having positive parity are also seen in this region but are not shown in the figure. Compared with the empirical results, the present calculations tend to concentrate the spectroscopic strength too low in energy. This is par-

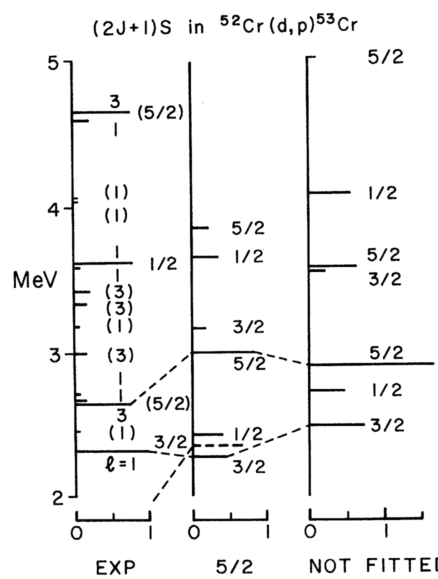

FIG. 5. Comparison of spectroscopic factors $(2J+1)S$ for the reaction $Cr^{52}(d,p)Cr^{53}$ in the energy region above 2 MeV in Cr^{53}. The theoretical results derive from the parameters listed in Table I and are labeled accordingly. Theoretical levels which are fitted to observed ones are so indicated by sloping dashed lines. In the central column, the $\frac{5}{2}$ level associated with the observed level at 1.96 MeV is indicated by means of a horizontal dashed line. Observed l values and spins are indicated in the experimental column. Theoretical spins, from which l values can readily be deduced, are also shown.

ticulaly true for calculations in which the observed level at 1.96 MeV is included in the fit.

A simple example will serve to illustrate the magnitude of this effect. The energy of the model state $|0+\text{ (ground)}, p_{3/2}, JM\rangle$ relative to the Cr^{53} ground state can be determined if all the contributions to the spectroscopic sum for the addition of a $p_{3/2}$ neutron to Cr^{52} can be located. In a recent paper,[18] this energy has been estimated to be about 1 MeV, which is considerably larger than the value 0.4 MeV obtained from the calculated spectrum when the level at 1.96 MeV is fitted with $J=\frac{5}{2}$, or the value 0.6 MeV obtained when the level at 1.96 MeV is not fitted. Similarly, the model state $|0+\text{ (ground)}, p_{1/2}, JM\rangle$ has been estimated[18] to lie at about 2.5 MeV above the Cr^{53} ground state. This estimate may be compared with the values 1.5 and 2.1 MeV, respectively, obtained from the same two theoretical calculations mentioned above. These discrepancies are symptomatic of the general failure of the present calculations to reproduce the observed spectroscopic factors. The presence of the strongly populated level at 4.66 MeV supplies further evidence that the amount of mixing may be considerably larger than indicated by our model.

A further remark concerns the nature of the level at 3.62 MeV. A recent study[19] using polarized deuterons has provided unambiguous evidence supporting a spin assignment of $J=\frac{1}{2}$ for this level rather than the value $J=\frac{3}{2}$ which had been suggested previously.[20,21] On the basis of our model, the level at 3.62 MeV would certainly more easily be accounted for as a (large) fragment of one of our higher-lying $J=\frac{1}{2}$ levels than as a fragment of a $J=\frac{3}{2}$ level. The model state $|0+\text{ (ground)}, p_{1/2}, JM\rangle$ normally lies higher in energy than the corresponding state built on the $p_{3/2}$ orbit and if the mixing is not too large, this state can be expected to dominate the higher-energy region.

V. FURTHER DISCUSSION

In this section we make a few comments on the model and its parameters. The model Hamiltonian has been decomposed into three parts in Eq. (1). The interaction between the core and the particle may be written generally[2] as

$$H_{\text{int}} = \sum_k T_c^k \cdot T_p^k , \tag{7}$$

where T_c^k and T_p^k are arbitary spherical tensors of rank k acting in the spaces of the core and the particle, respectively. In the model employed here, this interaction has been replaced by the very specific form given by Eq. (2).

Suppose we now imagine an ideal situation in which we are able to represent the observed properties – at least those of interest to us – of the core-plus-particle system by means of a set of model states constructed within our truncated basis, namely states built on the 0+ ground state and first excited 2+ state only. No matter how the observed eigenstates are formed from the allowed basis states or how they are located in energy, the interaction of Eq. (7) contains sufficient flexibility to allow all such eigenstates to be obtained as model eigenstates.

This conclusion no longer holds for the interaction of Eq. (2). If we restrict ourselves to the specific form of Eq. (2), a more or less satisfactory fit can be obtained by varying the parameters χ_1, χ_2, ξ. (We assume for the present that the spectrum of H_p has been determined from experiment.) It is important to realize, though, that when this has been done, the resulting numerical values taken on by the parameters will not necessarily bear any relationship to corresponding physical properties of the system. Thus, for example if the values of the matrix elements $\langle 0||Q_c^{(2)}||2\rangle$ and $\langle 2||Q_c^{(2)}||2\rangle$ for Cr^{52} were known from experiment, it does not necessarily follow that their ratio would be equal to the ratio of χ_1 to χ_2 as determined from the result of a searched fit to the Cr^{53} data. Any serious discrepancy between the two ratios could be interpreted as an indication that the form given in Eq. (2) was an oversimplification. On the other hand, the fact that the form assumed for the interaction is incomplete does not preclude the possibility that the optimum fit to the data when the parameters are allowed to vary may be very good.

Indeed, we have found in our present searches that there is a considerable degree of flexibility in the model even when we use the form given in Eq. (2). As a result, it would be hard to generalize the model in an orderly fashion. If more terms were added to Eq. (2) the flexibility of the model would be increased, and it would be hard to pin down the parameters without considerably more experimental data. Much of the data which might be helpful here are inaccessible to present experimental techniques. For example, there are no direct measurements of the amplitudes of the 2+ parentages of the Cr^{53} excited levels. The same comment applies even more strongly if the parametrization is increased by the inclusion in the model space of more highly excited states of the core.

Let us now turn to a consideration of the two additional parameters β_2 and e_p which were introduced in Sec. II. Figure 6 shows the variation of Δ as a function of β_2 in a series of one-parameter fits to the Cr^{53} $B(E2)$ data. The parameter e_p was

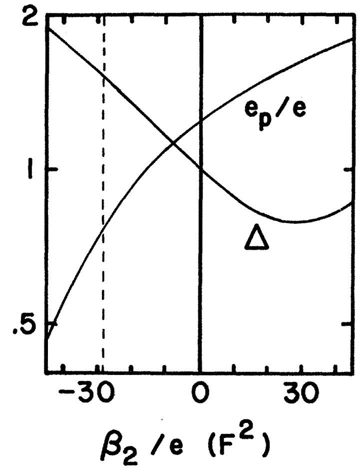

FIG. 6. Variation of the least-squares function Δ against the model parameter β_2 for the case in which the level at 1.96 MeV in Cr^{53} was not fitted. At each point on the curve, the function Δ was minimized with respect to the single parameter e_p, the resulting value of e_p being indicated in the figure. The value of β_2 consistent with Eq. (6) is indicated by means of a vertical dashed line.

allowed to vary, giving rise to the second curve shown in the figure. During this search, the other parameters were held constant at values given in Table I for the case in which the level at 1.96 MeV was not fitted. The value of β_2 at which Eq. (6) is obeyed is shown as a vertical dashed line. A similar search in which all the parameters other than β_2 were allowed to vary produced results scarcely distinguishable from those plotted in Fig. 6. It is clear from the figure that the best fit to the $B(E2)$ data occurs at a positive value of β_2, far removed from the value implied by Eq. (6). By virtue of the remarks made earlier in this section, we should perhaps be less surprised by this result than by the apparent compatibility[2] of Eq. (6) with the low-lying Cu^{63} data. Nevertheless, it is somewhat surprising that Eq. (6) does not even give the sign of the best-fit values of β_2 found in the Cr^{53} searches.

Another fit to the Cr^{53} data was attempted in which e_p was set equal to zero, and all the other parameters allowed to vary. The resulting value of Δ was 2.2 with $\beta_2/e = -52$ F^2, and the other parameters within a few precent of the values given in Table I. The corresponding point lies outside the range plotted in Fig. 6.

As a result of these searches, we see that both parameters are required if we wish to fit the Cr^{53} data. By contrast, if β_2 and e_p are varied about the solution given in Ref. 2 for the Cu^{63} data, we find that the fit to the $B(E2)$ values is considerably impaired if e_p/e is taken much different from unity (odd proton) or if β_2 is taken much different from the value implied by Eq. (6). We speculate that the different behavior of these two cases is attributable to the different structures of the core nuclei. The Cr^{52} core has four protons outside a Ca^{48} doubly magic core, whereas the Ni^{62} core has six neutrons outside a Ni^{56} doubly closed core. Any failure of the model to take proper account of the recoupling of the proton configuration in Cr^{52} is likely to be reflected in a renormalization of the $B(E2)$ parameters, whereas a similar failure in connection with the neutron configuration in Ni^{56} would have little effect on the $B(E2)$ parameters.

VI. CONCLUSION

In the present work we have attempted to fit the available experimental data concerning the low-lying levels of Cr^{53} with a restricted version of the core-plus-particle model. Although we have been successful in fitting a number of the observed properties, such as the energy spectrum, $B(E2)$ values, and branching ratios, we have encountered some trouble with other properties, such as the spectroscopic factors and the quadrupole moment of the ground state.

On the whole, the model appears less able to accomodate the Cr^{53} data than the Cu^{63} data, which also gave rise to difficulties in connection with the associated spectroscopic factors.[2] The comparison is not wholly fair, however, because in the present work we have taken more experimental information into account than was done in the parallel work on Cu^{63}. Indeed, if one performs a least-squares fit to the data on Cu^{63} considered in Ref. 2, one finds a somewhat improved fit with parameters considerably different from those given previously. This may be interpreted as an indication that more data should be employed in the Cu^{63} fit in an attempt to pin down the parameters of the model with greater precision for that case.

An attempt to predict the spin of the level at 1.96 MeV in Cu^{53} favors the assignment $J = \frac{5}{2}-$. Although the fit to the data is not entirely satisfactory here, it is possible that the residual discrepancies may be ascribed to the inherent limitations of the present model. This prediction is in agreement with earlier theoretical work.[8-11]

Note added in proof: After submitting this paper, the authors became aware of a similar calculation done by Larner[22] who has applied the model of Ref. 2 to several nuclei with $N=29$ or $Z=29$, including ^{53}Cr. Our values for the parameters χ_1, χ_2, ξ, $E_{3/2}$, and $E_{1/2}-E_{3/2}$ are consistent with the trends suggested by Larner's Table II.

APPENDIX

We give here a brief description of the least-squares function Δ which was used in the above calculations. Let the parameters of the model be symbolized by α and the set of empirical data which we desire to fit by $\{x_i\}$ where i is an index running over the individual items in the set. A theoretical number $y_i(\alpha)$, which can be calculated as a function of the model parameters was assigned to every piece of experimental data x_i. The function Δ can then be constructed from the functions $y_i(\alpha)$ as

$$\Delta(\alpha) = \sum_i \omega_i [1 - r_i(\alpha)]^2,$$

where ω_i is the weight associated with the ith piece of data and r_i was, with one exception, defined by $r_i \equiv y_i/x_i$. Thus our searches were designed to minimize the weighted sum of the squares of the *percentage* discrepancies between theory and experiment. An exception was made when it was desired to include in the search the requirement that a certain spectroscopic factor be zero. For this particular item of data, r_i was redefined as $r_i \equiv 1 - y_i^2$ where y_i was the associated spectroscopic amplitude.

The pattern of weights which was used in the searches reflects our prejudices concerning the relative importance of the various items of experimental information. Thus, the experimental energies of the excited levels were given a weight of 6, except for the level at 2.65 MeV, which was given a weight of 3. The ratios of spectroscopic factors and the $B(E2)$ values were each given a weight of 2. In the exceptional case that a zero spectroscopic factor was included (for the level at 1.96 MeV) it was associated with a weight of 10.

We note that the location of the least-squares minimum point in parameter space will vary somewhat if the relative values of the weights are altered. However, we do not expect reasonable manipulations of the weights to produce material changes from the results described above. In any case, multiplication of all the weights by a common factor has no effect on the location of the minimum point. Advantage has been taken of this fact to express all weights as integers.

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[1]A. de-Shalit, Phys. Rev. 122, 1530 (1961).

[2]V. K. Thankappan and W. W. True, Phys. Rev. 137, B793 (1965).

[3]V. K. Thankappan, Prog. Theoret. Phys. (Kyoto) 39, 985 (1968).

[4]C. A. Whitten, Jr., Phys. Rev. 156, 1228 (1967).

[5]M. N. Rao, J. Rapaport, A. Sperduto, and D. L. Smith, Nucl. Phys. A121, 1 (1968).

[6]C. F. Monahan, N. Lawley, C. W. Lewis, I. G. Main, M. F. Thomas, and P. J. Twin, Nucl. Phys. A120, 460 (1968); R. Chapman, S. Hinds, and A. E. Macgregor, Nucl. Phys. A119, 305 (1968); N. S. Freedman, F. Wagner, F. T. Porter, and H. H. Bolotin, Phys. Rev. 146, 791 (1966).

[7]B. L. Cohen, Phys. Rev. 130, 227 (1963).

[8]K. Ramavataram, Phys. Rev. 132, 2255 (1963).

[9]J. R. Maxwell and W. C. Parkinson, Phys. Rev. 135, B82 (1964).

[10]J. Vervier, Nucl. Phys. 78, 497 (1966); J. B. McGrory, Phys. Rev. 160, 915 (1967).

[11]H. Ohnuma, Nucl. Phys. 88, 273 (1966).

[12]L. S. Kisslinger and R. A. Sorensen, Kgl. Danske Videnskab. Selskab, Mat.-Fys. Medd 32, No. 9 (1960); A. S. Davydov, Nucl. Phys. 16, 597 (1960).

[13]J. R. Meriwether, I. Gabrielli, D. L. Hendrie, J. Mahoney, and B. G. Harvey, Phys. Rev. 146, 804 (1966).

[14]R. Bock, H. H. Duhn, S. Martin, R. Rudel, and R. Stock, Nucl. Phys. 72, 273 (1965).

[15]J. R. Meriwether, private communication.

[16]*Nuclear Data Sheets*, compiled by K. Way *et al.* (Printing and Publishing Office, National Academy of Sciences–National Research Council, Washington, D. C.).

[17]T. P. G. Carola, private communication.

[18]G. Delic and B. A. Robson, Nucl. Phys. A134, 470 (1969).

[19]T. J. Yule and W. Haeberli, Nucl. Phys. A117, 1 (1968).

[20]G. A. Bartholomew and M. R. Gunye, Can. J. Phys. 43, 1128 (1965).

[21]A. A. Rollefson, R. C. Bearse, J. C. Legg, G. C. Phillips, and G. Ŕoy, Nucl. Phys. 63, 561 (1965).

[22]D. Larner, following paper, Phys. Rev. C 2, 522 (1970).