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Influence of Nuclear Forces on the Coulomb Barrier in Heavy-Ion Reactions*

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Holm and Greiner have shown the effect of nuclear diffuseness on the barrier against fusion of two heavy nuclei. We show that the major part of this effect is a lowering of the barrier because of the diffuseness alone. There is an additional change in the barrier, due to deformation effects, which may be either positive or negative depending on the relative importance of the Coulomb and nuclear parts of the potential.

The effects of the Coulomb force on the shapes of two interacting heavy ions have been the subject of a number of discussions.[1-6] In general, it has been found that a nucleus tends to flatten in the Coulomb field of a nearby nucleus. Because of this flattening the two nuclei must be brought closer together than two spheres of equal volume before their surfaces touch and nuclear interaction occurs. The net result is an increase in the barrier for nuclear reaction over that for two rigid spheres; the effect is, however, calculated to be rather small.

Holm and Greiner[7] have extended this sort of calculation by including a Yukawa short-range nuclear force as well as the Coulomb force. They find an apparent substantial reduction in the barrier (5 to 10%) because of the Yukawa force. In interpreting these results it is important to distinguish between two effects. The first of these is that introducing a diffuse nuclear potential causes a lowering of the barrier even for spherical nuclei. The second is that the nuclear potential, being attractive, tends to induce prolate shapes, in opposition to the repulsive Coulomb potential, which tends to induce oblate shapes. The barrier against fusion will be less for prolate shapes than for spherical or oblate shapes.

The first of these effects is well known.[8-10] For instance, Huizenga and Igo[9] have shown that for a model in which the nuclear potential cuts off sharply, a radius parameter of 1.5×10^{-13} cm is required to get agreement between experimental and theoretical reaction cross sections. With a diffuse nuclear potential, however, they are able to get agreement between experiment and theory with a radius parameter of 1.17×10^{-13} cm. Thomas has come to similar conclusions in considering heavy-ion induced reactions.[8,10]

The effect of the Yukawa potential in reducing the barrier between two rigid spherical nuclei is easily illustrated. The energy of interaction V is given as

$$V=\frac{Z_1Z_2e^2}{D}+\frac{V_0}{4\pi}\int d\tau_1\rho(r_1)\int d\tau_2\rho(r_2)\frac{e^{-|\vec{r}_1-\vec{r}_2-\vec{D}|/\mu}}{|\vec{r}_1-\vec{r}_2-\vec{D}|},$$

where Z_1 and Z_2 are the nuclear charges and D is the separation between the two nuclei. V_0 and μ are parameters of the Yukawa potential. The nuclear density ρ is taken to be constant out to some radius $R=1.2A^{1/3}$ fm. The first term is the Coulomb interaction between the two nuclei and the second term is the Yukawa interaction. The second term is readily integrated, and, for nuclei of equal radius gives

$$\frac{V_0A^2}{4\pi}\left(\frac{3}{R^3}\right)^2\left[\mu^3\sinh\frac{R}{\mu}-R\mu^2\cosh\left(\frac{R}{\mu}\right)\right]^2\frac{e^{-D/\mu}}{D},$$

where A is the mass number of the nucleus. Using these expressions and values of V_0 and μ given by Holm and Greiner,[7] we have calculated the barrier for two rigid spherical ^{122}Sn nuclei and for two rigid spherical ^{148}Nd nuclei. The barrier is taken as the minimum energy required to bring the surfaces of the two nuclei (at $R=1.2A^{1/3}$ fm) into contact.

TABLE I. Fusion barriers for two different systems.

μ	V_0 (MeV fm)	E_{sphere} [a] (MeV)	E_{def} [b] (MeV)
	$^{122}Sn + ^{122}Sn$; $E_{Coul} = 302.4$		
0.6	−741	288.6	288.
0.8	−315	285.2	282.
1.0	−163	282.3	277.
1.2	−95.8	279.8	273.
	$^{148}Nd + ^{148}Nd$; $E_{Coul} = 408.4$		
0.6	−723.0	393.7	408
0.8	−307.0	390.0	401
1.0	−159.0	386.9	396
1.2	−93.1	384.1	392

[a] For rigid spherical nuclei.
[b] For deformable nuclei. From Ref. 7.

The results are given in the third column of Table I. Also given in the table, for comparison, are the values of the barrier if the nuclear potential cuts off sharply at a radius given by a radius parameter of 1.2×10^{-13} cm. We see that, in each case, the introduction of the diffuse nuclear potential causes a lowering of the potential barrier of 14 to 24 MeV as expected. (The reduction would be even greater with a Woods-Saxon nuclear potential and typical parameters.)

We turn now to the second point, the effect of the diffuse nuclear potential on the deformation. We recognize that any effect on the barrier due to deformation must be measured relative to the barrier for spheres calculated with the appropriate diffuseness. The fourth column of the table gives the results of the dynamic calculations of Holm and Greiner,[7] including deformation effects due to both Coulomb and nuclear interactions. To evaluate the effects of deformation alone, these results must be compared with the corresponding entries in column 3 of the table, which give the barriers calculated for the same degree of diffuseness, but without deformation.

On making this comparison for ^{122}Sn, we note that the barriers with deformation are very close to those without, indicating that only very slight deformation is taking place. This small deformation results either from the high rigidity of the closed-shell nucleus ^{122}Sn or from an approximate cancellation of the nuclear and Coulomb parts. For ^{148}Nd, the effects of deformation are much larger, possibly because the ^{148}Nd nucleus is easily deformed or because the Coulomb part is dominating the nuclear part.

For the two cases considered here, the effects of deformation on the barrier are opposite. For ^{122}Sn, the barrier is lowered by the deformation, indicating a prolate deformation. For ^{148}Nd the barrier is raised, indicating an oblate deformation (as is shown by Fig. 1 of Ref. 7). In either case, the effect of deformation is quite small and will be difficult to determine experimentally.

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