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Plane-Wave Model for the (^{7}Li, t) Alpha Transfer Reaction[*]

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Differential cross sections for the (^{7}Li,t) reaction on ^{16}O, ^{18}O, and ^{20}Ne at bombarding energies between 12 and 20 MeV have been analyzed with a finite-range plane-wave direct-reaction model. Sixteen triton angular distributions leading to strongly excited states in ^{20}Ne, ^{22}Ne, and ^{24}Mg have been selected so as to test the model over an extended range of reaction Q values, angular momentum transfers, and incident beam energies. The projectile form factor has been evaluated using a phenomenological p state α-plus-triton cluster-model wave function. Coulomb-distortion effects have been accounted for in the local-energy WKB approximation. Relative α cluster widths have been extracted and the effects of the structure of the projectile on the reaction process have been discussed. Results of the analysis show that the model gives an adequate description of the shape of the angular distributions and the kinematic dependence of cross sections.

I. INTRODUCTION

During the past few years a considerable body of data has been accumulated on the (^{7}Li, t) reaction at intermediate bombarding energies between 12 and 20 MeV.[1-5] The experimental evidence available indicates that the strongly excited states, especially for nuclei at the beginning of the $2s$-$1d$ shell, are predominantly populated by direct transfer of an α cluster with zero spin and isospin.[1-5] This view is supported by the stripping-like structure of the triton angular distributions which appear to be characteristic of the L value of the transferred α particle[1,3-5] and by the weak excitation of unnatural-parity states and states whose isospin differs from that of the target nucleus.[2,4]

The predominance of an α-particle transfer process is not unexpected in view of the fact that ^{7}Li is believed to possess an α-plus-triton cluster structure.[6-8] The binding energy between the α particle and the triton in ^{7}Li is only 2.47 MeV and, therefore, the (^{7}Li, t) reaction is expected to show features reminiscent of those of single-nucleon transfer reactions. There are major differences, however, between the (^{7}Li, t) reaction and conventional stripping reactions. Unlike the case of more conventional lighter projectiles, the α-particle and triton clusters exist in a relative p state.[6-9] Furthermore, the assumption of a short-range interaction between the two clusters is hard to justify. Both these characteristics make the application of conventional distorted-wave Born-approximation (DWBA) codes to evaluate the (^{7}Li, t) reaction amplitude questionable.

Attempts have been made to calculate the (^{7}Li, t) reaction cross section in the plane-wave Born approximation.[10-14] The attractive features of this treatment are the ease of calculation and the possibility of taking the internal wave function of the projectile into account in a proper fashion. The influence of the ^{7}Li form factor on the triton angular distribution has been clearly demonstrated.[12]

In this paper we wish to point out that the plane-wave model incorporating the ^{7}Li form factor and modified to take into account distortions provides an adequate description of the (^{7}Li, t) reaction process over a large range of bombarding energies, reaction Q values, and angular momentum transfers. Thus, the model presented in the following might prove useful in assigning L values and extracting relative α-particle widths. In addition, it could be helpful for the development of a more comprehensive reaction theory.

II. PLANE-WAVE MODEL

A complete evaluation of the (^{7}Li, t) four-particle transfer-reaction amplitude is of enormous complexity, and in addition would require complete knowledge of the wave functions of the particles involved. Therefore, it is necessary to simplify the problem by assuming that the projectile exists as a triton and an α-particle cluster in relative p-state motion. The orbital motion of the two clusters combines with the spin of the triton to yield the $\frac{3}{2}^-$ ground-state configuration of ^{7}Li. The transferred α cluster has zero spin and isospin. Following Glendenning,[15] the transition amplitude for the (^{7}Li, t) reaction can then be written as

$$T=\sum_{LM}(J_iM_i,\, LM|J_fM_f)(\tfrac{1}{2}\mu_t,\, 1\mu|\tfrac{3}{2}\mu_{\mathrm{Li}})(2L+1)^{1/2}\beta_L B_L^M . \tag{1}$$

The symbols J and L have been used for total and transferred angular momenta, respectively. The z components of the angular momenta are denoted by M and μ. The symbols m and m^* have been reserved for mass and reduced mass, respectively.

The subscripts t, Li, and α refer to the outgoing triton, the incoming lithium, and the transferred α cluster. The indices i and f specify the target and the residual nucleus. The β_L are the expansion coefficients of the final-state residual nuclear wave function $\Psi_{J_f}^{M_f}$ in a basis exhibiting the target plus the transferred α cluster.

$$\Psi_{J_f}^{M_f}=\sum_{J_cL}\beta_L(J_c)\sum_{M_cM}(J_cM_c, LM|J_fM_f)\Psi_{J_c}^{M_c}\Phi_L^M . \tag{2}$$

The index c specifies the target core. Assuming that the α-particle transfer process does not excite the core, the only contribution to the transition matrix element arises when the core is identical to the ground state of the target nucleus. Consequently, the index J_c on β_L has been dropped.

The quantity B_L^M is the probability amplitude for the absorption of an α cluster with quantum numbers L and M. In the plane-wave approximation it is given by

$$B_L^M=i^{-(L-1)}(2L+1)^{-1/2}\int e^{-i\vec{K}\cdot\vec{r}}\phi_{\rm Li}(r)Y_1^\mu(\hat{r})V(r)r^2drd\Omega$$
$$\times\int\mu_L(r_\alpha)Y_L^{M*}(\hat{r}_\alpha)e^{i\vec{q}\cdot\vec{r}_\alpha}r_\alpha{}^2dr_\alpha d\Omega_\alpha , \tag{3}$$

where $\vec{q}=\vec{k}_{\rm Li}-(m_i/m_f)\vec{k}_t$ is the momentum carried into the nucleus by the transferred α cluster, and $\vec{K}=\vec{k}_t-(m_t/m_{\rm Li})\vec{k}_{\rm Li}$ is the momentum transferred to the triton by its interaction with the α cluster. $k_{\rm Li}$ and k_t denote, respectively, the wave numbers for the relative motion in the initial and final states. $\vec{r}=\vec{r}_t-\vec{r}_\alpha$ is the separation between the triton and the α cluster. $\vec{r}_t$ and $\vec{r}_\alpha$ are the coordinates of the triton and the α cluster, respectively, referred to the center of mass of the target nucleus. $V(r)$ is the interaction between the projectile clusters. $\phi_{\rm Li}(r)Y_1^\mu(\hat{r})$ and $u_L(r_\alpha)Y_L^M(\hat{r}_\alpha)$ are, respectively, the wave function for the relative motion of the α and triton clusters, and the wave function for the motion of the transferred α cluster in the residual nucleus. Expanding the plane waves in Eq. (3) and using the orthonormality of the spherical harmonics, one obtains after some manipulations[15]

$$B_L^M=(2L+1)^{-1/2}[(\hbar^2/2m_\alpha^*)P(K)Y_1^\mu(\hat{K})]$$
$$\times 4\pi Y_L^{M*}(\hat{q})Ru_L(R)W_L(q,R) , \tag{4}$$

where R is the cutoff radius and m_α^* is the reduced mass for the α target system.

The form factor $P(K)$ accounts for the internal structure of the projectile and is given by the expression

$$P(K)=4\pi\int_0^\infty j_1(Kr)\phi_{\rm Li}(r)r^2dr , \tag{5}$$

where j_1 is the spherical Bessel function of the first order. In deriving this expression use has been made of the Schrödinger equation describing the internal motion of the projectile.

The quantity $W_L(q,R)$ is the usual Butler[16] expression and contains the dependence of the angular distributions on the L value of the transferred α cluster:

$$W_L(q,R)=R\left[\frac{d}{dR}j_L(qR)-\frac{j_L(qR)}{h_L^{(1)}(itR)}\frac{d}{dR}h_L^{(1)}(itR)\right] . \tag{6}$$

Here j_L and $h_L^{(1)}$ refer to the Lth-order spherical Bessel function and spherical Hankel function of the first kind, and t is related to the binding energy B_α of the α cluster in the residual nucleus by $\hbar^2t^2=2m_\alpha^*B_\alpha$. Equation (6) has been derived with the assumptions that $h_L^{(1)}$ vanishes at infinity and has the same radial dependence as u_L near the cutoff radius. These assumptions are only valid for bound states.

Inserting the definitions given above into the transition amplitude, one obtains for the differential cross section[15]

$$\frac{d\sigma}{d\Omega}=\frac{m_{\rm Li}^*m_t^*}{(2\pi\hbar^2)^2}\frac{k_t}{k_{\rm Li}}\frac{1}{4(2J_i+1)}\sum_{\substack{M_iM_t\\ \mu_t\mu}}|T|^2$$
$$=\frac{3}{16\pi^2}\frac{m_{\rm Li}^*m_t^*}{m_\alpha^{*2}}\frac{k_t}{k_{\rm Li}}\frac{(2J_f+1)}{(2J_i+1)}\frac{P^2(K)}{R}\sum_L\Theta^2(L)W_L{}^2(q,R). \tag{7}$$

The reduced width $\Theta^2(L)$ is related to the reduced α-cluster width $\Theta_0{}^2(L)$ by

$$\Theta^2(L)=\beta_L{}^2\Theta_0{}^2(L) ,$$

with

$$\Theta_0{}^2(L)=\tfrac{1}{3}R^3u_L{}^2(R) . \tag{8}$$

The statistical factors resulting from antisymmetrization between target and projectile, as well as residual nucleus and outgoing particle, have not been considered in this treatment. It is felt that they are best absorbed into the definition of the reduced widths.[14, 15]

The form factor $P(K)$ can be expressed in closed form by using an analytical expression for the α-plus-triton-cluster wave function in ^{7}Li. The proper functional form for this wave function is not obvious. An analysis of elastic and inelastic electron scattering on light nuclei within the framework of the nucleon-cluster model indicates that the cluster-model ground-state wave function should have no radial nodes, excluding the ones at zero and infinity.[7] This result is supported by a theoretical expansion of the shell-model ground-state wave functions of the $4n$ nuclei ^{8}Be, ^{12}C, and ^{16}O in terms of α clusters.[17] However, the oscilla-

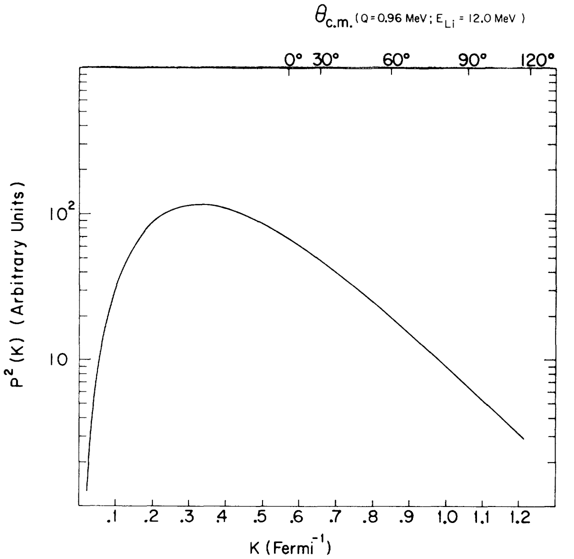

FIG. 1. The square of the form factor $P^2(K)$ versus K. The angular scale indicates the range pertinent to the 6.24-MeV state in ^{22}Ne at 12-MeV bombarding energy.

tor wave function which is generally used in variational cluster-model calculations[6-8, 17] does not have the correct asymptotic behavior at large separation radii. Noble[18] has suggested instead a radial wave function of the following form:

$$\phi_{\mathrm{Li}}(r) = N[1 - e^{-(r/R_0)}]^4 \frac{e^{-\alpha r}}{r}. \tag{9}$$

α is related to the binding energy B of the α and triton clusters in the projectile by $\hbar^2\alpha^2 = 2m^*B$. N is a normalization constant. A value of 1.2 F was chosen for the range parameter R_0 to give the best over-all fit to the angular distributions. This choice of R_0 corresponds to an rms separation of the α and triton cluster of 3.54 F. A plot of $P^2(K)$ versus K obtained with the above wave function is shown in Fig. 1.

So far distortion effects have not been included in the above treatment. However, Coulomb-distortion effects are expected to be nonnegligible at low and intermediate bombarding energies. Austern[19, 20] has suggested that the effects of distortion can be taken into account to first approximation in plane-wave theory by replacing the plane waves $\vec{k}_{\mathrm{Li}}$ and $\vec{k}_t$ by their localized WKB counterparts $\vec{K}_{\mathrm{Li}}$ and $\vec{K}_t$, which are directed along $\vec{k}_{\mathrm{Li}}$ and $\vec{k}_t$, respectively. Specializing to Coulomb distortion only and evaluating the Coulomb potential at the cutoff radius, the magnitudes of $\vec{K}_{\mathrm{Li}}$ and $\vec{K}_t$ are given by

$$K_{\mathrm{Li}} = \{(2m^*_{\mathrm{Li}}/\hbar^2)[E_{\mathrm{Li}} - V_i(R)]\}^{1/2},$$

$$K_t = \{(2m^*_t/\hbar^2)[E_t - V_f(R)]\}^{1/2},$$

where $V_i(R)$ and $V_f(R)$ are the Coulomb potentials in the incoming and outgoing channels, respectively. The inclusion of Coulomb-distortion effects leads to a striking improvement in the quality of the fits over an extended range of Q values and angles. These results indicate that Coulomb forces produce the dominant distortion effect for the (^{7}Li, t) reaction at intermediate bombarding energies.

III. RESULTS AND DISCUSSION

Figure 2 shows a collection of triton angular distributions from the (^{7}Li, t) reaction leading to α-particle bound states of known spin and parity in ^{20}Ne, ^{22}Ne, and ^{24}Mg. The angular distributions shown cover a wide range of bombarding energies E_{Li} from 12–20 MeV, reaction Q values from 7.20 to −1.99 MeV, and angular momentum transfers $L = 0$, 1, 2, 3, and 4. The data are taken from experiments performed at the University of Pennsylvania Tandem Accelerator Laboratory using a multiangle magnetic spectrograph to detect the outgoing tritons.[3, 4, 21, 22] Because of kinematical shifts, data at the extreme backward angles were not available for most of the angular distributions shown. Apart from the transition to the ^{22}Ne ground state, the selection in Fig. 2 has been restricted to strong transitions which are expected to be dominated by direct α-cluster transfer. All angular distributions show the strong forward-peaking characteristic of a direct process. In the case of weak transitions, such as the one leading to the ^{22}Ne ground-state, compound-nuclear contributions and/or higher-order direct processes appear to be nonnegligible, especially at the backward angles. The predictions of the plane-wave theory are shown as solid lines in Fig. 2.

The only variable parameter used in the plane-wave model is the cutoff radius R. The value of R exhibits only small variations from nucleus to nucleus. For a given nucleus, it decreases slightly with increasing bombarding energy. The comparatively large values of R, corresponding to $r_0 \approx 1.7$ F, presumably reflect the partial cancellation between Coulomb and nuclear distortions. Since the Coulomb potential is evaluated at the cutoff radius, the large values of R effectively simulate this cancellation. The over-all success of the plane-wave theory must be largely attributed to such cancellations. The inclusion of Coulomb distortions is primarily responsible for the good agreement between predicted and observed positions of maxima and minima in the angular distribution out to large angles and for the applicability of the plane-wave theory over a large range of reaction Q values, particularly for $L = 0$ transitions. The success of the model in predicting the relative magnitude of the

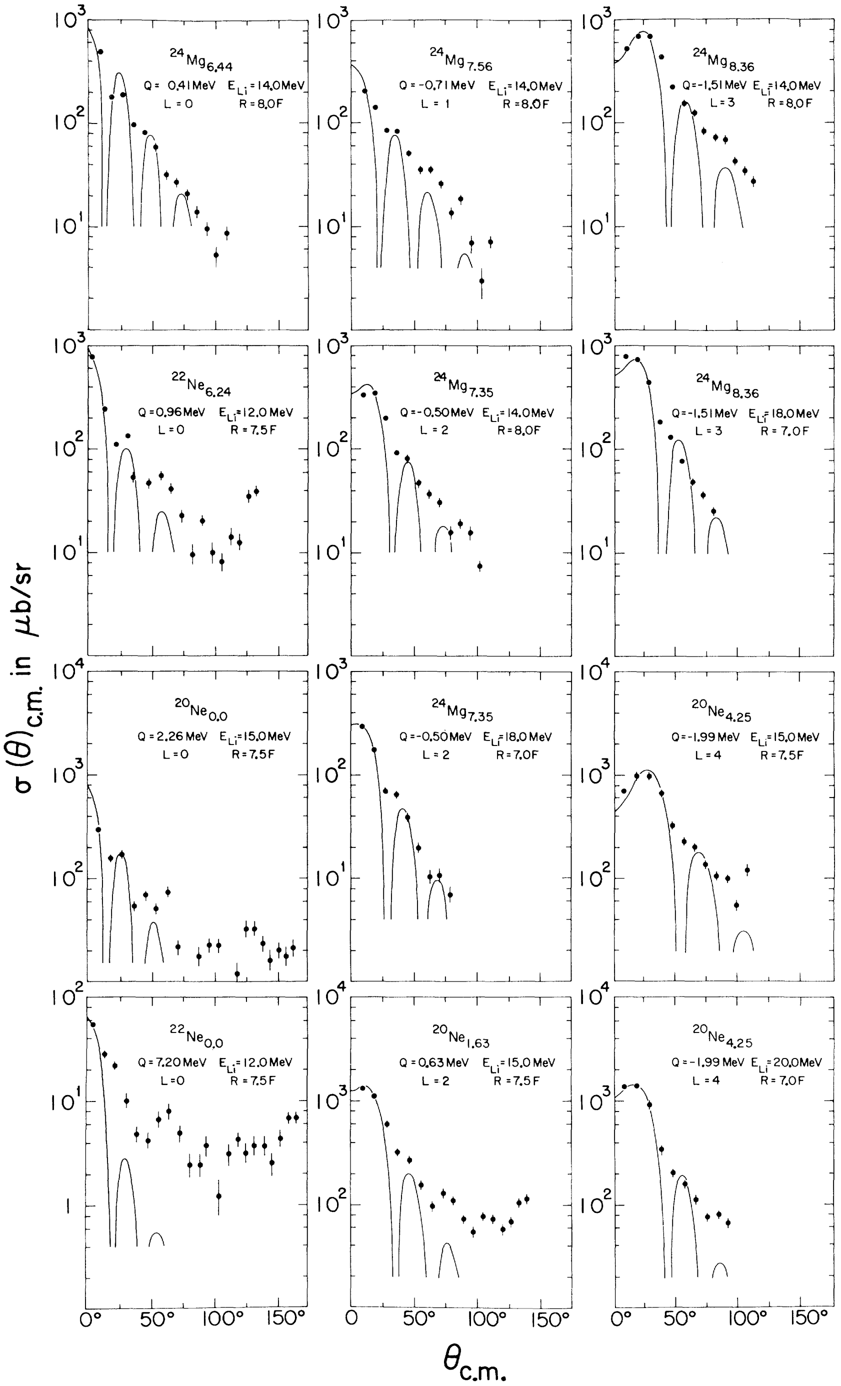

FIG. 2. Triton angular distributions from the (^{7}Li,t) reaction. The residual nuclear state, reaction Q value, bombarding energy, angular-momentum transfer, and cutoff radius are indicated in each case. The solid lines are the predictions of the plane-wave model.

second maximum must be attributed to the choice of the cluster-model wave function of Eq. (9). The steep falloff of the angular distributions towards larger angles is determined by the ^{7}Li form factor. This point is illustrated by the angular scale in Fig. 1, which expresses the momentum transfer K in terms of the angle of the outgoing triton for the 6.24-MeV state in ^{22}Ne shown in Fig. 2.

Reduced widths extracted with the help of Eq. (7) are listed in Table I. Because of the use of plane-

TABLE I. Summary of results for the (^{7}Li, t) reaction.

$E_{^7\mathrm{Li}}$ (MeV)	Target nucleus	Residual nucleus	E_x (MeV)	L	R (F)	Peak $\sigma_{\mathrm{exp}}(\theta)$ (μb/sr)	$\Theta^2(L)$ (relative)
15.0	^{16}O [a]	^{20}Ne	0.0	0	7.5	300	0.25
15.0	^{16}O [a]	^{20}Ne	1.63	2	7.5	1380	0.13
15.0	^{16}O [a]	^{20}Ne	4.25	4	7.5	1050	0.17
20.0	^{16}O [a]	^{20}Ne	4.25	4	7.0	1430	0.23
12.0	^{18}O [b]	^{22}Ne	0.0	0	7.5	55	0.0044
12.0	^{18}O [b]	^{22}Ne	6.24	0	7.5	780	0.088
14.0	^{20}Ne [c]	^{24}Mg	6.44	0	8.0	490	0.30
14.0	^{20}Ne [c]	^{24}Mg	7.56	1	8.0	205	0.045
14.0	^{20}Ne [c]	^{24}Mg	7.35	2	8.0	350	0.042
14.0	^{20}Ne [c]	^{24}Mg	8.36	3	8.0	725	0.10
18.0	^{20}Ne [d]	^{24}Mg	7.35	2	7.0	295	0.039
18.0	^{20}Ne [d]	^{24}Mg	8.36	3	7.0	815	0.12

[a] See Refs. 4 and 21. [b] See Ref. 3. [c] See Refs. 4 and 22. [d] See Ref. 22.

wave theory, only the relative values of $\Theta^2(L)$ are to be considered meaningful. The members of the ground-state rotational band in ^{20}Ne are expected to have comparable values of $\Theta^2(L)$ for the α-transfer reaction because, from theoretical arguments, the corresponding intrinsic state is nearly identical with the α-cluster-model state.[3,12,23] The values of $\Theta^2(L)$ extracted from the data for these states agree to within a factor of 2. A comparison of the $\Theta^2(L)$ extracted at two different bombarding energies for the 4.25-MeV state in ^{20}Ne and the 7.35- and 8.36-MeV states in ^{24}Mg serves as a test of the ability of the model to predict the kinematic dependence of cross sections. The agreement between the two sets of values obtained is a persuasive argument that the kinematic dependence is well reproduced and provides additional support for the view that the momentum dependence of the ^{7}Li form factor is essentially correct. In view of the rather simple assumptions underlying the theory, the over-all agreement with experiment appears to be very satisfactory.

IV. CONCLUSION

The good agreement between experiment and the plane-wave model described in this paper supports the idea that strong transitions observed in the (^{7}Li, t) reaction proceed by direct transfer of an α cluster. The success of the model depends crucially on the proper treatment of the projectile form factor, which has a rather marked effect on the angular distributions and the kinematical dependence of the cross section. Furthermore, it has been demonstrated that Coulomb interactions provide the dominant distortion effects in (^{7}Li, t) reactions at intermediate bombarding energies.

The encouraging success of the plane-wave model indicates that a finite-range DWBA code for the (^{7}Li, t) reaction which incorporates the projectile form factor would be extremely valuable in the identification of angular momentum transfers and the extraction of absolute spectroscopic factors. However, an exact evaluation of a p-state form factor in the distorted-wave scheme is a formidable computational problem. A promising alternative approach appears to lie in the evaluation of the form factor in the local-energy WKB approximation in a way analogous to the suggestions of Austern *et al.*, and Buttle and Goldfarb.[19,20] Another possibility might be to expand the finite-range DWBA in terms of the zero-range approximation as proposed by Sawaguri and Tobocman.[24]

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Levels of ^{61}Cu, ^{64}Cu, ^{67}Ga, and ^{68}Ga Excited by the (p, n) Reaction*

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Neutron time-of-flight techniques were used to study the (p,n) reaction on the nuclei ^{61}Ni, ^{64}Ni, ^{67}Zn, and ^{68}Zn. The locations of excited states in the residual nuclei were measured with an energy resolution of about 10 keV. The absolute excitation energies are accurate to ±2 keV for low-lying states and to ±5 to 7 keV for the highest states (2–3-MeV excitation). The following results were obtained: ^{61}Cu, 17 levels (0 to 2.6 MeV); ^{64}Cu, 66 levels (0 to 2.75 MeV); ^{67}Ga, 57 levels (0 to 3.3 MeV); and ^{68}Ga, 32 levels (0 to 1.6 MeV). Comparisons with other information on these nuclei indicate that the (p,n) reaction is a very effective tool for locating all the levels. The relative intensities of the neutron groups displayed distinctive patterns. However, quantitative yields of a succession of closely spaced proton energies showed typically 30% fluctuations. To maintain good energy resolution and yet reduce the fluctuations we averaged many individual runs in order to have more meaningful comparisons with the Hauser-Feshbach predictions. Spin assignments were made to 10–20 levels in ^{64}Cu, ^{67}Ga, and ^{68}Ga. The results are generally consistent with other available information on spin parities.

I. INTRODUCTION

The (p,n) reaction has not been extensively used as a spectroscopic tool. Cross sections for the (p,n) reaction do not exhibit a strong dependence on the properties of the residual state, such as the shell-model configuration in the case of transfer reactions or the collectivity of the state in the case of inelastic scattering reactions. This nonselectivity can be an advantage if one's purpose is to map the location of all the levels. However, adequate energy resolutions must be achieved if this is to be a useful endeavor. We have used the terminal pulsed proton beam from the Oak Ridge National Laboratory (ORNL) 6-MV Van de Graaff to measure (p,n) reactions by the time-of-flight method. The pulsed-beam quality, proton bursts of 1-nsec width and several mA peak current, was such that target thicknesses and flight paths could be used to achieve an energy resolution of 10 keV for the