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Shell-Model Structure of the p-Shell Hypernuclei*

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A systematic study of the effective two-body Λ-N interaction is made in an exact manner from the binding-energy data of the p-shell hypernuclei within the framework of the intermediate-coupling shell model. The binding energy is expressed in terms of the five independent potential integrals of the two-body Λ-N interaction, and the latter are determined by a least-squares fit to the experimental data. The energy levels and the γ transitions are calculated, and the structures of the p-shell hypernuclei are discussed. The potential integrals are then analyzed in terms of phenomenological potentials. A large contribution to the binding energy by noncentral forces, especially the tensor forces, is found. The effect of the existence of a three-body ΛNN force is also discussed.

1. INTRODUCTION

The early studies of the Λ-N interaction via the binding-energy data of the p-shell hypernuclei,[1] made by Dalitz and many others, follow strictly the analysis of the s-shell hypernuclei, where the two-body Λ-N interaction potential is assumed to be of the central form only. In the context of the intermediate-coupling shell model, the binding energy is expressed in terms of two parameters, which correspond to the expectation values of the radial part of the spin-independent and spin-dependent terms. The main assumption is that the oscillator parameters for the s and p orbitals are not appreciably changed, so that the two potential parameters can be regarded as constants through the p shell. However, by a least-squares fit to the experimental binding-energy data, it is found that these parameters cannot be consistently determined. In particular, the parameter for the spin-dependent term varies considerably through the p shell.

A more extended analysis of the p-shell hypernuclei has been made by Bodmer and Murphy,[2] in which the effect of core distortion and size variations of the core nucleus on the binding energy are considered. According to the authors, these effects should be particularly strong in the case of ${}^{9}_{\Lambda}$Be and ${}^{13}_{\Lambda}$C to be able to explain the exceptionally small binding energies of these two hypernuclei compared with those of ${}^{9}_{\Lambda}$Li and ${}^{13}_{\Lambda}$B, respectively. On the other hand, it has also been suggested by Bodmer and Murphy,[2] and more recently by Gal,[3] that the anomalously large binding-energy differences for the above two pairs may well be attributed to the existence of a strong three-body central ΛNN force of exchange character.

The importance of noncentral forces, such as the tensor force, in the two-body ΛN interaction has also been stipulated and considered. But no over-all consistent fitting of the p-shell hypernuclei using such forces has as yet been reported.

In the present work, we wish to reanalyze the p-shell hypernuclei by assuming only two-body forces in the Λ-N interaction, following an approach adopted by Cohen and Kurath[4] in the study of the two-body NN interaction in the p-shell nuclei. In this approach a specific form of the two-body Λ-N potential is not necessarily assumed. Instead, the information about the two-body Λ-N potential are essentially contained in five independent potential integrals, which correspond to the expectation values between a p-shell nucleon and the Λ. As will be shown, the binding energy is expressible in terms of these potential parameters, and a consistent determination of the latter is found in a fit to the experimental data. This indicates the important fact that the two-body forces alone are capable of explaining the binding-energy data of the p-shell hypernuclei. Furthermore, the analysis of the two-body interaction in terms of phenomenological potentials shows that the noncentral forces, including the tensor, two-body spin orbit, and Hamada-Johnston forces, play the essential role in accounting for the fluctuations in the binding energy of the p-shell hypernuclei about the average linear trend contributed from the spin-independent force. The tensor force is found to be especially important in lowering the binding-energy values of ${}^{9}_{\Lambda}$Be and ${}^{13}_{\Lambda}$C with respect to ${}^{9}_{\Lambda}$Li and ${}^{13}_{\Lambda}$B, respectively, contrary to the speculations in Refs. 2 and 3.

The computational method in our analysis is practically the same as that of the usual analysis by Dalitz and others,[1,2] but differs from it in a substantial detail. In fact, we will see that the effect of the excited core-nucleus states on the structure of the hypernucleus is very important, and therefore we take these contributions to the binding energy into account in an exact manner by diagonalizing the energy matrix elements, not just by taking some approximated expectation values.

In the next section, the general formalism for the evaluation of the energy matrix elements in terms of the five independent Λ-N potential integrals is presented. In Sec. 3 the potential integrals are determined by a least-squares fit to the experimental binding energies of some selected p-shell hypernuclei. In Sec. 4 the results from the parameter fitting are used to calculate the energy levels of the hypernuclei. Some calculations on the γ transitions are also presented. In Sec. 5 the potential integrals are analyzed in terms of some phenomenological potentials. The relative importance of the central three-body ΛNN forces is also discussed. Some remarks on our analysis of the p-shell hypernuclei are given in the final section.

2. FORMALISM

Consider a p-shell hypernucleus with mass number A in the state of isospin T and spin J_Λ. The Λ particle is assumed, as usual, to be in the s shell, to which are coupled various states of the core nucleus with isospin T and spin $J=J_\Lambda \pm \frac{1}{2}$. Denote the wave function of the hypernucleus by $|\alpha T J_\Lambda\rangle$, and the wave functions of the core nucleus (having n p-shell nucleons, $n=A-5$) by $|s^4p^n\beta TJ\rangle$, where α and β are additional quantum numbers needed to specify states with the same isospin and spin. We then write

$$|\alpha TJ_\Lambda\rangle = \sum_{\beta J} u(\alpha TJ_\Lambda, \beta TJ)|s^4p^n\beta TJ, \Lambda; TJ_\Lambda\rangle, \qquad (2.1)$$

with the coefficients u to be determined by the Λ-N potentials.

For the core-nucleus wave functions we shall use those calculated by Cohen and Kurath.[4] In the LS representation, in which the basic vectors $|s^4p^n[f]TSLJ\rangle$ are characterized by having the n p-shell nucleons coupled to an orbital symmetry $[f]$, isospin T, spin S, and orbital angular momentum L, we have

$$|s^4p^n\beta TJ\rangle = \sum_{fSL} v(\beta TJ, [f]\,TSL)|s^4p^n[f]\,TSLJ\rangle, \qquad (2.2)$$

with the coefficients v obtained from Ref. 4.

The energy levels of the hypernucleus are obtained by the diagonalization of the energy matrix elements

$$H_{TJ_\Lambda}(\beta TJ, \bar\beta T\bar J) = [E(\beta TJ) - B_s]\delta_{\beta\bar\beta}\delta_{J\bar J} + \mathcal{V}_{TJ_\Lambda}(\beta TJ, \bar\beta T\bar J), \qquad (2.3)$$

where $E(\beta TJ)$ are the excitation energies of the core nucleus, and B_s is the binding energy of ${}^5_\Lambda$He, whose experimental value is 3.08 MeV. $\mathcal{V}_{TJ_\Lambda}(\beta TJ, \bar\beta T\bar J)$ are matrix elements of the total Λ-N potential $\mathcal{V}$ between the p-shell nucleons and the Λ, i.e., it is the sum of the two-body Λ-N potentials, $V_{\Lambda N}$ between a p-shell nucleon and the Λ, viz.,

$$\mathcal{V}_{TJ_\Lambda}(\beta TJ, \bar\beta T\bar J) = \sum_{fSL}\sum_{\bar f\bar S\bar L} v(\beta TJ, [f]TSL)v(\bar\beta T\bar J, [\bar f]T\bar S\bar L)F_{TJ_\Lambda}(fSLJ, \bar f\bar S\bar L\bar J), \qquad (2.4)$$

where

$$F_{TJ_\Lambda}(fSLJ, \bar f\bar S\bar L\bar J) \equiv \langle p^n[f]TSLJ, \Lambda; TJ_\Lambda|\mathcal{V}|p^n[\bar f]T\bar S\bar L\bar J, \Lambda; TJ_\Lambda\rangle = n\sum_{f_1T_1S_1L_1}\langle p^n[f]TSL\{|p^{n-1}[f_1]T_1S_1L_1, p\rangle \times\langle p^n[\bar f]T\bar S\bar L\{|p^{n-1}[f_1]T_1S_1L_1, p\rangle K(SLJ, \bar S\bar L\bar J; S_1L_1), \qquad (2.5)$$

with

$$K(SLJ, \bar S\bar L\bar J; S_1L_1) = [(2S+1)(2L+1)(2J+1)(2\bar S+1)(2\bar L+1)(2\bar J+1)]^{1/2}\sum_{J_1j_1j_2j}(-)^{j_1-j_2}[(2j_1+l)(2j_2+1)]^{1/2} \times(2J_1+1)(2j+1)\begin{Bmatrix} S_1 & \frac{1}{2} & S\\ L_1 & 1 & L\\ J_1 & j_1 & J\end{Bmatrix}\begin{Bmatrix} S_1 & \frac{1}{2} & \bar S\\ L_1 & 1 & \bar L\\ J_1 & j_2 & \bar J\end{Bmatrix}\begin{Bmatrix} J_1 & j_1 & J\\ \frac{1}{2} & J_\Lambda & j\end{Bmatrix}\begin{Bmatrix} J_1 & j_2 & \bar J\\ \frac{1}{2} & J_\Lambda & j\end{Bmatrix} a_j(j_1, j_2), \qquad (2.6)$$

and

$$a_j(j_1, j_2) = \langle p(j_1), \Lambda; j\,|\,V_{\Lambda N}|p(j_2), \Lambda; j\rangle\,. \qquad (2.7)$$

In the above expressions, $\langle p^n[f]TSL\{|p^{n-1}[f_1]T_1S_1L_1, p\rangle$ are the coefficients of fractional parentage tabulated by Jahn and Wieringen[5] and the factors in curly brackets are the usual $9j$ and $6j$ symbols.[6] $a_j(j_1, j_2)$ are the potential integrals of the two-body Λ-N interaction with j_1 and j_2 being angular momenta of nucleons in the p shell, and j the total angular momentum of j_1 (or j_2) coupled to the Λ spin, $\frac{1}{2}$.

For computational convenience we adopt the hole-particle conjugation configurations for core nuclei with $n>6$. Therefore for $n>6$ or $A>11$, we use instead of (2.5) the following expression:

$$F_{TJ_\Lambda}(fSLJ,\bar{f}\bar{S}\bar{L}\bar{J}) = -B_p\delta_{f\bar{f}}\delta_{S\bar{S}}\delta_{L\bar{L}}\delta_{J\bar{J}} + (12-n)(-)^n \sum_{\substack{f_1T_1S_1L_1\\S_2L_2}} \langle p^{12-n}[f^+]TSL\{|p^{11-n}[f_1]T_1S_1L_1,p\rangle$$
$$\times\langle p^{12-n}[\bar{f}^+]T\bar{S}\bar{L}\{|p^{11-n}[f_1]T_1S_1L_1,p\rangle(2S_2+1)(2L_2+1)\begin{Bmatrix} S & \frac{1}{2} & S_1 \\ \bar{S} & \frac{1}{2} & S_2\end{Bmatrix}\begin{Bmatrix} L & 1 & L_1 \\ \bar{L} & 1 & L_2\end{Bmatrix} K(SLJ,\bar{S}\bar{L}\bar{J};S_2L_2), \quad (2.8)$$

where

$$-B_p = \sum_{j_1 j}(2j+1)\,a_j(j_1,j_2), \quad (2.9)$$

and $[f^+]$ denotes the conjugate symmetry of $[f]$.

The eigenvalues, $-B(\alpha TJ_\Lambda)$, of the energy matrix elements (2.3) correspond to the binding energies of the states with isospin-spin (T, J_Λ) with respect to the ground-state level of the core nucleus. In the conventional analysis,[1,2] they are expressed as the expectation values of the energy matrix elements (2.3), and hence we have

$$B(\alpha TJ_\Lambda) = -B_0(\alpha TJ_\Lambda) + B_s - \mathcal{V}(\alpha TJ_\Lambda), \quad (2.10)$$

where

$$B_0(\alpha TJ_\Lambda) = \sum_{\beta J} E(\beta TJ)u^2(\alpha TJ_\Lambda, \beta TJ), \quad (2.11)$$

and

$$\mathcal{V}(\alpha TJ_\Lambda) = \sum_{\beta\bar{\beta}J\bar{J}} \mathcal{V}_{TJ_\Lambda}(\beta TJ, \bar{\beta}T\bar{J}) \times u(\alpha TJ_\Lambda, \beta TJ)u(\alpha TJ_\Lambda, \bar{\beta}T\bar{J}). \quad (2.12)$$

For the ground-state of the hypernucleus, the first term in (2.10) is usually neglected in the approximation where the core nucleus is exclusively in the ground state. However, if some nearby excited state of the core nucleus contributes significantly to the formation of the ground state of the hypernucleus, then this term will be quite important. This happens to be the case for most p-shell hypernuclei, as we will see in the following sections.

3. LEAST-SQUARES FIT

In the preceding section we have expressed the energy matrix elements of a hypernucleus in a given state of isospin-spin(T, J_Λ) linearly in terms of the five potential integrals, $a_j(j_1, j_2)$. The basic assumption in our calculation is that the five potential integrals are constant throughout the p shell. They are to be determined by fitting the lowest eigenvalues of the energy matrix elements for the ground states of the p-shell hypernuclei to the experimental binding-energy data.

The experimental binding-energy data[7] chosen for our least-squares fits are only those from the hypernuclei with $A \geq 8$. For the $A=6$ hypernucleus, ${}^6_\Lambda$He, the Cohen-Kurath wave function of the exotic core nucleus ^{5}He is not reliable, whereas for $A=7$ hypernuclei the experimental situation concerning the anomalous distribution of the binding-energy data is still not clear. Hence, the data for $A=6$ and 7 hypernuclei will not be adopted.

The ground-state spins of all the selected hypernuclei are not as yet experimentally available. We tentatively assign the hypernuclear spin J_Λ to be $J_N - \frac{1}{2}$, where J_N is the ground-state core-nucleus spin. The assignment has been shown[8] to be valid in the case of ${}^8_\Lambda$Li. The validity of the spin assignments for other hypernuclei is to be confirmed *a posteriori* by the consistency of the results.

For the core-nucleus wave functions, as mentioned before, we shall adopt those provided by Cohen and Kurath.[4] For the $A=8$ hypernucleus, ${}^8_\Lambda$Li, we use that determined by the potential parameters denoted by (6–16) 2BME, whereas for the hypernuclei with $A>8$ we use those determined by the parameters denoted by (8–16) POT.

With the above prescription, we obtain the following results in a least-squares fit for the five potential integrals:

$$\begin{aligned} a_0(\tfrac{1}{2},\tfrac{1}{2}) &= 6.79 \pm 2.44 \text{ MeV},\\ a_1(\tfrac{1}{2},\tfrac{1}{2}) &= -2.12 \pm 0.62 \text{ MeV},\\ a_1(\tfrac{1}{2},\tfrac{3}{2}) = a_1(\tfrac{3}{2},\tfrac{1}{2}) &= 1.88 \pm 0.27 \text{ MeV},\\ a_1(\tfrac{3}{2},\tfrac{3}{2}) &= -0.57 \pm 0.21 \text{ MeV},\\ a_2(\tfrac{3}{2},\tfrac{3}{2}) &= -1.35 \pm 0.16 \text{ MeV}. \end{aligned} \quad (3.1)$$

We notice that the deviation from the average value in $a_0(\frac{1}{2},\frac{1}{2})$ is quite large. This is owing to the fact that the coefficient of $a_0(\frac{1}{2},\frac{1}{2})$ in the expression (2.3) for the energy matrix elements is extremely small so that a large variation of it does not affect appreciably the value of the binding energy. Apparently, the accurate determination of the potential integrals, and hence the precise

TABLE I. Level structure of the p-shell hypernuclei. Column 4 gives the calculated binding energies of the hypernuclear states with respect to the ground-state energies of the core nuclei. Columns 5 and 6 give the contributions to the binding energy due to the two-body Λ-N forces and the effect of the excitation of the core nuclei, respectively. In column 7 some dominant components of the core-nucleus states [denoted here by (J), with J being the spin] in the hypernuclear wave functions are given.

Hypernucleus	T	J_Λ	$B(\alpha TJ_\Lambda)$ (MeV)	$\mathcal{V}(\alpha TJ_\Lambda)$ (MeV)	$B_0(\alpha TJ_\Lambda)$ (MeV)	Wave function
${}^{8}_{\Lambda}$Li	$\frac{1}{2}$	1	6.79	−4.71	1.00	$0.70(\frac{3}{2}) - 0.69(\frac{1}{2})$
	$\frac{1}{2}$	2	6.31	−3.78	0.44	$0.98(\frac{3}{2})$
	$\frac{1}{2}$	1*	3.94	−1.63	0.77	$0.68(\frac{3}{2}) + 0.72(\frac{1}{2})$
	$\frac{1}{2}$	0	3.08	−3.31	3.31	$0.88(\frac{1}{2})$
${}^{9}_{\Lambda}$Li	1	$\frac{3}{2}$	8.20	−6.48	1.34	$-0.57(2) + 0.62(1)$
	1	$\frac{5}{2}$	8.22	−5.67	0.53	$0.93(2)$
	1	$\frac{1}{2}$	7.36	−6.02	1.74	$-0.83(1)$
${}^{9}_{\Lambda}$Be	0	$\frac{1}{2}$	6.63	−3.89	0.34	$0.99(0)$
	0	$\frac{3}{2}$	3.33	−5.26	5.01	$0.94(2)$
	0	$\frac{5}{2}$	3.07	−3.97	3.98	$1.00(2)$
${}^{10}_{\Lambda}$Be	$\frac{1}{2}$	1	9.07	−7.03	1.04	$-0.77(\frac{3}{2}) + 0.60(\frac{1}{2})$
	$\frac{1}{2}$	2	8.00	−5.32	0.40	$0.98(\frac{3}{2})$
	$\frac{1}{2}$	1*	6.67	−6.22	2.63	$0.42(\frac{3}{2}) + 0.68(\frac{1}{2}) + 0.58(\frac{1}{2}^*)$
	$\frac{1}{2}$	3	6.15	−6.28	3.21	$0.98(\frac{5}{2})$
${}^{11}_{\Lambda}$B	0	$\frac{5}{2}$	10.17	−8.34	1.25	$-0.88(3) + 0.32(2)$
	0	$\frac{1}{2}$	10.20	−9.54	2.41	$0.60(1) + 0.72(1^*)$
	0	$\frac{3}{2}$	9.93	−9.17	2.32	$0.72(1) - 0.41(1^*) + 0.42(2)$
	0	$\frac{7}{2}$	9.23	−6.64	0.49	$-0.97(3)$
	1	$\frac{1}{2}$	7.20	−6.03	1.91	$0.98(0)$
${}^{12}_{\Lambda}$B	$\frac{1}{2}$	1	11.14	−9.08	1.02	$-0.80(\frac{3}{2}) + 0.56(\frac{1}{2})$
	$\frac{1}{2}$	2	9.71	−7.23	0.60	$0.97(\frac{3}{2})$
	$\frac{1}{2}$	1*	8.15	−7.12	2.05	$0.51(\frac{3}{2}) + 0.80(\frac{1}{2})$
	$\frac{1}{2}$	3	7.33	−9.22	4.97	$0.92(\frac{5}{2}) - 0.32(\frac{7}{2})$
${}^{13}_{\Lambda}$B	1	$\frac{1}{2}$	12.51	−10.19	0.76	$0.94(1)$
	1	$\frac{3}{2}$	12.32	−10.30	1.06	$0.78(1) + 0.54(2)$
	1	$\frac{5}{2}$	11.05	−9.89	1.92	$0.97(2)$
${}^{13}_{\Lambda}$C	0	$\frac{1}{2}$	10.36	−7.88	0.60	$0.98(0)$
	0	$\frac{5}{2}$	6.72	−8.80	5.56	$0.98(2)$
	0	$\frac{3}{2}$	4.88	−8.34	6.54	$0.88(2) - 0.45(1)$

knowledge about the two-body ΛN interaction, requires more accuracy in the determination of the binding-energy data and, most of all, experimental information about the excited-state energies of the hypernuclei.

4. LEVEL STRUCTURE

With use of the potential parameters given in (3.1), we are able to find the energy levels of the p-shell hypernuclei. Some low-lying levels are listed in Table I. Column 4 gives the calculated binding energies with respect to the ground-state levels of the core nuclei. Column 5 gives the contributions from the total Λ-N forces between the Λ and all the p-shell nucleons, and column 6 gives the contributions due to the excitation of the core nuclei. The last column shows the structure of the hypernuclei. In many cases the hypernuclear levels are seen to have more than one dominant core-nuclear level coupled with the Λ. This increases the value of $B_0(\alpha, TJ_\Lambda)$, and hence diminishes the binding-energy contribution from the Λ-N interaction, $\mathcal{V}(\alpha TJ_\Lambda)$. An extreme example is found in the $\frac{3}{2}$ level of ${}^{9}_{\Lambda}$Li which happens to be practically overlapping with the first excited $\frac{5}{2}$ level. The core nucleus of the $\frac{5}{2}$ level is nearly in the pure (2) state, whereas that of the $\frac{3}{2}$ level has a large mixture from the first excited (1) state[9] at 0.91 MeV. The Λ-N interaction alone would have given an excitation energy to the $\frac{5}{2}$ level about 1 MeV above the $\frac{3}{2}$ level. However, the effect of mixing the excited state of the core nucleus for the $\frac{3}{2}$ level cancels this energy difference.

Cases of two overlapping hypernuclear levels are also found in the lowest states of ${}^{11}_{\Lambda}$B and ${}^{13}_{\Lambda}$B. For ${}^{13}_{\Lambda}$B, an interpretation similar to that in the case of ${}^{9}_{\Lambda}$Li may be applied to the two lowest $\frac{1}{2}$ and $\frac{3}{2}$ levels. But because of the large experimental in-accuracy in the determination of the binding energy, no definite conclusion should hastily be drawn from it. For the $\frac{5}{2}$ and $\frac{1}{2}$ levels of ${}^{11}_{\Lambda}$B, the situation is a little special. The ground state of the core nucleus, ^{10}B, has spin 3, above which there is a (1) state[9] at 0.91 MeV and a (1*) state[9] at 2.39 MeV. Due to the Λ-N forces the ground-state (3) level splits into the $\frac{5}{2}$ and the $\frac{7}{2}$ levels of ${}^{11}_{\Lambda}$B with a normal gap of 1–2 MeV. The $\frac{1}{2}$ state of ${}^{11}_{\Lambda}$B, however, is formed with nearly equal weight from both the excited (1) and (1*) levels of ${}^{10}_{\Lambda}$B in such a way that the Λ-N forces contribute an extremely large value (9.54 MeV) to the binding energy, exceeding the Λ-N contribution in the $\frac{5}{2}$ state by nearly 1 MeV. Hence, after the correction for the effect of the excitation energies of the core nucleus, both the $\frac{5}{2}$ and $\frac{1}{2}$ levels accidentally become close to each other.

From the preceeding discussions we have seen that the properties of the ground state of the p-shell hypernuclei depend not only on the lowest level but strongly on the adjacent excited levels of the core nuclei as well. In general, the structure of the hypernuclear levels is not simply the splitting of each core-nucleus level into a doublet. The complexity in the structure of the hypernuclei strongly suggests that the noncentral forces in the two-body Λ-N interaction play a more important role than the spin-dependent term of the central forces. This we will see in the next section.

From the calculated energy levels we have also evaluated the strengths and widths of various $M1$ and $E2$ γ transitions between some low-lying levels. The results are shown in Table II. In the calculation of the $M1$ γ transitions, the magnetic moment of the Λ is taken to have the value $\mu_\Lambda = -0.96\mu_p$ predicted by the SU_3 symmetry scheme of elementary particles.[10] Although there have been several experimental measurements[11] of the magnetic moment of the Λ, they do not seem to be in agreement with each other. The strengths and widths of the γ transitions, however, do not vary critically with the value of μ_Λ. In the calculation of the $E2$ γ transitions, we use the radiative width $\nu = 0.0480$ fm^{-1} for $A \leq 11$, and $\nu = 0.495$ fm^{-1} for $A > 11$.

From Table II it is seen that for ${}^{9}_{\Lambda}$Li, the excited $\frac{1}{2}$ state is isomeric with respect to the lowest $\frac{5}{2}$ state, whereas it has a large γ transition rate to the lowest $\frac{3}{2}$ state. Thus, we may expect that ${}^{9}_{\Lambda}$Li would be found mostly in the $\frac{3}{2}$ state. Likewise, for ${}^{11}_{\Lambda}$B the ground state has a better chance of having the spin $\frac{5}{2}$ than $\frac{1}{2}$. In the case of ${}^{13}_{\Lambda}$B it is more likely that the lowest $\frac{1}{2}$ state is the ground state.

Before concluding this section we wish to mention a few more interesting predictions resulting from our calculations of the potential integrals, which may be readily checked with a future experiment. The binding energies and the spins, which will be denoted by (B_Λ, J_Λ), for the heavier hypernuclei are found to be (13.42 MeV, 1) for ${}^{14}_{\Lambda}$N, (15.99 MeV, $\frac{3}{2}$) for ${}^{15}_{\Lambda}$N, (13.78 MeV, 1) for ${}^{16}_{\Lambda}$O, and (11.11 MeV, $\frac{1}{2}$) for ${}^{17}_{\Lambda}$O. It is noticed that the spins of these hypernuclei are $J_N + \frac{1}{2}$ instead of $|J_N - \frac{1}{2}|$. This point will be discussed in the last section.

We have also calculated the binding energies of the hypernuclei with $A \leq 7$. The results are expected to be poorer, since the Cohen-Kurath approach is not appropriate to be used here. For ${}^{6}_{\Lambda}$He, we have $J_\Lambda = 1$ and $B_\Lambda = 5.66$ MeV as compared with the experimental value[7] 4.09 ± 0.27 MeV. For ${}^{7}_{\Lambda}$Li, we have $J_\Lambda = \frac{3}{2}$ and $B_\Lambda = 6.43$ MeV as compared with the experimental value[7] 5.46 ± 0.12 MeV. The

TABLE II. Values of $M1$ and $E2$ transition strengths and widths. The strength refers to the quantity $(2J_\Lambda + 1) \times \Lambda(TJ_\Lambda \to \overline{T}\overline{J}_\Lambda)$. The width for an $M1$ transition is given by $\Gamma(TJ_\Lambda \to \overline{T}\overline{J}_\Lambda) = 0.419 \times 10^3 \times E^3 \Lambda(TJ_\Lambda \to \overline{T}\overline{J}_\Lambda)$ sec^{-1}. The width for an $E2$ transition is given by $\Gamma(TJ_\Lambda \to \overline{T}\overline{J}_\Lambda) = 1.22 \times 10^8 \times E^5 \Lambda(TJ_\Lambda \to \overline{T}\overline{J}_\Lambda)$ sec^{-1}. The values $a(x)$ denote $a \times 10^x$.

Hyper-nucleus	T	J_Λ	$\overline{T}$	$\overline{J}_\Lambda$	E (MeV)	Strength	Width (sec)$^{-1}$
${}^{8}_{\Lambda}$Li	$\frac{1}{2}$	2	$\frac{1}{2}$	1	0.48	271	2.55(13)
	$\frac{1}{2}$	1*	$\frac{1}{2}$	1	2.85	111	3.60(15)
	$\frac{1}{2}$	0	$\frac{1}{2}$	1	3.71	17.3	3.48(15)
	$\frac{1}{2}$	1*	$\frac{1}{2}$	2	2.37	31.6	5.84(14)
${}^{9}_{\Lambda}$Li	1	$\frac{1}{2}$	1	$\frac{3}{2}$	0.84	1.37	1.74(12)
	1	$\frac{1}{2}$	1	$\frac{5}{2}$	0.86	0.015	4.37(5)
^{9}Be	0	$\frac{3}{2}$	0	$\frac{1}{2}$	3.30	0.093	3.49(12)
	0	$\frac{5}{2}$	0	$\frac{1}{2}$	3.56	21 100	2.46(14)
${}^{10}_{\Lambda}$Be	$\frac{1}{2}$	2	$\frac{1}{2}$	1	1.07	76.4	7.87(13)
	$\frac{1}{2}$	1*	$\frac{1}{2}$	1	2.40	4.27	8.26(13)
	$\frac{1}{2}$	3	$\frac{1}{2}$	1	2.92	2700	9.84(12)
	$\frac{1}{2}$	1*	$\frac{1}{2}$	2	1.33	83.6	2.74(14)
	$\frac{1}{2}$	3	$\frac{1}{2}$	2	1.85	8.92	3.39(13)
${}^{11}_{\Lambda}$B	0	$\frac{3}{2}$	0	$\frac{5}{2}$	0.24	3.68	4.76(10)
	0	$\frac{3}{2}$	0	$\frac{1}{2}$	0.27	0.072	1.43(9)
	0	$\frac{7}{2}$	0	$\frac{5}{2}$	0.94	7.96	3.37(12)
	0	$\frac{7}{2}$	0	$\frac{3}{2}$	0.70	4500	1.15(10)
	1	$\frac{1}{2}$	0	$\frac{5}{2}$	2.97	467	6.53(12)
	1	$\frac{1}{2}$	0	$\frac{1}{2}$	3.00	27.0	1.53(15)
	1	$\frac{1}{2}$	0	$\frac{3}{2}$	2.73	166	7.13(15)
${}^{12}_{\Lambda}$B	$\frac{1}{2}$	2	$\frac{1}{2}$	1	1.43	29.5	7.22(13)
	$\frac{1}{2}$	1*	$\frac{1}{2}$	1	2.99	13.2	4.95(14)
	$\frac{1}{2}$	3	$\frac{1}{2}$	1	3.81	32.6	4.57(11)
	$\frac{1}{2}$	1*	$\frac{1}{2}$	2	1.56	92.5	4.97(14)
	$\frac{1}{2}$	3	$\frac{1}{2}$	2	2.38	94.1	7.64(14)
${}^{13}_{\Lambda}$B	1	$\frac{5}{2}$	1	$\frac{1}{2}$	1.46	2.27	3.06(8)
	1	$\frac{5}{2}$	1	$\frac{3}{2}$	1.27	6.17	8.82(12)
${}^{13}_{\Lambda}$C	0	$\frac{5}{2}$	0	$\frac{1}{2}$	3.64	115	1.49(12)
	0	$\frac{3}{2}$	0	$\frac{1}{2}$	5.48	0.57	9.89(13)
	0	$\frac{3}{2}$	0	$\frac{5}{2}$	1.84	7.39	4.87(13)

latter value, however, has not been well established because of the anomalously large distribution in the experimental data. A more careful analysis of the $A=7$ hypernuclei has been made by Bodmer and Murphy.[12]

5. PHENOMENOLOGICAL POTENTIALS

The potential integrals obtained in (3.1) may be analyzed in terms of phenomenological potentials. We will assume the same form for the two-body Λ-N potential as that adopted in the analysis of the nuclear forces, i.e.,

$$V_{\Lambda N}(\vec{r}) = V_1(r) + V_2(r)\tfrac{1}{4}\vec{\sigma}_\Lambda \cdot \vec{\sigma}_N + V_3(r)S_{\Lambda N} + V_4(r)\vec{l}\cdot\vec{S} + V_5(r)H_{\Lambda N}\,. \quad (5.1)$$

The first two terms are the central potentials that have been customarily used in the analysis of the Λ-N forces. The third term is the usual tensor potential with

$$S_{\Lambda N} = 3(\vec{\sigma}_\Lambda\cdot\vec{r})(\vec{\sigma}_N\cdot\vec{r})/r^2 - (\vec{\sigma}_\Lambda\cdot\vec{\sigma}_N)\,. \quad (5.2)$$

The fourth term represents the two-body spin-orbit interaction, with $\vec{l}$ being the relative angular momentum between the Λ and the nucleon, and $\vec{S}$ their combined spins. The fifth term is the Hamada-Johnston quadratic spin-orbit potential with

$$H_{\Lambda N} = (\vec{\sigma}_\Lambda\cdot\vec{\sigma}_N)l^2 - \tfrac{1}{2}[(\vec{\sigma}_\Lambda\cdot\vec{l})(\vec{\sigma}_N\cdot\vec{l}) + (\vec{\sigma}_N\cdot\vec{l})(\vec{\sigma}_\Lambda\cdot\vec{l})]\,. \quad (5.3)$$

$V_i(r)$ ($i=1, 2, \ldots, 5$) are the radial parts of the potentials.

We now note that the assumed potentials in (5.1) have the property that the total ordinary spin is conserved because of a symmetry in the exchange of the spin operators of the Λ and the nucleon. This gives rise to the following relation between the potential integrals for $j=1$:

$$\sqrt{2}\,[a_1(\tfrac{3}{2},\tfrac{3}{2}) - a_1(\tfrac{1}{2},\tfrac{1}{2})]/a_1(\tfrac{1}{2},\tfrac{3}{2}) = 1\,. \quad (5.4)$$

It is, of course, reasonable to suspect that the effect of the Λ spin in the Λ-N interaction may not be the same as that of the nucleon spin, because of the large differences in the property of the Λ and the nucleons. Hence, one may have to write, for example, the two-body spin-orbit potential as $V_4(r)\vec{l}\cdot\vec{\sigma}_\Lambda + V_4'(r)\vec{l}\cdot\vec{\sigma}_N$ with different radial parts. Using our least-squares-fitted values of the potential integrals in (3.1) we find, however, that the right-hand side of (5.4) has a value 1.16 ± 0.30. Therefore, within the present experimental accuracy, no definite conclusion can be drawn about the possible existence of the spin-nonconserving potentials. In the following discussions we shall therefore assume the relation (5.4) to hold, and discuss the contributions of the various potentials in (5.1) to the binding energies of the p-shell hypernuclei.

Through the Brody-Moshinski transformations[13] the potential integrals (now reduced to four independent ones) can be expressed linearly in terms of the four parameters $\overline{V}_i$ as follows:

$$\begin{aligned}
a_0(\tfrac{1}{2},\tfrac{1}{2}) &= \overline{V}_1 + \tfrac{1}{4}\overline{V}_2 - 2\overline{V}_3 - \overline{V}_4\,,\\
a_1(\tfrac{1}{2},\tfrac{1}{2}) &= \overline{V}_1 - \tfrac{1}{12}\overline{V}_2 + \tfrac{2}{3}\overline{V}_3 - \tfrac{1}{3}\overline{V}_4\,,\\
a_1(\tfrac{3}{2},\tfrac{3}{2}) &= \overline{V}_1 - \tfrac{5}{12}\overline{V}_2 + \tfrac{1}{3}\overline{V}_3 - \tfrac{1}{6}\overline{V}_4\,,\\
a_2(\tfrac{3}{2},\tfrac{3}{2}) &= \overline{V}_1 + \tfrac{1}{4}\overline{V}_2 - \tfrac{1}{5}\overline{V}_3 + \tfrac{1}{2}\overline{V}_4\,,
\end{aligned} \quad (5.5)$$

where $\overline{V}_i$ are related to the reduced integrals,

$$\overline{V}_i^{(l)} \equiv (0l\,|\,V_i(r)\,|\,0l)\,, \quad (5.6)$$

of the radial parts of the potentials in the relative frame (here the angular momentum is l, and the principal quantum number is zero), by

$$\begin{aligned}
\overline{V}_1 &= \tfrac{1}{2}(V_1^{(0)} + V_1^{(1)})\,,\\
\overline{V}_2 &= \tfrac{1}{2}(V_2^{(0)} + V_2^{(1)}) + \tfrac{8}{3}V_5^{(1)}\,,\\
\overline{V}_3 &= V_3^{(1)} + \tfrac{5}{6}V_5^{(1)}\,,\\
\overline{V}_4 &= V_4^{(1)}\,.
\end{aligned} \quad (5.7)$$

For $V_5^{(1)}$ equal to zero, $\overline{V}_1$, $\overline{V}_2$, $\overline{V}_3$, and $\overline{V}_4$ correspond to the usual expectation values of the radial parts of the central spin-independent, the central spin-dependent, the tensor, and the two-body spin-orbit potentials, respectively. Hence, if the Hamada-Johnston potential is important in the Λ-N interaction, we will not be able to obtain the information about the strengths of the central spin-dependent and the tensor forces from a binding-energy analysis of the p-shell hypernuclei.

The parameters $\overline{V}_i$ can be determined by (3.1) through (5.5). More accurately, they are determined by the least-squares method given in Sec. 3 in the same way as the $a_j(j_1, j_2)$, and have the following values:

$$\begin{aligned}
\overline{V}_1 &= -0.68\pm0.03\text{ MeV}\,,\\
\overline{V}_2 &= -2.00\pm0.37\text{ MeV}\,,\\
\overline{V}_3 &= -2.87\pm0.11\text{ MeV}\,,\\
\overline{V}_4 &= -1.50\pm0.22\text{ MeV}\,.
\end{aligned} \quad (5.8)$$

The estimated value of $\overline{V}_1$ in the ordinary analysis,[1,2] where only central forces are considered, is, in absolute value, 0.3 MeV larger than that evaluated here. This, according to our analysis, is obviously due to neglecting the contributions from strong tensor and two-body spin-orbit potentials in the ordinary analysis. The fact that $\overline{V}_2$ has a large negative value indicates also the existence of a rather strong Hamada-Johnston potential. Taking the contribution to $\overline{V}_2$ from the central spin-dependent potential (denoted conventionally by Δ) to be +0.2 MeV as an estimate from the s-shell analysis, we are led to the estimate of about −0.8 MeV for the strength $V_5^{(1)}$ of the Hamada-Johnston potential, and about −2.2 MeV for the strength $V_3^{(1)}$ of the tensor potential.

To see the contribution of each potential to the binding energy, we use the formula (2.10) and express $\mathcal{V}(\alpha TJ)$ in (2.12) in terms of the parameters $\overline{V}_i$. Considering only the ground-state level, the binding energy is written as

$$B_\Lambda = -B_0 + B_s + \sum_{i=1}^{4} B_i\,, \tag{5.9}$$

where

$$B_i = \alpha_i \overline{V}_i, \quad i = 1, 2, 3, 4\,, \tag{5.10}$$

and the α_i correspond to the expectation values of the spin-dependent parts of the potentials. Notice that $\alpha_1 = n$ is the number of the p-shell nucleons. B_1, B_2, B_3, and B_4 represent, respectively, the contributions from the spin-independent central force, the spin-dependent central force with the Hamada-Johnston force, the tensor force with the Hamada-Johnston force, and the two-body spin-orbit force. For simplicity, they will be referred to hereafter as the central, spin-spin, tensor, and spin-orbit terms, respectively. The B_0 term, as mentioned in a preceding section, accounts for the effect of the excitation of the core nucleus.

The calculated values of the contributing terms of the binding energy B_Λ in (5.9) are listed in Table III. For ${}^9_\Lambda$Li, ${}^{11}_\Lambda$B, and ${}^{13}_\Lambda$B the values for both of the overlapping doublets are provided. The central term B_1, which is proportional to the number of the p-shell nucleons, provides the general linearly increasing trend in the binding energies of the p-shell hypernuclei. On the other hand, the spin-dependent contributions from B_2, B_3, and B_4, which are to account for the fluctuation of the p-shell binding energies about the general trend, remain relatively constant throughout the p shell, and have a total contribution of around 4 MeV. Among them, however, the tensor term B_3 is found to contribute a dominant part, in most cases, as much as 3–4 MeV.

The large contribution of the tensor term to the binding energy further serves to explain the large experimental binding-energy difference within pairs of hypernuclei: (${}^9_\Lambda$Li, ${}^9_\Lambda$Be) and (${}^{13}_\Lambda$B, ${}^{13}_\Lambda$C), viz.,

$$\begin{aligned} &B_\Lambda({}^9_\Lambda\text{Li}) - B_\Lambda({}^9_\Lambda\text{Be}) \simeq 1.6 \text{ MeV}\,,\\ &B_\Lambda({}^{13}_\Lambda\text{B}) - B_\Lambda({}^{13}_\Lambda\text{C}) \simeq 2.0 \text{ MeV}\,. \end{aligned} \tag{5.11}$$

From Table III, it is immediately seen that these large binding-energy differences arise mostly from the fact that the contributions of the tensor term in ${}^9_\Lambda$Be and ${}^{13}_\Lambda$C are strongly suppressed owing to the fact that their core nuclei are spinless.

It has been pointed out by Bodmer and Murphy,[2] and lately by Gal[3] that the large binding-energy differences in (5.11) might be attributed to the existence of a strong central three-body ΛNN force with exchange character. We have also

TABLE III. Contributing terms to the binding energy. The binding energies in column 5 are calculated from the parameters $\overline{V}_i$ in (5.8). The values B_i with $i = 0, 1, 2, 3, 4$ denote, respectively, the contributions to the binding energy due to the effect of core excitation, the central spin-independent force, the central spin-dependent force mixed with the Hamada-Johnston force, the tensor force mixed with the Hamada-Johnston force, and the two-body spin-orbit force.

Hypernucleus	T	J_Λ	B_Λ(exp) (MeV)	B_Λ(cal) (MeV)	B_0 (MeV)	B_1 (MeV)	B_2 (MeV)	B_3 (MeV)	B_4 (MeV)
${}^8_\Lambda$Li	$\frac{1}{2}$	1	6.80 ±0.05	6.77	0.95	2.04	0.44	2.55	0.40
${}^9_\Lambda$Li	1	$\frac{3}{2}$	8.25 ±0.13	8.18	1.31	2.72	0.56	3.09	0.03
		$\frac{5}{2}$		8.17	0.48	2.72	0.97	0.87	1.01
${}^9_\Lambda$Be	0	$\frac{1}{2}$	6.63 ±0.04	6.61	0.31	2.72	0.05	0.48	0.60
${}^{10}_\Lambda$Be	$\frac{1}{2}$	1	9.10 ±0.64	9.04	0.99	3.40	0.42	2.95	0.18
${}^{11}_\Lambda$B	0	$\frac{5}{2}$	10.18 ±0.11	10.15	1.23	4.08	0.17	3.77	0.28
		$\frac{1}{2}$		10.09	2.42	4.08	0.51	4.69	0.15
${}^{12}_\Lambda$B	$\frac{1}{2}$	1	11.10 ±0.11	11.11	0.96	4.76	0.45	3.25	0.54
${}^{13}_\Lambda$B	1	$\frac{1}{2}$	12.5 ±0.7	12.36	0.77	5.44	−0.01	3.99	0.64
		$\frac{3}{2}$		12.17	1.06	5.44	0.33	3.17	1.21
${}^{13}_\Lambda$C	0	$\frac{1}{2}$	10.51 ±0.51	10.37	0.58	5.44	0.20	0.69	1.54

made a least-squares calculation by including the three-body force. The results are as follows:

The best fit to the experimental binding-energy data is obtained for the Slater integral F^0 (F^2 is neglected[2,3]) of the three-body force in a range of values between 0 and −0.1 MeV. The values of $\overline{V}_i$ for $i=2,3,4$ are found to decrease nearly linearly with F^0, while that of V_1 increases nearly linearly with F^0. The contribution of the three-body ΛNN force to the binding-energy difference in (5.11) is estimated to be about $-7F^0$ MeV for both pairs, hence at most, 0.7 MeV. Hence, the explanation of (5.11) still requires strong noncentral forces in the Λ-N interaction. The determination of the binding energies of heavier p-shell hypernuclei, such as ${}^{14}_{\Lambda}N$ or ${}^{15}_{\Lambda}N$, may shed further light on an understanding of the three-body ΛNN forces, because their contribution to the binding-energy increases almost quadratically with the number of the p-shell nucleons. If the three-body forces should exist, we may expect larger binding-energy values for the heavier p-shell hypernuclei than those estimated in the preceding section.

REMARKS AND CONCLUSIONS

In the preceding sections we have used the Cohen-Kurath approach to analyze the p-shell hypernuclei with a two-body Λ-N interaction, and have succeeded in obtaining a quite good binding-energy fit. The analysis was based on the assumption that the potential integrals of the Λ-N interaction are constants throughout the p shell. That is to say, the structure of the core nucleus is assumed not to be changed appreciably by the Λ-N interaction. This assumption is expected to hold for heavier core nuclei ($n>2$), as in the Cohen-Kurath analysis. Therefore, our calculations for hypernuclei with $A<8$ may not be very reliable. Special treatment for these hypernuclei therefore may require, for example, taking into account the size effect of the core nuclei.[12] The distortion of the unstable core ${}^8Be$ of the ${}^{9}_{\Lambda}Be$ has also been subjected to some speculation.[2] However, our results of a consistent fit do not indicate the need for such consideration.

The assumed rule of assigning the hypernuclear ground-state spin J_Λ as $|J_N-\frac{1}{2}|$, with J_N being the ground-state spin of the core nucleus, happens to be consistent with the results of the binding-energy fit. However, the existence of strong noncentral forces found in the Λ-N interaction renders the rule obsolete. In fact, the ground state of a hypernucleus depends very much on the spins of the excited levels of the core nucleus. This is illustrated in the cases of ${}^{9}_{\Lambda}Li$ and ${}^{13}_{\Lambda}B$, and applies also to the excited states of the hypernucleus, e.g., ${}^{11}_{\Lambda}B$. The complete manifestation of this effect is shown in the cases of ${}^{7}_{\Lambda}Li$, ${}^{14}_{\Lambda}N$, ${}^{15}_{\Lambda}N$, and ${}^{16}_{\Lambda}O$, where the ground-state hypernuclear spins are given by $J_\Lambda = J_N+\frac{1}{2}$.

The serious difficulty in our evaluation of the parameters of the two-body Λ-N interaction arises from the fact that the experimental information for the p-shell hypernuclei are meager. There are only eight suitable pieces of information available on the binding energy. In fact, for ${}^{10}_{\Lambda}Be$, ${}^{13}_{\Lambda}B$, and ${}^{13}_{\Lambda}C$ only one event has so far been identified in each case, with the binding energy not very accurately determined. Since the potential integrals are found to depend very sensitively on small variations of the binding-energy data, more accurate and more extended experimental data are certainly important in order to gain precise information about the two-body Λ-N forces. The presently available binding-energy data, however, can be interpreted by introducing in the two-body Λ-N forces, besides the central ones, strong noncentral forces consisting of the tensor, the two-body spin-orbit, and the Hamada-Johnston forces. The tensor force is found to be especially important. In particular, the especially low binding energies of ${}^{9}_{\Lambda}B$ and ${}^{13}_{\Lambda}C$ are seen to be due to the suppression of the contributions from the tensor force, because of the fact that the spins of the cores are zero.

After the present work was completed, it was brought to the authors' notice that a calculation similar to ours has also been attempted by Gal.[14] His approach, however, differs from ours in neglecting all the off-diagonal elements (with $J\neq\overline{J}$) of $H_{TJ_\Lambda}(\beta TJ, \overline{\beta}T\overline{J})$ in (2.3). These matrix elements are certainly important for obtaining the right spin assignment for ${}^{8}_{\Lambda}Li$. Nevertheless, it is interesting to mention that his calculated binding-energy values for the heavier p-shell hypernuclei from $A=14$ to 17, are very much compatible with our results, disregarding the completely opposite spin assignments for these hypernuclei in the two approaches.

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Single-Particle Resonances in the Renormalized Random-Phase-Approximation Treatment of Nucleon-Nucleus Scattering*

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The method of Garside and MacDonald is used to treat the single-particle resonances in the renormalized random-phase-approximation treatment of scattering. The resonances are shifted by an additional potential to single-particle bound states, which can be handled in the usual manner. The coupling to the continuum is done analytically using separable matrix elements.

I. INTRODUCTION

In previous work[1] the general theory of nucleon-nucleus scattering[2] was applied to a model in which the so-called bound-state problem or nuclear-structure problem was treated with the full renormalized effective particle-hole force, and the influence of the continuum was taken approximately into account by a schematic force. In the bound-state problem, all single-particle states belonging to the continuum have been neglected. In our model, even the amplitudes of the bound-state problem are expected to be a good approximation for the corresponding amplitudes with coupling to the continuum. But it is well known – for instance from random-phase-approximation (RPA) calculations – that in many cases one has to include into the bound-state problem certain single-particle resonances in order to obtain good agreement with the experimental situation.[3,4] An example is, for instance, the $1d_{3/2}$ resonance in ^{16}O necessary for the calculation of the excited negative-parity states of ^{16}O. Therefore, in order to complete the model[1] for calculating nucleon scattering by one-hole nuclei, one has to overcome this difficulty. Several methods for accomplishing this are known.[4] We are going to use the method of Garside and MacDonald,[5] since we can obtain with it an explicit solution for the nucleon-nucleus scattering amplitude. In this approach the single-particle potential is split into two parts

$$\tilde{v}_{\mu\nu}=\tilde{v}^{B}_{\mu\nu}+\Delta_{\mu\nu}\,, \tag{I.1}$$