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Pion Production and the Two-Nucleon Interaction*

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The distorted-wave Born approximation, justified in a previous study, is applied to π^0 production in proton-proton collisions. The production amplitude is studied as a function of the phenomenological two-nucleon potential used to describe the proton-proton interaction. For selected possible experiments it is shown that the pion-production amplitude depends on which phenomenological potential is used to describe the two-nucleon interaction.

INTRODUCTION

In this paper we study π^0 production in proton-proton collisions as a method of obtaining information about the nonrelativistic two-nucleon interaction. To perform this study we must have a method of treating pion production in the nonrelativistic limit. Such a method was established in an earlier study.[1]

Using an approach similar to the one Fubini and Furlan[2] have applied to pion-nucleon scattering, Banerjee *et al.*[1] (BLSZ) have obtained a dispersion relation in the pion-mass variable for the pion-production amplitude in two-nucleon collisions. Working in the rest frame of the pion the soft-pion limit (mass of the pion equal to zero) was obtained from current-field algebra. Since this soft-pion limit explicitly contains an off-shell nucleon-nucleon scattering amplitude factor, it differs from the soft-pion limit derived by Adler[3] and applied to this production process by Beder[4] and by Schillaci, Silbar, and Young.[5]

By considering the leading singularity in the mass-dispersion relation for the production amplitude, BLSZ were able to establish a connection between the mass-dispersion approach and nonrelativistic production theory. Once the validity of the nonrelativistic theory was established, the Schrödinger equation was used to extrapolate from the point at which the production amplitude is known (the soft-pion limit) to the value at the physical pion mass. This extrapolation involved only nonrelativistic two-nucleon wave functions. The final result was to obtain the distorted-wave Born-approximation (DWBA) amplitude in the rest frame of the pion as

$$T_{NN\to NN\pi}{}^a = -\frac{g_A m_\pi{}^2\sqrt{2}}{f_\pi}\left(\phi_\beta^{(-)}\right|\sum_{n=1}^{2} i\frac{m_\pi}{M}\vec{\sigma}_n\cdot\nabla_n\frac{\tau_n{}^a}{2}\left|\phi_\alpha{}^{(+)}\right), \qquad (1)$$

where

τ_n^a is the ath component of the isospin operator for nucleon n,

$\vec{\sigma}_n$ is the nucleon-spin operator for nucleon n,

f_π is the pion-decay constant,

g_A is the axial-vector-nucleon coupling constant,

m_π is the mass of the pion,

M is the mass of the nucleon,

ϕ_α is the distorted two-nucleon wave function for state α,

and $\vec{\nabla}$ acts on the nucleon space variables $\vec{x}_n$. The summation is over the nucleons. In the rest frame of the pion the transition operator shown above is the one frequently used in nonrelativistic studies of the pion-production process.[6] Equation (1) is shown graphically in Fig. 1.

In Sec. I we study the angular distribution of the final-state nucleons for an "ideal" experiment and for a realistic one. In Sec. II we study the total cross section for π^0 production in proton-proton collisions.

I. ANGULAR DISTRIBUTION OF FINAL NUCLEONS

Using the above result we are prepared to investigate the possibility of devising experiments which will depend strongly on the potential used to calculate the two-nucleon wave function. Since corrections to the DWBA depend on the pion-nucleon scattering potential,[1,7] we are not able to treat these corrections accurately until the nonrelativistic pion-nucleon scattering potential is established. Therefore, we must choose a process for which the corrections to the DWBA are as small as possible.

The low-energy π^0-nucleon elastic scattering amplitude is much smaller than either the charged-pion–nucleon elastic scattering or the charge-exchange scattering. This result arises because only the isoscalar amplitude contributes to π^0-nucleon elastic scattering. This amplitude is known to be much smaller than the isovector amplitude.[8]

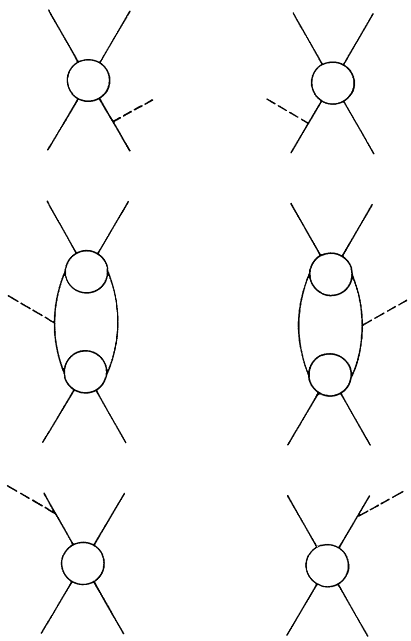

FIG. 1. Graphical description of $T(E)$ in the distorted-wave approximation.

For the process $pp \rightarrow pp\pi^0$, Fig. 2 shows that due to charge conservation π^0-proton elastic scattering is the only process which contributes to the first-order corrections to the DWBA. Thus, we will study the production of neutral pions in proton-proton collisions and be assured that the corrections to the DWBA amplitudes will be as small as possible.

As a further restriction we wish to avoid complications due to the (3, 3) pion-nucleon resonance. Recent calculations by Schillaci and Silbar[9] and an experiment by Cochran *et al.*[10] have shown that one does not avoid the resonance by simply considering pions at rest. Since the nucleons can recoil, one of the nucleons and the pion can be in a relative p state, and, thus, if the recoiling nucleon has sufficient energy, the two particles can resonate even though the pion is at rest. Therefore, we are required to consider energies very near threshold, where only s-wave pion production is important and insufficient energy is available to allow a resonant condition to exist. Another, more practical, consideration is the energy of accelerators available to perform the experiments to be suggested in this paper. The meson-physics facility being constructed at Los Alamos will be able to accelerate protons to laboratory energies of 300 to 800 MeV in increments of 10 MeV. Therefore, we will present the results of the following calculations at a proton energy of 300 MeV as well as at other energies which seem appropriate.

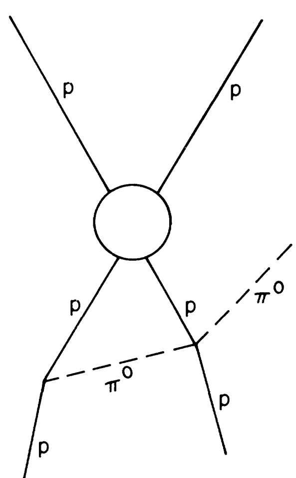

FIG. 2. The first-order correction to the DWBA for $pp \rightarrow pp\pi^0$. Note that charge conservation requires that the intermediate pion be π^0.

The experimental configuration which would display ideally the off-energy-shell two-nucleon scattering effects is shown in Fig. 3. In this situation the pion is emitted at rest. As they would in elastic scattering, the nucleons go off in opposite directions. Thus, we have an experimental situation which is very similar to an off-energy-shell two-body interaction. Unfortunately, since the velocity of the pion is zero, the phase-space factor vanishes, and therefore, so does the cross section. However, we are able to obtain information about the two-nucleon off-energy-shell scattering by a study of the production amplitude for this impossible experiment.

By studying this process we can determine at which energies we expect to see the largest differences in cross sections when different potentials are used to determine the two-nucleon Schrödinger wave functions. For this study we will use four nucleon-nucleon potentials: the Reid soft core[11] labeled RSC on graphs of the results of the calculation), the Reid hard core[11] (labeled RHC), the Hamada-Johnston hard-core[12] (H-J), and the Bressel-Kerman-Rouben finite core[13] (BKR). For the reasons discussed above we limit our discussion to laboratory energies of 300 MeV or less and to the process $pp \rightarrow pp\pi^0$.

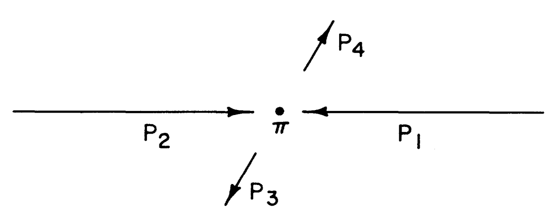

FIG. 3. Graphical description for pion produced at rest.

To evaluate the DWBA amplitude [Eq. (1)] we perform a partial-wave analysis of the initial and final two-nucleon relative wave functions. A semi-classical calculation (see Appendix) indicates that one must retain states for which the square of the orbital angular momentum quantum number (L) is less than or equal to the center-of-mass kinetic energy in MeV divided by 10. Since the final two-nucleon state is at a much lower kinetic energy than the initial energy, the highest value of angular momentum which we must consider is determined by the final state. For an incoming laboratory energy of 310 MeV the final nucleons would have (at most) an energy of 20 MeV in their center of mass. Thus, retaining states of orbital angular momentum of two and less would be more than sufficient. We have chosen to include all states of *total* angular momentum (J) less than or equal to two. Thus, we have retained states of $L \leq 3$.

The initial and final two-nucleon states used in all calculations in this paper are listed in Table I. The notation is the usual one ($^{2T+1,2S+1}L_J$, where T is the isospin, S is the total spin, L is the orbital momentum, and J is the total angular momentum). Since we consider only s-wave pion production, the parity of the initial and final two-nucleon states must differ (that is, the initial and final orbital angular momentum must differ by an odd number, while the total angular momentum remains fixed).

In Figs. 4 and 5 the square of the amplitude for production of pions at rest is shown as a function the laboratory energy of the incoming proton. Figure 4 shows the amplitude when the final nucleons are at an angle θ of 90° with the initial nucleons, while Fig. 5 shows the same calculation for the situation in which the final nucleons are moving on the same axis as the initial nucleons.

By examining Figs. 4 and 5 we can discover which off-energy-shell T matrices we are measuring in a pion-production experiment. If an energy region exists in which the $\theta = 90°$ (Fig. 4) and the $\theta = 0°$ (Fig. 5) amplitudes are essentially equal, then the $J = 0$ nucleon states must be dominating the production amplitude. (Of course, the comparison must be made for each nucleon-nucleon potential.) On the other hand, if these two production amplitudes are very different, then the $J = 2$ nucleon states must dominate the production. Very near the threshold the two amplitudes (for each potential) are essentially equal. However, the amplitude for $\theta = 90°$ drops off much faster than the $\theta = 0°$ amplitude. Thus, the higher angular momentum states must become important at energies very near the production threshold.

TABLE I. Nucleon states for s-wave pion production. $pp \rightarrow pp\pi^0$. (Notation: $2T+1,\ 2S+1L_J$.)

Initial state	Final state
$^{31}S_0$	$^{33}P_0$
$^{33}P_0$	$^{31}S_0$
$^{31}D_2$	$^{33}P_2$
$^{31}D_2$	$^{33}F_2$
$^{33}P_2$	$^{31}D_2$
$^{33}F_2$	$^{31}D_2$

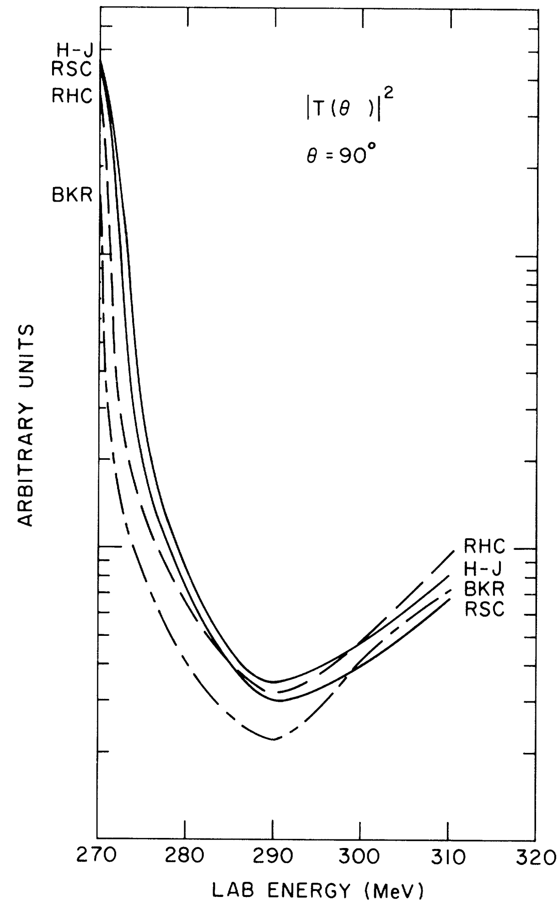

FIG. 4. Transition amplitudes squared for pions at rest and $\theta = 90°$.

Hence, at laboratory energies as low as 280 MeV (10 MeV above threshold) we expect to see structure in the angular distribution of the final-state protons. Figures 4 and 5 indicate that, for a theoretical calculation, this structure will be strongly dependent on the potential used to describe the nucleon-nucleon interaction. Thus, we are encouraged to look at the angular distribution of the protons as a method of determining the "true" nucleon-nucleon potential (or, at least, as a method of eliminating some of the presently used potentials).

Further, by studying Figs. 4 and 5 we can determine the energy region in which the different potentials will yield the greatest differences in the

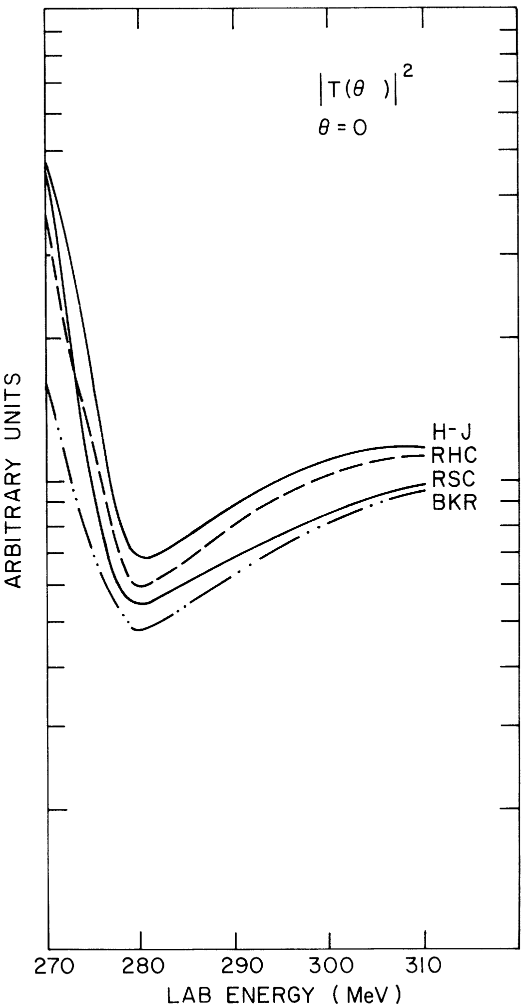

FIG. 5. Transition amplitudes squared for pions at rest and $\theta = 0°$.

calculated cross sections. First, we note the equality of the Hamada-Johnston and Reid soft-core amplitudes at threshold (270 MeV). Very near threshold the momenta of the final nucleons, $\vec{p}_3$ and $\vec{p}_4$, are essentially zero. Since from the previous study[1] we know that the amplitude for the emission of a pion is proportional to the velocity of the nucleon relative to the velocity of the pion, the amplitude for emission of a pion from the final nucleon state will be essentially zero (i.e., the two graphs at the bottom of Fig. 1 will not contribute to the production amplitude). Further, since the velocities of the final nucleons are very near zero, the two nucleons will be in a relative S state (i.e., in the state ${}^{31}S_0$). In addition, since the incoming nucleons are at a rather high energy, the initial-state distortion will be somewhat small. Thus, most of the contribution to the production amplitude will come from the processes associated with the top two graphs of Fig. 1 with the nucleons going from a ${}^{33}P_0$ state to a ${}^{31}S_0$ state. In this case, the off-energy-shell nucleon-nucleon scattering will be a scattering of two nucleons in a ${}^{31}S_0$ state. Thus, the equality of the DWBA production amplitudes implies the equality of the off-energy-shell scattering associated with the ${}^{31}S_0$ state. Redish, Stephenson, and Lerner[14] have studied this off-shell T matrix and have concluded that the T matrices of the Hamada-Johnston potential and the Reid soft-core potential are equal at this particular energy.

As we increase the energy of the incoming proton, the amplitudes calculated using different potentials become much different. The region in which the desired differences are most pronounced is from about 275 to 300 MeV. This result is very encouraging, since this region is precisely the energy region in which pions are produced predominately in an s state. Thus, all the ill effects arising from the p-wave production and the (3, 3) resonance can be avoided while we investigate the properties of the nucleon-nucleon potential.

In the next section we will discuss the total cross section for pion production. First, we will discuss an experiment which could reproduce as closely as possible the "ideal" experiment described above. The notation we use is shown in Fig. 6. To construct this possible experiment we restrict the magnitude of the momenta $|\vec{p}_3|$ and $|\vec{p}_4|$ to be equal, and we hold the angle between $\vec{p}_3$ and $\vec{p}_4$ fixed. (This angle will be called $\Theta = \theta_3 + \theta_4$ and Θ is held fixed.) By choosing Θ to be large enough (near 180°) we can be assured that the three-momentum of the pion is very small.

For $\Theta = 160°$, Figs. 7 and 8 show the differential cross section for 280 MeV and 300 MeV, respectively. As shown in these graphs, the differential cross sections, although very small, do show a reasonably strong dependence on the nucleon-nucleon potential used to perform the calculation. Table II shows some selected percentage differences between each of the potentials and the Hamada-Johnston potential. As is seen in Table II, many of these differences are quite large. Thus, we can conclude that this experiment is a reason-

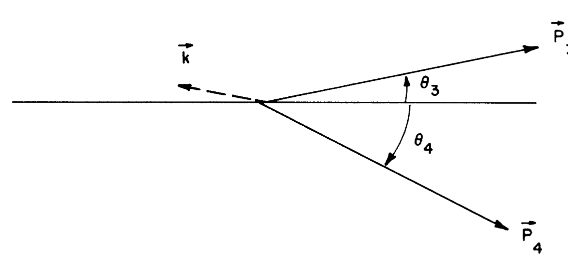

FIG. 6. Graphical description of notation.

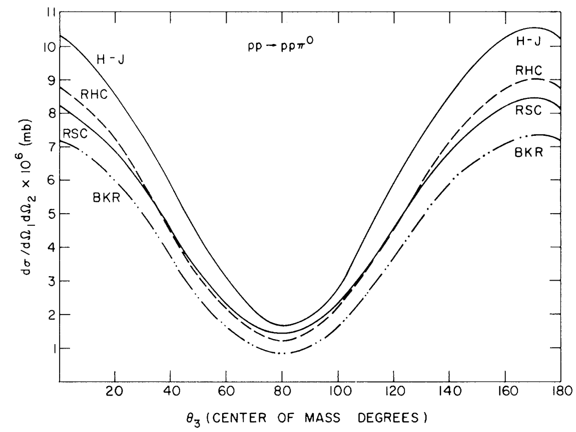

FIG. 7. Angular distribution of final protons for $\theta = 160°$ and a laboratory energy of 280 MeV.

able one to do to determine the "true" nucleon-nucleon potential.

II. TOTAL CROSS SECTION

Using the Hamada-Johnston potential Koltun and Reitan[7] have calculated the total cross section of π^0 production in proton-proton collisions. Their calculation was performed by approximating the energy dependence of the production amplitude as its value at the production threshold multiplied by a factor which depends on the energy and the nucleon-nucleon scattering length.[15] In the previous section we saw that the amplitude at threshold depends solely on the $^{31}S_0$ off-energy-shell scattering. By combining this term with the scattering length, Koltun and Reitan have used results which contain information about the singlet S nucleon-nucleon interaction only. The results of Sec. I seem to suggest that this approximation is not appropriate for pion production. We saw that at energies very near the threshold the higher angular momentum states enter the calculation in a very important

TABLE II. Comparison of the production amplitude for the potentials listed and the Hamada-Johnston potential. (The numbers are the percentage of the Hamada-Johnston cross section.)

	280 MeV		300 MeV	
	$\theta_3 = 0°$ (%)	$\theta_3 = 80°$ (%)	$\theta_3 = 0°$ (%)	$\theta_3 = 80°$ (%)
RSC	20	13	25	12
RHC	16	25	14	4
BKR	32	53	27	16

way. However, now we shall see that the total cross section is indeed dominated by the singlet S off-energy-shell scattering.

For the two-nucleon potentials discussed in Sec. I we have performed a DWBA calculation using the partial waves listed in Table I. The results of this calculation are shown in Fig. 9. Also shown in Fig. 9 are the curve obtained by Koltun and Reitan and the one available experimental measurement. (The datum is listed in Ref. 10 as less than 4 μb.)

The striking feature of Fig. 9 is the similarity near threshold of the cross sections using the Reid soft-core potential and the cross section using the Hamada-Johnston potential. At first, this result seems to contradict the results of the previous analysis (Sec. I). However, if we examine the energy distribution of the produced pions, the explanation of the results is clear. (Figure 10 shows this distribution for the Reid soft-core potential.) Since the pion is much less massive than the nucleons, it tends to carry most of the kinetic energy available to the pion two-nucleon system. This distribution requires that the nucleons usually have very little kinetic energy. Therefore, as we discussed in the previous section, the nucleons will be predominately in an S state. As we and Redish, Stephenso, Jr., and Lerner[14] have noted, the S-state off-energy-shell scattering amplitude in-

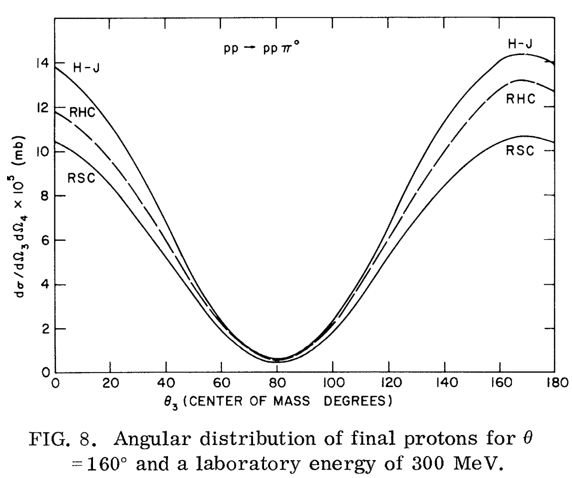

FIG. 8. Angular distribution of final protons for $\theta = 160°$ and a laboratory energy of 300 MeV.

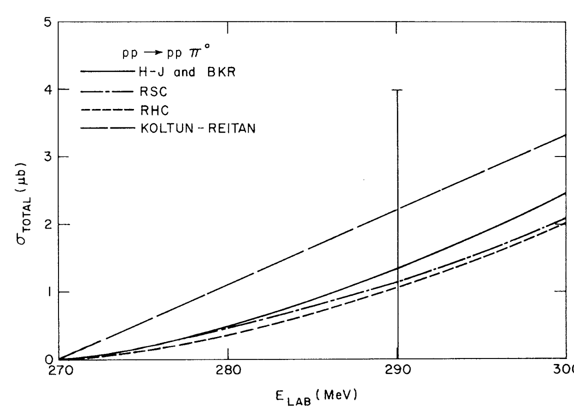

FIG. 9. Total cross section for s-wave pion production.

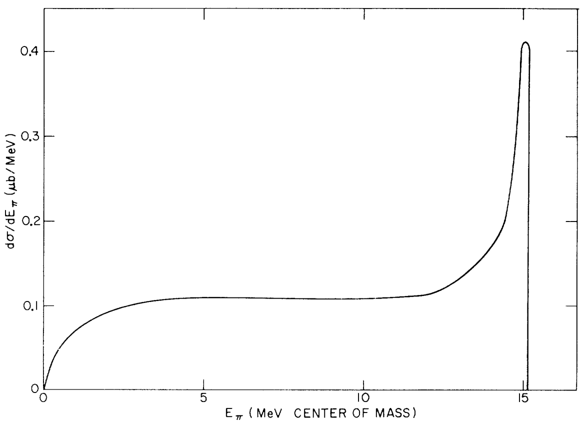

FIG. 10. Energy distribution of pions for a laboratory energy of 300 MeV.

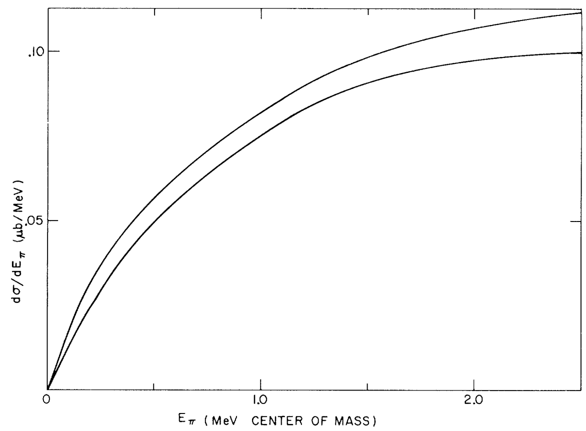

FIG. 11. Energy distribution of very low-energy pion for a laboratory energy of 300 MeV.

volved in this production process is very similar for the Reid potential and the Hamada-Johnston potential.

At threshold the slope of the cross section of this calculation indicates that in the neighborhood of threshold it may be written as

$$\sigma \approx (7\ \mu\text{b})|\vec{k}_\pi|^2/m_\pi^2 .$$

This result compares with

$$\sigma = (17\ \mu\text{b})|\vec{k}_\pi|^2/m_\pi^2$$

for the calculation of Koltun and Reitan, and with

$$\sigma = (32 \pm 7\ \mu\text{b})|\vec{k}_\pi|^2/m_\pi^2$$

for the experimental determination. Here, $\vec{k}_\pi$ is the maximum momentum of the pion. Since the experimental fit was obtained primarily from fitting measurements at energies greater than 310 MeV, it cannot be considered as a very good measurement of the slope at threshold. (We note that had we used the values of our calculation at 300 MeV to calculate the "threshold slope" we would have obtained

$$\sigma = (8\ \mu\text{b})|\vec{k}_\pi|^2/m_\pi^2$$

for the Reid potentials and $\sigma = (10\ \mu\text{b})|\vec{k}_\pi|^2/m_\pi^2$ for the Hamada-Johnston and Bressel-Kerman-Rouben potentials.)

The discussion of the total cross section reveals that, at least near threshold, we would find it useful to consider processes in which the nucleons have some kinetic energy. One such process was described in Sec. I. Another similar (and experimentally less difficult) measurement would be to study the energy distribution of the pion in the region where the pion has very little kinetic energy. A graph of this pion energy region for a bombarding energy of 300 MeV is shown in Fig. 11 for the Reid soft core and Hamada-Johnston potentials. Again, we see small (up to 15%) differences between the two potentials.

III. CONCLUSIONS

By applying the DWBA to the production of neutral pions in proton-proton collisions we have studied the two-nucleon interaction. The experiments described here would be very difficult to perform. However, the information obtained from them could shed much light on the nature of the two-nucleon interaction. Therefore, we hope that such experiments will be performed.

Further, we believe that this study has been only a first step in using pion production (and particle production in general) as a method of studying the two-nucleon interaction. The results have been encouraging, and we expect that an analysis of p-wave pion production (which must include a detailed description of the pion-nucleon final-state interaction) will yield even more information about the nucleon-nucleon interaction.

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APPENDIX

Using the classical equation for angular momentum, $L = Mvr$, we assume that the "radius" appropriate for the interacting nucleons in the inverse of the pion mass. Thus,

$$L^2 = \left(\frac{Mv}{M_\pi}\right)^2 = \frac{2ME}{M_\pi{}^2} \approx \frac{E \text{ (in MeV)}}{10 \text{ MeV}}.$$

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Elastic Scattering of 36.0- and 46.3-MeV Neutrons from Deuterium*

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The differential cross sections for elastic scattering of 36.0- and 46.3-MeV neutrons from deuterium were measured over an angular range of 15 to 170° c.m. The cross section agrees well in shape and absolute value with that for *p-d* scattering. A slightly more pronounced minimum seems to be present in the *n-d* results. From measured values of the total *n-d* cross sections, we obtain the values 154 ± 6 and 116 ± 5 mb at 36.0 and 46.3 MeV, respectively, for the *n-d* nonelastic cross section.

I. INTRODUCTION

The interaction of two neutrons and one proton is the simplest and the most fundamental three-body problem in nuclear physics. The lack of any Coulomb interaction among the particles simplifies the theoretical description of the system. The three-body problem was set on a sound mathematical basis by the work of Faddeev.[1] Independently, the introduction in 1962 of an S-wave separable potential in the Schrödinger equation by Mitra[2] and also by Amado[3] permitted an exact treatment of the dynamical aspects of the three-nucleon problem. This approach has been very successful in predicting low-energy properties of the three-nucleon system[4–6]; in particular, *n-d* differential cross sections up to 14 MeV. Inclusion of higher partial waves and tensor forces seems necessary to describe polarization and medium-energy, 20–60-MeV, *n-d* elastic scattering. Work in this direction is currently being attempted by several groups.[7,8]

Mainly because of the lack of intense neutron beams, no neutron *n-d* elastic scattering has as yet been published in the medium-energy region although incomplete work has been reported between 18.6 and 20.5 MeV at Los Alamos[9] and at 28 MeV at Lyon.[10] The present work involves measurements of *n-d* elastic scattering at 36.0 and 46.3 MeV. Preliminary results of some of this work have been reported.[11] Energies were selected to be close to those of the *p-d* measurements performed at the University of California, Los Angeles (UCLA) in order to have a more direct