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Two-Pion-Exchange Three-Body Force in Nuclear Matter*

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The contribution of the two-pion-exchange three-nucleon force to the binding energy of nuclear matter is evaluated using the modified pion-mass method of Brown and Green and by the conventional potential approach. Using nonrelativistic formulas, the relation between the two methods is clarified. It is shown that the Brown-Green prescription, as given in their paper, is incorrect when a cutoff in the *NN* distance is applied and the modified pion mass is momentum dependent. When the correct prescription is used, the first-order contribution becomes sensitive to the pionic form factor chosen. Using the same form factor as used by Brown and Green, the contribution with an *NN* cutoff of 1 F is only about 0.1 MeV instead of 1.25-MeV repulsion per particle as quoted by them. Another form factor leads to about 0.5-MeV repulsion. The second-order contribution, including the cross terms involving the two-body one-pion-exchange potential and three-body forces, has been recalculated in the Brown-Green spirit after correcting for their earlier error. It is attractive, much larger than the first-order term, and is about the same for the two form factors used. The net effect of the two-pion-exchange three-body force is rather sensitive to the *NN* cutoff and is about a 4- to 8-MeV per particle attraction in nuclear matter, depending on the cutoff.

I. INTRODUCTION

Recently, a number of authors have examined the effect of three-body *NNN* forces, as well as the very similar but somewhat simpler Λ*NN* forces on nuclear matter.[1-5] Most recently, Brown and Green[1] (hereafter referred to as BG) have estimated the effect of the two-pion-exchange (TPE) three-nucleon force on nuclear matter by modifying the pion mass in the one-pion-exchange two-nucleon potential (OPEP). In this method they do not need to derive the three-body potential in the configuration space and the calculation is simple. BG have concluded that whereas the effect of the lowest-order TPE three-nucleon force in nuclear matter is small, the second-order effects are larger, with the net result of about 2.5-MeV per particle additional attraction at the saturation density. They further claim that the ambiguities in their calculation are not very large. On the other hand, Bhaduri, Loiseau, and Nogami[2] (referred to as BLN) have calculated the first-order effect of the TPE Λ*NN* force in nuclear matter by explicitly deriving the three-body potential in the configuration space. BLN find that the calculation is beset with considerable ambiguities because the final contribution is very sensitive to the *NN* correlations and also to the short-range part of the force. Loiseau and Nogami[3] have considered the problem of the *NNN* potential in a similar way. They also find considerable ambiguities in this problem, although the effect of the *NNN* potential in nuclear matter is much smaller than the corresponding Λ*NN* case.

First, we mention the important differences between the approaches of BG and BLN. BG consider relativistic corrections while BLN work in the nonrelativistic static limit. BG do not derive the *NNN* potential in the configuration space, but, by examining its expression in the momentum space, show that its effect can be incorporated by modifying the pion mass in the OPEP. BLN use the TPE three-body potential, which is obtained explicitly in the coordinate space,[4] and estimate its effect in the first order only. BG have pointed out quite rightly that the second-order effect is substantial.

The purpose of the present paper is to clarify the relation between the two methods in detail, concentrating initially on the first-order effects. For simplicity we use the nonrelativistic formula for the pion-baryon interaction throughout, but include the pionic form factor. The nonrelativistic approximation should not be unreasonable, because the important range of the momentum transfer involved is around 1.3 F^{-1} for the first-order and around 2 F^{-1} for the second-order effect at the equilibrium density. We further concentrate on the single-exchange term of the expectation value of the TPE *NNN* force, which alone was considered by BG. This term has exactly the same form as the expectation value of the Λ*NN* force, which was investigated by BLN. We then go on to show the following:

(i) The connection between the modified pion-mass method of BG and the conventional potential approach of BNL. When there is no *NN* cutoff applied, the two methods are equivalent except that the BG approach includes some spurious contributions from δ-function-like contact interactions. Only for some specific forms of pionic form factor do the

two methods give identical results.
(ii) The BG prescription, as given in their paper, is incorrect when an NN cutoff is applied, and the modified pion mass is momentum dependent. It is shown how this mistake can be corrected. When this is done, the two methods of BG and BLN give practically identical results for the pionic form factors used. Using the same form factor as used by BG, the contribution is found to be only about 0.1 MeV for the NN cutoff of 1 F instead of 1.25-MeV repulsion per particle as quoted by BG.
(iii) When the corrected BG prescription is used, the first-order contribution, contrary to their claim, becomes quite sensitive to the pionic form chosen. This ambiguity is large, but we still find that the first-order contribution of the TPE three-body force is likely to be less than or about 0.5-MeV repulsion per particle.
(iv) Following the method of BG the second-order calculation is redone using the corrected procedure. This yields about 5.5-MeV attraction per particle in nuclear matter for the NN cutoff of 1 F and is about the same for two form factors chosen.

In Sec. II the basic formulas of the two methods are developed when there is no NN cutoff, and their relation clarified. Section III contains the more realistic case when an NN cutoff is applied and the second-order as well as the first-order contributions are worked out. Section IV contains the numerical results along with discussion and conclusions. Two Appendices are given for some of the mathematical derivations.

II. METHOD OF CALCULATION WITHOUT CUTOFF

We consider the TPE diagram of Fig. 1 whose S matrix element is given by[2,4,6] ($c=\hbar=1$)

$$S=\sum_{\alpha,\beta}\frac{4\pi f^2\tau_{1\alpha}\tau_{2\beta}}{(2\pi)^6\mu^2}\int d\vec{q}_1 d\vec{q}_2\,\frac{(\vec{\sigma}_1\cdot\vec{q}_1)(\vec{\sigma}_2\cdot\vec{q}_2)\langle\alpha,\vec{q}_1|S^{(3)}_{\pi N}|\beta,\vec{q}_2\rangle}{(q_1^2+\mu^2)(q_2^2+\mu^2)}K(q_1^2)K'(q_1^2)K(q_2^2)K'(q_2^2)e^{-i(\vec{q}_1\cdot\vec{r}_1-\vec{q}_2\cdot\vec{r}_2)}. \quad (1)$$

Here f^2 (=0.08) is the πN coupling constant, $\vec{q}_{1,2}$ are the momenta of the exchanged pions, $\vec{r}_{1,2}$ are the coordinates of the two nucleons 1 and 2, and μ is the pion mass. The Pauli matrix $\tau_{1\alpha}$ denotes the α component of the isospin operator of the nucleon 1, and likewise for the other nucleons. Similarly, $\vec{\sigma}_1$, $\vec{\sigma}_2$ are the Pauli spin matrices for the nucleons 1 and 2. The functions K and K' are the vertex and propagator corrections, and defined by

$$\bar{u}(p')\Gamma_{5\alpha}(p',p)u(p)=igK(q^2)\bar{u}(p')\gamma_5\tau_\alpha u(p), \quad (2)$$

$$\Delta'_F(q^2)=K'(q^2)(q^2+\mu^2)^{-1}. \quad (3)$$

Here g is the pseudoscalar πN coupling constant, p, p' and $q=p'-p$ are four-momenta, $\bar{u}$ and u are Dirac spinors, Γ and Δ' are the renormalized vertex part and propagator, respectively, and K and K' are normalized by $K(-\mu^2)=K'(-\mu^2)=1$. Let us consider the form factors of the following form[7];

$$\frac{K^2(q^2)K'(q^2)}{q^2+\mu^2}=\int_0^\infty dm^2\,\frac{\alpha(m^2)}{q^2+m^2} \quad (4)$$

with the "spectral function"

$$\alpha(m^2)=\delta(m^2-\mu^2)+\beta(m^2)\theta(m^2-9\mu^2). \quad (5)$$

Here $\beta(m^2)$ is a real function, and $\theta(x)$ is unity for $x\geq 0$ and zero otherwise. The "threshold" $m=3\mu$ is there because the lowest-mass intermediate state that modifies the propagator and the vertex is the three-pion state.

The πN scattering matrix $S^{(3)}_{\pi N}$ for a zero-energy pion is given by[4,8]

$$\langle\alpha,\vec{q}_1|S^{(3)}_{\pi N}|\beta,\vec{q}_2\rangle=2\pi i\delta(0)K(q_1^2)K(q_2^2)[(A\tau_{3\alpha}\tau_{3\beta}+B\tau_{3\beta}\tau_{3\alpha})(\vec{\sigma}_3\cdot\vec{q}_2)(\vec{\sigma}_3\cdot\vec{q}_1)+(B\tau_{3\alpha}\tau_{3\beta}+A\tau_{3\beta}\tau_{3\alpha})(\vec{\sigma}_3\cdot\vec{q}_1)(\vec{\sigma}_3\cdot\vec{q}_2)+2D\delta_{\alpha\beta}]e^{i(\vec{q}_1-\vec{q}_2)\cdot\vec{r}_3}, \quad (6)$$

where A and B are related to the p wave while D is related to the s-wave πN scattering, and are all constants in the nonrelativistic approximation.[4] The pionic form factor associated with $S^{(3)}_{\pi N}$ is assumed to be the same as that for the πNN vertex. The same assumption was made by BG.

Substituting (6) into (1) one gets the form $S=-2\pi i\delta(0)W$. Then W is interpreted as the TPE NNN force.[4] This W is our starting point. Now let us describe the two different ways of estimating the first-order effect of W on the binding energy of nuclear matter. The first of these corresponds to the BG method, while the second one follows the more conventional approach of BLN. For simplicity we consider only the contribution from the p-wave πN interaction, ignoring the term with D in (6).

A. Modified Pion-Mass Method of BG

Here we explain the modified pion-mass method in a very simplified manner. BG have used this

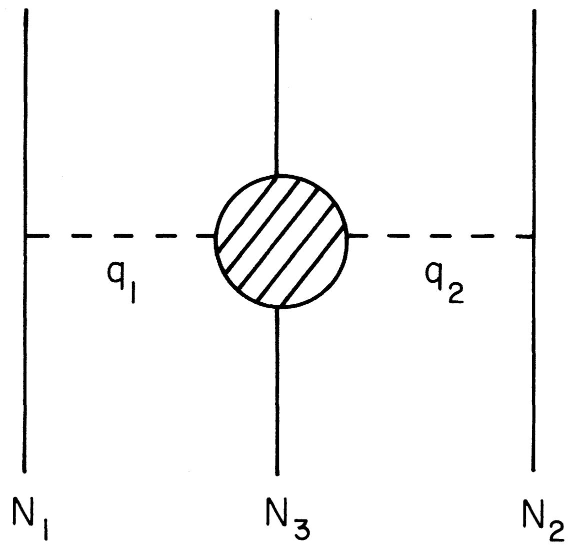

FIG. 1. The diagram for the two-pion-exchange three-body force.

method with a cutoff in the NN distance, while here we do not apply any cutoff. When no cutoff is applied the equations are simpler and it is easy to see its relation with the BLN approach. When there is no constraint on the coordinate $\vec{r}_3$, one can integrate (6) with respect to $\vec{r}_3$ at once. This gives a factor $(2\pi)^3\delta(\vec{q}_1-\vec{q}_2)$. Also we consider only the single-exchange term, which is obtained by taking the diagonal sum in (1) with respect to the spin and isospin of the third nucleon. Thus we obtain an effective NN force U_p due to the third nucleon (the subscript p denoting that only the p-wave part of the πN scattering is being considered)

$$U_p=\rho\int d\vec{r}_3\, W(\vec{r}_1,\vec{r}_2,\vec{r}_3)$$

$$=-\frac{\rho f^2\vec{\tau}_1\cdot\vec{\tau}_2}{\pi^2\mu^2}\int d\vec{q}\,\frac{(\vec{\sigma}_1\cdot\vec{q})(\vec{\sigma}_2\cdot\vec{q})}{(q^2+\mu^2)^2}\,q^2(A+B)\times[K^2(q^2)K'(q^2)]^2e^{i\vec{q}\cdot\vec{r}} \tag{7}$$

$$=-\frac{\rho f^2\vec{\tau}_1\cdot\vec{\tau}_2}{\pi^2\mu^2}\int d\vec{q}(\vec{\sigma}_1\cdot\vec{q})(\vec{\sigma}_2\cdot\vec{q})q^2(A+B)\times\left[\int_0^\infty dm^2\frac{\alpha(m^2)}{q^2+m^2}\right]^2e^{i\vec{q}\cdot\vec{r}}. \tag{8}$$

Here $\vec{r}=\vec{r}_1-\vec{r}_2$, and ρ is the density of nuclear matter. We have used (4) to go from (7) to (8). Note that the q integration in (8) is divergent, and hence U_p contains δ-function-like contact interactions with respect to $\vec{r}_1-\vec{r}_2$ in the coordinate space, unless[9]

$$\int_0^\infty dm^2\,\alpha(m^2)=0\,. \tag{9}$$

We now compare (7) with the two-nucleon OPEP,

$$V_{\text{OPEP}}=-\frac{f^2\vec{\tau}_1\cdot\vec{\tau}_2}{2\pi^2\mu^2}\int d\vec{q}\,\frac{(\vec{\sigma}_1\cdot\vec{q})(\vec{\sigma}_2\cdot\vec{q})}{q^2+\mu^2}K^2(q^2)K'(q^2)e^{i\vec{q}\cdot\vec{r}}\,. \tag{10}$$

We can rewrite U_p as

$$U_p=-\frac{f^2\vec{\tau}_1\cdot\vec{\tau}_2}{2\pi^2\mu^2}\int d\vec{q}K^2(q^2)K'(q^2)(\vec{\sigma}_1\cdot\vec{q})(\vec{\sigma}_2\cdot\vec{q})e^{i\vec{q}\cdot\vec{r}}\,\delta\mu^2(q^2)\frac{\partial}{\partial\mu^2}\left(\frac{1}{q^2+\mu^2}\right), \tag{11}$$

where

$$\delta\mu^2(q^2)=-2\rho(A+B)K^2(q^2)K'(q^2)q^2\,. \tag{12}$$

If $|\delta\mu^2|\ll\mu^2+q^2$, Eq. (11) can be further simplified as

$$U_p=-\frac{f^2\vec{\tau}_1\cdot\vec{\tau}_2}{2\pi^2\mu^2}\int d\vec{q}K^2(q^2)K'(q^2)(\vec{\sigma}_1\cdot\vec{q})(\vec{\sigma}_2\cdot\vec{q})e^{i\vec{q}\cdot\vec{r}}\,[(q^2+\bar{\mu}^2)^{-1}-(q^2+\mu^2)^{-1}]\,, \tag{13}$$

with $\bar{\mu}^2=\mu^2+\delta\mu^2$. BG used Eq. (13) with a further modification as follows: They use the OPEP without the factor K^2K' in (10) together with

$$\delta\mu^2=-2\rho(A+B)H^2(q^2)q^2\,,\quad H(q^2)=K^2(q^2)K'(q^2)\,. \tag{14}$$

The effect of the factor $H(q^2)$ in (10) which modifies the short-range part of the OPEP is taken care of by introducing an NN cutoff. Let us now calculate the first-order contribution of V_{OPEP} as given in (10), but omitting the form factors, to nuclear matter. Only the exchange term contributes and a simple calculation yields

$$\frac{\langle V_{\text{OPEP}}\rangle}{N}=\frac{3\rho f^2}{16\pi^2\mu^2}\int d\vec{q}\,\frac{q^2F(q)}{q^2+\mu^2}, \tag{15}$$

where N is the nucleon number and

$$F(q)=\frac{3\pi^2}{k_F^3}\left[2-\frac{3q}{2k_F}+\left(\frac{q}{2k_F}\right)^3\right]\theta(2k_F-q)\,. \tag{16}$$

The factor $q^2(q^2+\mu^2)^{-1}$ in (15) can be separated into two terms, $1-\mu^2(q^2+\mu^2)^{-1}$, which correspond to the potentials of the form $\delta(\vec{r})$ and $e^{-\mu r}/r$, respectively. $F(q)$ can also be written as

$$F(q)=\int d\vec{r}\, D^2(k_F r)e^{i\vec{q}\cdot\vec{r}}\,, \tag{17}$$

where

$$D(k_F r)=3j_1(k_F r)/(k_F r)\,, \tag{18}$$

j_1 being the spherical Bessel function of order 1. To get the contribution of U_p as defined in (7), all that one has to do is to replace

$$\frac{1}{q^2+\mu^2}\to\delta\mu^2(q^2)\frac{\partial}{\partial\mu^2}\left(\frac{1}{q^2+\mu^2}\right)$$

in (15) inside the integral with $\delta\mu^2$ given by (14). This yields

$$\frac{\langle U_p\rangle}{N}=\frac{3\rho f^2}{16\pi^2\mu^2}\int d\vec{q}\; F(q)\,q^2\delta\mu^2(q^2)\frac{\partial}{\partial\mu^2}\left(\frac{1}{q^2+\mu^2}\right) \tag{19}$$

$$=6C_p\rho^2\mu^{-6}\int_0^\infty dq\; q^6F(q)\left[\int_0^\infty dm^2\,\frac{\alpha(m^2)}{q^2+m^2}\right]^2, \tag{20}$$

where C_p is a constant given by[3,4,10]

$$C_p=\mu^4f^2(A+B)/4\pi$$
$$=\frac{\mu^4f^2}{9\pi^2}\int_0^\infty\frac{\sigma_{33}(p)}{p^2+\mu^2}\,dp=0.46\text{ MeV}\,, \tag{21}$$

where σ_{33} is the p-wave πN cross section in the $(I,J)=(\frac{3}{2},\frac{3}{2})$ state. The contribution from the contact interaction in (20) can be separated in the same manner as for (15).

B. Potential Method of BLN

We again start with (1) and (6), but instead of integrating (6) with respect to $\vec{r}_3$ we do the $\vec{q}_1$ and $\vec{q}_2$ integrations first, obtaining an NNN potential explicitly in the coordinate space.[2,4] This potential will be denoted by W_p and should be distinguished from the effective NN potential U_p as defined in (7). If we temporarily drop the factors K and K', W_p is given by

$$W_p=-\frac{C_p}{8\mu^4}[5(\vec{\tau}_1\cdot\vec{\tau}_3)(\vec{\tau}_3\cdot\vec{\tau}_2)+3(\vec{\tau}_2\cdot\vec{\tau}_3)(\vec{\tau}_3\cdot\vec{\tau}_1)]$$
$$\times(\vec{\sigma}_1\cdot\vec{\nabla}_x)(\vec{\sigma}_3\cdot\vec{\nabla}_x)(\vec{\sigma}_3\cdot\vec{\nabla}_y)(\vec{\sigma}_2\cdot\vec{\nabla}_y)Y(x)Y(y)$$
$$+(1\rightleftarrows 2, x\rightleftarrows y)\,, \tag{22}$$

where $\vec{x}=\vec{r}_2-\vec{r}_3$, $\vec{y}=\vec{r}_1-\vec{r}_3$, and $Y(x)=-h_0(i\mu x)=e^{-\mu x}/(\mu x)$. Note that the contact interactions with respect to x and y have been discarded in deriving (22). The single-exchange term of the expectation value of W_p in nuclear matter is given by[3]

$$\frac{\langle W_p\rangle}{N}=\tfrac{1}{4}C_p\rho^2\int D^2(k_F z)[1+(3\cos^2\theta-1)T(x)T(y)]$$
$$\times Y(x)Y(y)d\vec{x}\,d\vec{y}\,, \tag{23}$$

where $z=|\vec{x}-\vec{y}|$, $T(x)\,Y(x)=h_2(i\mu x)=[1+3(\mu x)^{-1}+3(\mu x)^{-2}]Y(x)$, and $\cos\theta=\vec{x}\cdot\vec{y}/xy$. The first term in the square brackets in (23), unity, is the contribution of the "central" part of W_p, while the second term is the "tensor" contribution. After some manipulations (see Appendix A) one can express the integrals in (23) as

$$I_c=\mu^6\int D^2(k_F z)Y(x)Y(y)d\vec{x}\,d\vec{y}$$
$$=8\mu^4\int_0^\infty dq\; q^2F(q)(q^2+\mu^2)^{-2}\,, \tag{24}$$

$$I_t=\mu^6\int D^2(k_F z)(3\cos^2\theta-1)Y(x)Y(y)T(x)T(y)d\vec{x}\,d\vec{y}$$
$$=16\int_0^\infty dq\; q^6F(q)(q^2+\mu^2)^{-2}\,. \tag{25}$$

Now if we include the factors K and K', the functions Y and T in (22)–(25) are replaced as follows:

$$\mu^3Y(x)\to-\int_0^\infty dm^2\alpha(m^2)m^3h_0(imx)\,, \tag{26}$$

$$\mu^3T(x)Y(x)\to\int_0^\infty dm^2\alpha(m^2)m^3h_2(imx)\,. \tag{27}$$

The expectation value of W_p then becomes

$$\frac{\langle W_p\rangle}{N}=2C_p\rho^2\mu^{-6}\int_0^\infty dq\,q^2F(q)$$
$$\times\left\{\left[\int_0^\infty dm^2\,\frac{m^2\alpha(m^2)}{q^2+m^2}\right]^2+2q^4\left[\int_0^\infty dm^2\frac{\alpha(m^2)}{q^2+m^2}\right]^2\right\}. \tag{28}$$

The two terms in the curly brackets correspond to the central and tensor contributions, respectively.

Now it is clear that the two methods give different answers in general. $\langle U_p\rangle$ of (20) and $\langle W_p\rangle$ of (28) agree only if (9) is satisfied. The difference is due to the contact interactions with respect to $\vec{r}_1-\vec{r}_3$ and $\vec{r}_2-\vec{r}_3$ included in U_p. Note that U_p, which is an effective two-body interaction between the nucleons 1 and 2, was derived from W by integrating it over all $\vec{r}_3$ [see (7)]. This results in the inclusion in U_p of some singular contact interactions with respect to $\vec{r}_1-\vec{r}_3$ and $\vec{r}_2-\vec{r}_3$. In W_p, however, such contact terms are discarded to start with. These contact interactions with respect to $\vec{r}_1-\vec{r}_3$ and $\vec{r}_2-\vec{r}_3$ should be clearly distinguished from those with respect to $\vec{r}_1-\vec{r}_2$ which appear in U_p and W_p. The latter disappear when an NN cutoff for $\vec{r}_1-\vec{r}_2$ is introduced, whereas the former still remain in U_p.

III. METHODS OF CALCULATION WITH A CUTOFF IN *NN* DISTANCE

A. First-Order Contribution

The first-order exchange contribution of an NN

potential $V(r)$ to nuclear matter is proportional to the integral

$$I=(2\pi)^3\int d\vec{r}\; V(r)D^2(k_F r)=\int d\vec{q}\; V(q)F(q)\,, \tag{29}$$

where the functions $F(q)$ and $D(k_F r)$ have been defined by (17) and (18). Here $V(q)=\int d\vec{r}\; V(r)e^{i\vec{q}\cdot\vec{r}}$. If now an NN cutoff is introduced, I in (29) should be replaced by

$$\tilde{I}=(2\pi)^3\int d\vec{r}\; V(r)D^2(k_F r)\theta(r-d)\,. \tag{30}$$

There are two ways of writing (30) in the q space. One can write

$$\tilde{I}=\int d\vec{q}\;\tilde{V}(q)F(q)\,,$$
$$\tilde{V}(q)=\int d\vec{r}\; V(r)\theta(r-d)e^{i\vec{q}\cdot\vec{r}}\,. \tag{31}$$

The alternative way is to introduce the θ function in $D^2(k_F r)$, and define

$$\tilde{F}(q)=\int d\vec{r}\; D^2(k_F r)\theta(r-d)e^{i\vec{q}\cdot\vec{r}}\,. \tag{32}$$

Then, if one takes care to throw out all singular contact terms from V explicitly, one can also write

$$\tilde{I}=\int d\vec{q}\; V(q)\tilde{F}(q)\,. \tag{33}$$

Following the latter prescription it is seen that $\langle U_p\rangle$ and $\langle W_p\rangle$ can be calculated in the presence of a cutoff by simply replacing F by $\tilde{F}$ in (20) and (28). The difference between $\langle U_p\rangle$ and $\langle W_p\rangle$ persists unless (9) for the pionic form factor is satisfied.

Alternatively, one can calculate $\tilde{I}$ from (31) by using $\tilde{V}(q)$. By inspecting (19) we see that (apart from a constant multiplicative factor)

$$V(q)\propto q^2\,\delta\mu^2(q^2)\frac{\partial}{\partial\mu^2}\left(\frac{1}{q^2+\mu^2}\right). \tag{34}$$

What one should do is to find the corresponding $V(r)$ from (34), cut it off at d, and then calculate $\tilde{V}(q)$. Once $\tilde{V}(q)$ is obtained (31) can be used to calculate $\langle U_p\rangle$. What BG did instead is to simply replace $(q^2+\mu^2)^{-1}$ by the Fourier transform of the cutoff Yukawa potential;

$$\frac{1}{q^2+\mu^2}\to\frac{1}{q^2+\mu^2}\,e^{-\mu d}\,\frac{\mu\sin qd+q\cos qd}{q} \tag{35}$$

in (34) to get $\tilde{V}(q)$. This procedure turns out to be correct only if $\delta\mu^2$ is q independent, as can be checked explicitly. When the prescription (35) is used, we get

$$\frac{\langle U_p\rangle_{\rm BG}}{N}=6C_p\rho^2\mu^{-6}e^{-\mu d}\int_0^\infty dq\,\frac{q^4F(q)H^2(q^2)}{q^2+\mu^2}\times\left[\left(1-\frac{\mu d}{2}-\frac{\mu^2}{q^2+\mu^2}\right)g(q,d)+\tfrac{1}{2}\mu d\, j_0(qd)\right], \tag{36}$$

where

$$g(q,d)=(\mu\sin qd+q\cos qd)/q\,. \tag{37}$$

The subscript BG denotes that we have used their prescription (35). Using (36) and the form factors that they have used, we shall reproduce, approximately, the numbers they have given for the first-order effect in their paper.[1] However, this procedure is incorrect when $\delta\mu^2$ is q dependent, and this is the realistic case. In general, the calculation of $\tilde{V}(q)$ from (34) is cumbersome, since the q dependence of $\delta\mu^2$ is complicated by the form factor $H(q^2)$. If, however, we ignore the form factors temporarily and use the fact that nonrelativistically $\delta\mu^2\propto q^2$, then we can easily calculate $\tilde{V}(q)$. Using this approach we give the expression for $\langle U_p\rangle/N$ just to show that it is very different from (36). We get

$$\frac{\langle U_p\rangle}{N}=6C_p\rho^2\mu^{-4}e^{-\mu d}\int_0^\infty dq\,\frac{q^2F(q)}{q^2+\mu^2}\times\left[\left(-2+\frac{\mu d}{2}+\frac{\mu^2}{q^2+\mu^2}\right)g(q,d)-\frac{\mu d}{2}j_0(qd)\right]+6C_p\rho^2\mu^{-6}\int_0^\infty F(q)q^2dq\,, \tag{38}$$

where the last term is the contribution from the contact interaction with respect to $\vec{r}_1-\vec{r}_2$ and should be added only if $d=0$. Equation (38) should be compared with (36) after putting $H(q^2)=1$ in (36). The two are very different and coincide only when $d=0$.

B. Second-Order Contribution

So far we have considered only the first-order effects. As was shown in Sec. II the effect of the three-body force can be simulated by an effective two-body potential, which consists of central and tensor parts. Only the central part contributes in the first order, but the tensor part will play a dominant role in the second order. If we follow the BG method the effective NN potential is given by U_p of (8). To obtain the NN potential due to W_p of (22) is more complicated. In Sec. II we took the spin and isospin summation of W_p before doing the $\vec{r}_3$ integration, thus picking up only the central part of the effective NN potential. To obtain the NN potential including its tensor part we have to first integrate W_p with respect to $\vec{r}_3$. For simplicity we follow the BG approach. We also assume, following BG, that the long-range part of the NN force in the absence of three-body forces is given by the OPEP, although this is true only at very large distances ($r \gtrsim 2$ F). The second-order contribution to the potential energy per nucleon is then given by

$$\Delta E^{(2)} = \left(2\left\langle V_{\mathrm{OPEP}} \frac{Q}{e} U_p \right\rangle + \left\langle U_p \frac{Q}{e} U_p \right\rangle\right) \Big/ N$$

$$\equiv \Delta E_1^{(2)} + \Delta E_2^{(2)} , \tag{39}$$

where the bracketed superscript denotes that these are second-order terms, Q is the Pauli operator, and e is an energy denominator to be specified later. The expectation value is to be taken with respect to plane-wave determinantal states. Each of the two terms in (39) consists of direct and exchange terms, which can be further split into central and tensor components. The potentials V_{OPEP} and U_p are cut off at a distance d. BG's prescription as given by their Eq. (36) is valid only when $\delta\mu^2(q^2)$ is q independent or $d = 0$.

It is now necessary to get the expression for U_p in the coordinate space. To do this explicitly let us assume the form factors of the form

$$H(q^2) = \xi + (1-\xi)\frac{\eta^2 - \mu^2}{q^2 + \eta^2} ,$$

which is obtained from the spectral function

$$\alpha(m^2) = \delta(m^2 - \mu^2) - (1-\xi)\,\delta(m^2 - \eta^2) . \tag{40}$$

Different pairs of (ξ, η) give rise to different form factors. The case $\xi = 1$ corresponds to no form factor, or $H = 1$, while $\xi = 0$ gives the special case when (9) is satisfied. The form factors which we use are displayed in Table I; the form factors II and III have been taken from Ref. 1.[11,12] BG gave their numerical values for the form factor II, although this form factor is not compatible with the requirement $\eta > 3\mu$.

Substituting (40) in (8), we get

$$U_p = -\frac{\rho f^2 \vec{\tau}_1 \cdot \vec{\tau}_2}{\pi^2 \mu^2}(A + B) \int d\vec{q}\, e^{i\vec{q}\cdot\vec{r}} (\vec{\sigma}_1 \cdot \vec{q})(\vec{\sigma}_2 \cdot \vec{q}) q^2$$

$$\times \left(\frac{1}{q^2 + \mu^2} - \frac{1-\xi}{q^2 + \eta^2}\right)^2 . \tag{41}$$

TABLE I. The parameters of the form factor $H(q^2)$ as defined by (38). The form factor II was used by BG in their numerical calculation.

Form factor	ξ	$(\eta/\mu)^2$	Comments
I	1.0	...	$H(q^2) = 1$
II	0.28	5.73	Eq. (29) of Ref. 1 given in Ref. 11
III	0.	10.	Eq. (29a) of Ref. 1 given in Ref. 12

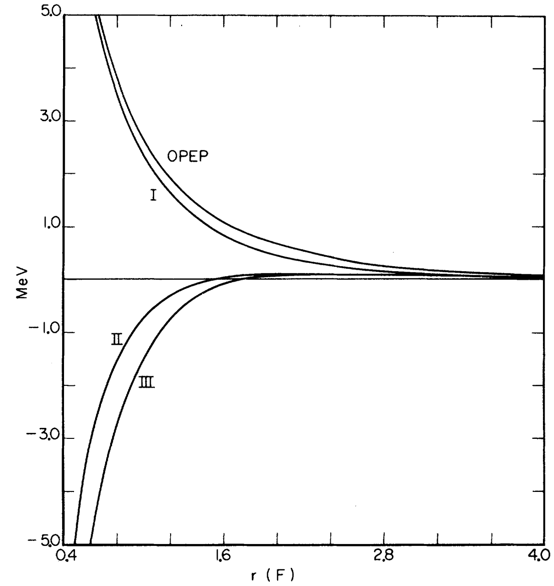

FIG. 2. The central part of U_p, $\lambda f_c(r)$ of (42), for the form factors I, II, and III, respectively. The curve "OPEP" shows the central part of the OPEP, $\frac{1}{3}f^2\mu g_c(r)$ of (43).

We use the standard procedure of replacing $(\vec{\sigma}_1 \cdot \vec{q}) \times (\vec{\sigma}_2 \cdot \vec{q})$ by $-(\vec{\sigma}_1 \cdot \vec{\nabla})(\vec{\sigma}_2 \cdot \vec{\nabla})$ and taking it out of the integral. This gets rid of contact interactions between the nucleons 1 and 2. Then U_p becomes

$$U_p(r) = \lambda \vec{\tau}_1 \cdot \vec{\tau}_2 [(\vec{\sigma}_1 \cdot \vec{\sigma}_2) f_c(r) + S_{12} f_t(r)] , \tag{42}$$

where $\lambda = \frac{4}{3}\pi C_p \rho \mu^{-3} = 0.980$ MeV, and $S_{12} = 3(\vec{\sigma}_1 \cdot \vec{r})$

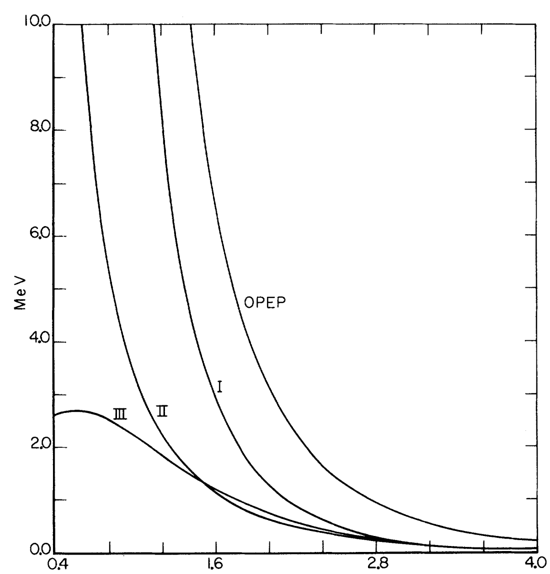

FIG. 3. The tensor part of U_p, $\lambda f_t(r)$ of (42), for the form factors I, II, and III, respectively. The curve "OPEP" shows the tensor part of the OPEP, $\frac{1}{3}f^2\mu g_t(r)$ of (43).

$\times(\vec{\sigma}_2\cdot\vec{r})/r^2-(\vec{\sigma}_1\cdot\vec{\sigma}_2)$. The functions $f_c(r)$ and $f_t(r)$ are defined in Appendix B. The two-nucleon OPEP is written in the form

$$V_{\text{OPEP}}(r)=\tfrac{1}{3}f^2\mu\vec{\tau}_1\cdot\vec{\tau}_2[(\vec{\sigma}_1\cdot\vec{\sigma}_2)g_c(r)+S_{12}g_t(r)]\,, \tag{43}$$

where $\frac{1}{3}f^2\mu=3.651$ MeV, and $g_c(r)=Y(r)$, $g_t(r)=T(r)Y(r)$. In Fig. 2 we have plotted the central part of U_p, $\lambda f_c(r)$, as a function of r for various form factors and compared it with the central part of OPEP, $\frac{1}{3}f^2\mu g_c(r)$. In Fig. 3 the tensor part $\lambda f_t(r)$ of U_p is plotted with the corresponding OPEP component for the various form factors. It is seen from these figures that U_p effectively modifies the long-range part of the two-nucleon potential. It turns out that the most important second-order contribution comes from the cross term between the tensor components of V_{OPEP} and U_p in $\Delta E_1^{(2)}$.

If one chooses the simple Rayleigh-Schrödinger propagator for e, then for the direct term we obtain

$$\frac{1}{N}\left\langle V_{\text{OPEP}}\frac{Q}{e}U_p\right\rangle=-\frac{6f^2\lambda\mu mk_F^{\,2}}{5\pi^2\hbar^2}\int_0^\infty dq\,qP\left(\frac{q}{2k_F}\right)\times[f_c(q)g_c(q)+2f_t(q)g_t(q)]\,, \tag{44}$$

where m is the nucleon mass, $P(q/2k_F)$ is the standard Euler function,[13] and

$$f_c(q)=\int_d^\infty f_c(r)j_0(qr)r^2dr\,,$$

$$f_t(r)=\int_d^\infty f_t(r)j_2(qr)r^2dr\,, \tag{45}$$

and likewise for $g_c(q)$ and $g_t(q)$. In an analogous way the direct contribution of $\langle U_p(Q/e)U_p\rangle/N$ may be written down. Alternatively, the second-order terms may be evaluated in the spirit of the reference-spectrum method by introducing a gap parameter Δ, as was done in Ref. 1. The only change in Eq. (44) is that the Euler function $P(q/2k_F)$ is replaced by a modified Euler function[14] $P(q/2k_F+0.6k_F/q)$, where we have used the same gap parameter Δ as used by BG in Ref. 1. The exchange second-order terms for potentials of the OPEP character are expected[15] to be relatively small (about 20% of the direct terms for $d=1$ F) and are not evaluated.

IV. NUMERICAL RESULTS AND DISCUSSION

In Table II we have given the numerical results for $\langle U_p\rangle_{\text{BG}}/N$ as calculated by using (36) with the incorrect procedure of BG. We take $k_F=1.36$ F^{-1} and $\mu=0.6939$ F^{-1} throughout. Case I (no form factor, or $H=1$) is unrealistic and has been given only for illustration to demonstrate that the contributions are suppressed by the introduction of form factors. The bracketed numbers under the form factors II are those quoted by BG for $k_F=1.35$ F^{-1} in their Table 8. These are somewhat larger, but the trend is well reproduced. The differences arise from small differences in C_p and also due to using a nonrelativistic formula for $\delta\mu^2$. Note specially that for the form factors II and III the results are not very different so that the ambiguity is rather small.

TABLE II. The values of $\langle U_p\rangle_{\text{BG}}/N$ in MeV calculated by using (36) for various values of the NN cutoff d, and $k_F=1.36$ F^{-1} and $\mu=0.6939$ F^{-1}. The numbers in parentheses in row II are those given by BG in their Table 8 for $k_F=1.35$ F^{-1}.

Form factor	$d=0.8$ F	1.0 F	1.2 F
I	3.34	2.03	1.05
II	1.37	0.90	0.54
	(1.79)	(1.25)	(0.79)
III	1.41	0.95	0.58

In Table III we have displayed $\langle U_p\rangle/N$ and $\langle W_p\rangle/N$ when these are correctly calculated by (20) and (28) with F replaced by $\tilde{F}$. The actual first-order contribution to nuclear matter is really $\langle W_p\rangle/N$, but the difference between $\langle U_p\rangle/N$ and $\langle W_p\rangle/N$ is very small. All the results in this table are given for $d=1$ F. Comparing Tables II and III it is observed that BG overestimated $\langle U_p\rangle/N$ by nearly a factor of 10. Furthermore, the correct results shown in Table III are now more sensitive to the particular form factor used, and thereby more ambiguous. From Table III it is seen that the first-order contribution of the TPE three-body force to nuclear matter is small and repulsive, probably less than 0.5-MeV per particle. Table IV summarizes the first- and second-order contributions of U_p to the binding energy per particle of nuclear matter. In the second order we have not shown the central and tensor contributions separately, but the major part comes from the latter. The results for the second-order contribution are nearly the same for the form factors II and III. This is so, since for $d=0.8$ F the tensor part of U_p, $\lambda f_t(r)$, is not much changed in going from

TABLE III. The values of $\langle U_p\rangle/N$ and $\langle W_p\rangle/N$ in MeV calculated by using (20) and (28) with F replaced by $\tilde{F}$, for $d=1$ F, $k_F=1.36$ F^{-1}, and $\mu=0.6939$ F^{-1}.

Form factor	$\langle U_p\rangle/N$	$\langle W_p\rangle/N$	$\langle U_p-W_p\rangle/N$
I	−2.51	−1.37	−1.14
II	0.109	0.105	0.04
III	0.540	0.540	0.

TABLE IV. Summary of the results obtained by using U_p. $\Delta E^{(1)} = \langle U_p\rangle/N$. $\Delta E_1^{(2)}$ and $\Delta E_2^{(2)}$ have been defined by (39). The total contribution is the sum of $\Delta E^{(1)}$, $\Delta E_1^{(2)}$, and $\Delta E_2^{(2)}$.

Form factor	I			II			III		
Cutoff d (F)	0.8	1.0	1.2	0.8	1.0	1.2	0.8	1.0	1.2
$\Delta E^{(1)}$ (MeV)	−3.3	−2.5	−1.8	0.42	0.11	−0.07	1.13	0.54	0.14
$\Delta E_1^{(2)}$ (MeV)	−24.4	−13.9	−8.2	−7.30	−5.06	−3.52	−6.56	−5.04	−3.78
$\Delta E_2^{(2)}$ (MeV)	−5.8	−3.2	−1.8	−0.56	−0.43	−0.33	−0.56	−0.47	−0.39
Total (MeV)	−33.5	−19.6	−11.8	−7.45	−5.38	−3.92	−5.99	−4.97	−4.03

form factors II to III, as can be seen from Fig. 3. It is clear from this Table IV that the effect of the form factors is to drastically suppress the contribution of the three-body force to the binding energy. The total contribution is fairly sensitive to the cutoff, and is in the range 4 ~ 8-MeV attraction per particle. Exchange effects will increase this by about 20%.

With the form factor II and $k_F = 1.35$ F^{-1}, BG obtained (BG's Table 9) $\Delta E^{(2)} = -4.11$, -3.51, and -2.87 MeV for $d = 0.8$, 1.0, and 1.2 F, respectively. Our corresponding numbers are, with $k_F = 1.36$ F^{-1}, $\Delta E^{(2)} = -7.86$, -5.49, and -3.85 MeV for $d = 0.8$, 1.0, and 1.2 F, respectively. There are two sources of the difference between the two results. Firstly, BG used their Eq. (36), which is not valid if $\delta\mu^2(q^2)$ is q dependent. Secondly, for very large q, their $\delta\mu^2(q^2)$ is suppressed compared with ours due to relativistic corrections.

All the above results and all formulas developed in the previous sections have been obtained by ignoring NN correlations due to the strong two-body NN forces. It is natural that the three-body effect should be sensitive to two-body NN correlations, since three particles are coming close together when giving the three-body contribution, and their wave functions are distorted due to the NN correlations. This is a separate effect from pionic form factors, although both in effect suppress the effect of the short-range part of the force. BLN[2] as well as Loiseau and Nogami[3] in their earlier calculations had ignored the pionic form factors, but introduced explicit cutoffs between all pairs of particles to simulate the NN correlation effect. For example, if we use no form factor but introduce an explicit NN cutoff of 1 F between pairs (1, 3) and (2, 3) in addition to the cutoff of 1 F that was put between nucleons (1, 2), then $\langle W_p\rangle/N$ changes from -1.37 (Table III) to 0.81 MeV. This is what we mean by saying that the results are very sensitive to NN corrections in Refs. 2 and 3. A careful calculation should take into account the pionic form factors, as well as NN corrections. It is possible to get even larger effects by using some other form factors,[16] but it should be remembered that NN correlations for the pairs (1, 3) and (2, 3) are likely to suppress the contribution of the three-body force.

The TPE part of the three-body ΛNN force has a very similar structure, but is about five times as strong as the corresponding NNN force.[10] Previously we estimated its first-order effect on the binding energies of Λ in $^3_\Lambda$H, $^5_\Lambda$He, and nuclear matter and pointed out that it might result in a considerable suppression of the binding energies. Now the second-order effect will completely change the situation. The second-order effect on the TPE ΛNN force in nuclear matter is given by $\Delta E_1^{(2)}$ for the NNN case multiplied by a factor 5. Note that there is no contribution of ΛNN which corresponds to $\Delta E_2^{(2)}$. The second-order effect may therefore be as large as 30-MeV attraction. This will pose a serious problem in hypernuclear physics, and the whole problem should be reexamined.

Finally an obvious remark: The two-pion-exchange is only one of a host of processes that give rise to three-body forces. One should remember that in the case of the two-body NN force, the two-pion-exchange potential is comparable with the OPEP even at a distance as large as $1\mu^{-1} = 1.4$ F.

We would like to acknowledge the valuable correspondences with Professor Jun-Ichi Fujita and Dr. Benoit Loiseau.

Note added in proof: The contribution of the TPE three-body force to the binding energy of nuclear matter has been reexamined by introducing cutoffs between all the pair of nucleons. The cutoffs are meant to simulate the NN correlation at short distances, and also to separate the effect of the better known long-range part of the force from that of the unknown short-range part. The contribution is indeed much reduced compared with that obtained in the present paper. With reasonable cutoffs, however, it can still easily be as large as 2-MeV attraction per particle. The details of this calculation will be published elsewhere.

APPENDIX A

We shall indicate here how (25) can be derived.

$$I_t = \mu^6 \int D^2(k_F|\vec{x}-\vec{y}|)(3\cos^2\theta - 1) \times h_2(i\mu x)h_2(i\mu y)d\vec{x}\,d\vec{y}\,,$$

where all the symbols have been defined in the text. Note that

$$3\cos^2\theta - 1 = \frac{8\pi}{5}\sum_m Y_{2m}^*(\vec{x})Y_{2m}(\vec{y})\,. \qquad \text{(A1)}$$

Also, according to (17) of the text,

$$D^2(k_F|\vec{x}-\vec{y}|) = (2\pi)^{-3}\int e^{i\vec{q}\cdot(\vec{x}-\vec{y})}F(q)d\vec{q}\,. \qquad \text{(A2)}$$

Using the formula

$$e^{i\vec{q}\cdot\vec{x}} = 4\pi\sum_{l,\nu} i^l Y_{l\nu}^*(\vec{q})Y_{l\nu}(\vec{x})j_l(qx)$$

and likewise for $e^{i\vec{q}\cdot\vec{y}}$, we can write

$$D^2(k_F|\vec{x}-\vec{y}|) = \sum_{l,\nu} f_l(x,y)Y_{l\nu}(\vec{x})Y_{l\nu}^*(\vec{y})\,, \qquad \text{(A3)}$$

where

$$f_l(x,y) = \frac{2}{\pi}\int_0^\infty F(q)j_l(qx)j_l(qy)q^2\,dq\,. \qquad \text{(A4)}$$

Substituting (A1) and (A2) in the integrand for I_t and doing the angular integrations, we get

$$I_t = 16\mu^6\int_0^\infty F(q)\left[\int_0^\infty j_2(qx)h_2(i\mu x)x^2\,dx\right]^2 q^2\,dq\,. \qquad \text{(A5)}$$

Substituting

$$\int_0^\infty j_2(qx)h_2(i\mu x)x^2\,dx = \frac{q^2}{\mu^3(\mu^2+q^2)} \qquad \text{(A6)}$$

in (A5) yields (25) of the text. Similarly (24) can be proved.

APPENDIX B

Here we give some details of arriving at Eq. (42). After replacing $(\vec{\sigma}_1\cdot\vec{q})(\vec{\sigma}_2\cdot\vec{q})$ with the differential operator and doing the q integration, (41) becomes

$$U_p = \rho\left(\frac{f}{\mu}\right)^2 \vec{\tau}_1\cdot\vec{\tau}_2(A+B)(\vec{\sigma}_1\cdot\vec{\nabla})(\vec{\sigma}_2\cdot\vec{\nabla})\left[\mu(2Y_\mu - E_\mu) + (1-\xi)^2\eta(2Y_\eta - E_\eta) - 4(1-\xi)\frac{\eta^3 Y_\eta - \mu^3 Y_\mu}{\eta^2-\mu^2}\right], \qquad \text{(B1)}$$

where $Y_\mu = e^{-\mu r}/\mu r$, $E_\mu = e^{-\mu r}$, and likewise for Y_η and E_η.

We now use the relations

$$(\vec{\sigma}_1\cdot\vec{\nabla})(\vec{\sigma}_2\cdot\vec{\nabla})Y_\mu = \tfrac{1}{3}\mu^2(\vec{\sigma}_1\cdot\vec{\sigma}_2 + S_{12}T_\mu)Y_\mu\,, \qquad \text{(B2)}$$

$$(\vec{\sigma}_1\cdot\vec{\nabla})(\vec{\sigma}_2\cdot\vec{\nabla})E_\mu = \tfrac{1}{3}\mu^2[(\vec{\sigma}_1\cdot\vec{\sigma}_2 + S_{12})E_\mu + (S_{12} - 2\vec{\sigma}_1\cdot\vec{\sigma}_2)Y_\mu] \qquad \text{(B3)}$$

to derive (42). After some simple algebra we find that U_p can be written as in Eq. (42) with

$$f_c(r) = c_1Y_\mu + c_2Y_\eta + c_3E_\mu + c_4E_\eta\,, \qquad \text{(B4)}$$

$$f_t(r) = b_1Y_\mu T_\mu + b_2Y_\eta T_\eta + c_3(E_\mu + Y_\mu) + c_4(E_\eta + Y_\eta)\,, \qquad \text{(B5)}$$

where

$$c_1 = 4\left[1 + (1-\xi)\frac{\mu^2}{\eta^2-\mu^2}\right],$$

$$c_2 = 4(1-\xi)\left(\frac{\eta}{\mu}\right)^3\left[(1-\xi) - \frac{\eta^2}{\eta^2-\mu^2}\right],$$

$$c_3 = -1, \quad c_4 = -(1-\xi)^2\left(\frac{\eta}{\mu}\right)^3,$$

$$b_1 = c_1 - 2, \quad b_2 = c_2 + 2c_4\,. \qquad \text{(B6)}$$

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[8]More precisely, for a pion with vanishing zeroth component of its four-momentum. The δ-function $\delta(0)$ in (6) actually stands for $\delta(q_{10}-q_{20})$ with $q_{10}=q_{20}=0$. All the vectors in this paper are three-vectors unless otherwise stated.

[9]This is so because A and B are constants in the nonrelativistic approximation. Relativistic effects may suppress the high-momentum contributions.

[10]Corresponding formulas for the TPE ΛNN force are obtained by replacing C_p by $3C_p(\Lambda NN)/2N$, where $C_p(\Lambda NN)\approx 1.43$ MeV. See Refs. 2 and 3. Hence the "conversion factor" is $3C_p(\Lambda NN)/2C_p(NNN)=4.7$.

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$$\frac{e^{-\mu r}}{r}\rightarrow\frac{1}{r}\left(e^{-\mu r}-\frac{U^2-\mu^2}{U^2-\eta^2}e^{-\eta r}+\frac{\eta^2-\mu^2}{U^2-\eta^2}e^{-\eta r}\right).$$

Since they take a very large value for U (= 20 m) the above formula is practically reduced to $(e^{-\mu r}-e^{-\eta r})/r$, which is obtained by using our spectral function (40) with $\xi=1-(\mu/\eta)^2$. The values they considered for η ranges from about 5μ to 7μ, hence $\xi\approx 1$. Therefore, using their form factor we would get very similar results to those with our form factor I.

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Tensor-Force Effects on the l-Forbidden $M1$ Transitions

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The reduced matrix elements for the l-forbidden $M1$ transitions are calculated in the framework of the pairing model. Three quasiparticle states are admixed to the seniority-one state by perturbation due to the short range δ force and the tensor force. It is shown that the mixing of the tensor force is essential to explain the observed values of the reduced matrix elements.

I. INTRODUCTION

According to the shell model, the l-forbidden magnetic dipole ($M1$) transitions between two states which differ in their orbital angular momenta are strictly prohibited, for the magnetic dipole interaction does not change the orbital angular momentum and parity. Hence, the $M1$ transitions are presumably allowed if the initial and the final states are assigned the same orbital angular momenta and parities, and vice versa. However, there have been observed many $M1$ transitions whose lifetimes are much longer than those expected from the shell-model estimate. Therefore, it has been suggested that either the $M1$ transition operator is not adequate, or there is a breakdown of the l-forbiddenness due to some nuclear effects. A theoretical explanation attributed the breakdown of the forbiddenness to the nucleon-nucleon interaction, and a modification of the form of the $M1$ operator was introduced.[1] This effect, however, is now believed to be too small to explain many of the large retardations actually observed. Another approach was made by Arima, Horie, and Sano (AHS)[2] by introducing the method of configuration mixing. Govil and Khurana[3] have investigated the systematic trend of the $M1$ transition matrix elements and they have found a shell effect in these matrix elements. They have also indicated that the calculated values of the matrix elements from the theory of AHS are sufficient to reproduce qualitatively those values deduced from the experimental transition rates.

Recently, the emphasis of the importance of the short-range residual interaction, which admixes a small amount of high-seniority configurations to the basic shell-model configuration, has led to the application of the pairing theory to this prob-