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PHYSICAL REVIEW C VOLUME 2, NUMBER 2 AUGUST 1970

Nuclear-Scattering Problem in the Generalized Hartree-Fock Approximation*

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(Received 18 February 1970)

The generalized Hartree-Fock approximation of Kerman and Klein is extended to include continuum states of a collective nature. The equations for scattering in the random-phase approximation are rederived.

I. INTRODUCTION

Theories of nuclear scattering which proceed from an atomistic viewpoint have recently been developed.[1,2] These owe their derivations to the techniques of nuclear many-body perturbation theory. The equations of the random-phase approximation (RPA) or quasiboson approximation also have been employed in the context of the nuclear-scattering problem.[3–5] Furthermore, calculations in the continuum using the RPA have been performed, and the analytic properties of the predicted S matrix have been examined.[6,7] The essential feature of these methods is that a unified picture is obtained which predicts both the bound and resonant states, and allows the calculation of an S matrix.

Typically, the problem under investigation is of the generic type $A(n,n)A^*$ where A is a nucleus one hole away from a doubly-closed-shell nucleus B, and A^* is the same nucleus in an excited state. In the RPA an S matrix for the processes $A(n,n)A^*$ is obtained and simultaneously the bound and resonant states of the nucleus B are obtained. Of course, both the structure and scattering information are correct to the degree that the RPA is correct in nuclear physics.

The purpose of this paper is to show that the generalized Hartree-Fock approximation (GHFA)) of Kerman and Klein,[8] which finds extensive application in nuclear structure, can be extended to the continuum problem with suitable modifications necessary to accomodate the scattering states. We shall rederive the equations of the RPA and further illustrate in the GHFA that one can calculate not only the $A(n,n)A^*$ scattering amplitude but also the S matrix $B(n,n)B^*$ in an approximation consistent with the RPA. In Sec. II we shall review the equations of Kerman and Klein of interest to us, and extract from them the Hartree-Fock equations. In Sec. III we shall rederive the RPA equations for nuclear scattering using the GHFA, and demonstrate the S matrix. Furthermore, in Sec. VI we shall calculate an approximate S matrix for scattering in the $B(n,n)B^*$ problem.

II. GENERALIZED HARTREE-FOCK APPROXIMATION

We start our considerations by displaying the Hamiltonian of our system.

$$H = t(x,x')\psi^\dagger(x)\psi(x') + \tfrac{1}{4}V(x,y;x',y')\psi^\dagger(x)\psi^\dagger(y)\psi(y')\psi(x'), \tag{1}$$

where

$$t(x,x') = t^*(x',x), \tag{2}$$

and

$$V(x,y;x',y') = V^*(x',y';x,y) = -V(y,x;x',y') = -V(x,y;y',x'). \tag{3}$$

Our interest resides in the following sets of states:
(i) The ground state of the N-particle system B, $|0\rangle$, and $|\omega\rangle$, the continuum states of the N-particle system at excitation energy ω;
(ii) the bound states of the $N-1$-nucleon system, $|a\rangle$, and $|a,\omega\rangle$, states of the $N-1$ system which correspond to the continuum states of the N system;
(iii) the particlelike states of the $N+1$-nucleon system, $|A\rangle$, and $|A,\omega\rangle$, states of the $N+1$ system corresponding to the continuum excited states of the N system.

The corresponding energies, when measured relative to the ground state of the N-nucleon system, are to $O(1/N)$:

(i) $$\omega = W_N(\omega) - W_N(0), \tag{4}$$

(ii) $$\epsilon_a = W_N(0) - W_{N-1}(a),$$

$$\epsilon_a - \omega = W_N(0) - W_{N-1}(a,\omega), \tag{5}$$

(iii) $$\epsilon_A = W_{N+1}(A) - W_N(0),$$

$$\epsilon_A + \omega = W_{N+1}(A,\omega) - W_N(0). \tag{6}$$

It is envisioned that among the energies ϵ_A there are two sets, $\epsilon_A < 0$ and $\epsilon_A = k_A^2/2m > 0$, corresponding to bound and scattering solutions, whereas the energies ϵ_a are such that $\epsilon_A < 0$. These energies are to emerge from the Hartree-Fock problem which comes about as follows[8]: One defines single-particle amplitudes

$$\psi_a(x) \equiv \langle a|\psi(x)|0\rangle, \tag{7}$$

and

$$\varphi_A(x) \equiv \langle 0|\psi(x)|A\rangle. \tag{8}$$

Then by studying the appropriate matrix elements of the equation of motion

$$[\psi(x),H] = t(x,x')\psi(x') + \tfrac{1}{2}V(x,y;x',y')\psi^\dagger(y)\psi(y')\psi(x'), \tag{9}$$

and by using (4) along with the GHFA one finds that

$$\epsilon_a\psi_a(x) = h(x,x')\psi_a(x'), \tag{10a}$$

and

$$\epsilon_A \phi_A(x) = h(x, x')\phi_A(x') . \tag{10b}$$

Here,

$$h(x, x') = t(x, x') + V(x, y; x', y')\sum_{a=1}^{N} \psi_a^*(y)\psi_a(y') . \tag{11}$$

In going from the matrix elements of (9) to (10), collective amplitudes such as $\langle a|\psi(x)|\omega\rangle$ are ignored and only ground-state quantities such as $\langle a|\psi(x)|0\rangle$ and $\langle 0|\psi^\dagger(y)\psi(y')|0\rangle$ are retained. Since h is Hermitian, one can take the set to be a complete set, i.e.,

$$\sum_a \psi_a(x)\psi_a^*(x') + \int_A \phi_A(x)\phi_A^*(x') \cong \langle 0|[\psi(x), \psi^\dagger(x')]|0\rangle = \delta(x - x') . \tag{12}$$

The symbol $\int_A$ is used to denote a sum on bound levels and an integral over continuum levels. Since we wish to keep the discussion as lucid as possible, we have deferred to the Appendix questions concerning angular momentum and continuum-state normalizations.

III. RANDOM-PHASE APPROXIMATION

Let us turn our attention to the following amplitudes:

$$\psi_a(x, \omega) \equiv \langle a|\psi(x)|\omega\rangle , \tag{13}$$

$$\bar{\psi}_a(x, \omega) \equiv \langle a, \omega|\psi(x)|0\rangle , \tag{14}$$

$$\phi_A(x, \omega) \equiv \langle \omega|\psi(x)|A\rangle , \tag{15}$$

$$\bar{\phi}_A(x, \omega) \equiv \langle 0|\psi(x)|A, \omega\rangle . \tag{16}$$

In the course of the following discussion the approximation for $\{\psi_a(x), \varphi_A(x)\}$ defined by (10) will be used; furthermore, it is assumed that the amplitudes

$$\langle a, \omega|\psi(x)|\omega'\rangle \cong \delta(\omega - \omega')\psi_a(x) , \tag{17}$$

and

$$\langle \omega'|\psi(x)|A, \omega\rangle \cong \delta(\omega - \omega')\phi_A(x) . \tag{18}$$

These latter approximations are the extension to the continuum of the RPA assumption that the static ground- and excited-state densities are the same and is perhaps the most suspect approximation.

The linearized equations of motion obeyed by the off-diagonal amplitudes (13) and (14) can be derived as follows. By taking appropriate matrix elements of the equation of motion (9), we find

$$(\omega + \epsilon_a)\psi_a(x, \omega) = t(x, x')\psi_a(x', \omega) + \tfrac{1}{2} V(x, y; x', y')\langle a|\psi^\dagger(y)\psi(y')\psi(x')|\omega\rangle , \tag{19}$$

and

$$(-\omega + \epsilon_a)\bar{\psi}_a(x, \omega) = t(x, x')\bar{\psi}_a(x, \omega) + \tfrac{1}{2} V(x, y; x', y')\langle a, \omega|\psi^\dagger(y)\psi(y')\psi(x')|0\rangle . \tag{20}$$

The GHFA[8] approximates the matrix elements of three operators appearing in (19), for example, by

$$\begin{aligned}\langle a|\psi^\dagger(y)\psi(y')\psi(x')|\omega\rangle &\cong 2\int_{E=\{0,\omega\}} \psi_a(x'E)\langle E|\psi^\dagger(y)\psi(y')|\omega\rangle \\ &= 2[\psi_a(x')\langle 0|\psi^\dagger(y)\psi(y')|\omega\rangle + \int \psi_a(x', \omega')\langle \omega'|\psi^\dagger(y)\psi(y')|\omega\rangle] ,\end{aligned} \tag{21}$$

where

$$\langle \omega'|\psi^\dagger(y)\psi(y')|\omega\rangle \cong \delta(\omega - \omega')\sum_a \psi_a(y')\psi_a^*(y) , \tag{22}$$

and

$$\langle 0|\psi^\dagger(y)\psi(y')|\omega\rangle \cong \sum_a [\psi_a(y', \omega)\psi_a^*(y) + \psi_a(y')\bar{\psi}_a^*(y, \omega)] . \tag{23}$$

Equations (22) and (23) are accurate to first order in a density fluctuation. Both (22) and (23) require the use of (17) and (18). With these crucial approximations we may rewrite (19) and (20) in a linearized form, so that

$$[(\omega+\epsilon_a)\delta(x-x')-h(x,x')]\psi_a(x',\omega)=\sum_a V_{a'a}(x,x')\psi_{a'}(x',\omega)+\sum_{a'}U_{a'a}(x,x')\bar{\psi}_{a'}{}^*(x',\omega)\,, \tag{24}$$

and

$$[(-\omega+\epsilon_a)\delta(x-x')-h^t(x,x')]\bar{\psi}_a{}^*(x',\omega)=\sum_{a'}U_{a'a}{}^*(x,x')\bar{\psi}_{a'}{}^*(x',\omega)+\sum_{a'}V_{a'a}{}^*(x,x')\psi_a(x',\omega)\,, \tag{25}$$

where

$$h^t(x,x')=h(x',x)\,, \tag{26}$$

$$V_{a'a}(x,x')\equiv V(x,a';a,x')\equiv V(x,y;y',x')\psi_{a'}{}^*(y)\psi_a(y')\,, \tag{27}$$

$$U_{a'a}(x,x')\equiv V(x,x';a,a')\equiv V(x,x';y,y')\psi_a(y)\psi_{a'}(y')\,. \tag{28}$$

Notice in (24) and (25) that although the integrals over x' correspond to the complete set of Hartree-Fock single-particle wave functions, only the particle states contribute a nonvanishing result.[8] This result can be obtained by looking at the effects of the Pauli principle. Specifically, since to the degree of approximation used above

$$0=\psi_a{}^*(x)\psi_{a'}(x')\{\langle 0|[\psi(x)\psi^\dagger(x')+\psi^\dagger(x')\psi(x)]|\omega\rangle\}$$
$$\cong\psi_{a'}(a,\omega)+\bar{\psi}_a{}^*(a',\omega)\,,$$

then

$$\sum_{a',a''}V(x,a';a,a'')\psi_{a'}(a'',\omega)+V(x,a'';a,a')\bar{\psi}_{a'}{}^*(a'',\omega)=0\,.$$

We have used the notation

$$\psi_{a'}(a,\omega)\equiv\psi_a{}^*(x)\psi_{a'}(x,\omega)\,. \tag{29}$$

Thus only the projections of $\psi_a(x,\omega)$ and $\bar{\psi}_a(x,\omega)$ on

$$\tau(x,x')=\int_A\phi_A(x)\phi_A{}^*(x') \tag{30}$$

are determined self-consistently by (24) and (25). With the understanding that, for example, the $\psi_a(x,\omega)$ in (24) and (25) mean $\tau(x,x')\psi_a(x',\omega)$, we shall proceed further.

Let us now specify the states $|\omega\rangle$ more closely by appending an additional subscript b and a superscript + or − to ω, i.e.,

$$|\omega\rangle\rightarrow|\omega_b{}^\pm\rangle\,,$$

and correspondingly

$$\psi_a(x,\omega)\rightarrow\psi_a{}^\pm(x,\omega_b)\equiv\langle a|\psi(x)|\omega_b{}^\pm\rangle\,. \tag{31}$$

Such states are to be interpreted as continuum states which at time $t=\mp\infty$ consist of the target ($N-1$ nucleon system) in the state b, and the projectile in the continuum state $\varphi_{\epsilon(k_b)}{}^\pm(x)$, where

$$\epsilon(k_b)=\omega_b+\epsilon_b\,. \tag{32}$$

With this identification, we display this implied boundary condition in a more convenient form, i.e.,

$$\mathcal{L}_{t\rightarrow\mp\infty}\exp\{i[\epsilon(k_a)-\epsilon_a-\omega_b]t\}\varphi_{\epsilon(k_a)}{}^\pm{}^*(x)\psi_a{}^\pm(x,\omega_b)=\delta(\omega_a-\omega_b)\delta_{a,b}\,. \tag{33}$$

We defer questions of normalization and angular momentum to the Appendix. The S matrix elements of interest may now be calculated as follows:

$$\langle k'_a,a|S|k_b,b\rangle={}_{\text{out}}\langle k'_a,a|\omega_b{}^+\rangle=\mathcal{L}_{t\rightarrow+\infty}\varphi_{\epsilon(k'_a)}{}^-{}^*(x,t)\psi_a{}^+((x,t),\omega_b)$$
$$=\delta_{a,b}\langle k'_b|S^{\text{HF}}|k_b\rangle-2\pi i\,\delta(\omega_a-\omega_b)\mathcal{L}_{\omega_b\rightarrow\epsilon(k_a)-\epsilon_a}[\omega_b-\epsilon(k_a)+\epsilon_a]\psi_a{}^+(k'_a,\omega_b)\,. \tag{34}$$

From (24) and (34) for $\psi_a{}^+(x,\omega_b)$ we find

$$\langle k'_a,a|S|k_b,b\rangle=\delta_{a,b}\langle k'_a|S^{\text{HF}}|k_b\rangle-2\pi i\,\delta(\omega_b-\omega_a)\sum_{a'}\{[\varphi_{k'_a}{}^-,V_{a'a}\psi_{a'}{}^+(\omega_b)]+[\varphi_{k'_a}{}^-,U_{a'a}\bar{\psi}_{a'}{}^*(\omega_b)]\}\,. \tag{35}$$

In (34) and (35) $\langle k'_a|S^{\mathrm{HF}}|k_b\rangle$ is the S matrix for scattering off the Hartree-Fock potential, and is described in more detail in the Appendix.

This system of Eqs. (24), (25), and (35) is particularly transparent in the schematic model where one takes the two-body potential to be

$$V(x,y',x',y') = g\, v(x,x')v(y,y')\,, \tag{36}$$

so that, for example,

$$V_{a'a}(x,x') = g\langle x|v|a\rangle\langle a'|v|x'\rangle\,. \tag{37}$$

Upon solving (24) and (25) subject to the boundary condition (33) we find

$$\langle k'_a, a|S|k_b, b\rangle = \delta_{a,b}\langle k'_a|S^{\mathrm{HF}}|k_b\rangle - 2\pi i\,\delta(\omega_a-\omega_b)\,g\,\frac{\langle \varphi_{k'_a}{}^-|v|a\rangle\langle b|v|\varphi_{k_b}{}^+\rangle}{d(\omega_b)}, \tag{38}$$

where

$$d(\omega_b) = 1 - g\int_{A,a}|\langle a|v|A\rangle|^2\left[\frac{1}{\omega_b + i\epsilon - (\epsilon_A-\epsilon_a)} + \frac{1}{-\omega_b+\epsilon_A-\epsilon_a}\right]. \tag{39}$$

The first term in (38) represents the potential scattering term and the second term the resonant scattering term. The multiple-cut structure of $d^{-1}(\omega)$ reflects the coupled-channel aspect of the problem; the branch points in the physical region for scattering $\omega > |\epsilon_b|$ occur at $\omega = |\epsilon_b|$ and at each of the other hole energies $|\epsilon_a|$, reflecting the fact that as ω gets large enough new exit channels open up. One may verify that the S matrix is unitary, for example, for $\omega_b > |\epsilon_b|$,

$$\int_c \langle k_c, c|S|k_b', b'\rangle^*\langle k_c, c|S|k_b, b\rangle = \delta(\omega_b-\omega_{b'})\delta_{b,b'} - 2\pi i g\Big\{\langle \varphi_{k'_b}{}^+|v|b'\rangle\Big[\frac{1}{d(\omega_b)} - \frac{1}{d(\omega_b{}^*)} - \frac{2\pi i g}{|d(\omega_b)|^2}$$
$$\times\int_c |\langle k_c|v|c\rangle|^2\,\theta(\omega_b - \epsilon(k_c)+\epsilon_c)\Big]\langle b|v|\varphi_{k_b}{}^+\rangle\Big\}\delta(\omega_b-\omega_{b'})$$
$$= \delta_{b,b'}\,\delta(\omega_b-\omega_{b'})\,,$$

since

$$\frac{1}{d(\omega)} - \frac{1}{d(\omega^*)} = \frac{2\pi i g}{|d(\omega)|^2}\int_c \theta(\omega_b-\epsilon(k_c)+\epsilon_c)\,|\langle k_c|v|c\rangle^2\,.$$

Furthermore, zeros of $d(\omega)$ which in a discrete single-particle basis would have yielded bound states now yield resonating continuum states for $\omega > |\epsilon_b|$. For $0 < \omega_b < |\epsilon_b|$ and $g < 0$, one can have zeros of $d(\omega)$ which can be interpreted as physical bound states E_n of the N-particle system. Notice that because of the inclusion of ground-state correlations in the RPA $d(\omega)$ is an even function of ω, and there exists an unphysical image set of singularities of $d(\omega)$ for $\omega < 0$.

IV. ODD-MASS SYSTEM

Let us now turn our attention to the scattering in the $N+1$-nucleon system. Since the scattering of a particle from a resonating state is in reality a three-body problem, we shall ignore such complications and only ask for the S matrix for a nucleon of momentum p impinging upon the ground state of the N-nucleon system going into a nucleon of momentum p', with the target possibly excited to a discrete state E_n, i.e., $\langle p', E_n|S|p, 0\rangle$. Further, we shall ask that the amplitude be consistent with previous quantities calculated. To see what is involved, let us rewrite $\langle p', E_n|S|p, 0\rangle$ remembering that the states $|E_n\rangle$ have the interpretation of being "bound eigenstates of H." Thus,

$$\langle p', E_n|S|p, 0\rangle = \mathcal{L}_{t\to+\infty}\langle E_n|\psi(x,t)|p, 0\rangle_{\mathrm{in}}\varphi_{p'}{}^{-*}(x,t) = -2\pi i\,\delta(\epsilon_p - \epsilon_{p'} - E_n)\mathcal{L}_{\epsilon_p\to\epsilon_{p'}+E_n}(\epsilon_p - \epsilon_{p'} - E_n)\phi_p{}^+(p'^-, E_n)\,, \tag{40}$$

where in accord with (15)

$$\phi_p{}^+(x, E_n) = \langle E_n|\psi(x)|p^+\rangle\,, \tag{41}$$

and satisfies the equation

$$\langle x|(\epsilon_p - E_n - h)|x'\rangle \varphi_p(x', E_n) = \sum_a \int_A [V(xA;x',a)\psi_a^*(A,E_n) + V(x,a;x',A)\bar{\psi}_a(A,E_n)]\phi_p(x'), \tag{42}$$

where ψ and $\bar{\psi}$ are known from the previous calculation in the $\boldsymbol{B}$ system, and are normalized so that

$$\sum_a \int_A [|\psi_a(A,E_n)|^2 - |\bar{\psi}_a(A,E_n)|^2] = 1. \tag{43}$$

One may use (41) and a similar equation for $\varphi_p(x')$ to define a coupled-channel calculation for the $N+1$ nucleon system. However, we shall not do this, and merely calculate the direct reaction inelastic amplitude using (42) so that

$$\langle p', E_n|S|p, 0\rangle = -2\pi i \delta(\epsilon_p - \epsilon_{p'} - E_n) \sum_a \int_A [V(p'^-, A; p^+, a)\psi_a^*(A,E_n) + V(p'^-, a; p^-, A)\bar{\psi}_a(A,E_n)], \tag{44}$$

which in the schematic model equals

$$-2\pi i \delta(\epsilon_p - \epsilon_{p'} - E_n) \mathcal{G} \langle p'^-|v|p^+\rangle A(E_n), \tag{45}$$

where

$$A(E_n) = \mathfrak{N}/\mathcal{G}, \tag{46}$$

and

$$\mathfrak{N}^2 = \mathcal{G} \Big/ \left[\frac{\partial d(\omega)}{\partial \omega}\Big|_{\omega = E_n}\right]. \tag{47}$$

In summary we have seen how the GHFA can be applied to scattering problems when augmented by boundary conditions such as (33). In particular, in the case of collective motion of a vibrational type, the results of Lemmer and Veneroni are reproduced for the scattering process $A(n,n)A^*$. Furthermore, the GHFA predicts an S matrix for $B(n,n)B^*$, albeit here in a simple approximation. It would be interesting to investigate further the well-known coupling of particle and hole amplitudes in the GHFA in the context of the scattering problem.

ACKNOWLEDGMENTS

The author would like to thank A. Klein for interesting discussions with regard to this problem.

APPENDIX

Angular Momentum and Continuum Normalization Considerations

In practice it is important to fix the normalization of the continuum solutions and also to decouple the RPA equations in the spin space of the compound nucleus (referred to as B in the text). We indicate below in a schematic manner what additional considerations will affect the decoupling for the simpler problem of the Tamm-Dankoff approximation (TDA).

Suppose the compound nucleus $\boldsymbol{B}$ has spin zero so that $h(x,x')$ is spherically symmetric and possibly has a spin orbit term. The eigenstates of h then are of two types: bound states, i.e.,

$$\psi_a(\vec{r}) = R_{n_a,l_a,j_a}(r)\mathcal{Y}_{j_a,\mu_a,l_a}(\hat{r})i^{-l}, \tag{A1}$$

and continuum states which we may write as

$$\varphi_{\vec{k},\sigma}^{\pm}(\vec{r}) = \sum_{j,\mu,l} R_{l,j}^{\pm}(k,r)\mathcal{Y}_{j,\mu,l}(\hat{r})\mathcal{Y}_{j,\mu,l}^{\dagger}(\hat{k})\chi_\sigma^{1/2} i^{-l}, \tag{A2}$$

where

$$\mathcal{Y}_{j,\mu,l}(\hat{x}) = \sum_\sigma \begin{bmatrix} l & \frac{1}{2} & j \\ \mu-\sigma & \sigma & \mu \end{bmatrix} i^l Y_{l,\mu-\sigma}(x')\chi_\sigma^{1/2}, \tag{A3}$$

and

$$R_{l,j}^{\pm}(k,r) \xrightarrow[r\to\infty]{} \left(\frac{2mk}{\pi\hbar^2}\right)^{1/2} \frac{\sin}{kr}(kr + \delta_{l,j} - \tfrac{1}{2}l\pi)e^{\pm i\delta_{l,j}}. \tag{A4}$$

The asymptotic condition (A4) implies that

$$\int_0^\infty R^{\pm *}(k,r)R^{\pm}(k',r)dr = \delta(\epsilon(k)-\epsilon(k'))\,, \tag{A5}$$

where

$$\epsilon(k) = \hbar^2 k^2/2m\,. \tag{A6}$$

The expansions (A1) and (A2) allow one to write a typical potential matrix element of interest, such as $V((\vec{k}^+,\sigma),a',a,(\vec{k}',\sigma'))$ (Direct),

as

$$\sum_{\substack{j,\mu,l\\ j',\mu',l'\\ I,M}} \chi_{\sigma'}{}^{\dagger}\mathcal{Y}_{j,\mu,l}(\hat{k})\begin{bmatrix} j & j_a & I\\ \mu & -\mu_a & M\end{bmatrix}\begin{bmatrix} j' & j_{a'} & I\\ \mu' & \mu_{a'} & M\end{bmatrix} S_a S_{a'}\mathcal{Y}_{j',\mu',l'}{}^{\dagger}(\hat{k})\chi_\sigma F^I((j,l),j_a;(j',l'),j'_a;\epsilon(k),\epsilon(k'))\,, \tag{A7}$$

where

$$S_{ja} = (-)^{j_a-\mu_a}\,.$$

If one further sets

$$\psi_a{}^{+}((\vec{k}^+,\sigma),\omega_b) \equiv \int \phi_{\vec{k},\sigma}{}^{+*}(x)\psi_a{}^{+}(x,\omega_b)d\vec{x}$$

$$= \sum_{\substack{j,\mu,l\\ j',\mu',l'\\ I,M}} \chi_\sigma{}^{\dagger}\mathcal{Y}_{j,\mu,l}(\hat{k})\begin{bmatrix} j & j_a & I\\ \mu & -\mu_a & M\end{bmatrix}\begin{bmatrix} j' & j_b & I\\ \mu' & -\mu_b & M\end{bmatrix} S_a S_b \mathcal{Y}^{\dagger}(\hat{k}')\chi_{\sigma'}\psi^I((j,l),j_a;(j',l'),j_b;\epsilon(k),\epsilon(k'))\,, \tag{A8}$$

where

$$\epsilon(k') = \omega_b + \epsilon_b\,, \tag{A9}$$

then ψ^I obeys the equation in the TDA

$$[\omega-\epsilon(k)+\epsilon_a]\psi^I((j,l),j_a;(j',l'),j_b;\epsilon(k),\omega+\epsilon_b) = \sum_{j'',l'',j_c}\int_\epsilon F^I((j,l),j_a;(j'',l''),j_c;\epsilon(k),\epsilon) \times\psi^I((j'',l''),j_c;(j',l'),j_b;\epsilon,\omega+\epsilon_b)\,. \tag{A10}$$

The boundary condition in (33) reads

$$\mathcal{L}_{t\to-\infty}\exp\{i[\epsilon(k)-\epsilon_a-\omega_b]t\}\psi^I((j',l'),j_a;(j,l),j_b;\epsilon(k),\omega_b+\epsilon_b) = \delta_{j,j'}\delta_{l,l'}\delta_{j_a,j_b}\delta(\epsilon(k)-\epsilon_a-\omega_b)\,, \tag{A11}$$

so that

$$\mathcal{L}_{t\to-\infty}\exp\{i[\epsilon(k)-\epsilon_a-\omega_b]t\}\psi_a{}^{+}((\vec{k}^+,\sigma),\omega_b) = \delta_{\mu_a,\mu_b}\delta_{j_a,j_b}\delta(\epsilon(k)-\epsilon_a-\omega_b)\delta(\hat{k}-\hat{k}')\delta_{\sigma,\sigma'}\,, \tag{A12}$$

and the S matrix in TDA is

$$\langle(\vec{k}',\sigma'),a'|S|(\vec{k},\sigma),a\rangle$$

$$= \sum_{\substack{j,\mu,l\\ j',\mu',l'\\ I,M}} \chi_{\sigma'}{}^{\dagger}\mathcal{Y}_{j',\mu',l'}(\hat{k}')\begin{bmatrix} j' & j_{a'} & I\\ \mu' & -\mu_{a'} & M\end{bmatrix}\begin{bmatrix} j & j_a & I\\ \mu & -\mu_a & M\end{bmatrix} S_a S_{a'}\mathcal{Y}_{j,\mu,l}{}^{\dagger}(\hat{k})\chi_\sigma S^I((j',l'),j_{a'};(j,l),j_a;\epsilon(k'),\epsilon(k))\,. \tag{A13}$$

Here

$$S^I((j',l'),j_{a'};(j,l),j_a;\epsilon(k'),\epsilon(k)) = \delta(\omega_a-\omega_{a'})[\,\delta_{j_a,j_{a'}}\delta_{l,l'}\delta_{j,j'}S^{\mathrm{HF}}_{j,l}(\epsilon(k)) - 2\pi i \times\int_\epsilon \sum_{j'',l'',j_{a''}} F^I((j';l'),j_{a'};(j'',l''),j_{a''};\epsilon(k'),\epsilon)\times\psi^I((j'',l''),j_{a''};(j,l),j_a;\epsilon,\epsilon(k))\,] \tag{A14}$$

and

$$S^{\mathrm{HF}}_{j,l}(\epsilon(k)) = 1 - 2\pi i\, T_{j,l}(\epsilon(k)), \tag{A15}$$

where

$$T_{j,l} = \int dr\, dr'\, r^2 r'^2 \left(\frac{2mk}{\pi\hbar^2}\right)^{1/2} j_l(k, r) v^{\mathrm{HF}}_{l,j}(r, r') R_{l,j}{}^{\dagger}(k, r'). \tag{A16}$$

Here the total Hartree-Fock Hamiltonian is assumed to have a form

$$v^{\mathrm{HF}}_{l,j}(\vec{r}, \vec{r}') = \sum_{j,\mu,l} \mathcal{Y}_{j,\mu,l}(\hat{r}) \mathcal{Y}_{j,\mu,l}{}^{\dagger}(\hat{r}') v^{\mathrm{HF}}_{j,l}(r, r'). \tag{A17}$$

Further Clebsch-Gordan algebra useful in simplifying (A13) may be done; however, we shall not pursue this avenue.

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*Work supported in part by a City University of New York grant.

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PHYSICAL REVIEW C VOLUME 2, NUMBER 2 AUGUST 1970

Brueckner Reaction Matrix and Separable Potentials

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(Received 19 January 1970)

The binding energy of He^4 and O^{16} was calculated using reaction matrix elements of Tabakin's potential. A separable form of the potential has been used to reduce the basic equation to a simple linear algebraic system. The Pauli operator Q defined in terms of harmonic-oscillator intermediate states permits an easy and accurate calculation. Our numerical results for the binding energies include the first- and second-order contributions. A very resonable agreement between the experimental and theoretical values has been obtained, since the occupied-state energies are made nearly self-consistent and a cancellation of other important higher-order contributions has been achieved by a shift of the entire harmonic-oscillator spectra.

I. INTRODUCTION

A uniform description of the free nucleon-nucleon scattering and nuclear-structure data belongs apparently to the most important and very popular problems in present-day nuclear theory. It is now well known that the Brueckner reaction matrix t should be introduced in the nuclear-structure calculations rather than the free N-N interaction v. The idea is to treat the interaction inside the particle pairs ("two-body cluster") to all orders before letting any of the particles from the pair interact with the remaining particles. New progress is understanding the nature of Brueckner's perturbation expansion was provided recently in articles by Rajaraman and Bethe[1] and Brandow,[2] where earlier references can also be found.

The t matrix can be defined by the operator