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[8]More precisely, for a pion with vanishing zeroth component of its four-momentum. The δ-function $\delta(0)$ in (6) actually stands for $\delta(q_{10}-q_{20})$ with $q_{10}=q_{20}=0$. All the vectors in this paper are three-vectors unless otherwise stated.

[9]This is so because A and B are constants in the nonrelativistic approximation. Relativistic effects may suppress the high-momentum contributions.

[10]Corresponding formulas for the TPE ΛNN force are obtained by replacing C_p by $3C_p(\Lambda NN)/2N$, where $C_p(\Lambda NN)\approx 1.43$ MeV. See Refs. 2 and 3. Hence the "conversion factor" is $3C_p(\Lambda NN)/2C_p(NNN)=4.7$.

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$$\frac{e^{-\mu r}}{r}\rightarrow\frac{1}{r}\left(e^{-\mu r}-\frac{U^2-\mu^2}{U^2-\eta^2}e^{-\eta r}+\frac{\eta^2-\mu^2}{U^2-\eta^2}e^{-\eta r}\right).$$

Since they take a very large value for U (= 20 m) the above formula is practically reduced to $(e^{-\mu r}-e^{-\eta r})/r$, which is obtained by using our spectral function (40) with $\xi=1-(\mu/\eta)^2$. The values they considered for η ranges from about 5μ to 7μ, hence $\xi\approx 1$. Therefore, using their form factor we would get very similar results to those with our form factor I.

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Tensor-Force Effects on the l-Forbidden $M1$ Transitions

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The reduced matrix elements for the l-forbidden $M1$ transitions are calculated in the framework of the pairing model. Three quasiparticle states are admixed to the seniority-one state by perturbation due to the short range δ force and the tensor force. It is shown that the mixing of the tensor force is essential to explain the observed values of the reduced matrix elements.

I. INTRODUCTION

According to the shell model, the l-forbidden magnetic dipole ($M1$) transitions between two states which differ in their orbital angular momenta are strictly prohibited, for the magnetic dipole interaction does not change the orbital angular momentum and parity. Hence, the $M1$ transitions are presumably allowed if the initial and the final states are assigned the same orbital angular momenta and parities, and vice versa. However, there have been observed many $M1$ transitions whose lifetimes are much longer than those expected from the shell-model estimate. Therefore, it has been suggested that either the $M1$ transition operator is not adequate, or there is a breakdown of the l-forbiddenness due to some nuclear effects. A theoretical explanation attributed the breakdown of the forbiddenness to the nucleon-nucleon interaction, and a modification of the form of the $M1$ operator was introduced.[1] This effect, however, is now believed to be too small to explain many of the large retardations actually observed. Another approach was made by Arima, Horie, and Sano (AHS)[2] by introducing the method of configuration mixing. Govil and Khurana[3] have investigated the systematic trend of the $M1$ transition matrix elements and they have found a shell effect in these matrix elements. They have also indicated that the calculated values of the matrix elements from the theory of AHS are sufficient to reproduce qualitatively those values deduced from the experimental transition rates.

Recently, the emphasis of the importance of the short-range residual interaction, which admixes a small amount of high-seniority configurations to the basic shell-model configuration, has led to the application of the pairing theory to this problem.[4,5] Since there had been no criterion for determining how much configuration admixture to allow to explain the breakdown of the l-forbiddenness of the $M1$ transitions, it was expected that the pairing theory would provide a reasonable measure for the admixing of excited configurations. Freed and Kisslinger[5] have examined the effects of pairing correlations coupled to quadrupole phonon vibrations on the l-forbidden $M1$ transition rates, while Sorensen[4] has also calculated these rates using wave functions resulting from the shell model with a residual pairing-plus-$P^{(2)}$ force. The former authors show that the projected single quasiparticle state with a mixture of higher-seniority states generated by the quadrupole-quadrupole interaction gives almost satisfactory values for the transition rates for many nuclei in a wide mass-number region. They observed, however, that not all of the known l-forbidden $M1$ transition data are explained, and suggested that a part of the residual interactions, such as the tensor force, may also be important in generating configuration mixing.[5]

The effect of the tensor force has been examined by Shikata[6] for nuclei in the medium-heavy-mass region. He has pointed out that the tensor force by itself provides better agreement with the observed data than the central force does, but that these two forces cannot be combined together unless the relative sign of these two forces is altered to the negative of the commonly used one.

It is the purpose of this paper to show that the central and tensor forces can be combined, without introducing the awkward relative sign, by changing the interaction range of the tensor force, as was mentioned by Shikata,[6] and that the residual interaction, thus obtained, can give very reasonable results for the l-forbidden $M1$ transition rates for nuclei of all mass numbers.

II. ASSUMPTIONS AND CHOICE OF PARAMETERS

For the magnetic dipole transition, the radiative transition probability is written as

$$w_\gamma = \frac{1}{\tau_\gamma} = \frac{mc^2}{\hbar}\left(\frac{E}{mc^2}\right)^3\left(\frac{m}{M}\right)^2 \frac{e^2}{\hbar c}\frac{\mathfrak{M}^2}{2j+1}$$

$$= 0.419\times 10^{13}\frac{E^3\mathfrak{M}^2}{2j+1}\ \mathrm{sec}^{-1}, \tag{1}$$

where τ_γ is the mean lifetime, E is the photon energy measured in MeV, and $\mathfrak{M}$ is the reduced transition matrix element

$$\mathfrak{M}^2 = \sum_{qmm'} |(j'm'|\textstyle\sum\mu_q|jm)|^2 = (j'\|\textstyle\sum\mu\|j)^2, \tag{2}$$

where j and j' are the angular momenta of the initial and final states, respectively, and the sum $\sum\mu$ is taken over all nucleons in the nucleus.

The initial- and final-state wave functions are expressed in the form

$$|\psi(jm)\rangle = \alpha_{jm}^\dagger|\psi_0\rangle + \sum_J a_J \alpha_{j_1}^\dagger \alpha_{j_2}^\dagger(J)\alpha_{j'}^\dagger(jm)|\psi_0\rangle, \tag{3}$$

and

$$|\psi(j'm')\rangle = \alpha_{j'm'}^\dagger|\psi_0\rangle + \sum_J a_J \alpha_{j_1}^\dagger \alpha_{j_2}^\dagger(J)\alpha_j^\dagger(j'm')|\psi_0\rangle,$$

in terms of the quasiparticle creation operators $\alpha_{jm}^\dagger$ and $\alpha_{j'm'}^\dagger$, and $|\psi_0\rangle$ is the quasiparticle vacuum state. The additional three quasiparticle terms in the wave functions contribute to the reduced matrix element of the l-forbidden $M1$ transition, just like those admixed terms do in the configuration-mixing calculation. j_1 and j_2 must have the same orbital angular momentum,[7] and J takes only the value of unity because of the angular momentum selection rule. The coefficients a_J are calculated in the first-order perturbation

$$a_J = -\langle\psi_0|\alpha_{jm} V \alpha_{j_1}^\dagger \alpha_{j_2}^\dagger(J)\alpha_j^\dagger(jm)|\psi_0\rangle/\Delta E, \tag{4}$$

where ΔE is the energy difference between the zeroth order and excited configurations.

The residual interaction inducing the above configuration mixing is the sum $H_{\rm res} = H_\delta + H_T$, where H_δ and H_T are the short-range δ-interaction and tensor-interaction Hamiltonians, respectively.

The short-range interaction H_δ has the form

$$H_\delta = [V_0 + V_1\vec\sigma(1)\cdot\vec\sigma(2)]\,\delta(\vec r_1 - \vec r_2)$$

$$= \{\tfrac14[1 - \vec\sigma(1)\cdot\vec\sigma(2)]V_s\}$$

$$+ \{\tfrac14[3 + \vec\sigma(1)\cdot\vec\sigma(2)]V_t\}\,\delta(\vec r_1 - \vec r_2), \tag{5}$$

which involves two parameters, the singlet strength V_s and the triplet strength V_t. The tensor force interaction H_T is of the form

$$H_T = V_T(1-\alpha\vec{\tau}_1\cdot\vec{\tau}_2)e^{-(r_{12}/r_0)^2}\{[Y^{(2)}(1)\times\vec{\sigma}(1)]^{(1)}\cdot[Y^{(2)}(2)\times\vec{\sigma}(2)]^{(1)}+[Y^{(2)}(1)\times\vec{\sigma}(1)]\cdot\vec{\sigma}(2)+[Y^{(2)}(2)\times\vec{\sigma}(2)]\cdot\vec{\sigma}(1)\}\,. \tag{6}$$

Here V_T is the strength parameter, α is the isospin singlet-triplet mixing parameter, r_0 is the range parameter, $\vec{\sigma}(i)$ and $Y^{(2)}(i)$ are the spin operators and the spherical harmonics for the ith particle, respectively. One can, then, readily calculate the reduced transition matrix elements for the l-forbidden $M1$ transitions. The result is

$$\mathfrak{M}=\sum_m \mathfrak{M}_m = (U_jU_{j'}+V_jV_{j'})\left[\frac{l(l'+1)}{2(l+l'+1)}\right]^{1/2}(g_s-g_l)\sum_m F_m\,, \tag{7}$$

$$F_m=(F_\delta)_m+(F_T)_m\,,$$

where

$$(F_\delta)_m=(U_{j_1}V_{j_2}-V_{j_1}U_{j_2})^2\left[\frac{l_1(l_1+1)}{(2l_1+1)}\right]\begin{cases} V_sI_\delta/(-\Delta E_m)\\ \tfrac{1}{2}g(V_t-V_s)I_\delta/(-\Delta E_m)\end{cases} \tag{8}$$

and

$$(F_T)_m=\left(\frac{2}{\sqrt{\pi}}-\frac{1}{2\pi\sqrt{2}}\right)(U_{j_1}V_{j_2}-V_{j_1}U_{j_2})^2\left[\frac{l+l'}{l'+1}\right]^{1/2}\left\{\left[\frac{l_1(l_1+1)}{2l_1+1}\right]-\delta_{l_1l'}\left[\frac{3l_1}{2(2l_1+1)}\right]-\delta_{l_1l}\left[\frac{3(l_1+1)}{2(2l_1+1)}\right]\right\}\frac{V_TI_T}{-\Delta E_m}\,. \tag{9}$$

Here, U and V are the coefficients of the Bogoljubov-Valatin transformation.[8] The upper line in the brace of Eq. (8) corresponds to the even pair – the quasiparticles in the orbits j_1 and j_2 are of the same kind (proton or neutron) as the one in the orbit j or j' – and the lower line in the brace corresponds to the odd pair. g_s-g_l in (7) is $4.585\mu_N$ for odd-proton nuclei and $-3.826\mu_N$ for odd-neutron nuclei, while g in (8) is the ratio 3.826/4.585 = 0.834 for the creation of neutron quasiparticles in the odd-proton transition, and is 4.585/3.826 = 1.199 for the creation of proton quasiparticles in the odd-neutron transition. I_δ is the Slater integral for the δ-function interaction, whereas I_T is the one for the tensor interaction. It is rather interesting to note that the first term $U_jU_{j'}$ in (7) corresponds to the like-core transition of AHS,[2] while the second term $V_jV_{j'}$ corresponds to the unlike-core transition, if we write

$$\begin{aligned} U_j^2&=(2j+1-p)/(2j+1)\,,\\ V_j^2&=p/(2j+1)\,,\\ U_{j'}^2&=(2j'+1-q)/(2j'+1)\,,\\ V_{j'}^2&=q/(2j'+1)\,. \end{aligned} \tag{10}$$

In the numerical calculations, we use the harmonic-oscillator wave functions in order to evaluate the Slater integrals. For the δ force, we introduce

$$-V_sI_\delta(j_1j_2;jj')=C_Tf(j_1j_2;jj')A^{-1/2}\,, \tag{11}$$

where the interaction strength C_T is chosen to fit the experimental values of the $M1$ transition rate, and the mass-number dependence of the integral I_δ is determined in the same way as in the work of Noya, Arima, and Horie,[9] while the relative magnitude V_t/V_s of the strengths for the triplet and singlet interaction is assumed to be 1.5, the same as used by AHS.[2] The strength parameter for the tensor interaction V_T is assumed to be -25 MeV, and the value of the isospin singlet-triplet parameter α is approximately equal to unity. The range parameter λ is introduced in place of r_0,[9] defined by the equation $\lambda=r_0(\nu/2)^{1/2}$, where ν is the oscillator wave-function parameter.

The single-particle energy levels used in solving the gap equation of the pairing model are essentially the same as those which are currently used.[10] At both ends of the major shells, we only included the relevant pairs of spin-orbit partners, whose separations have been readjusted, and the strengths of the pairing interaction are $G_n=19.5/A$ MeV and $G_p=21/A$ MeV throughout the whole mass region.[11]

Since we noticed during our preliminary calculations that the effect of the tensor force was rather small, we first investigated the effect of the central force so that we were able to choose the strength parameter C_T in Eq. (11) to reproduce almost all experimental data as closely as possible. We found that most of the data can be fitted, except for the heavy-mass nuclei, by choosing the parameters $C_n=50$ MeV for neutrons and $C_p=25$ MeV for protons. These values are consistent with those of the parameters used by Freed and Kisslinger.[5] (These authors used $k_n=60$ MeV and $k_p=30$ MeV.) Then the values of the range parameter λ and the isospin singlet-triplet parameter α of the tensor interaction were varied, including also a minor adjustment of the other parameters, and we have found that the value of 0.3 for the range parameter λ provided the best fit for the whole mass region, whereas the parameter α is found to be 0.8.

III. RESULTS AND DISCUSSION

The results of the calculation are shown in Fig. 1 for the odd-neutron nuclei, and in Fig. 2 for the odd-proton nuclei, respectively. Experimental data are mostly taken from the compilation of Geiger *et al.*[12] To show explicitly the effect of the tensor force, the figures show squares of the calculated reduced $M1$ transition matrix elements with and without the tensor-force contribution.

For the odd-neutron nuclei, over-all good agreement is obtained. For Ni, Cd, Sn, and Hg isotopes, the central force gives better results. This is because these nuclei are interpreted to have almost pure quasiparticle nature.[5] On the other hand, improvement is observed for Te and Xe isotopes when the spin-dependent quadrupole force, tensor force, is also included. Since the spin-dependent quadrupole force has a resemblance to a collective force,[5,13] these soft-vibrational nuclei can be better described by the inclusion of the collective effect.[5]

For the odd-proton nuclei, the effect of the tensor force is more important, although no sizable effect is clearly observed up to the medium-heavy region. Except for the heavy-mass nuclei, the pure quasiparticle component is dominant in this calculation and, therefore, among the soft-vibrational nuclei the softest-vibrational I isotopes seem to be more collective than Cs isotopes, which are described quite well. The evidently deformed Eu isotopes are certainly not described by the present model, which gives very small values as seen from the figure. As and Rb in the relatively light-mass region are also not explained very well. These nuclei may be soft vibrational, or they may require an effectively stronger interaction, such as the inclusion of the neutron-proton interaction, and are subject to a more careful study. The most important result of this calculation is seen in the heavy-mass region, where experimental data show very small reduced $M1$ transition matrix elements for Ir, Au, and Tl isotopes. As can be seen from the figure, the central force alone gives much higher values of the matrix elements, whereas the inclusion of the tensor force drastically reduces the magnitudes of these matrix elements. This is because of the interference effect between the ten-

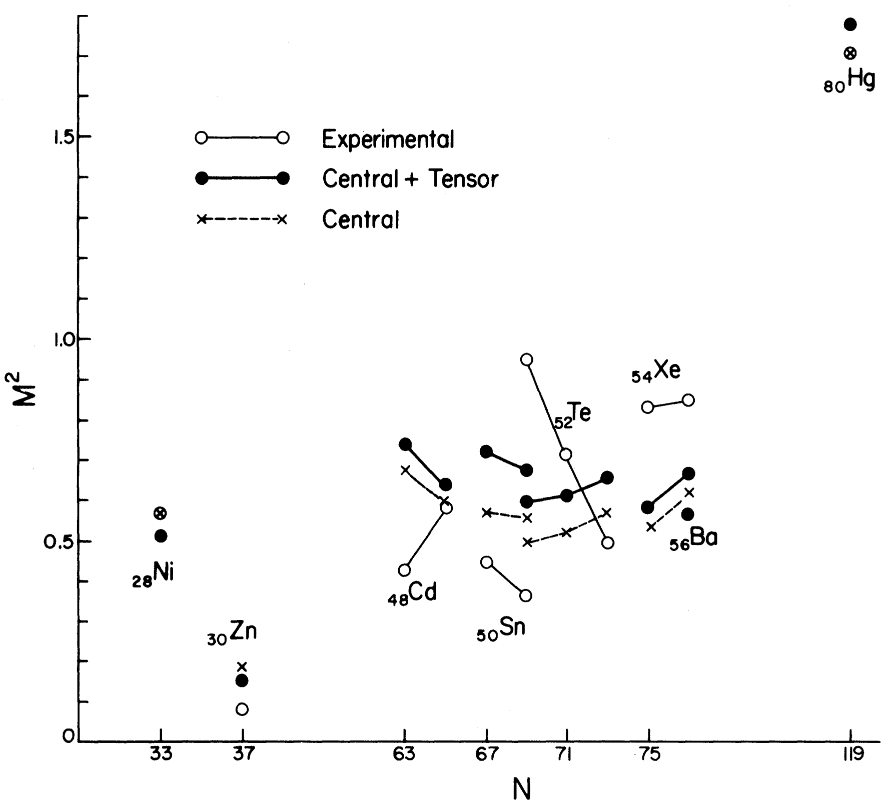

FIG. 1. The square of the reduced transition matrix elements of the l-forbidden $M1$ transitions for odd-neutron nuclei versus the neutron number N.

sor force and the central force, giving rise to opposite signs and comparable magnitudes for the matrix elements $\mathfrak{M}_\delta$ and $\mathfrak{M}_T$. Although the result for Tl isotopes looks as if it does not indicate the effect, the interference minimum is slightly shifted to the lower-mass side according to the presently chosen value of the range parameter, and a minor change of the range parameter will make the agreement much better because the dependence on the choice of the tensor-force parameter is very sensitive, particularly in this mass region. It is interesting to note that the spin-dependent quadrupole force gives better results than the spin-independent quadrupole force does, as pointed out by Kisslinger,[13] and this may be because the spin-independent quadrupole force is not strong enough in this mass region.[5] It seems a bit strange that the calculated results for Ir agree so well with experimental values, because one might think that these isotopes were still in the deformed region, but the phonon component of these isotopes is not more than 50% and, therefore, these isotopes may not be well deformed.[14]

We have seen that our calculation agrees with that of Freed and Kisslinger,[5] both in its trends and in the choice of the strength parameters for the short-range interaction, as far as the central force is concerned. On the other hand, the improvement due to the mixing of the collective force is different, and we see that the spin-dependent quadrupole force is quite important for the heavy-mass isotopes. The effect of the spin-dependent quadrupole force has been examined by Shikata in the medium-heavy-mass region.[6] He has pointed out that the tensor force is more important than the central force, but that the tensor force cannot be combined with the central force. Our results show not only that the tensor force can be mixed with the central force without violating the normal choice of the relative sign, but also that the importance of the tensor force is essential in the hevay-mass region rather than in the medium-heavy-mass region. We have chosen a stronger interaction strength of −25 MeV, for which he uses −10 MeV, and a shorter range parameter of 0.3, which he takes as 1.0. The tensor-force effect is more significant for the isospin singlet as he also observed.

As a conclusion, we stress that the interference effect between the central and tensor forces is

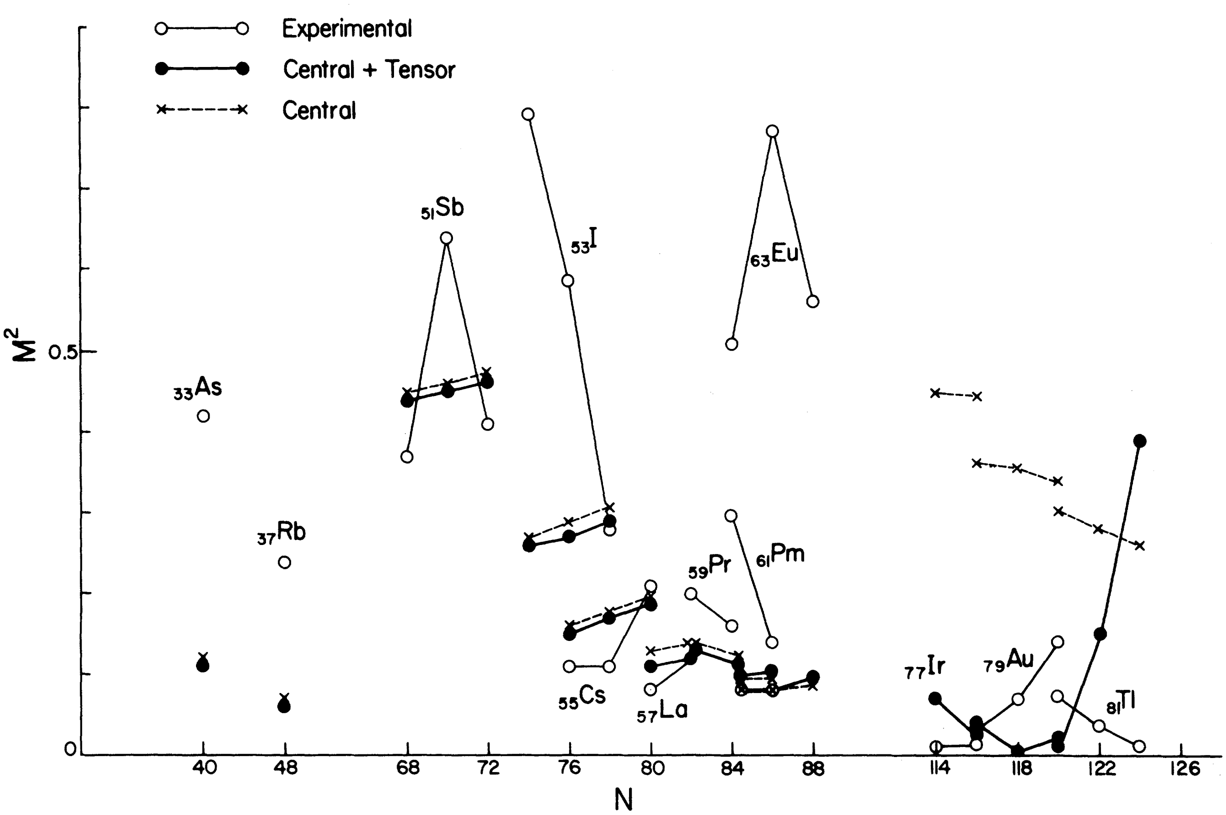

FIG. 2. The square of the reduced transition matrix elements of the l-forbidden $M1$ transitions for odd-proton nuclei versus the neutron number N.

quite important for the heavy-mass isotopes in order to explain the small observed values of the l-forbidden $M1$ transition rates for the odd-proton nuclei.

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