Study of the Odd Strontium Isotopes with Stripping and Pickup Reactions

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The (d,t) and (d,p) reactions induced by 20.65-MeV deuterons have been used to study the levels of ^{87}Sr and ^{85}Sr up to an excitation of 3 MeV. The mass defect of ^{83}Sr has been determined by the (d,t) reaction, and its level structure below 3 MeV has been investigated. The cross sections have been analyzed with the distorted-wave Born approximation (DWBA) to determine the l values of the neutron transfers and the spectroscopic factors. Tentative spin assignments have been made on the basis of sum rules and the j dependence of the cross sections.

I. INTRODUCTION

Nuclei with closed proton (or neutron) shells presumably have simple structures involving only neutron (or proton) configurations. There are four nearly degenerate shells in the region immediately following the $1f_{7/2}$ shell (see Fig. 1) which closes at N or $Z = 28$. Nuclei with closure at $Z = 28$ and $N = 50$ have been used extensively to study single-nucleon configurations for $28 < (N \text{ or } Z) \lesssim 44$. The present experiment on the structure of the odd strontium isotopes provides similar information for the less well-known region $45 \leq N < 50$.

We have investigated[1] the structure of ^{83}Sr with the (d,t) reaction and those of ^{85}Sr and ^{87}Sr with both the (d,t) and (d,p) reactions. The (d,t) reaction was considered the primary tool since the neutron shells of interest are full, or nearly so, and neutron hole states based on them should be strongly populated. Since the next major shell (beginning with the $2d_{5/2}$, $1g_{7/2}$, $3s_{1/2}$, and $2d_{3/2}$ orbitals) starts only about 2 MeV above the $N = 50$ shell, states based on these orbitals may complicate the level spectra. The (d,p) studies identify such states, and measure more accurately the degree of emptiness of the nearly full states.

The strontium isotopes were studied because previous work on the proton configurations in $N = 50$ nuclei has shown that the proton shells are rather well closed at strontium ($Z = 38$). Theoretical studies[2,3] have assumed that the $1f_{5/2}$ and $2p_{3/2}$ proton shells are closed, and that only the $2p_{1/2}$ and $1g_{9/2}$ shells need be used as a basis. These studies have been relatively successful in explaining the available data. Experimental evidence comes from studies of proton stripping[4,5] and pickup[6] on ^{88}Sr.

Yttrium-89 has a $J^\pi = \frac{1}{2}^-$ ground state and a $\frac{9}{2}^+$ state at 0.9 MeV which are usually interpreted as $2p_{1/2}$ and $1g_{9/2}$ single-particle states outside the ^{88}Sr core. It also has states at 1.49 MeV ($\frac{3}{2}^-$) and 1.73 MeV ($\frac{5}{2}^-$) that have been interpreted as hole states formed by the promotion of a $2p_{3/2}$ or $1f_{5/2}$ particle to the $3s_{1/2}$ orbital. The reaction ^{88}Sr(^{3}He, d)^{89}Y, being a one-step process, cannot populate either of these states if this model is valid, and so the cross sections to these states are a direct measure of the emptiness of the $2p_{3/2}$ and $1f_{5/2}$ shells. The transition to the 1.49-MeV state was observed to have one quarter of the strength of the ground-state transition in one (^{3}He, d) study,[4] but was not observed in the other[5]; in neither study was the 1.73-MeV level populated. These states have also been postulated to be formed from the weak coupling of the $\frac{1}{2}^-$ ground state to a 2^+ vibration of the ^{88}Sr core, but the available evidence favors the hole-state model.[6]

The shell closure at ^{88}Sr also has been studied by looking for admixtures of the higher orbitals in its ground state.[6] No $l = 4$ transitions ($g_{9/2}$) have been observed in the (d, ^{3}He) reaction, but a second weak $l = 1$ transition to an 0.84-MeV state in ^{87}Rb was seen. The spin of this state is unknown and could be $\frac{3}{2}$, like the ground state, but it has roughly the correct excitation to be a $p_{1/2}$ state. Its spectroscopic factor is about $\frac{1}{4}$ that of the ground state. In summary, the above data indicate that the $2p_{3/2}$, $1f_{5/2}$ shell closure is rather good, but that there may be as much as 25% admixture of $2p_{1/2}$ in the $2p_{3/2}$ shell.

The strontium isotopes are probably best for studies of the single-particle states for $N < 49$, since heavier elements are unstable for $N < 50$ and the lighter isotopes of krypton are not available in enriched form. Selenium and other lighter elements have been studied, but their proton config-

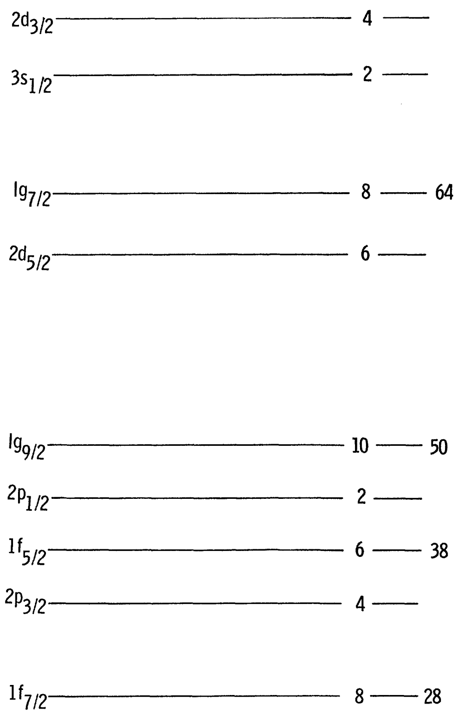


FIG. 1. Single-particle levels in the neighborhood of 50 nucleons. The numbers of nucleons in the shells and their totals are given at the right.

urations are believed to be quite complex.

II. EXPERIMENT AND DATA REDUCTION

Most of the experimental apparatus has been described by Stewart, Baron, and Leonard.[7] A 20.65-MeV deuteron beam with 40-keV energy spread and 0.3° angular divergence in the scattering plane was obtained from the National Aeronautics and Space Administration–Lewis cyclotron. Targets were prepared from enriched strontium nitrate obtained from Oak Ridge National Laboratory; in the ^{88}Sr, ^{86}Sr, and ^{84}Sr targets, the principal isotope concentrations were 99.8, 97.6, and 75.6%, respectively. For the early runs, 0.2 to 0.4 mg/cm^2 of metallic Sr was evaporated[8] onto 40-μg/cm^2 VYNS backings.[9] Since the metallic Sr oxidized during use, later runs employed carbon-backed SrO targets[10] obtained by evaporating Sr onto carbon-backed slides, allowing it to oxidize, then floating it off the slides.

The silicon-detector telescope[11] used in early runs subtended 0.307 msr, contained a 5-mm stopping detector, and gave a resolution of 90 to 100 keV full width at half maximum (FWHM). Data were taken in the angular range 16 to 100°. Several weak transitions observed in the 84 to 90° range with ^{86}Sr and ^{88}Sr targets could not be analyzed because of poor statistics, so these measurements were repeated using a 1.5-mm stopping detector which yielded 70 keV FWHM resolution.

Proton, deuteron, and triton signals were distinguished by a power-law particle identifier,[12] which gave nearly perfect mass resolution, and were stored in three different segments of the multichannel-analyzer memory. Since the targets were nonuniform, scattered deuterons at 45° were monitored. Analyzer live time was measured and used to correct the yield. Consistent energy calibration data were obtained from (d,p) and (d,t) transitions to strontium ground states and to strontium excited states of well-known energy and from (d,p) and (d,t) reactions on the ^{16}O and ^{12}C target contaminants. The ground-state Q was not known for ^{84}Sr$(d,t)^{83}$Sr, so the calibration was obtained from the contaminant reaction ^{88}Sr$(d,t)^{87}$Sr. For those transitions which were kinematically attributable to Sr, excitation energies were obtained at several angles and averaged to produce our quoted values. These have an estimated over-all uncertainty of about 1%, of which about 0.6% arises from statistical uncertainties and the rest from energy-calibration uncertainty. The areas of the peaks were extracted by the computer assuming a semi-logarithmic background, adjusted to pass through the minima between well-separated peaks.

Absolute normalizations were obtained by comparing deuteron elastic scattering yields between 20 and 50° with optical-model predictions, after first verifying that the elastic angular distributions for the three isotopes had nearly identical shapes. Since no suitable optical-model studies have been made at 21 MeV, we interpolated between the very similar optical-potential parameters at 26 MeV (Tjin, Djie, and Brockman, Jr.,[13] set 2) and 15 MeV (Perey and Perey,[14] set B). The resulting fits to the Sr elastic-scattering data were quite satisfactory. The normalization procedure was checked by comparing the optical-model predictions with the ^{90}Zr$(d,d)^{90}$Zr absolute cross sections at 20.4 MeV measured[15] with a target of accurately known density. The results agreed within 5%, and we believe the normalization of the data to be accurate to 10% or better.

A search for weak (d,t) transitions was made by summing the spectra obtained at many angles[16] to produce a composite spectrum having excellent statistics, and a few new levels were found. The excitation energies found for the stronger peaks were consistent with those determined from the individual-angle data. The (d,p) data were not treated in this way because of the large oxygen and carbon contaminations.

III. DWBA ANALYSIS

Angular distributions for the (d,p) reactions were calculated with the distorted-wave code DRC,[17] a general reaction code using spin-independent optical potentials. The (d,t) angular distributions were obtained from detailed balance after calculating the stripping cross section for the inverse (t,d) reaction with DRC.

The optical potentials used were of the conventional form:

$$-V_S f(r, r'_S, a_S) - i\,W_S f(r, r'_W, a_W) - 4ia_d W_d \frac{d}{dr}[f(r, r'_d, a_d)] \ . \qquad (1)$$

The function f is the Woods-Saxon form factor

$$f(r, r', a) = (1 + e^{(r - r' A^{1/3})/a})^{-1} \ , \qquad (2)$$

where A is the atomic mass number and the various parameters are listed in Table I. A diffuse Coulomb potential is also included. The proton parameters are derived from potential set number 1 of the 30-MeV analysis of Satchler,[18] neglecting the spin-orbit term. The triton parameters are those used for ^{90}Zr in the 20-MeV Los Alamos studies.[19] The deuteron potential used to generate elastic cross sections for the normalization of the data was also used for the DWBA calculations.

The bound-state wave functions were those of a single particle in a Woods-Saxon well [Eq. (2)] containing a spin-orbit term of the Thomas-Fermi type, i.e.,

$$-V_{so}\lambda_\pi^2 \vec{\sigma}\cdot\vec{l}\,\frac{1}{r}\frac{d}{dr}[f(r, r_S, a_S)] \ . \qquad (3)$$

Since there are no universally accepted parameters for the bound-state potential well, two quite different sets were used in order to estimate the uncertainty in the analysis. Potential set A in Table I came from an optical-model study of 20-MeV proton scattering,[20] and set B was taken from a study of the systematics of neutron single-particle levels.[21] The depth V_S was adjusted to reproduce the experimental binding energy of the state being analyzed.

The spectroscopic factors (S) listed in Tables II, III, and IV were obtained by evaluating

$$\frac{d\sigma}{d\Omega}(\text{exp}) = CS\frac{d\sigma}{d\Omega}(\text{DRC}) \qquad (4)$$

at the first stripping peak. The constant C is unity for the (d,p) calculation. In the (d,t) calculations it is taken to be 3.33 to account for the internal structure of the triton.[22]

TABLE I. Parameters used in the DWBA analysis of the data. The potential depths are in MeV and the geometrical dimensions are in fermis.

	Proton	Deuteron	Triton	Neutron (A)	Neutron (B)
V_S	54.43	96.6	152.0	···	···
r_S	1.12	1.134	1.24	1.25	1.295
a_S	0.75	0.864	0.684	0.675	0.62
W_S	···	···	19.6	···	···
r_W	···	···	1.48	···	···
a_W	···	···	0.771	···	···
W_d	6.36	17.83	···	···	···
r_d	1.33	1.377	···	···	···
a_d	0.62	0.633	···	···	···
V_{so}	···	···	···	6.0	9.5
β_{NL}	0.85	0.54	0.2	0.85	0.85
β_{FR}	1.25	···	1.69	···	···

TABLE II. Summary of the results for the levels of ^{87}Sr obtained by the (d,t) and (d,p) reactions. The ground-state Q values are −4.86 and 6.21 MeV, respectively.

E	Level	l		(d,t) S	(d,p) S	(d,p) $(2j+1)S$
0.0	1	4	9/2	9.3	0.127	1.27
0.38	2	1	1/2	1.85	0.09	0.18
0.87	3	1	3/2	2.71	0.064	0.25
1.22	4	2	5/2	···	0.11	0.64
1.25	5	3	5/2	4.15	···	···
1.76	6	2	5/2	<0.1	0.46	2.77
2.11	7	1	(1/2)	0.35	···	···
			(3/2)	0.31	···	···
2.16	8	0	1/2	···	0.35	0.70
2.23	9	(4)	(9/2)	1.7	···	···
2.41	10	3	5/2	0.70	···	···
2.54	11	nondirect			···	···
2.69	12	1	(3/2)	0.53		
2.84	13	(1)	(3/2)	0.42	···	···
2.88	14	···	···	···	···	···
3.07	15	(2)	···	···	···	···

TABLE III. Summary of the results for the levels of ^{85}Sr obtained from the (d,t) and (d,p) reactions. The ground-state Q values are −5.27 and 6.26 MeV, respectively.

E	Level	l	j	(d,t) S	(d,p) S	(d,p) $(2j+1)S$
0.0	1	4	9/2	7.62	0.21	2.09
0.24	2	1	1/2	1.51	0.27	0.54
0.75	3	2	5/2	···	0.14	0.86
0.76	4	1	3/2	2.11	···	···
0.91	5	3	5/2	0.59	···	···
1.15	6	1	3/2	0.68	···	···
1.36	7	2	5/2	···	0.33	1.99
1.67	8	1	(1/2)	0.84	···	···
			(3/2)	0.74	···	···
1.82	9	2	5/2	···	0.10	0.59
1.93	10	1	(3/2)	0.20	···	···
2.09	11	3	5/2	1.83	···	···
2.36	12	(2)	···	···	···	···

TABLE IV. Summary of the results for the levels of ^{83}Sr obtained by the (d,t) reaction. The ground-state Q value is −5.75 MeV.

E	Level	l	j	S
0.0	1	4	9/2	6.10
0.25	2	1	(1/2)	0.96
0.47	3	3	(5/2)	3.66
0.68	4	1	(3/2)	1.34
0.96	5	(1)	(3/2)	0.87
1.23	6	···	···	···
1.41	7	···	···	···
1.75	8	···	···	···

The basic DRC uses local optical potentials and a zero-range stripping interaction. We markedly improved the fits to the (d,p) distributions by using the local-energy approximation to correct for nonlocality[23] and finite range.[24] The stripping interaction ranges β_{FR} and nonlocalities β_{NL} (see Table I) were taken from Bassel.[22] The calculated (d,t) cross sections contain no such corrections, since we found that their only effect was to change the normalization by about 30%.

The ^{88}Sr$(d,t)^{87}$Sr reaction provides a good test of the DWBA, since ^{88}Sr is a closed-shell nucleus, and the levels of ^{87}Sr provide a wide range of l values. The ground-state transfer is $l=4$, $j=\frac{9}{2}$; while the first two excited-state transfers have $l=1$ and are believed to have different j values, $\frac{1}{2}$ and $\frac{3}{2}$. These three states lie within 1 MeV of each other, minimizing effects of the energy dependence of the reaction. Figure 2 shows that the DWBA fits to the stripping peaks are nearly perfect, and the agreements at larger angles are generally better than about 25%. The shape of the reaction cross section was found to be insensitive to variation of the optical potential, because of the strong absorption in both the deuteron and triton channels. Most of the contribution from the interior is eliminated, and the reaction is dominated by the surface, where the shapes of the wave functions are unaffected by small changes in the potentials.

It was found that changes in the bound-state potential caused considerable variations in the normalization of the cross section, but only slight variations in the shape. In order to reduce the uncertainty in the spectroscopic factors, we required that the potential yield S values consistent with the sum rules. The experimental ^{88}Sr$(d,t)^{87}$Sr spectroscopic factors should obey the sum rules for a closed shell,[25]

$$\sum S = N - Z/(2T+1)\ , \tag{5}$$

where Z and N are the numbers of protons and neutrons in the l, j shell of the target nucleus, and T is the isotopic spin. Table V shows a comparison of the theoretical and experimental sums. The j values listed in Tables II, III, and IV and potential A were used in finding the experimental sums. The sum rules for the $l=4$ and $l=1$ shells are quite uniformly satisfied, indicating that potential A is nearly correct. Potential B was rejected because it decreases the $l=4$ spectroscopic factors by about 30% relative to those for $l=1$. It is unlikely that much of the $l=4$ strength is missing, since there is little splitting. Renormalizing the $g_{9/2}$ sum to its theoretical value (10.0) results in excessive values for both of the $l=1$ sums.

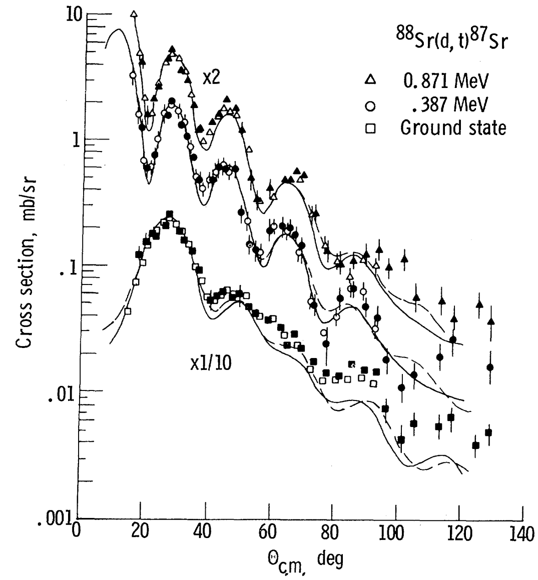


FIG. 2. Comparison of DWBA calculations to the angular distributions of the first three triton groups in the ^{88}Sr$(d,t)^{87}$Sr reaction. The solid symbols are from the first set of experiments, and the open are from the second. The solid and dashed curves are calculated with the use of potential sets A and B, respectively.

TABLE V. Comparison of the sums of the spectroscopic factors for given l and j for the different isotopes of strontium as obtained by the (d,t) reaction. The last column gives the sum-rule limit for a closed shell. The numbers in the parentheses are the sums assuming that all $l=1$ states except the first are $j=\frac{3}{2}$.

	S			
	88	86	84	Full shell
$g_{9/2}$	11.0	7.62	6.1	10.0
$p_{1/2}$	2.20	2.35		2.0
	(1.85)	(1.51)	(0.96)	
$p_{3/2}$	3.66	2.99	2.21	3.64
	(3.97)	(3.73)		
$f_{7/2}$	4.85	2.42	3.66	5.45
$g+p$	16.8	12.9	9.3	15.64

Figure 2 shows a significant difference between the depths of the minima of the $j=\frac{1}{2}$ and $\frac{3}{2}$ data near 80 and 100°. This phenomenon is similar to the j dependence found in other studies, and was used in determining the j values for the other $l=1$ transitions.

A similar study of the ^{88}Sr$(d,p)^{89}$Sr reactions was made. The strongly populated low-lying states of ^{89}Sr are based on the $2d_{5/2}$, $3s_{1/2}$ or $2d_{3/2}$ single-particle orbitals. The zero-range DWBA was much less accurate in describing these data than it was for the (d,t) studies, especially so for the 1.031-MeV state which is populated by an $l=0$ transfer. The calculation predicted a peak at either 20 or 40°, depending on whether a conventional cutoff was used or not, but it did not predict the double peak seen in the data. When the calculations were repeated including nonlocal and finite-range corrections, very satisfactory fits were produced for both the $l=0$ and $l=2$ distributions (Fig. 3). Potential set A was used for the final calculations, but set B produced essentially the same results.

Little can be said about the normalization. There is considerable fragmentation of the $l=2$ states, and Cosman, Enge, and Sperduto (CES)[26] have shown that about 30% of the $l=0$ strength lies in states not observed in this experiment. If our observed $3s_{1/2}$ strength is increased by 30%, the sum-rule limit is exceeded by 15%. The $2d_{5/2}$ sum rule, however, is only 60% filled by the ground-state transition. It is most likely that some of the higher $l=2$ states should be assigned $j=\frac{5}{2}$. The two higher $l=2$ groups that we observe, which comprise only about half of the strength seen by CES, already exceed the $j=\frac{3}{2}$ sum-rule limit.

IV. DISCUSSION OF INDIVIDUAL LEVELS

Levels of ^{87}Sr

The (d,p) and (d,t) cross sections are shown in Figs. 4 through 7. The ^{87}Sr results are summarized in Table II, and the observed levels are compared with data from other sources in Fig. 8. The *Nuclear Data Sheets*[27] show that the ^{87}Sr ground state has $J^{\pi}=\frac{9}{2}^{+}$, the first excited state at 388 keV has $\frac{1}{2}^{-}$, and the next state at 874 keV probably has $\frac{3}{2}^{-}$. A 1.22-MeV state has been identified.[28] Subsequent studies using the (d,α) reaction[29] and the photoexcitation of isomeric states[30] have yielded several new states, but little information about their nature. Additional spin assignments were made in a recent paper on the $(^3\text{He},\alpha)$ reaction.[31,32]

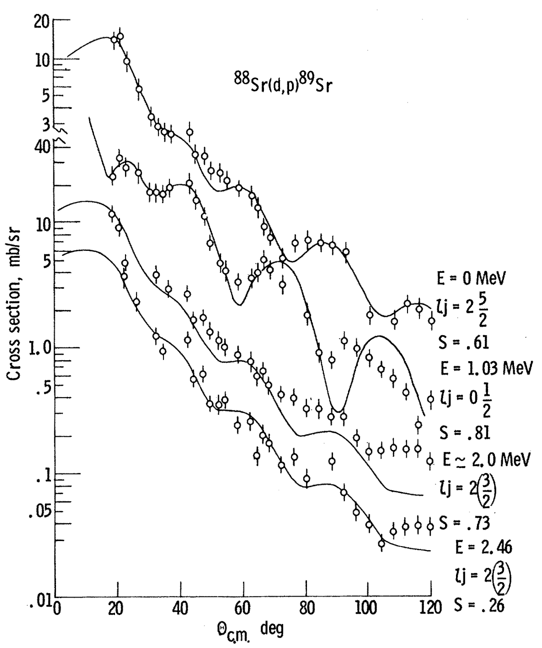


FIG. 3. Comparison of DWBA calculations to the angular distributions of the proton groups in ^{88}Sr$(d,p)^{89}$Sr reaction. The group at 2.0 MeV is the unresolved sum of two strong $l=2$ transitions to states at 1.931 and 2.000 MeV plus two weak nonstripping transitions to states at 2.059 and 2.071 MeV. These states have been resolved in Ref. 26.

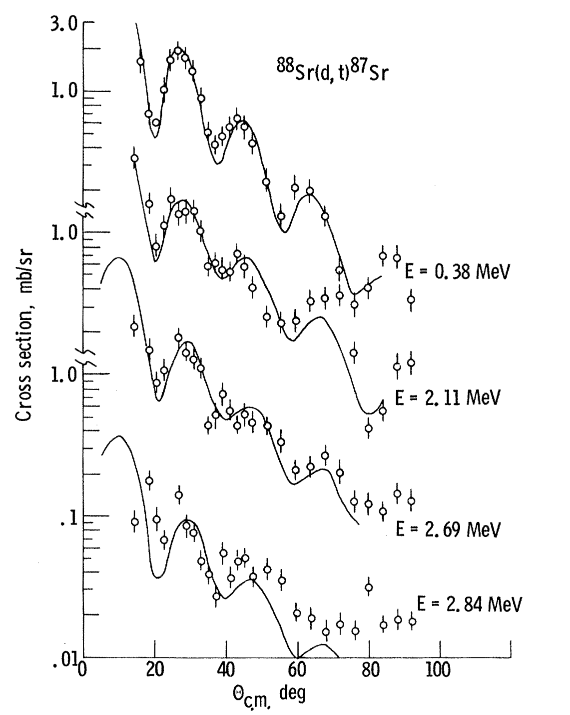


FIG. 4. Angular distributions of the triton groups in the ^{88}Sr$(d,t)^{87}$Sr reaction assigned as $l=1$ transitions. The 2.84-MeV-state assignment is tentative. The lines through the data points are from DWBA calculations.

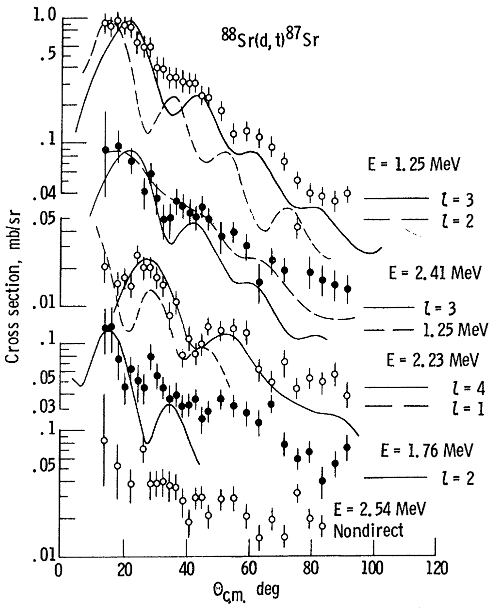


FIG. 5. Angular distributions of the triton groups in the ^{88}Sr$(d,t)^{87}$Sr reaction assigned l = values other than 1. All curves through the data points are from DWBA calculations with the exception of the dashed curve through the 2.41-MeV-state data. It is representative of the 1.25-MeV-state data, and shows the great similarity of the two cross sections. The 2.23-MeV data are probably a mixture of $l=4$ and $l=1$ transitions to two unresolved states.

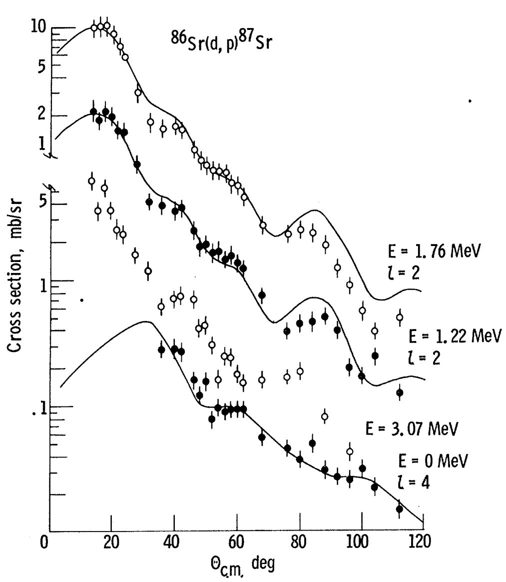


FIG. 6. Angular distributions of the high l transitions in the ^{86}Sr$(d,p)^{87}$Sr reaction. Curves through the data points are DWBA calculations.

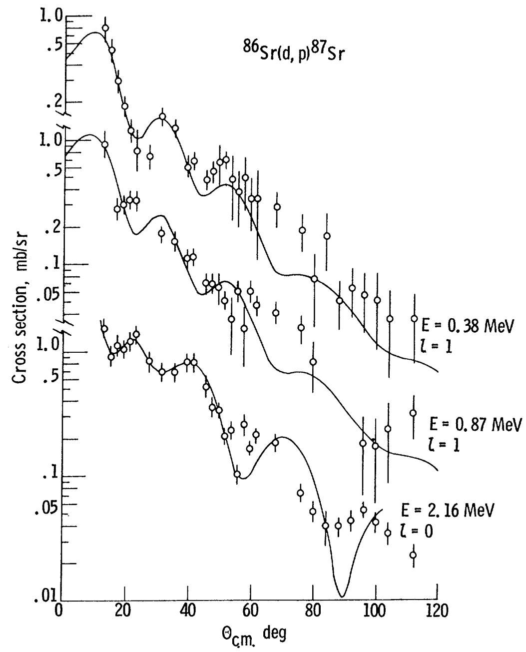


FIG. 7. Angular distributions of the low l transitions in the ^{86}Sr$(d,p)^{87}$Sr reaction. Curves through the data points are DWBA calculations.

The data on the first three states presented in the previous section are in good agreement with the *Nuclear Data Sheets*.[27] They strongly support a $\frac{3}{2}^-$ assignment for the 874-keV state. An assignment of $\frac{1}{2}^-$ for both it and the 388-keV state would greatly violate the $l=1$, $j=\frac{1}{2}$ sum rule.

The 1.25-MeV state is strongly populated by the (d,t) reaction, and the data indicate an $l=3$ transfer corresponding to the expected $f_{5/2}$ neutron pickup. The large spectroscopic factor exhausts two thirds of the total $f_{5/2}$ strength. Similar results were obtained from the (^{3}He, α) reaction.[31] Since the (d,t) DWBA fit was not good (see Fig. 5), it was suspected that the level was a doublet. Therefore a ^{90}Zr$(d,t)^{89}$Zr angular distribution which populates an isolated $f_{5/2}$ state at nearly the same energy[33] was measured. The cross sections were essentially identical, showing that the failure is in the DWBA, and there is no need to postulate a doublet. It is not clear why the theory should be poorer for $l=3$ transfers than for others.

The (d,p) reaction populates a state at 1.22 MeV, with an $l=2$ distribution almost identical to the ground-state distribution in ^{88}Sr$(d,p)^{89}$Sr. It has about one fifth of the strength expected of an empty $2d_{5/2}$ shell. Thus there are two $j=\frac{5}{2}$ levels near 1.25 MeV; one is nearly full ($l=3$) and is populated only in the (d,t) reaction, while the other is nearly empty ($l=2$) and is populated only in the (d,p) reaction. A second $l=3$ state is seen at 2.41 MeV.

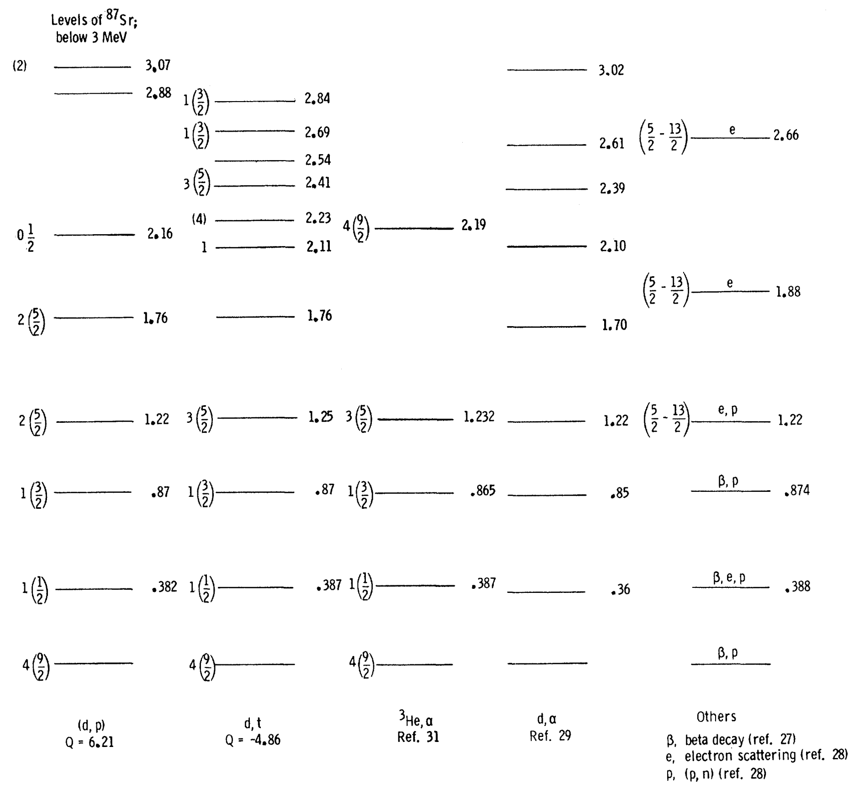


FIG. 8. Comparison of the levels of ^{87}Sr obtained in the present experiment with previous work.

The strongest observed (d, p) transition is an $l=2$ transfer to a state at 1.76 MeV. The combined strengths for the 1.22- and 1.76-MeV-state (d, p) transitions almost equals that for the ^{88}Sr$(d, p)^{89}$Sr ground-state transition. A (d, t) transition to this state also is observed, but is very weak and does not have a pickup pattern. This could indicate that the $N=50$ shell is not fully closed in ^{88}Sr, containing an admixture of about 0.1 particle in the $2d_{5/2}$ orbital. A more likely interpretation is that the reaction is not direct and represents a small breakdown of simple stripping theory. There is no evidence that the level is a doublet.

The 2.23-MeV state is observed only by a (d, t) transition, best described by $l=4$. The most forward datum is definitely above the curve, and could be caused either by an undetected contaminant or by an unresolved level with smaller l. The level was also assigned $g_{9/2}$ in a $(^{3}\text{He}, \alpha)$ experiment,[31] which gave $S=1.6$, in good agreement with our value of 1.7. This agreement suggests that only one state is populated; since the $(^{3}\text{He}, \alpha)$ reaction preferentially populates high-spin states, contaminants would affect its results less than ours.

Three levels above 2 MeV were identified in the (d, t) spectra as having $l=1$ transfers: 2.11, 2.69, and 2.84 MeV. The data on the 2.84-MeV level are poor, and the $l=1$ assignment is tentative. The angular distribution for the 2.11-MeV state indicates that it should be assigned $j=\frac{1}{2}$, since its minimum near 80° is as deep as that for the 388-keV $j=\frac{1}{2}$ state. Since the $2p_{1/2}$ strength is nearly exhausted by the 388-keV transition (Table II), the 2.69 and 2.84 states are probably $2p_{3/2}$. The $2p_{1/2}$ and $2p_{3/2}$ sum rules would be most uniformly filled if one of the three levels had $j=\frac{1}{2}$, but within the experimental uncertainty all could have $j=\frac{3}{2}$.

Levels of ^{85}Sr

The only existing information about the levels of ^{85}Sr comes from studies[34,35] of the positron and K-capture decay of the ground $(\frac{9}{2}^{+})$ and 40-keV metastable $(\frac{1}{2}^{-})$ states of ^{85}Y. We refer mainly to the work of Horen and Kelly[35] (HK), which is the most recent and complete, as the other studies are in basic agreement. The levels seen in the different experiments are shown in Fig. 9; our results are shown in Table III and Figs. 10 to 13.

The ground state was populated by both the (d, p) and (d, t) reactions. The l value (4) and spectroscopic factors were in good agreement. Decay studies give levels of 0.231 and 0.237 MeV with J^{π} of $\frac{7}{2}^{+}$ and $\frac{1}{2}^{-}$. Both (d, p) and (d, t) reactions yield clean $l=1$ diffraction patterns, indicating a negligible population of the 0.231-MeV state.

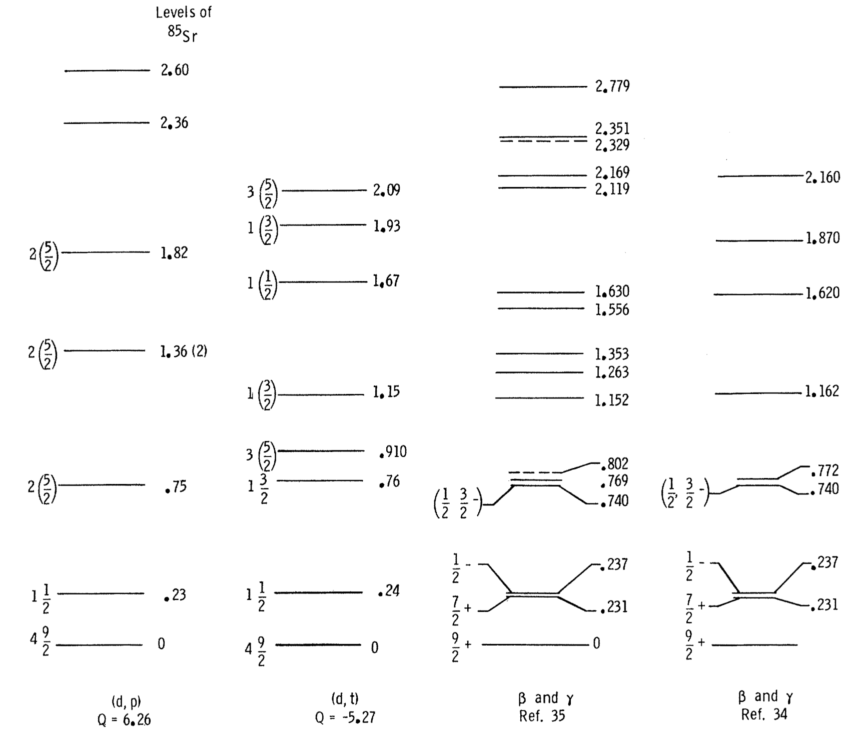


FIG. 9. Comparison of the levels (in MeV) of ^{85}Sr obtained in the present experiment with previous work.

This state has been described[2] as a seniority-3 combination of three $g_{9/2}$ holes. Since single-particle transfer reactions can only change seniority by one, it is not surprising that it is not populated.

The (d, t) reaction strongly populates an $l = 1$ level at 0.76 MeV. It is assigned $j = \frac{3}{2}$, since $j = \frac{1}{2}$ would violate the $p_{1/2}$ sum rule. This is probably

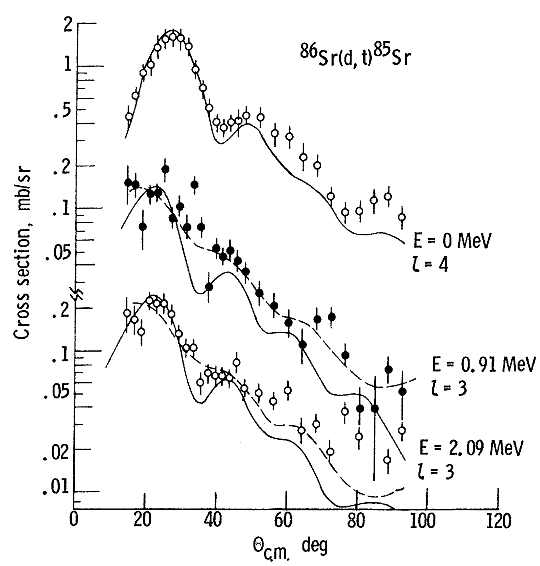


FIG. 10. Angular distributions of the high l transitions in the ^{86}Sr$(d,t)^{85}$Sr reaction. The DWBA calculations are shown by the solid lines; the dashed lines are taken from the $l = 3$ (d,t) transition to the 1.25-MeV state in ^{87}Sr.

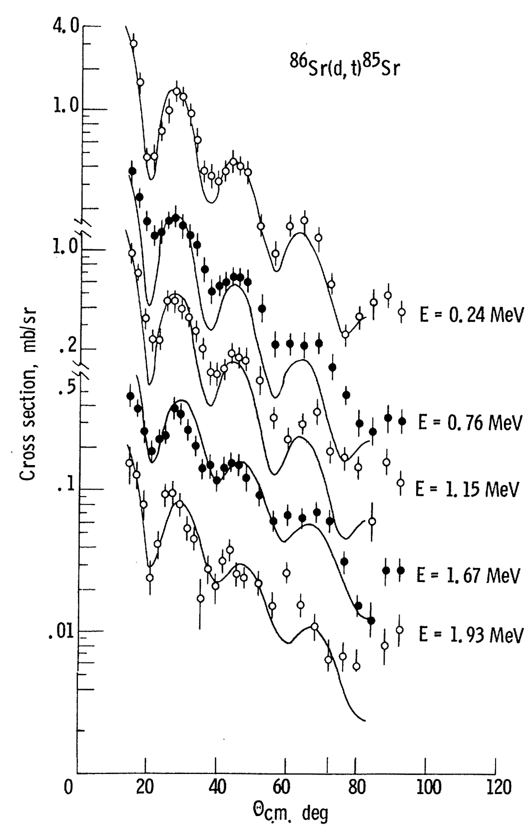


FIG. 11. Angular distributions of the transitions assigned as $l = 1$ obtained in the ^{86}Sr$(d,t)^{85}$Sr reaction.

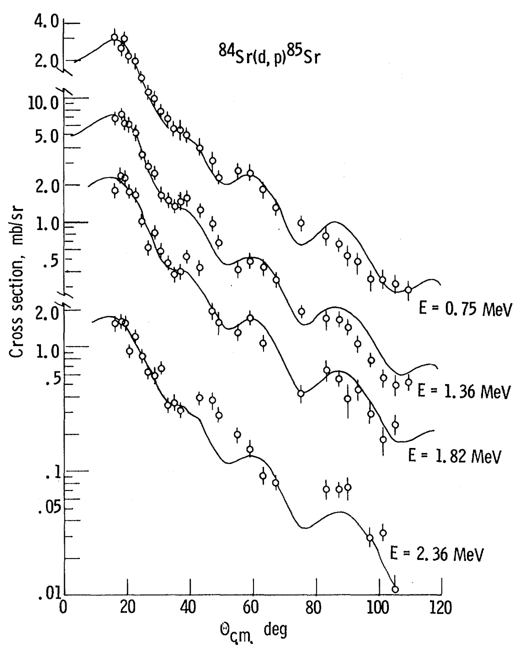


FIG. 12. Angular distributions of the triton transitions assigned as $l = 2$ in the ^{84}Sr$(d,p)^{85}$Sr reaction.

the state observed by HK at 740 keV. The (d, p) reaction also populates a level at this energy which must be $2d_{5/2}$, since the stripping pattern is clearly $l = 2$. The $2d_{5/2}$ level probably is not the state observed by HK at 769 keV, since the ft of a transition to a $\frac{5}{2}^+$ level should be about 9, in contrast to the observed value of 7.1. Thus there may be three nearly degenerate states. There is further evidence of this; the diffraction pattern in the (d, t) reaction is washed out, and the peak in the summed spectrum is significantly wider than the adjacent peaks. It is unlikely that these effects

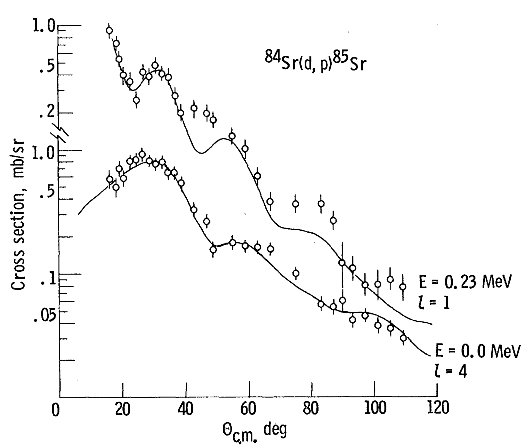


FIG. 13. Angular distributions of the first two transitions in the ^{84}Sr$(d,p)^{85}$Sr reaction. Smooth curves are the experimental distributions observed in ^{88}Sr$(d,t)^{87}$Sr.

are due to the ^{88}Sr contamination in the target or to population of the $d_{5/2}$ level, since the summed spectrum shows only a small yield for comparable levels of ^{87}Sr and for the other $d_{5/2}$ levels of ^{85}Sr. An interpretation consistent with all the data would be to assume that the (d, t) reaction populates both the 740- and 769-keV levels seen by HK, and that the latter has $J^{\pi} = \frac{9}{2}^+$. The (d,p) reaction would only weakly populate these two states.

The states at 0.91 and 2.09 MeV are assigned $l = 3$ and, tentatively, $j = \frac{5}{2}$. Probably neither of these was observed by HK. The weak 910-keV state required special handling to extract the cross section, since it is near the strong 760-keV group. Since its width was required to be the same as the 760-keV peak, and its position 14 channels below the 760-keV peak, its energy uncertainty is about 60 keV.

The 1.36 and 1.82-MeV states are observed only in the (d,p) reaction, and are identified as fractions of the $2d_{5/2}$ orbital. The sum of $(2j+1)S$ for these states plus the 760-keV state is 3.44, compared with 3.66 for the ground state of ^{89}Sr. Possibly our 1.36-MeV state is the 1.35-MeV state seen by HK, but again the ft value should be higher than they reported (approximately 9 against 6.9).

Three states at 1.15, 1.67, and 1.93 MeV are observed only in the (d, t) reaction, and all have $l = 1$ transfers. The sum rule suggests that all have $j = \frac{3}{2}$, but one state could have $j = \frac{1}{2}$ without violating it badly. The j dependence indicates that the 1.67-MeV state has $j = \frac{1}{2}$ and that the others have $j = \frac{3}{2}$.

Transitions at 2.36 and 2.60 MeV are seen in the (d,p) reaction, but the data do not allow an l-value determination.

Levels of ^{83}Sr

^{83}Sr may only be studied via the (d, t) reaction, since ^{82}Sr is unstable. The (d, t) angular distributions (Fig. 14) are of rather poor quality, since the target had only 75.69% of ^{84}Sr. It was felt worthwhile to obtain them, however, since there are no existing data on the ^{83}Sr level structure. The three most-energetic peaks observed in the spectra resulted from the ^{88}Sr and ^{86}Sr contaminants. Angular distributions for two of them were compared with those from the other experiments. The shapes were identical, and the yields indicated that the concentration of ^{88}Sr was 20%. This value was used as a normalization in subtracting the contaminants from the other angular distributions.

The lowest ^{83}Sr state populated by the (d, t) reaction had $l = 4$. This is not necessarily the ground state, since the systematics in this region are ambiguous. The heavier strontium isotopes have $\frac{9}{2}^+$ ground states with low-lying $\frac{1}{2}^-$ excited states, but

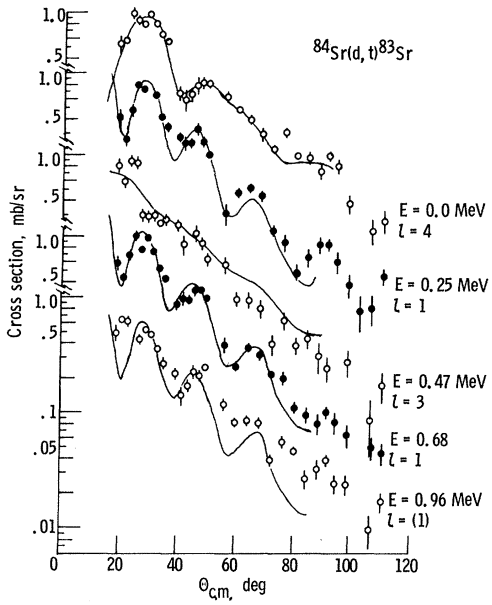


FIG. 14. Angular distributions of the triton groups in the ^{84}Sr$(d,t)^{83}$Sr reaction. All curves are DWBA calculation with the exception of the one for the 0.47-MeV state. It is taken from the $l=3$ transition to the 1.25-MeV state in ^{87}Sr.

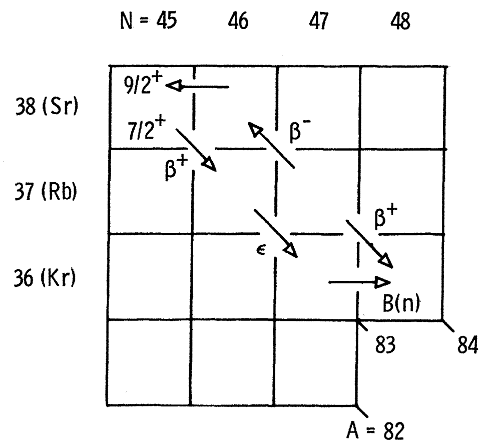


FIG. 15. Mass chain used in evaluating the energy difference between the $\frac{7}{2}^+$ and $\frac{9}{2}^+$ states of ^{83}Sr.

all the other nearby $N=45$ isotones have $\frac{7}{2}^+$ ground states. The $\frac{7}{2}^+$ states must be rather complex, since the $g_{7/2}$ single-particle orbital is believed to lie considerably above the $g_{9/2}$ orbital. Talmi and Unna[2] have predicted a $\frac{9}{2}^+$ ground state with a $\frac{7}{2}^+$ state at 320 keV. The decay[36] of ^{83}Sr originates from a $\frac{7}{2}^+$ state, which is probably the ground state. Alternatively, the ground state may be $\frac{1}{2}^-$, and the $\frac{7}{2}^+$ state an isomer, but no evidence of an isomeric transition was found.

The present experiment excites only seniority-1 states and hence would not populate the complex $\frac{7}{2}$ state described by Talmi and Unna. It does show that the $\frac{9}{2}^+$ state lies below any simple $\frac{1}{2}^-$ or $\frac{3}{2}^-$ states, and so eliminates the choice of $\frac{1}{2}^-$. The $\frac{9}{2}^+$ state must lie somewhat above the $\frac{7}{2}^+$ state, since the latter is not an isomer.

Using the mass chain shown in Fig. 15, we found that the mass excess of the $\frac{9}{2}^+$ state is -76.70 MeV, and that its excitation energy above the $\frac{7}{2}^+$ state is 0.014 ± 0.033 MeV. The Q value of the ^{84}Sr $(d,t)^{83}$Sr ($\frac{9}{2}^+$ state) was determined in this experiment to be -5.755 ± 0.030 MeV. The ^{83}Rb K-capture decay energy was taken to equal its lower limit.[37] The ^{83}Sr decay energies were taken from Etherton *et al.*,[36] and the other energy differences from the Berkeley tables.[38]

The cross sections for the 0.25-MeV state are well described by the $l=1$ curve, and the deep minima near 80 and 100° indicate $j=\frac{1}{2}$, in view of the j dependence observed in ^{88}Sr$(d,t)^{87}$Sr. This accords with the systematics of the other isotopes.

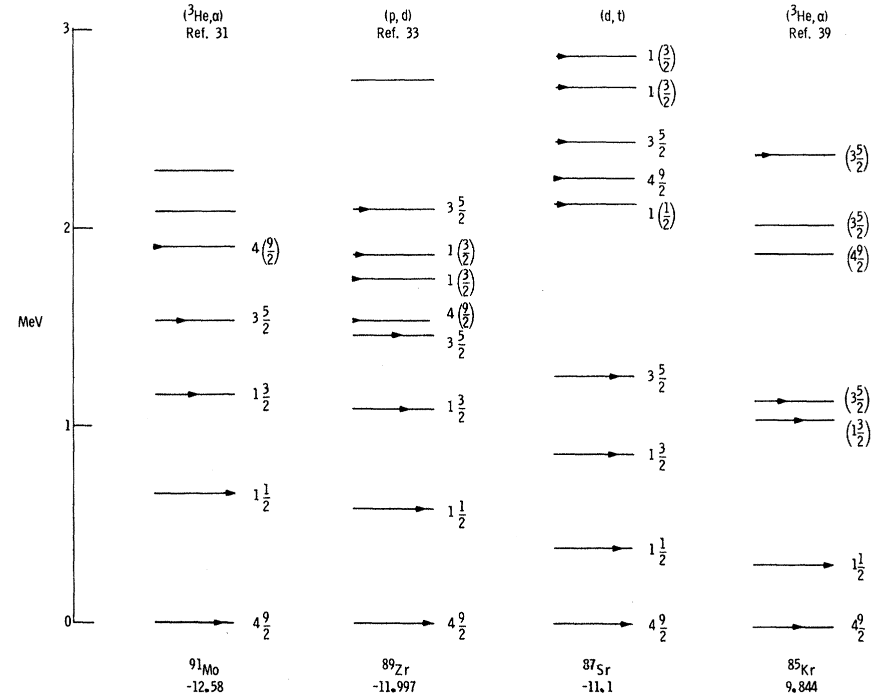
The angular-distribution data for the 0.47-MeV state are well described by an $l=3$ distribution except for a few discrepancies believed to be caused by unidentified light contaminants. The state has a major fraction of the expected $f_{5/2}$ shell strength. The angular distribution for the 0.68- and 0.96-MeV levels have been tentatively assigned to $l=1$ transfers. The 0.96-MeV level appears to have the first minimum of the $l=1$ angular distribution filled in; it also resembles an $l=4$ curve, but this assignment would violate the $g_{9/2}$ sum rule. The j dependence of the 0.68-MeV state's angular distribution suggests $j=\frac{3}{2}$. Another reason for our tentative assignment of $j=\frac{3}{2}$ to both these states is the uniform filling of the sum rules; if the 0.25 and 0.68 levels were both $j=\frac{1}{2}$, their combined reduced widths would exceed the limit for a full $p_{1/2}$ shell.

CONCLUSIONS

The shell model suggests that the structures of ^{87}Sr and the other $N=49$ isotones are very simple. Each should have four low-lying states corresponding to single holes in the $g_{9/2}$, $2p_{1/2}$, $2p_{3/2}$, and $f_{5/2}$ shells. Below 1.5 MeV, the levels of these nuclei populated by the (d,t) reaction (shown in Fig. 16) agree with the simple picture. The order of the spins and orbital angular momenta of the levels are correct, and usually vary smoothly with mass. The spectroscopic factors are also consistent with the model within their uncertainties.

This interpretation is compromised, however, by the results at higher excitation. A significant amount of the pickup strength for all the orbitals lies in the transitions to the higher states. This is true for all the isotones in Fig. 16, and the rela-

FIG. 16. Comparison of the levels of ^{87}Sr excited by the (d,t) reaction with the levels in the other $N=49$ isotones. Weakly populated levels not believed to be associated with either the p, f, or g holes have been omitted. The lengths of the arrows on the bars indicate the ratio of the spectroscopic factor of the transition to the sum rule for a closed shell. The binding energy of the neutrons removed in the reactions are given below the diagrams.



tive simplicity of ^{91}Mo and ^{85}Kr probably results from reduced experimental sensitivity.[31,39] This fragmentation may result from coupling of the holes with core vibrations,[40] and an explicit calculation for ^{87}Sr has been made by Zawischa and Werner.[41] States consisting of a hole coupled to a vibration normally would not be populated by the (d, t) reaction, a one-step process, but they can receive some strength by mixing with a single-hole state having the same spin and parity. A detailed calculation of the locations and strengths of the resulting states is beyond the scope of this paper, but some feeling for these quantities can be gained from the locations of the unperturbed states which can couple. One expects greater coupling the closer the states are in energy, and the larger the number of possible configurations. The core state of ^{88}Sr, which is expected to be most important, is the quadrupole vibration at 1.84 MeV, with the octupole state at 2.74 MeV having less influence. The single-hole states all lie below the vibrations, and it is expected that the higher ones will be affected the most. In addition, the $2p_{1/2}$ hole can only couple to the single configuration $(2^+ \times p_{3/2})$. The $p_{3/2}$ and $f_{5/2}$ holes can couple to more configurations and are expected to have more fragmentation, in accord with the data (Table II).

The calculations of Zawischa and Werner[41] show the states of ^{87}Sr to be badly fragmented, with significant pieces extending to 5 MeV. Our data show less splitting than the calculation, both in energy spread and in the number of states. Even though the experiment may miss weak states, the states which are excited are not in agreement with the calculation. Nevertheless, both theory and experiment have splittings which are characterized by a single low-lying state containing most of the transition strength, and several weakly excited states all well separated from it.

Cohen[42] has perceived a great similarity between the relative positions of the neutron and proton single-particle levels. One would also expect the filling of the levels to be similar. Since the $2p_{3/2}$ and $1f_{5/2}$ proton shells are rather well filled at Sr $(Z=38)$, the corresponding neutron shells in the strontium isotopes studied $(N \geqq 45)$ should be filled, and only the $2p_{1/2}$ and $1g_{9/2}$ shells should be filling. The (d,p) spectroscopic factors are proportional to the emptiness of the target nucleus and allow us to check this conjecture. Table II shows that the $2p_{3/2}$ state at 0.87 MeV is about half as strongly populated as the ground state. Either the neutron and proton shell-filling orders are not similar, or the $2p_{3/2}$ shell fills at $N, Z=38$ and then empties as particles are added.

The $N=50$ shell appears to have good closure. Since (d,p) results show that the 1.76-MeV state in ^{87}Sr corresponds to a $d_{5/2}$ particle, it should be populated in the (d, t) reaction only if there is a $d_{5/2}$ admixture in the ^{88}Sr gound state. The weak transition that is seen indicates an admixture of

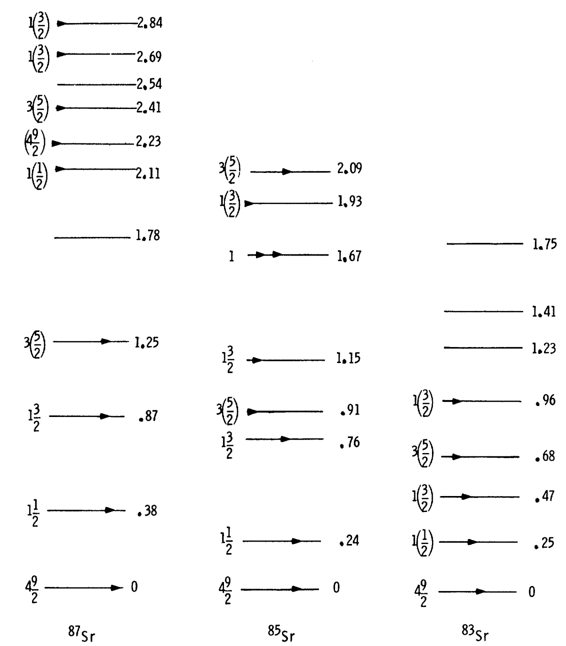


FIG. 17. Comparison of the levels (energies in MeV) of the different strontium isotopes excited by the (d,t) reaction. The lengths of the arrows indicate the ratios of the spectroscopic factors of the transitions to the sum rule for a closed shell.

0.1 particle or less. This accords with other results on the $N=50$ isotones.

In Table V all the spectroscopic factors for each j and l have been summed, assuming the most likely values of j for the $l=1$ transitions. These should equal the number of particles in the shells, and the remarkable (and probably fortuitous) agreement for ^{87}Sr has been mentioned. We can probably conclude that most of the fragments of the orbitals, except for $f_{5/2}$, have been found. The sums over l and j for ^{85}Sr and ^{83}Sr should be two and four less. Since this is obviously untrue, not all the fragments have been located. Even if we discard the $f_{5/2}$ states, assuming that they have the same fullness in all of the isotopes, the sums over the p and g states are deficient by 1.9 and 3.5 particles in ^{86}Sr and ^{84}Sr, respectively. This provides evidence of the expected increased fragmentation of the single-particle states as one leaves a closed shell.

Figure 17 summarizes the systematics of the states populated by the (d,t) reaction. Clearly, the locations of the lowest states of each spin vary smoothly, but it is not clear whether their descent as the neutrons are removed results from a shift of the single-particle states or their increased fragmentation.

The results of the (d,p) studies are summarized in Fig. 18. The results for the p and g shells are of limited scope, but are in good agreement with the (d,t) work. Only limited conclusions can be made about the splitting of the d and s states, since even in ^{89}Sr there is probably a good deal of strength which is missed in this experiment. The S sums for the low-lying $l=2$ states in ^{87}Sr and ^{85}Sr are both equal to 0.57. This is quite close to the S value (0.61) for the ground state of ^{89}Sr. There is substantial fragmentation of the ground state as neutrons are removed, but only to a few pieces. The ^{87}Sr data show that the $d_{5/2}$ single-

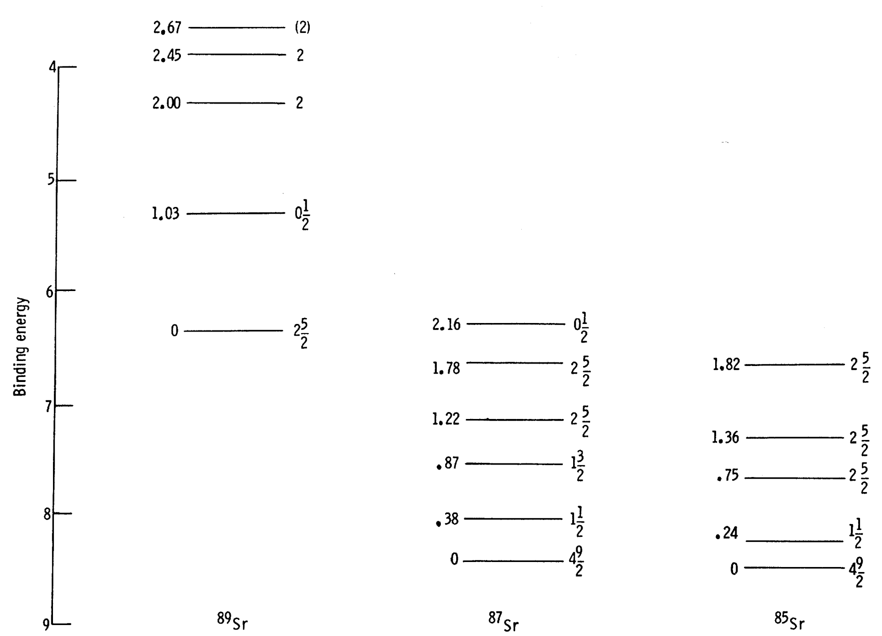


FIG. 18. Comparison of the levels of the different strontium isotopes excited by the (d,p) reaction. The scale on the left gives the binding energy (in MeV) of the last neutron in the nucleus.

particle state is 1.6 MeV or more above the $g_{9/2}$ state.

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