PHYSICAL REVIEW C VOLUME 2, NUMBER 6 DECEMBER 1970

# Discontinuity in Isotopic Mass Dispersion in Thermal-Neutron-Induced Fission of $^{235}U$

M. Talât-Erben and Binay Dağsöz (Güven)
*Department of Physical Chemistry, Faculty of Chemical Engineering, Technical University of Istanbul, Istanbul, Turkey*
(Received 13 July 1970)

The isotopic mass dispersion curves have been calculated by using a universal isobaric-charge dispersion function and the revised fission yield data for individual nuclides reported by Wahl *et al.* The mass dispersion curves are discontinuous at the most probable fragment mass value, so that a different dispersion curve applies on either side of the most probable mass, both branches being Gaussian, except at $Z = 50$ where the dispersion law deviates from the Gaussian in the region of mass numbers larger than the most probable mass. For some $Z$ values the dispersion is narrower on the light side and broader on the heavy side of the most probable mass, whereas the situation is the opposite for other $Z$ values, without apparent regularity. For a very few $Z$ values only, the two branches differ so little that they may practically be considered as defining a single curve.

## INTRODUCTION

Among the various fission processes, the thermal-neutron-induced fission of $^{235}U$ is best known. Best estimates of normal yields of fission products have been calculated on the basis of a universal Gaussian charge dispersion function[1]; a universal charge dispersion is used because no systematic variation of the width with the mass number has been established.

The mass dispersion curve, on the other hand, which represents the formation probability of isotopic fragments as a function of mass number, is experimentally unobtainable for the same obvious reason that makes the isotopic fission yield curve directly unobtainable. The mass dispersion law can, however, be deduced indirectly from data in the same manner as the isotopic yield curve, which has been deduced indirectly by calculations based on experimental facts.[1,2]

It would be of interest to calculate the mass dispersion curve for different $Z$ values and to see how its width varies with $Z$. All the data needed for this purpose are contained in a revised form in a recent paper by Wahl *et al.*[1]

## CALCULATIONS

(1) *Mass dispersion curve*. To obtain this curve for a given $Z$, each individual isotopic yield is normalized by dividing it by the sum of the isotopic yields for this particular $Z$. A smooth mass dispersion curve is constructed by plotting the normalized yield values versus mass number. The most probable mass $A_p$ is read off as the abcissa of the maximum normalized yield on this curve. However, a closer examination discloses that in most cases the curve thus constructed is composed of two different, though Gaussian, branches on either side of the most probable mass $A_p$: in other words, the mass dispersion curve has a discontinuity at $A_p$. In fact, if the logarithms of the normalized isotopic yields are plotted against $(A - A_p)^2$, two intersecting straight lines are obtained instead of one for a particular $Z$ (Fig. 1), indicating that two different Gaussian dispersion expressions are operative for the same $Z$ value.

(2) *Width parameter*. The equation for the normalized isotopic mass dispersion curve is

$$P_Z(A) = (1/\pi c)^{-1/2}\, e^{-(A-A_p)^2/c} ,$$

or

$$\log_{10} P_Z(A) = -\tfrac{1}{2}\log_{10}\pi c - (0.4343/c)(A - A_p)^2 .$$

The last equation is of the form

$$Y = aX + b ,$$

where

$$Y = \log_{10} P_Z(A), \quad X = (A - A_p)^2 ,$$
$$a = -0.4343/c, \quad b = -\tfrac{1}{2}\log_{10}\pi c .$$

The width parameter of the isotopic mass dispersion curves can, therefore, be calculated from the slopes or intercepts of the straight lines whose equations are, in turn, obtained by a least-squares calculation.

## RESULTS

It is very interesting that the mass dispersion does not follow a single Gaussian but two different Gaussian curves instead, one of which is valid for mass numbers smaller than the most probable fragment mass, while the other is operative for mass numbers larger than the most probable mass for a given $Z$ (Fig. 2). This fact is observed al-

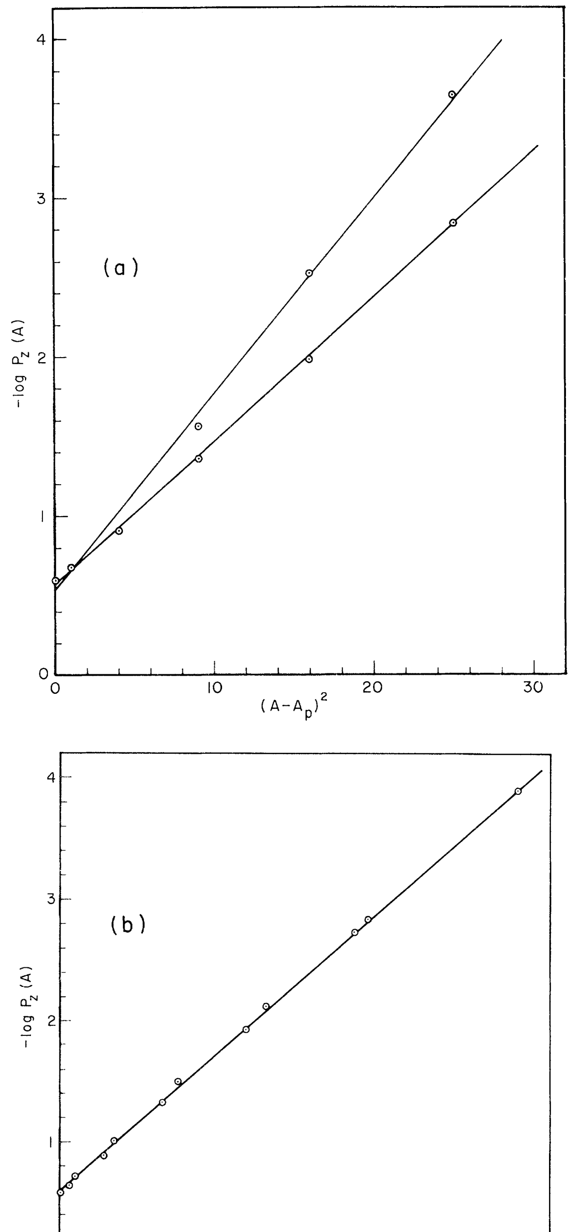


FIG. 1. (a) A plot of the logarithm of the formation probability against $(A-A_p)^2$ for $Z=53$, indicating that two different Gaussian laws are operative on either side of the most probable mass. (b) A plot of the logarithm of the formation probability against $(A-A_p)^2$ for $Z=38$. In this case the two Gaussians almost coincide and a single dispersion curve may be considered.

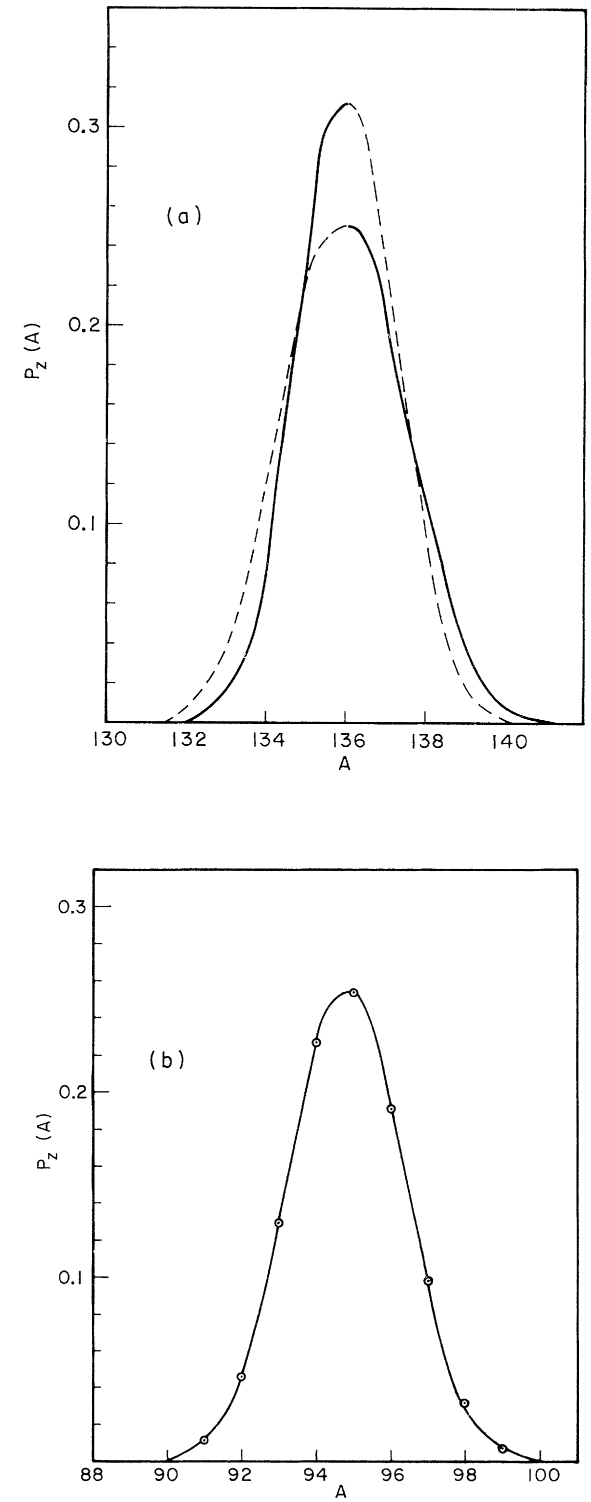


FIG. 2. (a) Isotopic mass dispersion curve for $Z=53$. As seen, different curves apply on either side of the most probable fragment mass. (b) Isotopic mass dispersion curve for $Z=38$. In this case the two branches are not clearly distinct and a single Gaussian may be considered.

most in all cases; although for a few $Z$ values the two curves are very close to each other, they still cannot be considered as strictly coincident. For some particular $Z$ values the curves are very

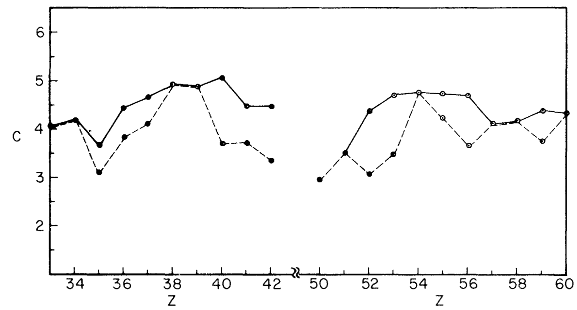


FIG. 3. The width parameter of the isotopic mass dispersion curve as a function of $Z$. The upper continuous curve represents the broader side whereas the lower dotted one corresponds to the narrower side of the dispersion. As seen, the two sides differ, especially at $Z = 52$ and 56.

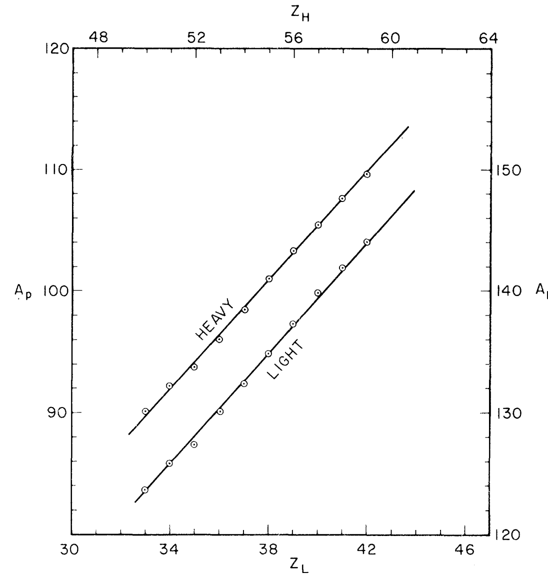


FIG. 4. The most probable fragment mass as a function of $Z$.

markedly distinct. This situation occurs at $Z = 50$, 52, 53, 55, 56, and 59 in the heavy group, and at $Z = 35$, 36, 37, 40, 41, and 42 in the light group of fission products. It should be noted that at $Z = 50$ the mass dispersion is Gaussian only for fragments lighter than the most probable fragment; in the region of heavier fragments the dispersion curve clearly deviates from the Gaussian form for $Z = 50$, and this is the only case in which a systematic deviation of the mass dispersion from the Gaussian law is observed.

Consequently, the width parameter $c$ as a function of $Z$ is a double curve composed of two branches, one extending in the light group, the other in the heavy group of fission products. The general shape of the curve is that of ram's horns as seen in Fig. 3.

The mass $A_p$ corresponding to maximum formation probability as a function of $Z$ gives two straight lines which are almost parallel (Fig. 4). The fact that two different mass dispersion curves apply for the same $Z$ value does not affect the linearity of the $A_p$ versus $Z$ curve, because, as stated above, the maxima of the curves occur at the same mass value.


[1]A. C. Wahl, A. E. Norris, R. A. Rouse, and J. C. Williams, in *Proceedings of the Second International Atomic Energy Symposium on Physics and Chemistry of Fission, Vienna, Austria, 1969* (International Atomic Energy Agency, Vienna, Austria, 1969).

[2]M. Talât-Erben and B. Güven, Phys. Rev. 129, 1762 (1963).