

$^{58}\text{Ni}(p, ^3\text{He})^{56}\text{Co}$ and $(p, t)^{56}\text{Ni}$ Reactions at 45 MeV*

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The reactions $^{58}\text{Ni}(p, ^3\text{He})$ and $^{58}\text{Ni}(p, t)$ have been studied at a proton energy of 45 MeV. An average energy resolution of 45 keV has been achieved. Angular distributions have been obtained for 26 levels of ^{56}Co and 20 states of ^{56}Ni . The experimental results have been compared with distorted-wave Born-approximation calculations, using spectroscopic amplitudes given by a microscopic calculation.

I. INTRODUCTION

In the nuclear shell model both ^{56}Co and ^{56}Ni may be thought of as having relatively simple structures. In ^{56}Ni ($Z = N = 28$) both neutrons and protons have just filled the $1f_{7/2}$ shell, while ^{56}Co may be regarded as a ^{56}Ni core plus a single $1f_{7/2}$ proton hole and one neutron outside that shell. Consequently, microscopic calculations of wave functions describing the states of these nuclei are available for both.¹⁻³ Experimentally, however, neither of these nuclei may be studied by the familiar methods of inelastic scattering or single-particle transfer reactions. Consequently, the only available results have been obtained from decay experiments,⁴⁻⁸ two-particle transfer, or charge-exchange reactions. ^{56}Co , for example, has been investigated by means of the $^{54}\text{Fe}(^3\text{He}, p)^{56}\text{Co}$ reaction⁹⁻¹² at low energy, and by means of the $^{58}\text{Ni}(d, \alpha)$,^{9, 10, 13-15} $^{54}\text{Fe}(\alpha, d)$,¹⁶ and $^{56}\text{Fe}(^3\text{He}, t)$ ¹⁷⁻²⁰ reactions. To date, however, no study of ^{56}Co with the $(p, ^3\text{He})$ reaction has been reported, primarily because of the large negative Q value for the reaction (-11.839 MeV).²¹ The previous experiments have yielded considerable information on the structure of low-lying ^{56}Co states and have located some two-particle, two-hole states, including the $T = 2$ isobaric analogs of the ground and first excited (2^+) states of ^{56}Fe .^{9, 10, 20}

The nature of these two-particle-two-hole states has also been qualitatively confirmed by the $^{56}\text{Fe}(^3\text{He}, t)$ reaction, which is particularly well suited because of the ^{56}Fe structure, for their formation. In this same study,^{18, 19} however, a number of states were observed whose angular distributions do not display the expected shapes. This failure may be due to a poor description of either the nuclear structure or the reaction mechanism. In either case, a study of the $(p, ^3\text{He})$ reaction should be helpful in understanding more ^{56}Co states.

The ^{56}Ni nucleus is more difficult to study. To date, only two kinds of experiments have been

performed: $^{54}\text{Fe}(^3\text{He}, n)$,¹¹ which measured the ^{56}Ni mass and excitation energies for three excited states; and three previous $^{58}\text{Ni}(p, t)$ experiments.²²⁻²⁴ The last of these, by Davies *et al.*,²⁴ had an energy resolution of 70 keV and identified a number of states up to excitation energies of 8.5 MeV, including the $T = 1$ isobaric analogs of several ^{56}Co levels. In view of the better energy resolution obtainable here, it seemed worthwhile to re-study the (p, t) reaction. In addition, since it was done simultaneously with the $(p, ^3\text{He})$ reaction, a comparison of the cross sections for formation of pairs of isobaric analog levels is possible.

Finally, there are new microscopic wave functions now available for both nuclei.³ They will be compared with the experimental results. The sensitivity of the calculated angular distributions to the wave functions used will provide a more quantitative test for these wave functions.

II. EXPERIMENTAL

The 45-MeV proton beam of the 88-in. Berkeley cyclotron has been used to bombard a self-supporting ^{58}Ni foil. The target was isotopically enriched ($>99\%$), and its thickness was $200 \pm 50 \mu\text{g}/\text{cm}^2$. Outgoing tritons and ^3He particles were detected by means of two solid-state silicon $E-\Delta E$ telescopes ($\Delta E = 250 \mu\text{m}$, $E = 3 \text{ mm}$), coupled to a Goulding particle identifier. Data were taken between 14 and 70° c.m. The cross-section errors which are indicated in tables or graphs are only statistical. An error of approximately 25% in the absolute cross sections results from the uncertainty in target thickness. Energy calibration of the system was accomplished by means of the ground-state Q values for $^{58}\text{Ni}(p, t)$ and $^{58}\text{Ni}(p, ^3\text{He})$ (Ref. 21) together with Q values for the same reactions on ^{16}O (Ref. 21), which was present as a target contaminant.

III. RESULTS

A. ^{56}Co

Figure 1 displays an experimental spectrum.

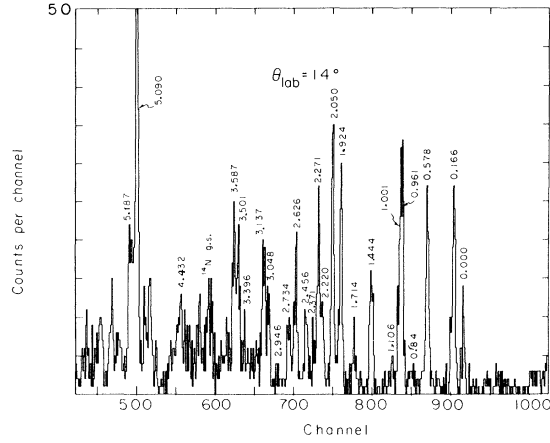


FIG. 1. Experimental spectrum for the $^{58}\text{Ni}(p, ^3\text{He})^{56}\text{Co}$ reaction.

The over-all resolution is 50 ± 5 keV. Angular distributions have been measured for 26 ^3He groups corresponding to levels in ^{56}Co up to 5.2-MeV excitation energy. In a first step, using either empirical curves relative to previously known levels in ^{56}Co , or rough distorted-wave Born-approximation (DWBA) calculations, 21 of these can be classified according to the strongest component of the angular momentum (L) of the transferred nucleon pair. Four $L=0$ transitions have been observed, eleven $L=2$, five $L=4$, and one $L=6$. The angular distributions are shown in Figs. 2–4. Table I summarizes the excitation energies and L values measured in the present experiment and compares them with other available experimental results.

B. ^{56}Ni

Triton groups corresponding to 24 levels of ^{56}Ni have been detected. The highest excitation energy observed was 8.896 MeV. A typical triton energy spectrum is shown in Fig. 5. Angular distributions were measured for 20 states. For 17 of these, L values have been confirmed or assigned. A summary of the energy levels, spins, parities, and total cross sections (integrated from 14 to 70°) is given in Table II. Also listed there are the results of the experiment of Davies *et al.*²⁴ All of the angular distributions measured in the present experiment are shown in Fig. 6.

IV. CROSS-SECTION CALCULATIONS

A. General Formalism

The general formalism for two-particle transfer reactions has been extensively described.^{25,26} Assuming a spin-isospin-dependent zero-range interaction, the two-particle transfer cross section can be written

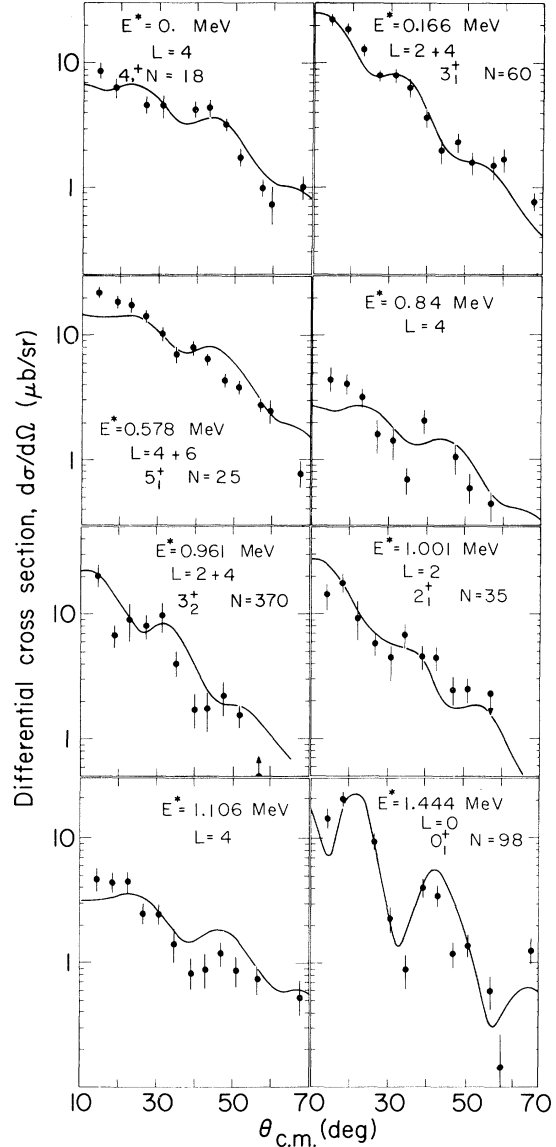


FIG. 2. Angular distribution measured for the $^{58}\text{Ni}(p, ^3\text{He})^{56}\text{Co}$ reaction. The curves are calculated using McGrory's spectroscopic amplitudes when a spin is indicated. The others must be considered as DWBA curves. N is the normalization constant as defined in the text.

$$\frac{d\sigma}{d\Omega} = \frac{\mu_A \mu_b k_b}{(2\pi\hbar^2)^2 k_a} \left(\frac{2S_b + 1}{2S_a + 1} \right) \sum_{JM\sigma_a\sigma_b} \left| \sum_{LST\Lambda} b_{ST}(T_B T_B T_T T_A T_A) D(S, T) \sum_N g_N(LSJT) B_{L\Lambda}(\theta) \right|^2, \quad (1)$$

where

$$g_N(LSJT) = \sum_{\substack{n_1 l_1 j_1 \\ n_2 l_2 j_2}} S_{AB}^{1/2} (n_1 l_1 j_1; n_2 l_2 j_2) G_N(LSJT). \quad (2)$$

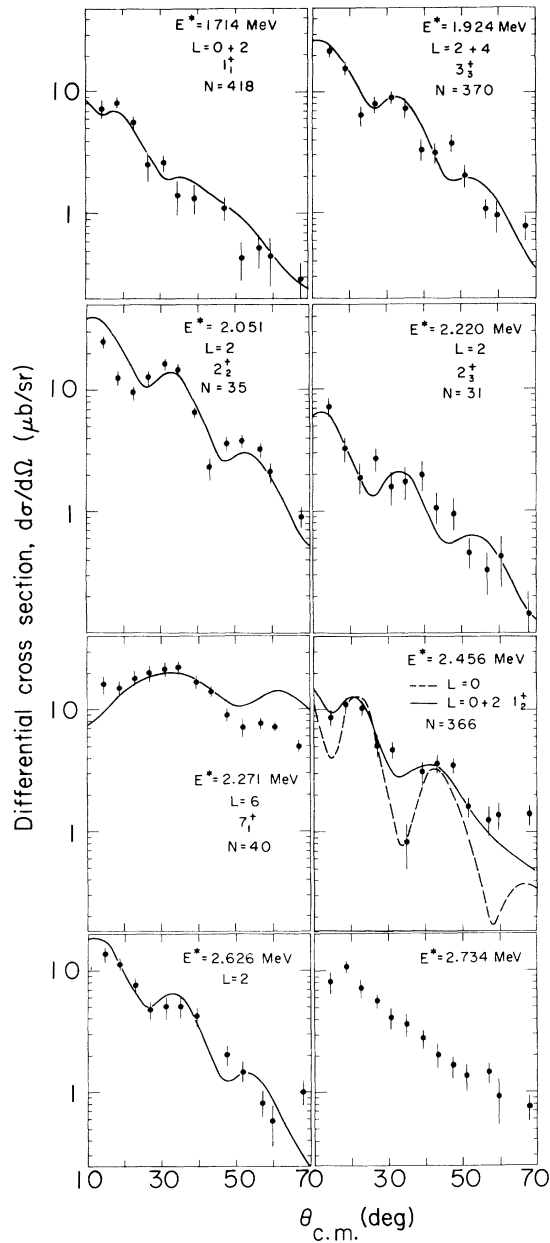


FIG. 3. Angular distributions measured for the $^{58}\text{Ni}(p, ^3\text{He})^{56}\text{Co}$ reaction. The curves are calculated using McGrory's spectroscopic amplitudes when a spin is indicated. The others must be considered as DWBA curves. N is the normalization constant as defined in the text.

Here b_{ST}^2 is essentially a spectroscopic factor for the light particle. Also, $D(S, T)$ is a spin-isospin exchange term introduced by Hardy and Towner.²⁷ It can be deduced from the force mixture which is used. In the following calculations, we have taken the value 0.42 for the ratio R :

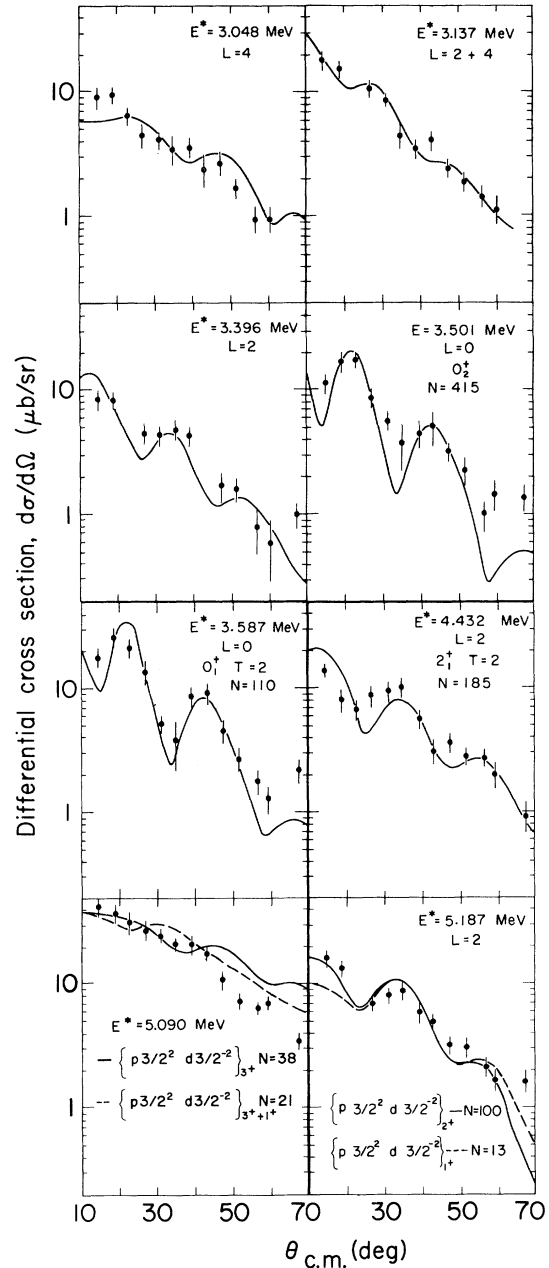


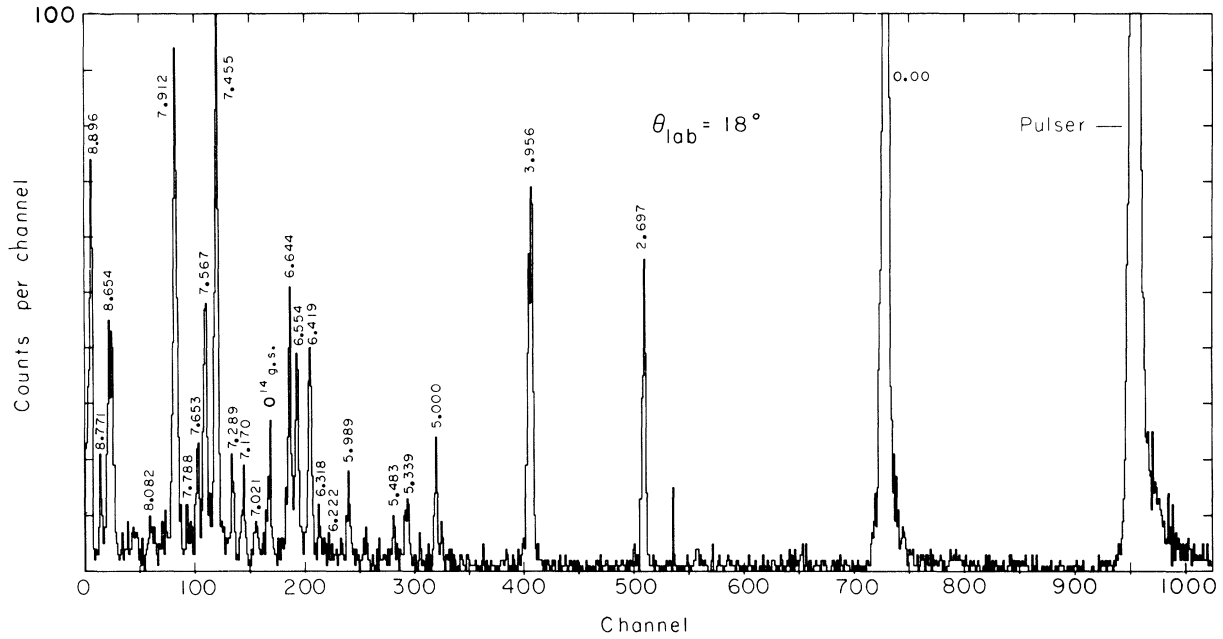
FIG. 4. Angular distributions measured for the $^{58}\text{Ni}(p, ^3\text{He})^{56}\text{Co}$ reaction. The curves are calculated using McGrory's spectroscopic amplitudes when a spin is indicated. The others must be considered as DWBA curves. The curves relative to the 5.090 and 5.187 levels have been calculated assuming pure shell-model wave functions for the ^{58}Ni ground-state and $[d_{3/2}^{-2} p_{3/2}^2]$ configurations for ^{56}Co . N is the normalization constant, as defined in the text.

$$R = \left| \frac{D(1, 0)}{D(0, 1)} \right|^2. \quad (3)$$

This corresponds to the force mixture of Gillet and

TABLE I. Experimental determinations of the ^{56}Co levels.

This work E^* J^π	Decay Ref. 8 E^* J^π		$^{54}\text{Fe}(^3\text{He}, p)^{56}\text{Co}$ Ref. 9 E^* L Ref. 10 E^* L Ref. 11 E^*		$^{58}\text{Ni}(d, \alpha)^{56}\text{Co}$ Ref. 13 E^* L Ref. 14 E^* L Ref. 9 E^* L Ref. 10 E^* L Ref. 15 E^* L		$^{56}\text{Fe}(^3\text{He}, t)^{56}\text{Co}$ Ref. 19 E^* J^π Ref. 20 E^* J^π		$^{54}\text{Fe}(\alpha, d)$ Ref. 16 E^*			
	E^* J^π	E^* J^π	E^* L	E^* L	E^* L	E^* L	E^* L	E^* J^π		E^* J^π		
0	0.0	4 ⁺	0.0	0.0	0.0	4	0	0	4			
0.166	0.1583	3 ⁺	0.155	0.167	0.166	2	0.160	0.165	0.157	2	0.160	3
0.578	0.592	5 ⁺	0.592	0.575	0.576	4	0.578	0.578	0.572	2+4	0.590	5
0.840	0.840	4 ⁺ , 5 ⁺		0.832	0.832	(4, 2)	0.829	0.810	0.827	(0.824) 4		
0.961	0.9705	2 ⁺	0.976	1.000	0.978	2	0.965	0.972	0.964	2		
1.001		2 ⁺					1.008	1.013	1.003	2	1.000	2
1.106		4 ⁺ , 5 ⁺			1.111	4	1.026					
					1.111	2	1.111	1.121	1.107	2+4		
					1.246							
1.444	1.4512	(1 ⁻ , 2 [±])	1.453	1.453	1.445		1.445				1.470	
1.714	1.7209	(1 ⁺)	1.721	1.721	1.723	(0)	1.714	1.730	(1.670)		1.720	1.720
1.924		3 ⁺	1.929	1.927	1.934	2	1.925	1.946	1.930	1.921	1.940	1.960
2.050		2 ⁺	2.067	2.051	2.087	(2)	2.056	(2.040)	2.058	2.049	2.070	
2.220		2 ⁺	2.244		2.225		2.222		2.224	(2.214) 2		
2.271		7 ⁺	2.283	2.296	2.312	6	2.291	2.300	2.285	2.284	2.272	2.290
									2.285	2.272	2.290	
2.371					2.381		2.358		2.359	2.361		2.350
2.456		1 ⁺ ?		2.471	4		2.481	(0)	2.469	2.473	2.460	
									2.608	2.597		
2.626		2 ⁺ , 3 ⁺								2.623		
					2.647				2.636	2.635	(2.636)	
									2.667	(2.655)		
2.734		1 ⁺ ?	2.744	2.740			2.650	4	2.725	2.717	2.720	
2.946			2.932	2.963	2		2.780	(0)	2.773			
									2.927	(2.917)		
3.048		4 ⁺ ?	3.069				2.971		2.971	(2.960)	3.000	3.000
									3.048	(3.035)		
										(3.050)		
										3.061	3.061	
3.137		3 ⁺ ?		3.081	2		3.080		3.077			
									3.129	3.129		
									3.176	3.166		
									3.244	3.245		
									3.295	3.286		
3.396		3 ⁺ , 2 ⁺		3.385	2				3.373	3.373		3.362
									3.431	3.422		3.423
									3.505	(3.499)		3.489
3.501		0 ⁺	3.511	0					3.514	3.514		3.522
										3.536		3.539

FIG. 5. Experimental spectrum for the $^{58}\text{Ni}(p, t)^{56}\text{Ni}$ reaction.TABLE II. Summary of results of $^{58}\text{Ni}(p, t)$ experiments.

Excitation energy, E (MeV)		Spin and parity, J^π		Integrated cross section σ (μb)
Present work	Ref. 24	Present work	Ref. 24	Present work
0.00	0.000	0^+	0^+	192.9 ± 19
2.697 ± 0.015	2.64	2^+	2^+	23.6 ± 2.8
3.956 ± 0.015	3.90	4^+	4^+	37.1 ± 4.1
5.000 ± 0.020	4.95	0^+	0^+	6.49 ± 1.50
5.339 ± 0.020	5.33	6^+	(2^+)	12.7 ± 1.7
5.483 ± 0.025		3^-		4.80 ± 1.12
5.989 ± 0.020	5.90	(4^+)	(4^+)	7.48 ± 1.37
6.222 ± 0.035		(2^+)		2.24 ± 0.74
6.318 ± 0.025		2^+		5.80 ± 1.25
6.419 ± 0.015	6.38	4^+	4^+	17.5 ± 1.93
6.554 ± 0.020		(2^+)		21.4 ± 2.55
6.644 ± 0.020	6.58	0^+	0^+	16.8 ± 2.12
7.021 ± 0.025	7.00	—		4.36 ± 1.0
7.170 ± 0.030	7.12	1^-	1^-	6.99 ± 1.37
7.289 ± 0.025		0^+		7.55 ± 1.50
7.455 ± 0.020	7.42	2^+	2^+	43.4 ± 4.3
7.567 ± 0.015	7.56	3^-	3^-	25.7 ± 2.55
7.653 ± 0.030		($1^-, 2^+$)		6.99 ± 1.37
7.788 ± 0.030				5.86 ± 1.75
7.912 ± 0.020	7.92	0^+	0^+	25.2 ± 2.7
(8.082) ± 0.100				
	8.48		2^+	
8.654 ± 0.020				
8.771 ± 0.025				
8.896 ± 0.020				

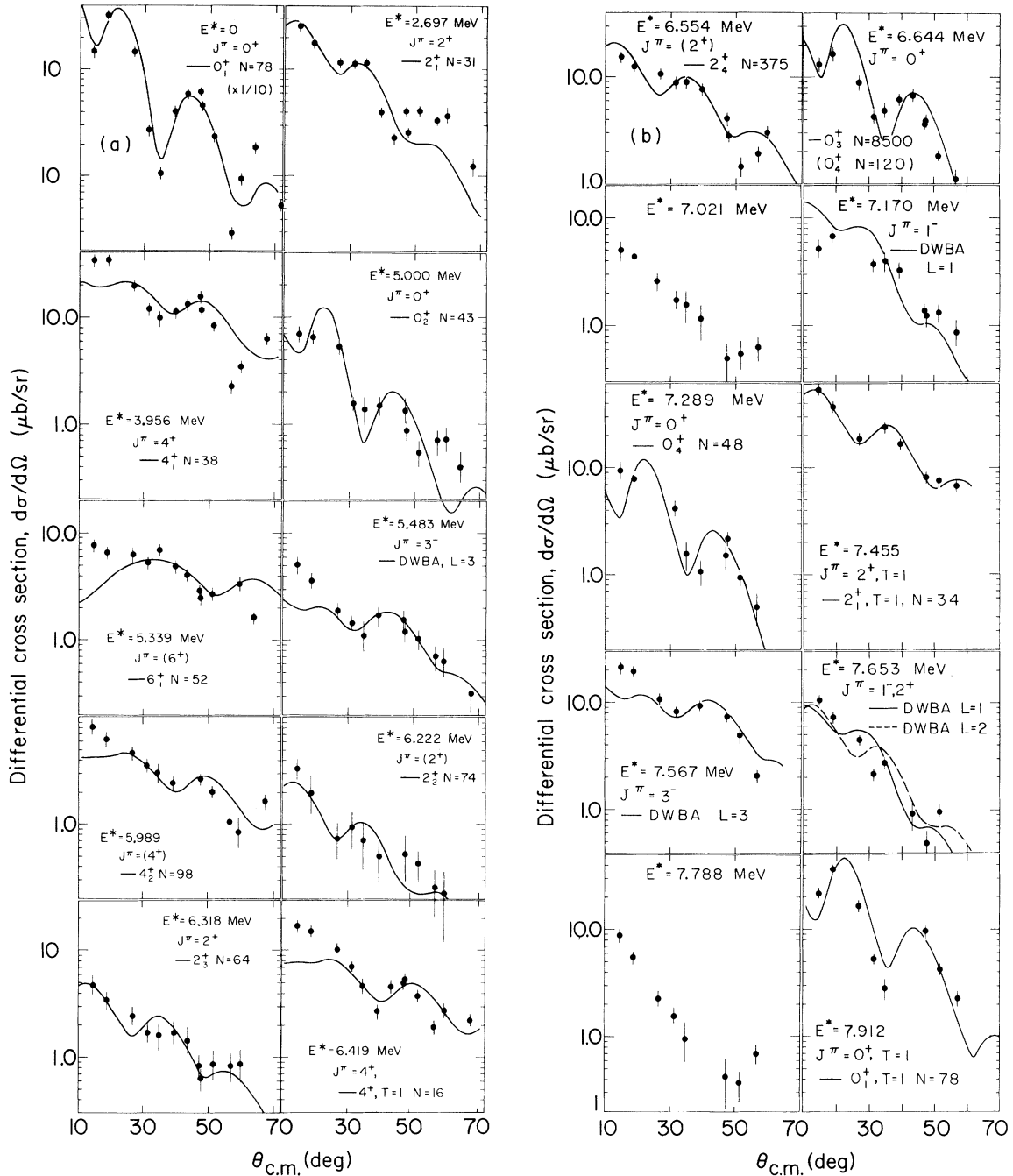


FIG. 6. (a),(b) Angular distributions measured for the $^{58}\text{Ni}(p,t)^{58}\text{Ni}$ reactions. The curves correspond to DWBA predictions as explained in the text.

and Vinh-Mau for ^{16}O .²⁸ All the quantum numbers are defined as follows:

$J(M)$ = total angular momentum of the final nuclear state (and its z component).

σ_a, σ_b = z components of spin of incoming and outgoing particles, respectively.

$L(\Lambda, S, \text{ and } T(\tau))$ = orbital angular momentum, total spin, and isospin of the c.m. of the transferred nucleon pair (and their Z components).

N = principal quantum number of the c.m. motion of the nucleon pair.

$[n_1 l_1 j_1], [n_2 l_2 j_2]$ = quantum numbers of nuclear shell from which nucleons 1 and 2 are picked up.

The spectroscopic amplitude $S_{AB}^{1/2}$ expresses the parentage of the target nucleus A , based on the final nucleus B plus two nucleons in the shells $[n_1 l_1 j_1], [n_2 l_2 j_2]$, and is exactly the quantity defined by Towner and Hardy.²⁶ The quantities $G_N(LSJT)$ are the structure amplitudes tabulated by Glendenning.^{29, 30} $B_{L\Lambda}(\theta)$ includes geometrical factors and the radial integral of the form factor.

B. Selection Rules

Assuming that in both ${}^3\text{He}$ and t , the three nucleons have a relative angular momentum 0, one finds $S+T=1$ and $\pi_i \pi_f = (-)^L$.

The selection rules can be summarized as follows for a zero-spin target:

1. $(p, {}^3\text{He})$ reaction. For the transitions to the positive-parity states in ${}^56\text{Co}$, the L values will be even; thus in the general case where the two particles are transferred from different shells,

$$\text{if } J \text{ is even, } J=L \quad \begin{cases} S=0, & T=1, \\ S=1, & T=0, \end{cases}$$

if J is odd, $S=1, T=0$, and $L=J \mp 1$.

When both of the particles are from the same shell, $J+S$ must be even and for J even only the $(S=0, T=1)$ possibility remains.

2. (p, t) reaction. The isospin of the removed neutron pair is $T=1$. Thus the selection rules are

$$L=J \text{ and } \pi_i \pi_f = (-)^L.$$

Under these conditions the (p, t) reaction allows the observation of only natural-parity states.

C. Calculation of the Cross Section

The calculation of the cross section has been carried out using Glendenning's values for the structure factors G ,^{29, 30} and the computer code DWUCK³¹ modified by J. C. Hardy to perform two-nucleon transfer calculations, including coherence effects caused by spin-orbit terms in the optical potentials.

D. Optical Potentials

Several optical potentials have been tested in each of the particle channels. Three different proton potentials have been used, two triton, and two ${}^3\text{He}$. These are listed in Table III. Various combinations of proton-triton and proton- ${}^3\text{He}$ potentials have been tested on the (p, t) experiment to determine which gave the best fit to the ground-state angular distribution. The best fit was obtained using Set 1 for the protons and Set 6 for the tritons. The use of potential 5 for the tritons and ${}^3\text{He}$ nuclei gave the same shapes and the same relative strengths for all states, although calculated absolute cross sections were changed. Potentials 4 and 7 gave significantly poorer fits to experimental angular distributions. The results presented here used the same potentials for both the (p, t) and $(p, {}^3\text{He})$ reactions.

E. Spectroscopic Factors and Shell-Model Wave Functions

It can be seen in Eq. (1) that the expression for the cross section involves a coherent sum over various single-particle components (nlj). Consequently it is impossible, given experimental cross sections, to determine spectroscopic amplitudes.

TABLE III. Optical potentials used in the DWBA calculations.

Particle	Volume real part			Imaginary part						Spin orbit			Ref.	Set no.
	V (MeV)	r (fm)	a (fm)	W_I (MeV)	r_I (fm)	a_I (fm)	W_D (MeV)	r_D (fm)	a_D (fm)	V_{SO} (MeV)	r_{SO} (fm)	a_{SO} (fm)		
Proton	-49.49	1.109	0.782	-6.5	1.477	0.495	-2.08	1.477	0.495	5.53	1.071	0.641	a	1
	-45.05	1.16	0.75	-6.63	1.37	0.63	-1.22	1.37	0.63	6.04	1.064	0.738	b	2
	-42.7	1.211	0.707	-1.5	1.067	0.545	-10.3	1.067	0.545	7.67	1.211	0.707	36	3
${}^3\text{He}$	-137.4	1.082	0.805	-16.1	1.66	0.795							36	4
	-172.6	1.147	0.712	-20.16	1.562	0.802							c	5
t	-154.3	1.24	0.677	-26.11	1.431	0.85							d	6
	-169.7	1.16	0.732	-22.8	1.510	0.796							d	7

^aM. P. Fricke, E. E. Gross, B. J. Morton, and A. Zucker, Phys. Rev. **156**, 1207 (1967).

^bM. P. Fricke and G. R. Satchler, Phys. Rev. **139**, B567 (1965).

^cP. E. Hodgson, Advan. Phys. **17**, 563 (1968), and included references.

^dE. R. Flynn, D. D. Armstrong, J. G. Berry, and A. G. Blair, Phys. Rev. **182**, 1113 (1969).

However, this also implies that the calculation of two-particle transfer cross sections provides a test of not only the magnitude but the signs of various components of any theoretical wave function.

For the present work, wave functions and two-particle spectroscopic factors have been calculated by McGrory³ and used for the ⁵⁸Ni ground state, and all the states of ⁵⁶Co and ⁵⁶Ni. These were calculated assuming an inert ⁴⁰Ca core and configurations involving up to two holes in the $1f_{7/2}$ shell, with the remaining nucleons distributed over the $2p_{3/2}$, $2p_{1/2}$, and $1f_{5/2}$ shells. The single-particle energies used were those which best reproduced the ⁵⁷Co spectrum.³² Matrix elements were calculated using the interaction of Kuo and Brown.³³

The states which resulted from this calculation are shown and compared with experiment in Figs. 7 and 8. It is clear that an unambiguous comparison of theory with experiment based on excitation

energy alone is not possible. In addition, determinations based on the shape of the angular distributions are also sometimes ambiguous. For instance, we have observed in ⁵⁶Co 11 $L=2$ transitions, which may correspond to either 2^+ or 3^+ states. The microscopic calculations do not predict very dramatic changes in the angular distributions between 2^+ and 3^+ , as shown in Fig. 9. Consequently, the criteria adopted for this comparison are based on: first, the shape (dominant L -value) of the cross section; second, the order in excitation energy; and third, the strength of the calculated cross section compared with experiment, expressed in terms of the normalization factor N , where

$$N = \sigma(\text{experimental})/\sigma(\text{theoretical}).$$

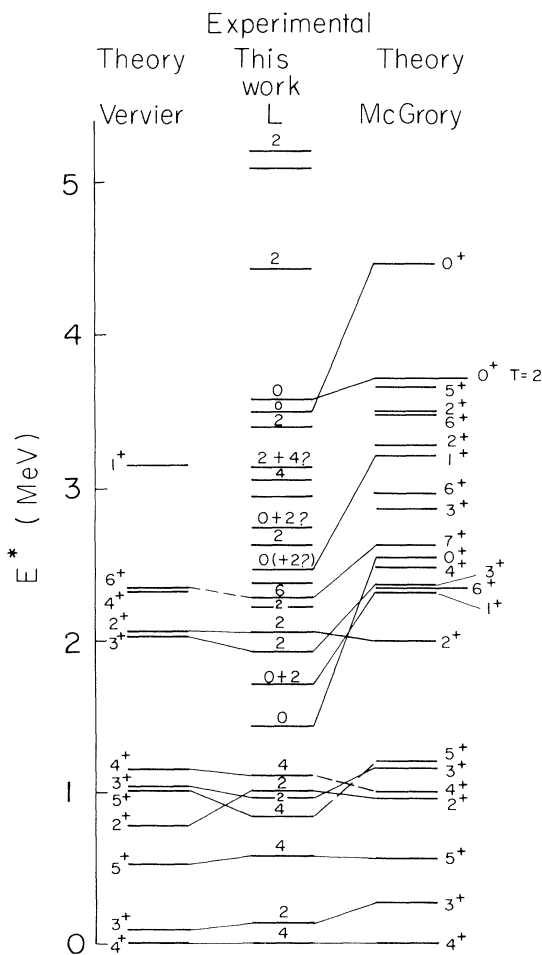


FIG. 7. Comparison of the ⁵⁶Co experimental spectrum to the theoretical predictions of Vervier and McGrory.

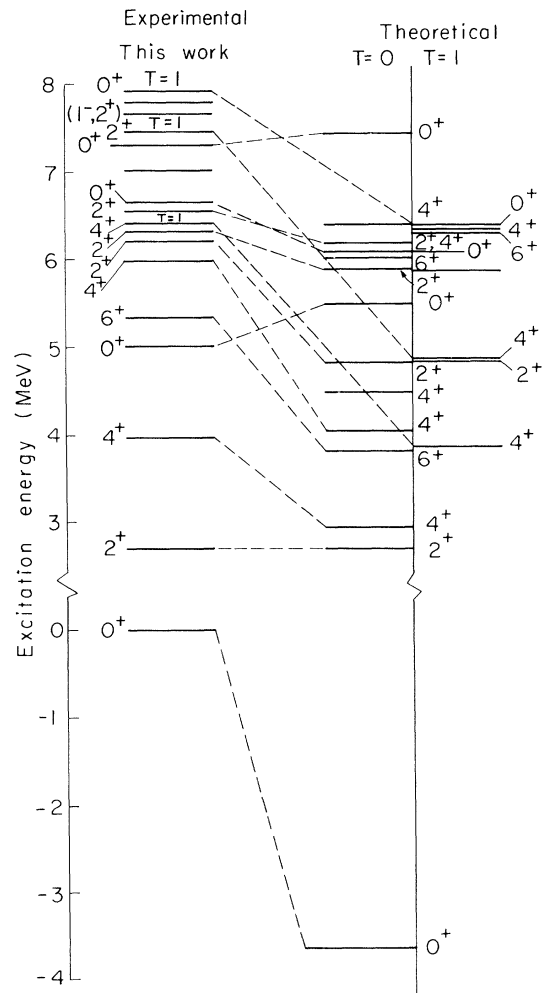


FIG. 8. Comparison of the ⁵⁶Ni experimental spectrum to the predictions of McGrory. Only the natural parity states are shown. For a more convenient presentation, the energies of the levels predicted by McGrory have been aligned on the first experimental 2^+ state.

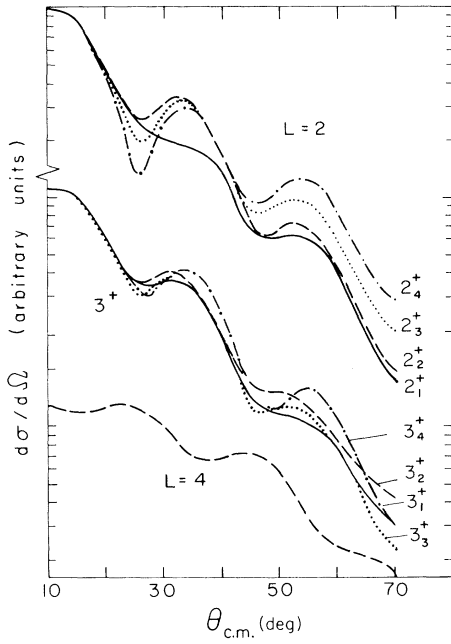


FIG. 9. Calculated angular distributions for the $^{58}\text{Ni}(p, ^3\text{He})^{56}\text{Co}$ reaction. The angular distributions corresponding to the first four 3^+ states in McGrory's calculations, are compared to those for first four 2^+ ($L=2$) and first 4^+ states.

V. DISCUSSION

A. ^{56}Co

Below 2.5 MeV, the level density of ^{56}Co is such that the present experiment should be able to resolve most states. That this was accomplished is indicated by the excellent agreement of excitation energies with other high-resolution experiments. Above 2.5 MeV, however, the level density increases so that, except for the selectivity of the reaction mechanism, very few single states should be resolved with our resolution. From this point of view a comparison of various experimental results will be given.

Within simple shell-model considerations, the lowest-lying states of ^{56}Co are expected to be one-particle-one-hole states of the form

$$[(\pi f_{7/2})^{-1}(\nu p_{3/2})]_{2,3,4,5}, [(\pi f_{7/2})^{-1}(\nu p_{1/2})]_{3,4},$$

$$[(\pi f_{7/2})^{-1}(\nu f_{5/2})]_{1,2,3,4,5,6}.$$

Vervier's wave functions¹ indicate that these levels should lie below approximately 3.5 MeV.

The two-particle-two-hole states should begin to appear at higher energies, the lowest group being of the configuration

$$[(\pi f_{7/2})^{-2}(\pi p_{3/2})(\nu p_{3/2})]_{\delta \dots 1}^{T=1};$$

the centroid for this configuration is expected at

approximately 2.5 MeV.¹⁰ Two-particle-two-hole levels may be pictured in terms of a "weak-coupling" model used by Arima, Horiuchi, and Sebe³⁴ for ^{16}O and Sherr *et al.* for ^{42}Sc .³⁵ Let us adopt the following notations to represent a two-particle-two-hole state:

$$\frac{\alpha|\beta}{\gamma|\delta} (T_p, J_p) / (T_h, J_h),$$

where α and β represent the number of protons and neutrons, respectively, outside a closed $f_{7/2}$ shell; γ and δ the number of protons and neutrons in the $f_{7/2}$ shell; $T_{p(h)}$ and $J_{p(h)}$ represent the total isospin and spin of the particles (holes). Any two-particle-two-hole state of ^{56}Co may then be represented by the following configurations:

$$|1\rangle = \frac{1}{6} \left| \frac{1}{8} T_p=0, T_h=1 \right\rangle,$$

$$|2\rangle = \frac{2}{7} \left| \frac{2}{7} T_p=1, T_h=0 \right\rangle,$$

$$|3\rangle = \left\{ \left(\frac{1}{\sqrt{2}} \left| \frac{2}{7} - \frac{1}{\sqrt{2}} \frac{1}{6} \frac{1}{8} \right\rangle T_p=1 \right) / \left(\frac{1}{\sqrt{2}} \left| \frac{2}{7} - \frac{1}{\sqrt{2}} \frac{1}{6} \frac{1}{8} \right\rangle T_h=1 \right) \right\}_{T=1},$$

$$|4\rangle = \left\{ \left(\frac{1}{\sqrt{2}} \left| \frac{2}{7} + \frac{1}{\sqrt{2}} \frac{1}{6} \frac{1}{8} \right\rangle T_p=1 \right) / \left(\frac{1}{\sqrt{2}} \left| \frac{2}{7} + \frac{1}{\sqrt{2}} \frac{1}{6} \frac{1}{8} \right\rangle T_h=1 \right) \right\}_{T=2}.$$

Each of these configurations, however, is similar to a known nucleus. The outer particles in $|1\rangle$, for example, can exist in states which are already known, namely the ($T=0, J=1, 3$) states of ^{58}Cu and, similarly, the holes in $|1\rangle$ form the ($T=1, J=0, 2, 4, 6$) levels of ^{54}Fe . Assuming a J_p, J_h, J independent particle-hole interaction, one may construct a series of $T=1$ states of ^{56}Co by coupling all the $T=0$ ($J=1, 3$) states³⁶ of ^{58}Cu to the $T=1, J=0, 2, 4, 6$ states³⁷ of ^{54}Fe . Using Zamick's formulation,³⁸ the energies for levels of configuration $|1\rangle$ corresponding to the coupling of the ^{58}Cu 1^+ and 3^+ states to the ^{54}Fe ground state will be given by

$$E_{1^+,3^+} = M(^{58}\text{Cu}_{1^+,3^+}) - M(^{56}\text{Co}) + M(^{54}\text{Fe}) - M(^{56}\text{Ni}) - 4a + 2c.$$

This energy will be compared with that of the previously known 0^+ analog (3.587) and antianalog (1.444-MeV) states, which are

$$E(0^+)_{T=1,2} = \frac{1}{2} \{ M(^{58}\text{Cu})_0 + M(^{56}\text{Co}) + M(^{54}\text{Fe}) - M(^{56}\text{Ni}) + 2c \}$$

$$+ \frac{1}{2} \{ M(^{58}\text{Ni}) - M(^{56}\text{Co}) + M(^{54}\text{Co}) - M(^{56}\text{Ni}) \} - 4a \mp b,$$

where a is the center of gravity of particle-hole states, c is the Coulomb particle-hole energy, and b characterizes the separation of the $T=1$ and $T=2$ centers of gravity.³⁸ The sign $+$ refers to the

analog state. In ^{56}Co the difference of the energies of the ($T=1$) and ($T=2$) 0^+ states equals $2b$ and yields $b=1.07$ MeV. This value is smaller than usual.³⁸ The Coulomb energy c will be taken as -0.4 .³⁸ Taking as reference the 0^+ ($T=1$) 1.444-MeV state, we can eliminate the a parameter. This yields finally for the 1^+ and 3^+ states of configuration $|1\rangle$ predicted energies which are 2.17 and 2.62 MeV, respectively. In the same way, states of the form $|2\rangle$, $|3\rangle$, and $|4\rangle$ can be constructed by coupling the known levels of ^{58}Ni (Ref. 8) and ^{54}Co ,³⁹ or ^{58}Cu (Ref. 36) and ^{54}Fe (Ref. 37), with the proper isospin. In particular, 7^+ and 1^+ levels of configuration $|2\rangle$ can be predicted at 2.86 and 3.78 MeV, respectively.

One should expect to see only a few of these states, since most of them are capable of mixing with some nearby level. In particular, these 0^+ , 1^+ , 3^+ , 7^+ , and second 1^+ states might be relatively pure. Our discussion in terms of this model will be restricted to these levels.

Examination of the configurations shows immediately that states of the form $|1\rangle$ will be preferentially excited by ($^3\text{He}, p$) reactions, $|2\rangle$ will be enhanced in ($p, ^3\text{He}$) reactions, while $|3\rangle$ and $|4\rangle$ should be equally strong in both reactions.

To compare the observed cross section with theory in a quantitative way, we shall use McGrory's wave functions and distorted-wave Born approximation (DWBA) as outlined in Sec. IV. It is necessary before proceeding, however, to point out several remarks on this calculation. First, at the proton energy available ($E_p=45$ MeV), it is quite possible to observe pickup of nucleons from deeper shells, exciting states such as

$[(d_{3/2})^{-2}(p_{3/2})^2]$. These configurations are not included in McGrory's calculations. Secondly, since the absolute value of the ($p, ^3\text{He}$) cross section is not calculated, agreement with observed strengths will be judged as reasonable if the normalization constant $N = \sigma(\text{experiment})/\sigma(\text{theoretical})$ agrees within a factor of 2 with an average normalization $\langle N \rangle = 37$, determined from the four states which are assumed to be well known as described by the $[(\pi f_{7/2})^{-1}(\nu p_{3/2})]_{2+3+4+5+}$ configuration. These levels are the ground state (4^+), 0.166-MeV (3^+), 0.578-MeV (5^+), and 1.001-MeV (2^+) levels. The calculated wave functions for these states include effectively a strong component of this configuration. There is, however, some ambiguity for the 1.001-MeV level, since the 0.961-MeV state also displays an $L=2$ angular distribution and has nearly the same strength. Assignment of $J^\pi=2^+$ to the 1.001-MeV state is based on a better agreement with the shape of the angular distribution. A summary of the experimental results compared to the McGrory calculation is presented in Table IV. The 0.961-MeV state is tentatively assigned as the second level with $J^\pi=3^+$, although the rather large normalization makes this questionable. Two weak states, principally $L=4$, are seen at 0.84 and 1.106 MeV. In (d, α) experiments,¹⁵ a state was seen at 0.824 MeV with $L=4$ and another at 1.107 MeV with $L=2+4$. Both of these states give rather poor results when compared with either 5_2^+ or 4_2^+ states of McGrory's calculation.

The transition to the 1.444-MeV state is principally $L=0$. Its spin has previously^{6, 7, 10, 18} been limited to be 1^- or 0^+ , so that the assignment $J^\pi=0^+$, $T=1$ may now be made. This state would

TABLE IV. Numerical results for the $^{58}\text{Ni}(p, ^3\text{He})^{56}\text{Co}$ experiment. Given for each level are the energy (MeV), strongest transferred angular momentum (L), cross sections integrated between 14 and 62° c.m., the possible spin and parity referred to McGrory's predictions, and the corresponding normalization constant N , as defined in the text.

E (MeV)	L	σ (μb)	J_n^π	N	E (MeV)	L	σ (μb)	J_n^π	N
0.0	4	9.79 ± 1.68	4_1^+	18	2.371 ± 0.015	...	7.61 ± 2.3
0.166 ± 0.010	2	15.8 ± 1.90	3_1^+	60	2.456 ± 0.015	0	12.8 ± 2.2	1_2^+	366
0.578 ± 0.010	4	23.4 ± 2.80	5_1^+	25	2.626 ± 0.015	2	$11.8 \pm 2.$	$2^+, 3^+$	
0.84 ± 0.015	4	3.74 ± 1.25	4_2^+	170	2.734 ± 0.015	0+2?	9.54 ± 1.68	$1^+?$	
			5_2^+	8					
0.961 ± 0.015	2	10.9 ± 2.2	3_2^+	370	2.946 ± 0.020	...	2.2 ± 0.9		
1.001 ± 0.015	2	14.4 ± 2.9	2_1^+	35	3.048 ± 0.020	4	9.8 ± 1.7	4_3^+	14
1.106 ± 0.015	4	4.06 ± 1.0	4_2^+	210	3.137 ± 0.015	2+4?	14.3 ± 2.3	$3^+?$	
			5_2^+	10					
1.444 ± 0.015	0	11.9 ± 1.8	0_1^+	98	3.396 ± 0.015	2	9.42 ± 1.9	$2^+, 3^+$	
1.714 ± 0.015	0+2	5.67 ± 1.56	1_1^+	418	3.501 ± 0.015	0	15.7 ± 3.4	0_2^+	415
1.924 ± 0.015	2	15.2 ± 2.3	3_3^+	370	3.587 ± 0.015	0	19.5 ± 3.7	$0_1^+ T=2$	110
2.050 ± 0.015	2	22.6 ± 2.3	2_2^+	35	4.432 ± 0.020	2	15.8 ± 2.4	$2_1^+ T=2$	185
2.220 ± 0.015	2	4.49 ± 1.12	2_3^+	31	5.090 ± 0.020		51.4 ± 6.4		
2.271 ± 0.010	6	41.2 ± 4.1	7_1^+	40	5.187 ± 0.020	2?	15.4 ± 6.2	$1^+, 2^+?$	

correspond to a one-phonon pairing vibration state taking ^{56}Ni ground state as reference.⁴⁰ As expected for a 0^+ , the 1.444-MeV level is not excited in (d, α) , but is reasonably populated in both $(^3\text{He}, \rho)$ and $(\rho, ^3\text{He})$, as shown in Fig. 10, which gives a comparison of the integrated cross sections for these two reactions, as well as for the $^{56}\text{Fe}(^3\text{He}, t)^{56}\text{Co}$ reaction at 30° . The relative strength of the 1.444-MeV state in the two reactions is a further indication that this level is the $0^+ T=1$ state with the configuration $|3\rangle$ in the weak-coupling model. Also, as predicted for such a state, it is very weakly excited in $(^3\text{He}, t)$ reactions,^{18,19} where, in fact, it exhibits an angular distribution of $L=1$ shape.¹⁸ This is not surprising, since several known 0^+ antianalog states⁴¹ exhibit angular distributions with $L=1$ shapes. The cross section predicted for this state (0_1^+) by McGrory is somewhat small.

The 1^+ nature of the 1.714-MeV level has been previously established. It furnishes an example of well-mixed L values (0, 2) for an unnatural parity state. It is also assigned 1^+ in the most recent (d, α) experiment.¹⁵ The fact that this state is strongly excited in the $(^3\text{He}, \rho)$ experiments suggests that it might correspond to the weak-coupling 1^+ of configuration $|1\rangle$ predicted at 2.17 MeV. How-

ever, this state is also populated in $(\rho, ^3\text{He})$, indicating that it also contains some admixture of other states, such as the 1^+ of form $|2\rangle$ predicted at 3.78 MeV. McGrory's wave function for this state (1_1^+) contains many terms, the strongest of which are types $|1\rangle$ and $|3\rangle$. Unfortunately, the calculated cross section is somewhat small, indicating perhaps that a stronger type- $|2\rangle$ component is needed.

We have measured three $L=2$ angular distributions (1.924, 2.050, and 2.220 MeV) in agreement with previous experiments. The 1.924-MeV state is observed in both $(^3\text{He}, \rho)$ and $(\rho, ^3\text{He})$ experiments, it has been proposed as the 2p-2h antianalog 3^+ state.¹⁰ Angular distributions calculated using McGrory's wave functions suggest that these three states should be 3_3^+ , 2_2^+ , and 2_3^+ , respectively.

The 2.271-MeV level is the only $L=6$ transition observed in the present experiment. Selection rules allow $J=5, 6, 7$. The state has also been reported as strongly excited by Schneider and Daehnick¹⁵ at 2.272 MeV. Since (d, α) particularly favors configurations such as $(f_{7/2})^{-2}_{J=7}$, the 7^+ assignment seems most probable. This level corresponds to the 7^+ of form $|2\rangle$ predicted at 2.86 MeV by the weak-coupling model. This is particularly favorable for pickup reactions which are the only ones to report this state. Calculations with McGrory's wave functions yield a reasonable fit for 6^+ or 7^+ . However, the strength calculated for the 6_1^+ ($N=220$) is much more reasonable than for the 7_1^+ is mainly $[(f_{7/2})^{-2}_7(\rho_{3/2})^2_{0,2}]_{7^+}$ which is type $|2\rangle$, as expected.

We have observed no $L=2$ transition near 2.3 MeV. Our resolution did not allow us to separate this state from the observed 2.271-MeV level, but if the $(\rho, ^3\text{He})$ experiment allows the excitation of an $L=2$ level here, it is weakly excited. This suggests that the strong ($L=2$) 2.296-MeV level observed in Ref. 10 can be the 3^+ state of configuration $|1\rangle$ predicted at 2.62 MeV by the weak-coupling model. The 2.371-MeV state is weakly excited and yields a structureless angular distribution.

The well-separated 2.456-MeV level displays an $L=0$ angular distribution but can be compared with the calculated curves corresponding to either the 0_2^+ ($T=1$) or 1_2^+ ($T=1$) states with comparable normalizations (Fig. 2). No possible 1^+ state has been observed in the previous $(^3\text{He}, \rho)$ experiments (Table I). This level could be compared with the predicted 3.78-MeV 1^+ state described by the component $|2\rangle$, but it is unexpectedly weakly excited, and its excitation energy is rather low. On the other hand, this state should not be observed at all in the $(^3\text{He}, \rho)$ experiment (Fig. 10), whereas the observation of an eventual 0^+ state is allowed.

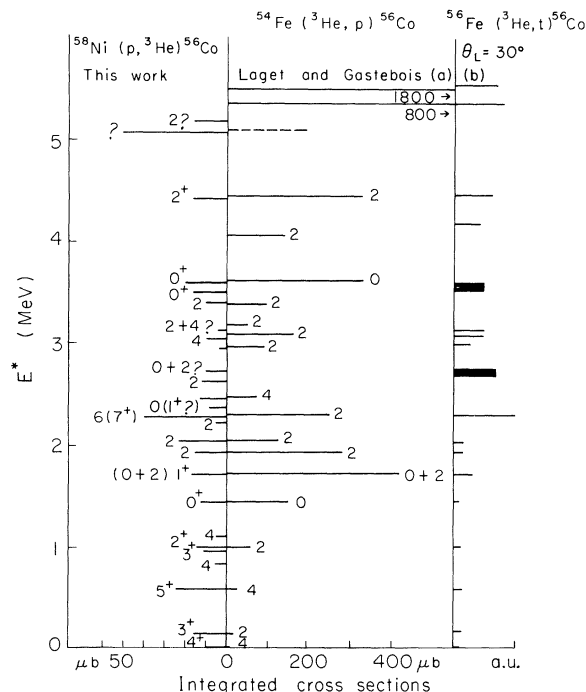


FIG. 10. The integrated cross sections for the levels observed in the $^{58}\text{Ni}(\rho, ^3\text{He})^{56}\text{Co}$ reaction (this work) are compared to (a) those measured in the $^{54}\text{Fe}(^3\text{He}, \rho)^{56}\text{Co}$ experiment of Ref. 10 and (b) to a ^{56}Co spectrum of the $^{56}\text{Fe}(^3\text{He}, t)$ reaction (Ref. 19). The lines are proportional to the strengths.

The observation of a single 2.460-MeV level in the Pittsburgh (d, α) experiment¹⁵ is a good indication that this level is 1^+ and not 0^+ , but this state corresponds probably to a more complicated structure.

Levels Above 2.5 MeV

Above 2.5 MeV, the level density is increased, making more questionable any further identification with McGrory's predicted states, except for a few selected states.

The 2.626-MeV level displays an $L = 2$ angular distribution which suggests a 2^+ or 3^+ spin assignment, if the (d, α) experiment had not indicated that there is a doublet at 2.597–2.623 MeV.

The 2.736-MeV level displays a straight-line angular distribution which might suggest a 1^+ assignment. But once more the most recent (d, α) experiment¹⁵ reports a doublet at corresponding energy, just as for the 3.048 ($4^+?$), 3.137 ($3^+?$), and 3.396 ($2^+, 3^+?$) levels.

Analog States

The existence of an $L = 0$ doublet around a 3.55-MeV excitation energy has been observed by Belote, Dorenbusch, and Rapaport.⁹ This doublet has also been investigated by the ($^3\text{He}, t$) experiments.^{17–20} Levels are reported at 3.587 and 3.585 MeV in (d, α) experiments,^{9,15} but they are weakly excited and can be other states. Dzubay *et al.*²⁰ have reported eight states between 3.362 and 3.614 MeV (Table I). They definitely assign the 3.522- and 3.592-MeV states spins 0^+ ($T = 1$) and 0^+ ($T = 2$), respectively, with a certain isospin impurity. We have measured $L = 0$ angular distributions for levels at 3.501 and 3.587 MeV.

If one refers to the weak-coupling model, dealing with pure configurations, the ($p, ^3\text{He}$) reaction is expected to select the term $\frac{0}{7} | \frac{2}{7}$ in components $|3\rangle$ and $|4\rangle$. Then the ratio of the expected cross sections for the analog $T = 2$ and antianalog $T = 1$ states is

$$R = \frac{d\sigma/d\Omega(T=2)}{d\sigma/d\Omega(T=1)} = 1.$$

Matching the experimental angular distributions for the 3.587- and 1.444-MeV states, we find $R = 1.6 \pm 0.4$, which is compatible with the previous determinations of 1.67¹² and 1.2⁹ in ($^3\text{He}, p$). It is difficult to draw any significant conclusion from this ratio with respect to the pairing vibration model, since in our experiment the cross-section ratio can be modified by the presence of additional unresolved states close to the 3.587-MeV level. In addition, however, a small amount of four-particle-two-hole components in the ^{58}Ni ground state can drastically affect the experimental value for this ratio.

The ratio of the cross sections for the 3.587- and 3.501-MeV levels has been found to be 1.4 ± 0.5 in the present experiment. In the same way this ratio has to be compared with the value of 2 found by Dzubay²⁰ in ($^3\text{He}, t$), and the value of 2 drawn from the Belote, Dorenbusch, and Rapaport ($^3\text{He}, p$) experiment.⁹ McGrory's wave functions do not take into account the isospin mixing observed for these states by Dzubay *et al.*²⁰ They lead to calculated angular distributions which are both too weak, but the large normalization we get for the 3.501-MeV state suggests this level is not the 0^+ , $T = 1$ of McGrory. The calculated cross section for the previously known analog 2^+ , $T = 2$ state is slightly small.

Additional States

The 5.090- and 5.187-MeV levels are strongly excited, especially the first one. The angular distribution for the 5.187-MeV level can be classified as $L = 2$, but that corresponding to the 5.090 MeV does not display any significant pattern. Considering that the most strongly excited state in the low-energy spectrum corresponds to an $(f_{7/2}^{-2}, p_{3/2}^2)_{7^+}$ configuration, the strength of the 5.090-MeV level suggests that it might result from the pickup of a deeper $d_{3/2}$ nucleon pair, leading to the 3^+ state in the $(d_{3/2}^{-2}, p_{3/2}^2)$ configuration.

Simple considerations allow an estimate for the excitation energy of this 3^+ state. It is possible to evaluate the spacing ΔE between the $(f_{7/2}^{-2}, p_{3/2}^2)_{7^+}$ state at 2.271 MeV and the $(d_{3/2}^{-2}, p_{3/2}^2)_{3^+}$ state:

$$\Delta E = \epsilon_p + \epsilon_n + M_1 - M_2,$$

where ϵ_p and ϵ_n are the proton and neutron single-particle energy differences between the $1f_{7/2}$ and $1d_{3/2}$ shells, respectively, which can be taken from the work of Gillet, Giraud, and Rho,⁴² and are $\epsilon_p = \epsilon_n = 2.1$ MeV. M_1 and M_2 are the residual interaction matrix elements between, respectively, two $f_{7/2}$ holes coupled to 7^+ , $T = 0$ and two $d_{3/2}$ holes coupled to 3^+ , $T = 0$. M_1 can be calculated in ^{38}K using the code PHYLLIS⁴³ with True's⁴⁴ potential and is found to be 2.4 MeV. M_2 is already known to be 2.6 MeV.¹⁶ Thus the 3^+ state is expected to be about 6.3-MeV excitation and the 1^+ at 7 MeV. These are not too far from the measured energies. Assuming then that both levels have $T = 1$ and a simple shell-model wave function for the ^{58}Ni ground state, we have calculated angular distributions for spins 0^+ , 1^+ , 2^+ , and 3^+ .

Figures 2–4 show that the calculated angular distribution for the 3^+ state does not agree very well with the experimental results for the 5.090-MeV state. A better agreement is obtained if the

curve calculated for the 1^+ state of the same configuration is added (dashed curve). That could indicate that either this level is not resolved from another $L=2$ state, or that there are in the wavefunction configuration mixtures which provide a strong $L=2$ component.

In contrast, the 5.187-MeV angular distribution is well reproduced by any $L=2$ calculation ($J^\pi = 2^+$ or 1^+). The renormalizations for these calculations are given with the figures and do not allow any further identification. If the 5.090-MeV state is a 3^+ with the proposed structure, then it should be observed in the $^{58}\text{Ni}(d, \alpha)$ experiment. Hjorth¹⁴ observed a strong level at 5.18-MeV excitation energy which could be one of these states. Furthermore, such a level must be observed in other $(\rho, {}^3\text{He})$ or (d, α) experiments, on the iron isotopes for example. The excitation energy of these states can be predicted either from the ground state of the target nucleus or from the first strong 7^+ if it is known. In ^{52}Mn , for instance, the 7^+ state is known⁴⁵ at 0.85 MeV, and one would expect such a state about 2.75 MeV above the 7^+ (as in ^{56}Co), which is an excitation energy of about 3.6 MeV.

B. ^{56}Ni

The states observed in the present experiment include all but one of those reported by Davies *et al.*²⁴ In addition, the better energy resolution of the present experiment permitted detection of 10 new levels as indicated in Table II. Many of the excitation energies reported here appear to be inconsistent with those reported in Ref. 24. However, they agree very well with the result of Miller and Kavanagh,¹¹ and are consistent with our energy determination for ^{56}Co .

L -value assignments agree with Ref. 24 with one exception. The 5.339-MeV state has been reported to be 2^+ , while the present work measures an angular distribution which is flatter than a 2^+ , suggesting perhaps a 6^+ .

Of the other five states for which spin assignments have been made, two (6.222, 6.554) must be considered only tentative, inasmuch as agreements with experimental data are rather poor. The assignments of 3^- to the state at 5.483 MeV, 2^+ to the 6.318-MeV state, and 0^+ to the 7.289-MeV state appear more certain.

The possible energy levels of ^{56}Ni may be constructed by considering ^{56}Ni to consist of a closed shell plus two paired valence neutrons in the $\rho_{3/2}$, $f_{5/2}$, or $\rho_{1/2}$ shells. A (ρ, t) experiment would then be expected to excite states of the form $(f_{7/2})^{16}_{J=0}$ by pickup of the two valence neutrons; $[(f_{7/2})^{15}(\rho_{3/2})]_{J=2^+, 4^+}$; $[(f_{7/2})^{15}(f_{5/2})]_{J=2^+, 4^+, 6^+}$; and $[(f_{7/2})^{15}(\rho_{1/2})]_{J=4^+}$ by the pickup of one core and

one valence neutron. In addition, a large number of two-particle-two-hole states may be constructed which result from pickup of two core nucleons.

The two-particle-two-hole states may be discussed in terms of weak coupling as was done for ^{56}Co . However, its complete construction should require a knowledge of the levels of ^{54}Ni and ^{58}Zn which are not available, and in addition the level scheme which can be deduced from the known ^{54}Co and ^{58}Cu states is much more complicated than for ^{56}Co . Discussion will, therefore, be limited to some 0^+ states. The reference level is assumed to be the first observed 0^+ at 5.000 MeV. Other $T=0$ states are then predicted at 7.91, 8.08, 9.4, and 10.14 MeV. Only the 5.000-MeV state has a configuration which allows it to be observed through the $^{58}\text{Ni}(\rho, t)$ reaction unless there is some level mixing.

Perhaps the most serious flaw in this picture is that it ignores the presence of more complicated configurations in the ^{56}Ni ground state. Wong and Davies² have shown that some two-particle-two-hole and four-particle-four-hole configurations are necessary in order to obtain any agreement with the experimentally observed energy levels. Unfortunately, the inclusion of four-particle-four-hole configurations would make a calculation such as McGrory's completely impractical.

As a result, McGrory's calculations yield an energy spectrum in which the ground-state- 2^+_1 spacing is much too large. This is just the effect observed by Wong and Davies. In addition, since the configuration space is limited to the $1f_{7/2}$,

TABLE V. Comparison of experimental and theoretical levels in ^{56}Ni . Odd-spin states and states of undetermined spin have been omitted.

J^π_n	$E_{\text{experimental}}$ (MeV)	$E_{\text{theoretical}}$ (MeV)	$N = \frac{\sigma(\text{experiment})}{\sigma(\text{theory})}$
$T=0$ states			
0^+_1	0.	0.	78
2^+_1	2.697	6.363	31
4^+_1	3.956	6.614	38
0^+_2	5.000	9.150	43
6^+_1	5.339	7.479	52
(4^+_2)	5.989	7.716	98
2^+_2	6.222	8.465	74
2^+_3	6.318	9.567	64
(2^+_4)	6.554	9.860	375
0^+_3	6.644	9.760	8500
0^+_4	7.289	11.168	48
$T=1$ states			
4^+_1	6.419	7.537	16
2^+_1	7.455	8.496	34
0^+_1	7.912	10.062	78

$2p_{3/2}$, $1f_{5/2}$, $2p_{1/2}$ shells, negative-parity states cannot be described. Experimentally, at least three negative-parity states are observed, presumably resulting from pickup from $d_{3/2}$ shell.

In spite of this, however, the positive-parity states have been compared with McGrory's calculations, as shown in Fig. 8 and listed in Table V. The first five excited $T=0$, $J \neq 0$ states are described as mainly $[(f_{7/2}^{-15})_{7/2}(l_j)_j]_J$. Examination of the normalizations obtained using this description indicates that it is fairly reasonable. In addition, this accounts for all but one (4_3^+) of the states which one can form by means of simple one-particle-one-hole configurations.

At higher excitation energies, one expects to see the more complicated two-particle-two-hole states. Comparison of states at $E = 6.318$ and 6.554 MeV with McGrory's wave functions yields the normalizations shown in Table V. The value $N = 64$ is quite reasonable and compares very well with those obtained for one-particle-one-hole states. The normalization for the 2_4^+ ($N = 375$) is quite large; however, the fit is rather poor and perhaps the level is not really a 2^+ .

Also observed in the present experiment were four 0^+ , $T=0$ states ($E = 0, 5.000, 6.644$, and perhaps, 7.289 MeV). Again this is just the number which one would expect to result from the configurations $(f_{7/2}^{-16})_{J=0}$, $(f_{7/2}^{-2}p_{3/2}^2)_{J=0}$, $(f_{7/2}^{-2}, f_{5/2}^2)_{J=0}$, and $(f_{7/2}^{-2}p_{1/2}^2)_{J=0}$. In terms of weak coupling, three excited 0^+ states are expected at $5.000, 7.91$, and 10.14 MeV. This is somewhat different from the experimental values. Except for the first, however, which is the ground state, the calculated wave functions for the 0^+ states exhibit considerable configuration mixing. Nevertheless, the normalizations for all but the 0_3^+ state ($E = 6.644$ MeV) are quite reasonable, in comparison with the one-particle-one-hole states. The large normalization constant ($N = 8500$) for the 6.644 -MeV level suggests that this level is not the 0_3^+ level predicted by McGrory. If this level is compared with the following 0_4^+ state, a much more reasonable N value ($N = 120$) is obtained. The apparent failure of the weak-coupling model can be due to either a strong mixture of these 0^+ states or to a poor choice for the reference 0^+ state.

Wong and Davies² showed previously that a strong four-particle-four-hole component is needed to account for the observed energies of the low-lying states in ^{56}Ni . Pure two-particle-two-hole components give higher excitation energies. The 5.000 -MeV 0^+ level may not be the suitable reference for the ($T=0$) two-particle-two-hole states. This energy can be estimated if one considers that the 0^+ states deduced from the coupling of the 0^+ $T=1$ levels in ^{58}Cu and ^{54}Co may have isospin

$T=0, 1$, or 2 . The energy separations between these states can be evaluated through a simplified relation deduced from Zamick's formalism³⁸:

$$E = E_0 + \frac{1}{2}bT(T+1),$$

where E_0 depends upon the ^{56}Ni structure and b has been defined earlier. We have established in ^{56}Co that $b = 1.07$ MeV. We shall see that the 0_1^+ $T=1$ state lies at 7.912 MeV in ^{56}Ni . This allows us to expect the corresponding $T=0$ and $T=2$ states at 6.84 - and 10.05 -MeV excitation energies, respectively. The 6.644 -MeV level is a reasonable candidate to be the former state. In addition, in the same way as we did for the ($T=1$) and ($T=2$) 0^+ states in ^{56}Co , it is possible to predict relative strengths for these $T=0, 1$, and 2 states. In the (p, t) reaction, they are expected to be proportional to $\frac{1}{3}$, $\frac{1}{2}$, and $\frac{1}{6}$, respectively. The experimental value of the ratio

$$R_o = \frac{\sigma(E = 7.912 \text{ MeV}) T=1}{\sigma(E = 6.644 \text{ MeV}) T=0} = 1.5 \pm 0.3$$

is a further indication that the 6.644 -MeV state is the right reference state for the two-particle-two-hole levels predicted by the weak-coupling model in ^{56}Ni .

The wave function predicted by McGrory for the 0_4^+ state has effectively the suitable structure. The 7.289 -MeV level has been tentatively assigned 0^+ and could be another candidate. However, the corresponding R_o value, smaller by a factor of 2 , is less convincing. The predicted energy value for the ($T=2$) 0^+ state is in good agreement with the previous experimental determination: 9.90 ± 0.10 MeV.⁴⁶ Three negative-parity states have been

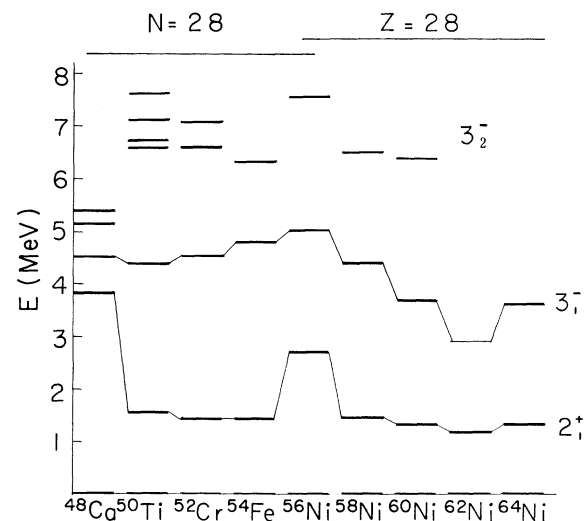


FIG. 11. First 2^+ and first and higher 3^- levels in the even $N=28$ and $Z=28$ nuclei.

observed here. Drawn in Fig. 6 are calculated curves which correspond to simple shell-model wave functions for these states, assuming the pickup of an s - d shell neutron. The 3^- assignment for the 7.567-MeV state is consistent with that of Davies *et al.*²⁴ The 5.483-MeV level is a good candidate to be the first 3^- state, as suggested by Fig. 11 where the previous energies measured by (α, α') inelastic scattering for the first 2^+ and higher 3^- states in the $Z=28$ and $N=28$ nuclei are displayed.^{47, 48}

C. Isobaric Analog States

A number of pairs of states have been seen in the (p, t) and $(p, ^3\text{He})$ reactions which appear likely to be isobaric analogs. Figure 12 compares all the states observed in the two reactions. In this figure the ^{56}Co ground state is aligned with the 6.419 MeV 4^+ , whose energy is very close to the value (6.40 MeV) estimated from the work of Sherr⁴⁹ and using the $^{56}\text{Co}(p, n)$ Q value calculated by Mattauch *et al.*⁵⁰ It appears likely then that only two excited state analogs have been observed, namely, the 2^+ at 1.001 MeV and the 0^+ at 1.444 MeV in ^{56}Co . Selection rules for (p, t) , of course, forbid the excitation of unnatural parity states, or if they are excited they may not have shapes characteristic of any angular momentum, or the appropriate strengths.

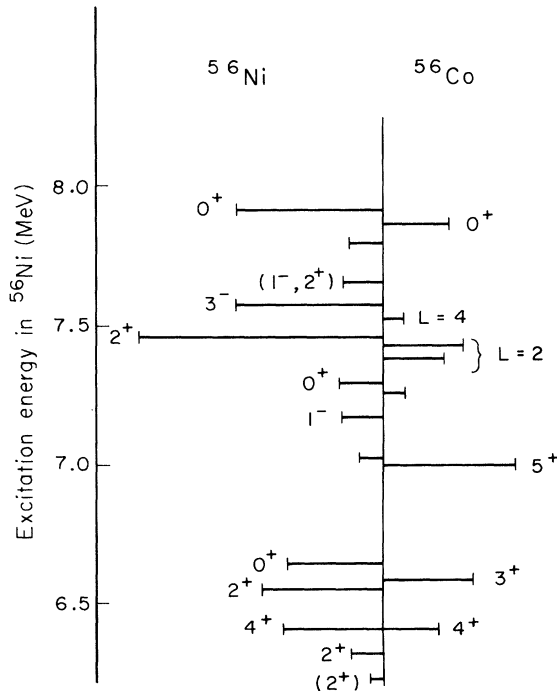


FIG. 12. Comparison of the ^{56}Co energy levels to the states above 6.419 MeV in ^{56}Ni .

If the states in question are isobaric analog pairs, then one should be able to estimate the ratio of the (p, t) to $(p, ^3\text{He})$ cross sections (R). This is given by Hardy, Brunnader, and Cerny⁵¹ when both nucleons are picked up from the same shell, as

$$R_0 = (k_t/k_{^3\text{He}})(2/T_f),$$

where T_f is the isospin of the final level. Here, T_f equals 1. Otherwise,

$$R_1 = R_0 \left[1 + \frac{1}{3} \frac{\lambda^2 (T_f + 1)^2}{T^2 L(L+1)} \right]^{-1},$$

where

$$\lambda = [l_1(l_1 + 1) - l_2(l_2 + 1)] - [j_1(j_1 + 1) - j_2(j_2 + 1)],$$

$$T = [t_1(t_1 + 1) - t_2(t_2 + 1)] - [t'_1(t'_1 + 1) - t'_2(t'_2 + 1)],$$

if the nucleons are removed from shells $n_1 l_1 j_1$ which have isospin t_1, t_2 , respectively, before the transition, t'_1 and t'_2 after the transition.

The values for this ratio, both experimental and calculated are given in Table VI. It is clear that both the 4^+ and 0^+ have just the expected strength. The 0^+ can only be formed by removal of two $f_{7/2}$ nucleons, so that R must have the value 1.7. Similarly, if the ground state of ^{56}Co is $[(f_{7/2})^{-1}(p_{3/2})_4]_4$ then it is formed mainly through pickup of an $f_{7/2} - p_{3/2}$ nucleon pair and R should be 1.59, which is consistent with the observed value. For the 2^+ , however, R experimental is considerably larger than the predicted upper limit (R_0) even after taking account of the larger error which results from the fact that the ^{56}Co peak is one of a poorly resolved doublet. This large value for R might result from the presence of an unresolved state in the ^{56}Ni , or it might indicate that the 2^+ state in nickel is not really the analog of the cobalt level. A better resolution experiment would help to resolve this question.

CONCLUSION

Both the $(p, ^3\text{He})$ and (p, t) reactions have been studied on the ^{58}Ni target nucleus. Some of the observed levels have been discussed in terms of a weak coupling model. As far as possible, the ex-

TABLE VI. Energies and intensities of analog state pairs in ^{56}Ni and ^{56}Co . R_0 is the expected ratio $\sigma(p, t)/\sigma(p, ^3\text{He})$ if both transferred nucleons are from the same shell, R_1 is for pickup from the $f_{7/2}$ and $p_{3/2}$ shells, R_2 is for pickup from the $f_{7/2}$ and $f_{5/2}$ shells.

Spins	$E(^{56}\text{Co})$ (MeV)	$E(^{56}\text{Ni} - 6.419)$ (MeV)	$R_{\text{experiment}}$	R_0	R_1	R_2
4^+	0	0.	1.55 ± 0.3	1.7	1.59	0.94
2^+	0.960	1.036	4.5 ± 2.0	1.7	1.39	0.46
0^+	1.444	1.493	1.7 ± 0.3	1.7		

perimental results have been compared with microscopic shell-model calculations showing a reasonable over-all agreement. There is experimental evidence, at least in ^{56}Co , that above 2.5-MeV excitation energy the experimental resolution has limited the possibilities for interpretation. In the $(p, ^3\text{He})$ experiment, two levels are preferentially excited which can be described in terms of pickup of a nucleon pair coupled to maximum J . Most of the other observed levels have comparable strengths. This does not favor the comparison to either a weak-coupling model or McGrory's calculations.

Our experimental results seem to indicate that any model which can predict energies more accurately will be helpful because of the possible ambiguities due to the similarities of experimental angular distributions.

The present experiment provides best angular momentum matching at $L = 2.4\hbar = [(\vec{k}_i - \vec{k}_f)R]$ and therefore forms $L = 2$ components preferentially. It would be valuable to compare the results of this experiment, not only to other types of experiment, but also to other $(p, ^3\text{He})$ investigations at higher energy. For example, using a 70-MeV proton beam, one might expect changes in angular distributions corresponding to mixed L transitions, since the higher energy would favor the higher ($L = 4$) value.

In the (p, t) reaction, the selection rules remove these ambiguities and allow unique spin assign-

ments once an L value is determined. Consequently, in the present experiment, six new spin assignments have been suggested. In comparing experimental results with McGrory's calculations, it is clear that his wave functions give very poor predictions for excitation energies. On the other hand, employing the criteria which we have adopted for comparing theoretical and experimental results, one can obtain reasonably good estimates for the cross sections, especially the one-particle-one-hole states.

Finally, the relative strengths of states excited via $(p, ^3\text{He})$ and (p, t) reactions provides additional evidence for the assignment $T = 1$ to the 6.419- and 7.912-MeV levels in ^{56}Ni . However, a further experiment is required for the 7.456 MeV 2^+ .

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Study of the Energy Levels of ^{74}Ge Using Nuclear Photoexcitation

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Elastic and inelastic scattering of monochromatic photons were used for studying the energy levels of ^{74}Ge ; the photons were produced by thermal-neutron capture in iron. The energy of the resonance level was found to be 6.018 MeV. Assuming the high-energy lines to be primary transitions deexciting the resonance level, eight energy levels were found from the ground state up to 3 MeV, four of which are believed to be new levels in ^{74}Ge . The angular distribution of the elastic and inelastic lines were measured and the following spin-parity determinations were made: $0.596(2^+)$, $1.206(2^+)$, $1.486(0^+)$, $2.200(2^+)$, $2.229(0^+)$, and $6.018(1^-)$. The parities of the first and last levels were directly determined by polarization measurements using a Compton polarimeter. The parities of the other levels were inferred from the radiation strengths of the corresponding high-energy transitions. The total and partial radiative widths of the 6.018-MeV level were also measured. The $M2/E1$ mixing ratios for the high-energy transitions were obtained. For one transition, this ratio was $x^2 = 0.017$, which is much higher than that predicted by the simple theory.

I. INTRODUCTION

The energy level structure of ^{74}Ge has been investigated by way of the $^{74}\text{Ge}(\gamma, \gamma')$ reaction, using a Ge(Li) detector. The γ source was obtained from thermal-neutron capture in iron. The basic idea

in the present technique¹⁻³ is that a photoexcitation in the target is obtained only when a chance overlap exists between at least one of the incident γ lines and one level in any isotope of the target studied. It turned out that only the 6.018-MeV iron capture γ line happened to overlap a level in