Comments and Addenda

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Off-Shell Continuation of the Two-Body T Matrix with Fixed On-Shell Behavior*

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The off-shell continuation of the two-body T matrix is written in terms of a T matrix that is correct on-shell plus a term that effects only the off-shell behavior. The work of Baranger *et al.* and of Haftel is used to show what is arbitrary in the off-shell term for fixed on-shell data.

The off-shell two-body transition matrix or T matrix is the dynamical input in a wide range of nuclear-physics problems. In fact, one of the prime reasons for studying these problems is to obtain the off-shell information that cannot be obtained from the (on-shell) two-nucleon system. Recently Baranger *et al.*[1] (BGMS) have shown how to continue the two-body T-matrix off shell without passing through the rather cumbersome intermediate step of constructing a potential. Their work was for a partial-wave T matrix without a bound state. More recently, Haftel[2] has extended that work to include a bound state. The major result of the work of BGMS is a general expression for the off-shell T matrix in terms of a function of two variables $\varphi(k,k')$, the diagonal part of which can be written in terms of the scattering phase shift. They show that the symmetric part of φ is arbitrary and that the antisymmetric part may be calculated in terms of it. Haftel has shown how to modify this procedure when there is a bound state present. In this case one must include information on the binding energy and bound-state wave function as well as the scattering phase shifts. These forms of the off-shell two-body T matrix, although allowing in principle for arbitrary off-shell behavior, are not particularly suited to direct use in nuclear-physics calculations, since they are too complex and rather too arbitrary. In particular the off-shell arbitrariness contained in the off-diagonal parts of φ is mixed in with the diagonal part, which contains the on-shell information. One would like a form in which the off-shell part could be varied at will but the on-shell part kept explicitly fixed. Thus for many applications one would like to write the off-shell T matrix as the sum of a term that is easily managed, easily calculated, and gives the exact phase shifts and bound states on shell, plus a "correction" term that vanishes on shell and can be parametrized to test the dependence of the particular calculation on off-shell effects. In this note we provide such a form.

The essence of how to separate the off-shell arbitrariness from the on-shell restrictions in the T matrix is already in Haftel's paper. In order to include a bound state in the BGMS formulation he introduces a model potential V_m, which has the same bound-state energy and wave function as the full amplitude. We suggest making it have the same scattering phase as well. One can then write for the general off-shell T matrix[3]

$$\langle k'|T(\omega)|k\rangle=\langle k'|T_m(\omega)|k\rangle+C(k',k;\omega)\,, \tag{1}$$

where T_m is the off-shell T matrix for V_m and C is the "correction" term. Since T_m has the scattering phase of T by construction, C vanishes on shell ($k^2=k'^2=\omega$). Clearly the existence of any number of bound states is irrelevant so long as they are all fully contained in T_m.

We now obtain an explicit expression for C. Following BGMS and Haftel, we define $|\psi_k^0\rangle$ to be the real wave-function solution of the actual (not the model) Schrödinger equation. The half-shell function $\varphi(k,k')$ is defined by

$$\varphi(k,k')=\langle k'|V|\psi_k^0\rangle . \tag{2}$$

The diagonal part of φ is related to the scattering phase shift δ by

$$\varphi(k,k)=-(2k/\pi)\sin\delta(k) . \tag{3}$$

BGMS show that if no bound state exists, the off-shell T matrix can be expressed entirely in terms of φ as follows:

$$\langle k'|T(\omega)|k\rangle=\varphi(k,k')\cos\delta(k)+\int_0^\infty dq\left(\frac{1}{\omega-q^2}-\mathrm{P}\frac{1}{k^2-q^2}\right)\varphi(q,k')\varphi(q,k) \tag{4}$$

for any complex ω. The symbol P means the principal value should be taken at the singularity. Now we introduce a model potential V_m and the corresponding model half-shell function $\varphi_m(k,k')$. The condition that the model phase shift and the actual phase shift agree is

$$\varphi_m(k,k)=\varphi(k,k) . \tag{5}$$

We can therefore write

$$\varphi(k,k')=\varphi_m(k,k')+(k^2-k'^2)\Delta(k,k') , \tag{6}$$

where we have used the factor $k^2-k'^2$ to display explicitly that φ and φ_m agree when $k=k'$. If we substitute (6) into (4), we obtain

$$\langle k'|T(\omega)|k\rangle=\langle k'|T_m(\omega)|k\rangle+(k^2-k'^2)\Delta(k,k')\cos\delta(k)+\int_0^\infty dq\left(\frac{1}{\omega-q^2}-\mathrm{P}\frac{1}{k^2-q^2}\right)[\varphi_m(q,k')\Delta(q,k)(q^2-k^2)+(q^2-k'^2)\Delta(q,k')\varphi_m(q,k)+(q^2-k'^2)(q^2-k^2)\Delta(q,k')\Delta(q,k)] , \tag{7}$$

where T_m is the model T matrix and is obtained from (4) with φ replaced everywhere by φ_m. It is the off-shell T matrix corresponding to the model potential V_m. It is easy to see that the terms after T_m on the right of (7) vanish on shell. At this point the restriction to no bound states may be lifted so long as any and all bound states are fully contained in T_m.

Given the model potential V_m and the associated real solutions to the model Schrödinger equation $|\chi_k^0\rangle$ (corresponding to the $|\psi_k^0\rangle$ for the actual potential V), we can construct the model half-shell function

$$\varphi_m(k,k')=\langle k'|V_m|\chi_k^0\rangle . \tag{8}$$

The difference function Δ may then be constructed in terms of the quantity

$$\varphi_W(k,k')=\langle\chi_{k'}^0|V-V_m|\psi_k^0\rangle \tag{9}$$

defined by Haftel, as he shows in his Eq. (17). It is simpler and closer to the spirit of BGMS not to form Δ in this way but to take it as arbitrary, or nearly so. BGMS show that if $\varphi(k,k')$ is written as the sum of a symmetric and an antisymmetric part, the antisymmetric part may be determined from the symmetric, which is in turn arbitrary. Because of the factor of $k^2-k'^2$, it is the antisymmetric part of Δ which is arbitrary; the symmetric part may be obtained from it and φ_m using the prescription of BGMS.

The procedure for use of this formalism we envisage is as follows. One picks a model potential or T matrix that is easily solved, and fits the two-body data (phase shifts and bound states, if any) to it. The model T matrix should be chosen so that it is easy to use in the nuclear-physics application under study. For many problems this may mean choosing a separable interaction, but it certainly need not. The algorithm for constructing the separable potential that has a given phase shift and bound state is well known.[4] One then constructs φ_m and T_m from that potential. From φ_m and a suitably parametrized Δ one constructs C in (1) from (7). One then solves the problem at hand with various choices of the parameters in Δ to test the off-shell sensitivity of the answers. Hopefully, C could be relatively small and its effects on the $C=0$ case ($V=V_m$) can be studied by some perturbative method. In any case, one will always be sure that no matter how one treats Δ, the on-shell two-body results are staying fixed. The precise way to handle C, and the related question of the most interesting way to parametrize the antisymmetric part of Δ are interesting questions that remain to be answered. They are probably model dependent.

It is clear that the method outlined above is very close in spirit to the quasiparticle ideas used long ago to justify the use of separable potentials in the three-body problem.[5]

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Decay of ^{81}Rb

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The energies and relative intensities of γ rays emitted in the decay of ^{81}Rb have been measured for mass-separated sources using a 15-cm^3 Ge(Li) detector. New weak-intensity γ-rays have been observed at 244, 1041, 1069, 1109, 1429, and 1555 keV. The half-life of ^{81}Rb has been measured as 4.580 ± 0.009 h.

I. INTRODUCTION

The γ-ray spectrum of ^{81}Rb (reported[1] half-life 4.7 h) has been studied using a Ge(Li) detector by Li-Scholz and Bakhru.[2] However their samples were contaminated with 6.4-h ^{82m}Rb, which interfered with the determination of the intensities of some of the ^{81}Rb γ rays. To overcome this difficulty we have carried out a similar investigation using mass-separated samples of ^{81}Rb.

II. EXPERIMENTAL METHODS

Samples were prepared by bombarding NaBr targets with 28-MeV α particles in the external beam of the Medical Research Council cyclotron. Using a beam current of 35 μA for 1.5 h, about 125 mCi of ^{81}Rb and about 25 mCi ^{82m}Rb were produced from ^{79}Br$(\alpha, 2n)$ and (α, n) reactions together with lower activities of ^{83}Rb and ^{84}Rb from ^{81}Br$(\alpha, 2n)$ and (α, n) reactions. The rubidium radionuclides were chemically separated from the target material using a zirconium phosphate ion-exchange column, and about 30-mg RbCl carrier was added. The ^{81}Rb was then mass-separated at the National Physical Laboratory, with a collection efficiency of about 2%. Altogether six samples were prepared in this manner: Three, each of about 30 μCi, were used in the γ-spectrum studies; three more, each of about 10 μCi, were used for half-life determinations.

The γ-ray spectrometer used in this investigation consisted of a 15-cm^3 Ge(Li) detector, standard electronics units (Nuclear Enterprises, Edinburgh Series) and a 4000-channel analyser (Inter-technique). An energy calibration was obtained for the range 122 to 2754 keV using 20 standard γ-ray energies. Each calibration point was derived from the experimental data using the method of Onno,[3] and all the individual experimental points up to that at 1173 keV were combined by computer analysis to give an energy-calibration curve as a poly-

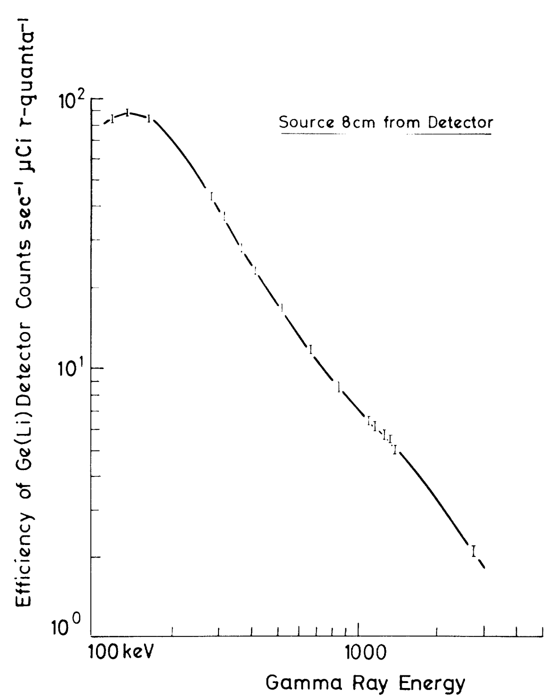

FIG. 1. Full-energy peak efficiency of Ge(Li) detector.