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Comparison of Asymmetric Shapes for Deformed Negative-Parity States in Heavy Even Nuclei*

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The predictions of asymmetric collective models with pure quadrupole, pure octupole, and mixed quadrupole-plus-octupole shapes are developed for negative-parity states in deformed even nuclei in an effort to determine the usefulness of the asymmetric model as a tool to distinguish surface shapes. The models are applied to the data for ^{228}Th, ^{232}U, and ^{234}U, and the comparison indicates that the energy and electric quadrupole branching-ratio predictions are independent of surface-shape multipolarity.

I. INTRODUCTION

One of the problems with the asymmetric collective model for excited states in even nuclei is the connection between surface shape and eigenstate parity. It is assumed that parity is associated with shape multipolarity, which means that odd-order-multipole mass moments give rise to negative-parity states and even-order deformations give rise to positive-parity states. There is nothing in the collective Schrödinger equation itself, however, to fix this association. It is not clear whether a nucleus in an excited negative-parity state should have a purely odd order equilibrium surface or whether the shape has small odd-order contributions superposed on a stable even-order deformed ground state. Competition between states occurs in spherical nuclei[1] and it is probable that some deformed even nuclei have pure octupole negative-parity states, while others have states with mixture. This work is an attempt to see whether the simple asymmetric model can be used to determine surface shape on the basis of some available negative-parity data.

The positive-parity collective levels in deformed even nuclei are fairly well described by several models.[2,3] Collective rotations and vibrations of a stable quadrupole-shaped surface describe most energy levels and $E2$ transitions (the predominant collective decay mode) with accuracy. The information on shapes is not so extensive for negative-parity states, however, partly because of the small number of negative-parity states thus far investigated.

The first asymmetric model for negative-parity states assumed the nucleus had a pure octupole shape[4,5] and was developed in a manner directly parallel to the model of Davydov, Fillippov, and Chaban (DFC)[6,7] for positive-parity deformed states. The model assumed that basis states (symmetric rotor functions) had only even projections K of angular momentum on the body-fixed z axis. A similar model was developed by the author,[8] but it made use of contributions from both even- and odd-K basis states. This model, not surprisingly, led to twice as many predicted levels. Both the above models suffer from fundamental difficulties. The nuclei in the mass regions of interest have highly stable deformed quadrupole shapes, and there is still some debate over the stability of pure octupole deformations in heavy nuclei.[9] Further, electric quadrupole transitions are the dominant transitions even between negative-parity states, and electric quadrupole moments and transition probabilities of an octupole-shaped nucleus are approximately an order of magnitude smaller than those of a quadrupole-shaped nucleus.[10] It is also difficult to show that shape-vibration (β vibration) levels are of negative parity in such a model.

The nucleus apparently does at least have a different shape for positive- and negative-parity states, since one finds that the shape parameters used to fit the positive-parity levels will not directly fit the negative-parity levels also.[11] A few simple possibilities remain. The first and most likely is that a negative-parity excitation gives rise to a slight octupole deformation superimposed on the quadrupole shape. This was examined by Lipas and Davidson[12] using a symmetric model, and a similar model has recently been applied to light nuclei by Castel and Svenne.[13] The author has also applied a similar but asymmetric model to heavy nuclei.[11] Another possibility is that a negative-parity intrinsic excitation changes the quadrupole shape slightly but the shape can still be described as quadrupole. In this paper we develop the pure quadrupole model for negative-parity states and compare it with the results of Refs. 4 and 11. If negative-parity data can be described by a pure quadrupole model, then the simple asymmetric collective model can probably not be used to distinguish surface shapes.

The basic formalism of the asymmetric model is not new, and Sec. II and Sec. III are brief reviews of what is now a standard development of the model used and of $E2$ transition predictions. Section IV is a comparison of results of the various models, and Sec. V is the conclusion.

II. MODELS

This section is a combined summary of the asymmetric quadrupole model for positive-parity states developed for even nuclei by DFC,[6,7] the pure octupole model for negative-parity states developed by Davidson,[4] and the mixed quadrupole-plus-octupole model developed by the author for negative-parity states.[11] The first two models are discussed in more detail in Sec. II of Ref. 10.

In their model for positive-parity states DFC[6,7] separated the collective motion into two parts; rotation of the nucleus relative to a laboratory reference frame, and a breathing mode or β vibration of the nuclear shape relative to the rotating coordinate system.

In their asymmetric model the γ-vibration band of the symmetric model ($k=2$ band) arose from the rotation of the asymmetric surface, and asymmetry vibrations (γ vibrations) were not discussed. They wrote the Schrödinger equation for the rotation part as

$$\left[\tfrac{1}{2}\sum_{k=1}\frac{L_k^{\,2}}{i_k}-\mathcal{E}_{LN}\right]\psi_{LN}=0\,, \tag{1}$$

where L_k is the kth component of the angular momentum operator. The quantum numbers L and N are spin and spin ordinal numbers, respectively. The corresponding vibration equation was

$$\left[-\frac{\hbar^2}{2B}\frac{1}{\beta^3}\frac{d}{d\beta}\beta^3\frac{d}{d\beta}+\frac{\hbar^2}{4B\beta^2}\mathcal{E}_{LN}+\tfrac{1}{2}C(\beta-\beta_0)^2-E_{LNn}\right]\Phi_{LNn}=0\,, \tag{2a}$$

where β is the deformation parameter, B and C are mass and spring parameters, respectively, for the vibrational motion, and n is the vibration ordinal number. The kth component of the (diagonal) moment-of-inertia tensor is i_k of Eq. (1) multiplied by $4B\beta^2$.

The second and third terms of Eq. (2) were treated as an effective potential and expanded for each $\mathcal{E}_{LN}$ about the root (β_{LN}) of the first derivative from $\beta_{LN}{}^4-\beta_{LN}{}^3\beta_0-\frac{1}{2}\beta_0{}^4\mu^4\mathcal{E}_{LN}=0$. The parameter $\mu=\beta_0{}^{-1}\hbar^{1/2}(CB)^{-1/4}$ was treated as a free parameter ($0<\mu\lesssim1$) and appears in the fitting data for various nuclei. They then defined a new independent variable $y=z_1(\beta-\beta_{LN})/\beta_{LN}$, where $z_1{}^4=(\beta_{LN}/\beta_0\mu)^4+\frac{3}{2}\mathcal{E}_{LN}$, to finally obtain Weber's equation,

$$[(d/dy)^2-y^2+(2\nu+1)]D_2(\sqrt{2}y)=0\,, \tag{2b}$$

where ν gives the total energy.

This approach to shape vibrations, common to all the negative-parity models discussed here, ignores asymmetry (γ) vibrations in an effort to treat deformation (β) vibrations exactly and assumes that such vibrations are sufficiently high in energy that they do not couple strongly to the lower collective levels. A perturbation treatment of asymmetry vibrations was later discussed by Davydov.[7]

The nuclear shape appears only in the moments of inertia $\mathfrak{g}_k=4B\beta^2 i_k$. The shape can be expanded in the body-fixed system by writing the radius as a sum over spherical harmonics

$$R(\theta,\phi)\sim1+\sum_{\lambda,\mu}a_{\lambda\mu}Y_{\lambda\mu}(\theta,\phi)\,. \tag{3}$$

In the quadrupole model ($\lambda=2$) the coefficients are written

$$\begin{aligned}a_{20}&=\beta_2\cos\gamma,\\ a_{2\pm2}&=(\beta_2/\sqrt{2})\sin\gamma,\\ a_{2\pm1}&=0\,.\end{aligned} \tag{4}$$

This choice has the same number of degrees of freedom (three Euler angles plus two vibration variables) as before parametrization ($2\lambda+1=5$). The moments of inertia are given by

$$i_1 = \sin^2(\gamma - \tfrac{2}{3}\pi),$$
$$i_2 = \sin^2(\gamma + \tfrac{2}{3}\pi), \quad (5)$$
$$i_3 = \sin^2\gamma.$$

In the octupole model ($\lambda = 3$) the coefficients are written analogously:

$$a_{30} = \beta_3 \cos\eta,$$
$$a_{3\pm 2} = (\beta_3/\sqrt{2})\sin\eta, \quad (6)$$
$$a_{3\pm 1} = a_{3\pm 3} = 0.$$

Here the number of variables is still five (three Euler angles plus two vibration variables) while the original number of degrees of freedom was $2\lambda + 1 = 7$. It is alternatively possible to parametrize the coefficients with $a_{30} = a_{3\pm 2} = 0$, $a_{31} \neq 0 \neq a_{3\pm 3}$, but this leads to asymmetric shapes for all values of the parameters. The simultaneous parametrization of all seven coefficients (neither $a_{30} = a_{3\pm 2} = 0$ nor $a_{3\pm 1} = a_{3\pm 3} = 0$) was investigated earlier[8] in an attempt to explain the complicated spectrum of ^{182}W, but the nuclear shape could not be uniquely determined. Soloviev, Vogel, and Kornichuk[14] have shown in a quasiparticle calculation that the $a_{3\pm 1}$ and $a_{3\pm 3}$ degrees of freedom are not in general collective, and since the extra states do not appear at low energies to mix with the collective levels in the nuclei examined here, the octupole model of Eq. (6) would appear to be the physically reasonable version to use. The moments of inertia are given by

$$i_1 = \sin^2\eta + \tfrac{3}{2}\cos^2\eta + \tfrac{1}{2}\sqrt{15}\,\sin\eta\,\cos\eta,$$
$$i_2 = \sin^2\eta + \tfrac{3}{2}\cos^2\eta - \tfrac{1}{2}\sqrt{15}\,\sin\eta\,\cos\eta, \quad (7)$$
$$i_3 = \sin^2\eta.$$

If we assume the nucleus is predominantly quadrupole in shape, but has some octupole deformation in addition, then the moments of inertia, again assuming $a_{3\pm 1} = 0 = a_{3\pm 3}$, can be written approximately as[11]

$$i_1 = \sin^2(\gamma - \tfrac{2}{3}\pi) + D(\sin^2\eta + \tfrac{3}{2}\cos^2\eta + \tfrac{1}{2}\sqrt{15}\,\sin\eta\,\cos\eta),$$
$$i_2 = \sin^2(\gamma + \tfrac{2}{3}\pi) + D(\sin^2\eta + \tfrac{3}{2}\cos^2\eta - \tfrac{1}{2}\sqrt{15}\,\sin\eta\,\cos\eta),$$
$$i_3 = \sin^2\gamma + D\sin^2\eta, \quad (8)$$

where $D = (B_3\beta_{30}^2/B_2\beta_{20}^2)$. The subscripts 2 and 3 refer to $\lambda = 2$ and $\lambda = 3$, respectively, and the subscripts 0 refer to equilibrium values. For small deformations, $B_3\beta_3^2 = B_2\beta_2^2 D$. Here D is treated as a free parameter. The quadrupole parameters are obtained from the fit to positive-parity levels, and two free parameters (η and D) remain to fit the negative-parity states. The octupole-deformation vibrations are assumed coupled directly to the quadrupole vibrations, which then give rise to an excited negative-parity band. This is the negative-parity analog to the positive-parity β-vibration band and can be thought of as arising from a quadrupole phonon coupled to a negative-parity state. This contrasts with the pure octupole description, which treats this band as the combination of two octupole excitations.

The total wave function is the product of the rotation factor ψ_{LN} of Eq. (1) and the vibration factor Φ_{LNn} of Eq. (2), so the over-all parity depends on the parities of the two factors. The parity of Φ_{LNn} has not been well discussed, but is usually assumed to be unaffected by deformation vibrations, since the deformation parameter β is unaffected by the parity transformation. On the other hand, it has been argued that the parity of a deformation-vibration phonon should be positive (negative) for even- (odd-) order deformations, respectively, in which case the octupole model should predict the one-phonon band to be of positive parity (i.e., negative relative to the zero-phonon band). In the present work, however, when discussing the octupole model we will follow the original assumptions of that model[4] that the parity is unaffected by deformation vibrations. This allows us to take the parity of the asymmetric rotor function ψ_{LN} as the over-all parity in all the models discussed here. Asymmetric rotor states can be expanded in sums over symmetric rotor function $D^{L}_{MK}{}^*$ as

$$\psi^M_{LN} = \sum_{K>0} A_{KN}(D^L_{MK}{}^* \pm (-1)^{L-K} D^L_{M-K}{}^*), \quad (9)$$

where $M(K)$ are the projections of angular momentum L onto the laboratory- (body-) fixed z axes, respectively. Positive- (negative-) parity states are formed[15] by assuming the positive (negative) sign, respectively. The shape of the nucleus does not directly determine this choice. The sum is over even or odd K (but not both), since the Hamiltonian connects basis states with K differing by $\Delta K = 0$, ± 2 only. For negative-parity states the choice of either even or odd K leads to no spin-$L = 0$ level; one each of $L = 1, 2$; two each of $L = 3, 4$; etc., as may be seen by writing out the sum of Eq. (9) in detail.

To construct a quadrupole model for negative-parity states we simply adopt the quadrupole parametrization of Eq. (4) and choose the minus sign in Eq. (9). We have then three negative-parity models to compare; the quadrupole ($\lambda = 2$) model developed here, the pure octupole ($\lambda = 3$) model of Ref. 4, and the mixed quadrupole-octupole model ($\lambda = 2, 3$) of Ref. 11.

III. E2 BRANCHING RATIOS

The electric quadrupole transition branching ratios are calculated by methods described by Davidson and the author[10] which will be briefly summarized here. The reduced transition probability can be written

$$B_2(LNn \to L'N'n') = (2L+1)^{-1}\sum_{\mu\mu'} |\langle L'N'n'M'|Q_{2\mu}^{\text{lab}}|LNnM\rangle|^2, \tag{10}$$

where $Q_{2\mu}^{\text{lab}}$ is the quadrupole operator seen in the laboratory coordinate system. If we assume a constant nuclear charge density proportional to the mass density, we can write the quadrupole operator in the body-fixed system as an integral over the nuclear volume,

$$Q_{2\mu}^{\text{body}} = \left(\frac{4\pi}{5}\right)^{1/2}\int_0^R r^2 Y_{2\mu}(\theta,\phi)\rho_e(r)d\tau, \tag{11}$$

where $\rho_e(r)$ is the charge density and R is the nuclear surface described by Eq. (5). Upon integration we obtain

$$Q_{2\mu}^{\text{body}} = \left(\frac{3ZeR_0^2}{4\pi}\right)\left(\frac{4\pi}{5}\right)^{1/2} a_{2\mu} \tag{12}$$

for the quadrupole case, while for the octupole case we have

$$Q_{2\mu}^{\text{body}} = -\left(\frac{ZeR_0^2}{\pi}\right)\left(\frac{3}{5}\right)^{1/2}\sum_{\mu\mu'} C(233\mu,\mu'-\mu,\mu') \times a_{3\mu}a_{3\mu'-\mu} \,. \tag{13}$$

The operator for the mixed quadrupole-plus-octupole surface would be the sum over both the above expressions, but since the octupole part is smaller by a factor $a_{3\mu}$, we ignore that contribution here. The accuracy of the experimental data does not warrant keeping these terms.

IV. RESULTS

Since the quadrupole model has not been applied in detail to negative-parity states in deformed even nuclei before, the variation of energy levels with the asymmetry parameter is presented in Fig. 1 for the stiffness parameter $\mu = 0$, and the variation of the levels with μ is shown in Fig. 2 for the value $\gamma = 11.888°$ used to fit ^{228}Th below.

The quadrupole-model predictions for positive-parity levels of ^{228}Th, ^{232}U, and ^{234}U are compared with experiment[16-20] in Table I for completeness. The negative-parity fits are shown in Table II for the quadrupole model ($\lambda = 2$), octupole model ($\lambda = 3$), and quadrupole-plus-octupole model ($\lambda = 2, 3$).

^{228}Th

In the fit to the negative-parity levels of this nu-

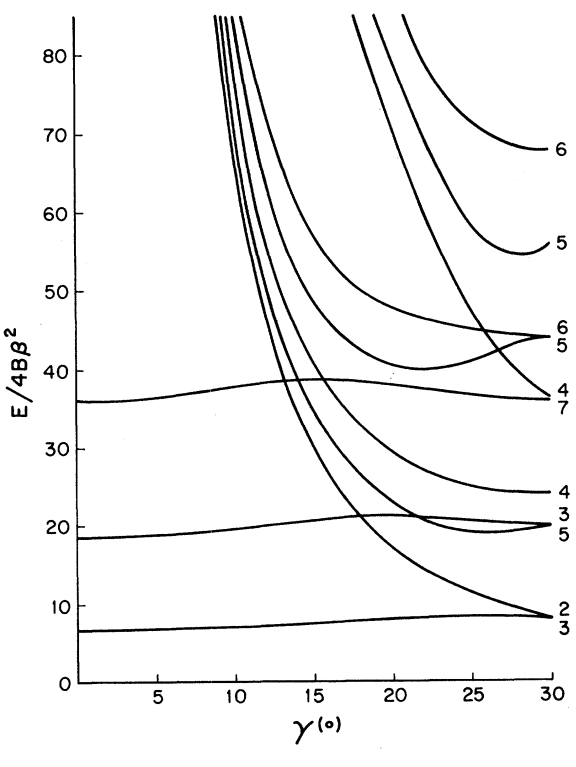

FIG. 1. The negative-parity energy eigenvalues in units of $\hbar^2/4B_2\beta^2$ relative to the spin-$L=1$ level for a rigid asymmetric top with a quadrupole surface ($\lambda = 2$) as a function of the asymmetry parameter γ. The hydrodynamic moments of inertia of Eq. (7) have been used and the stiffness parameter set to $\mu = 0$.

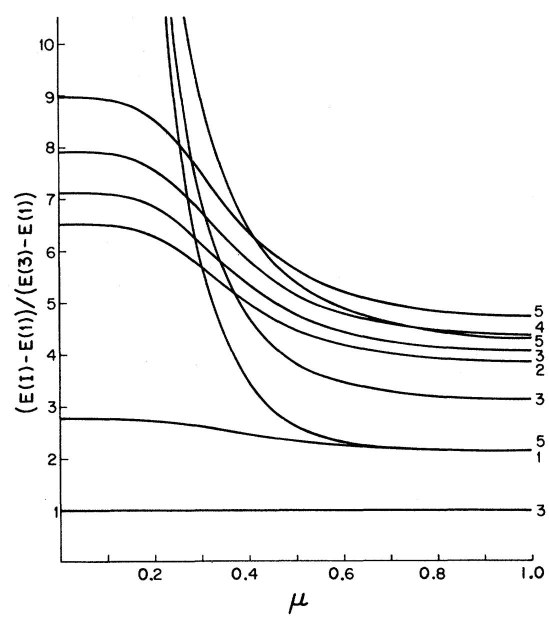

FIG. 2. The negative-parity energy eigenvalues in units of the 3^- to 1^- spacing for a rigid asymmetric top with a quadrupole surface ($\lambda = 2$) as a function of the stiffness parameter μ. The asymmetry parameter is set to the value $\gamma = 11.888°$ used to fit ^{228}Th.

TABLE I. The quadrupole-model fits to the positive-parity energy levels of ^{228}Th, ^{232}U, and ^{234}U. The parameter λ is the asymmetry parameter, μ is the stiffness parameter, and rms is the rms deviation of all the predicted levels for which there are experimental equivalents.

	^{228}Th Exp. (Refs. a,b)	Theory	^{232}U Exp. (Ref. c)	Theory	^{234}U Exp. (Refs. d,e)	Theory
γ		9.328°		8.678°		8.125°
μ		0.221		0.249		0.221
rms (keV)		279.		9.8		10.1
Energies (keV)						
$I=0$	0.	0.	0.	0.	0.	0.
2	57.5	57.5	47.6	47.6	43.4	43.4
4	186.6	188.6	156.6	154.9	143.1	142.5
6	378.	387.	321.	315.	295.5	292.9
8		644.		518.	496.1	488.8
2	969.	970.	876.	877.	927.1	949.4
3	1023.	1018.	912.	913.	969.1	984.8
4	1092.	1083.	971.	962.	1023.5	1031.8
5		1162.		1021.	1092.7	1089.5
6		1258.		1092.	1172.2	1159.5
7		1364.		1172.	1261.9	1237.6
0	830.	1045.	693.	676.	810.0	801.7
2		1110.	735.	732.	851.4	851.0
4	1433.	1258.	832.	855.	947.3	962.7
2	1620.	2118.2		1658.		1845.5
3	1650.	2171.		1697.		1883.6
4	1690.	2241.		1749.		1934.0

[a] See Ref. 16.
[b] See Ref. 17.
[c] See Ref. 18.
[d] See Ref. 19.
[e] See Ref. 20.

cleus (Table II) the 3^- level at 1123 keV is taken to be a member of the one-phonon β-vibration band. In the quadrupole-plus-octupole model this level is fit at the cost of fitting the 0^+ level tentatively placed at 830 keV as shown in Table I. For comparison the last column $(\lambda=2,\pi^+)$ of Table II shows the negative-parity fit predicted by the quandrupole model using the positive-parity parameters (except for an over-all scale factor) from Table I. This fit is not appreciably different from those of the other models for ^{228}Th. The only significantly better fit is obtained by using the same quadrupole model with different shape parameters $(\lambda=2,\pi^-)$. The model predictions for three $E2$ branching ratios are shown in Table III. All three models make the same prediction.

For interest we have plotted the approximate shapes of the three models for ^{228}Th in Fig. 3. They are, of course, quite different for the three models; however, it is the moment of inertia rather than the shape that appears in the Hamiltonian. Even the magnitudes of the moments are unimportant; only the ratios of the three principal moments determine the structure of the state functions and consequently the energy-level predictions (within a scale factor) and the $E2$ branching ratios. As can be seen in Table IV all three models predict similar ratios of the moments of inertia for ^{228}Th. Therefore only knowledge of static moments of negative-parity states and absolute transition probabilities will enable us to use the above information to determine the shape multipolarity for these levels.

^{232}U, ^{234}U

The number of levels available for these nuclei is not sufficient to yield much information about shapes except that it is clear from Table II that the positive-parity parameters do not give a good fit to negative-parity states. In fitting these nuclei the value of the stiffness parameter μ from the positive-parity fits was also used in fitting the negative-parity levels, since there are no data available for negative-parity β-vibration levels. The actual value for μ is expected to be within 15 or 20% of this value. In the quadrupole (octupole) models the free parameters are the over-all scale factor and $\gamma(\eta)$, respectively, while in the quadrupole-plus-octupole model the free parameters are the octupole asymmetry parameter (η) and D, a measure of the relative octupole content. So there are

TABLE II. A comparison of theory with experiment for the negative-parity states of ^{228}Th, ^{232}U, and ^{234}U. The first column gives the experimental data, and the next three columns give the prediction of the pure quadrupole ($\lambda=2$) model proposed here, the pure octupole model ($\lambda=3$) of Ref. 4, and the mixed quadrupole-octupole model ($\lambda=2,3$) of Ref. 11, respectively. To show the difference in quadrupole nuclear shape between positive- and negative-parity states, the last column gives the quadrupole predictions using the fitting parameters taken directly from Table I for the fit to positive-parity levels. In the quadrupole model the spacing of the lowest two levels is taken directly from experiment, and the asymmetry parameter γ and stiffness parameter μ are adjusted to fit the spacing of the remaining levels. Similarly in the octupole model the spacing of the lowest two levels is taken directly from experiment and μ and the asymmetry parameter η are adjusted. In the mixed model, however, the octupole asymmetry parameter η and mixing parameter D are adjusted to fit the spacing of all levels. The rms deviations are for all predicted levels having experimental equivalents.

	Exp. (Refs. a–e)	Theory ($\lambda=2$)	Theory ($\lambda=3$)	Theory ($\lambda=2,3$)	Theory ($\lambda=2,\pi^+$)
			^{228}Th		
γ		8.758°		9.328°	9.328°
μ		0.189	0.259	0.221	0.221
η			11.888°	9.4°	
D				0.19	
rms (keV)		3.5	6.7	5.7	6.3
Energies (keV)					
$I=1$	327.5	327.5	327.5	327.5	327.5
3	396.0	396.0	396.0	396.5	396.0
5	514.	517.	515.	518.	516.
7		688.	679.	687.	684.
2	1123.	1132.	1136.	1134.	1128.
3	1169.	1169.	1171.	1171.	1164.
4	1227.	1228.	1215.	1217.	1211.
5		1280.	1273.	1277.	1271.
6		1352.	1337.	1347.	1340.
1		1376.	1372.	1373.	1366.
3	1451.	1452.	1452.	1451.	1443.
5		1585.	1591.	1588.	1579.
			^{232}U		
γ		11.126°		8.678°	8.678°
μ		0.249	0.249	0.249	0.249
η			15.619°	34.6°	
D				0.06	
rms (keV)		0.	1.	0.	199.
Energies (keV)					
$I=1$	564.	564.	564.	564.	564.
3	630.	630.	630.	630.	630.
5		741.	743.	743.	744.
7		894.	895.	896.	898.
2	1017.	1017.	1016.	1017.	1291.
3	1031.	1031.	1032.	1032.	1321.
4		1094.	1099.	1095.	1362.
5		1149.	1162.	1151.	1412.
6		1211.	1226.	1214.	1471.
1		1119.	1588.	1187.	1137.
3		1195.	1665.	1263.	1213.
5		1324.	1796.	1393.	1344.
			^{234}U		
γ		16.165°		8.125°	8.125°
μ		0.220	0.220	0.220	0.220
η			22.351°	67°	
D				0.09	
rms (keV)		15.2	15.4	13.8	474.3

TABLE II (Continued)

	Exp. (Refs. a–e)	Theory ($\lambda=2$)	Theory ($\lambda=3$)	Theory ($\lambda=2,3$)	Theory ($\lambda=2,\pi^+$)
			U^{234} (Continued)		
Energies (keV)					
$I=1$		786.9	786.9	786.9	786.9
3	849.8	849.8	849.8	849.8	849.8
5	962.4	944.5	949.4	957.0	959.8
7		1105.1	1073.8	1104.2	1112.3
2	989.8	993.0	986.4	993.2	1107.0
3	1064.2	1030.6	1028.0	1030.0	1638.1
4	1069.4	1077.0	1075.0	1077.1	1679.1
5	1127.6	1143.9	1156.6	1139.8	1730.4
6	1194.7	1203.7	1208.2	1205.2	1790.3
1		1428.6	1924.5	1572.3	1491.9
3		1498.6	1995.7	1643.4	1563.1
5		1617.0	2107.9	1764.9	1686.5

[a] See Ref. 16 for ^{228}Th.
[b] See Ref. 17 for ^{228}Th.
[c] See Ref. 18 for ^{232}U.
[d] See Ref. 19 for ^{234}U.
[e] See Ref. 20 for ^{234}U.

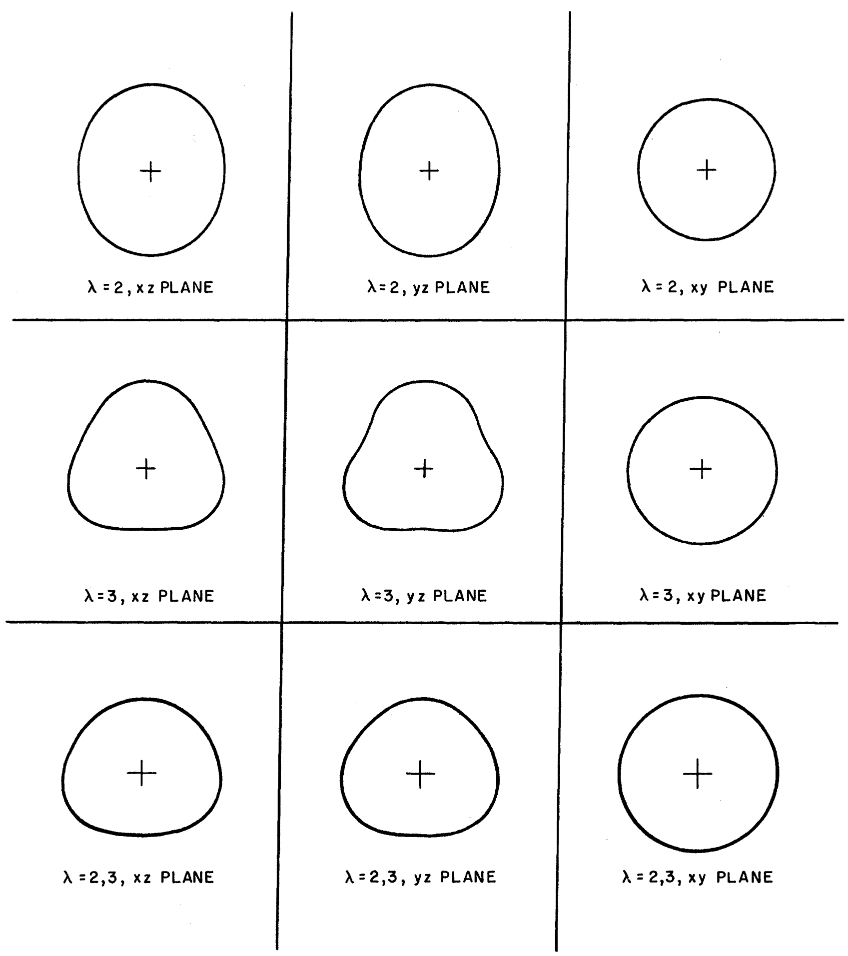

FIG. 3. A comparison of shapes predicted for ^{228}Th by the quadrupole ($\lambda=2$), octupole ($\lambda=3$), and mixed quadrupole-octupole ($\lambda=2,3$) models. The deformation parameter was arbitrarily set at $\beta=0.25$ for the quadrupole, and mixed quadrupole-octupole models, and at $\beta=0.15$ for the octupole model.

two free parameters in each of the three models. The predictions for one branching ratio of ^{232}U are shown in Table III, and once again the three models make similar predictions.

TABLE III. A comparison of moments of inertia in the quadrupole, octupole, and mixed quadrupole-octupole models for ^{228}Th.

Moment	$\lambda=2$ model	$\lambda=3$ model	$\lambda=2,3$ model
$2i_1/(i_1+i_2)$	1.176	1.266	1.194
$2i_2/(i_1+i_2)$	0.824	0.734	0.806
$2i_3/(i_1+i_2)$	0.031	0.029	0.031

V. CONCLUSION

We have compared experimental data for negative-parity states in ^{228}Th, ^{232}U, and ^{234}U with the predictions of three possible asymmetric rotor models; pure quadrupole, pure octupole, and mixed quadrupole plus octupole. There seems to be little difference between pure quadrupole and pure octupole model predictions, so it is not surprising that mixing the two together does not offer different predictions either. The significant point here is that from the available data, the collective model appears incapable of determining whether the nucleus has any octupole-shape content at all when in an excited negative-parity state.

TABLE IV. A comparison of model predictions with experimental data for $E2$ branching ratios of the negative-parity states of ^{228}Th and ^{232}U, for the quadrupole, octupole, and mixed quadrupole/octupole models.

Transition	Exp.	Theory ($\lambda=2$)	Theory ($\lambda=3$)	Theory ($\lambda=2,3$)
^{228}Th				
$\frac{211-311}{211-111}$	<0.3 [a]	0.22	0.22	0.22
$\frac{321-111}{321-311}$	0.36 ± 0.02 [a]	0.49	0.49	0.49
$\frac{411-511}{411-311}$	0.75 ± 0.02 [a]	0.39	0.39	0.38
^{232}U				
$\frac{211-311}{211-111}$	4.00 ± 1 [b]	0.20	0.19	0.18

[a]See Ref. 16.
[b]See Ref. 18.

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Proton Decay of Isobaric Analog Resonances in ^{95}Nb†

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Differential cross sections for elastic and inelastic proton scattering on ^{94}Zr were measured between 6.0- and 8.5-MeV bombarding energy. The decay of analogs of states in ^{95}Zr to the ground state and the 0.92 MeV (2^+), 1.30 MeV (0^+), 1.66 MeV (2^+), and 2.06 MeV (3^-) states in ^{94}Zr was investigated. In the analysis of the inelastic proton groups, the enhancement of the Hauser-Feshbach background, as well as the interference of the resonance amplitude with direct nuclear excitation were taken into account. Spins, spectroscopic factors, and possible weak-coupling configurations in the parent system are derived and compared with results previously obtained for ^{91}Zr and ^{93}Zr.

1. INTRODUCTION

Since the pioneering work of Jones, Lane, and Morrison,[1] inelastic proton scattering through isobaric analog resonances (IAR) has become an efficient tool in nuclear spectroscopy for determining, in a qualitative way, correlations between low-lying core states $|c\rangle$ of the target and the states $|nc\rangle$ of the parent system. Conclusive analyses, however, resulting in numerical values for partial widths and spectroscopic factors, have been performed only for two categories of resonances; namely, single-particle analog resonances (1) built upon the ground state of a target with magic neutron number and decaying to excited neutron-particle–neutron-hole states,[2] and (2) built upon excited core states (weak-coupling model).[3,4]

The partial widths and angular momenta of the outgoing proton partial waves manifest themselves in the shapes of the on-resonance angular distributions. So far, only resonances above a very small background[2] or superimposed on a Hauser-Feshbach background[5-7] have been studied in detail, as those angular distributions feature near symmetry about 90°. However, IAR occur in general at proton energies where Coulomb and direct excitation compete with the resonance process, particularly in exciting collective states. The interference between different modes of excitation can affect an angular distribution to such an extent that details of the resonant part (like the coefficients of the P_4 and higher terms in a Legendre polynomial expansion) are masked. It thus seemed worthwhile to investigate a reaction where the effects of direct scattering have to be taken into account. We chose ^{94}Zr as target and measured the decay of analogs of parent states in ^{95}Zr to target states below 3.0-MeV excitation. From inelastic proton scattering at 12.7 MeV,[8] these states are known to be weakly "deformed," except for the lowest quadrupole and octupole vibrations at 0.92 and 2.06 MeV, respectively, which indeed will show a pronounced nonresonant excitation above 7-MeV proton energy. Because of the very low (p,n) threshold at 1.69 MeV and the high level density in ^{94}Nb, Hauser-Feshbach contributions in the (p,p') cross section from $T_<$ compound states are expected to be small.

The second object of this experiment was to measure spins of IAR and partial widths for the decay to low-lying target states. As was pointed out previously,[5,9] the differential cross section of the protons leaving ^{94}Zr in its excited 0^+ state at 1.30 MeV is most appropriate for this task, since the on-resonance angular distributions depend very sensi-